Galvanomagnetic effects in a spatially inhomogeneous system

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Investigation is made of the galvanomagnetic properties of an electron gas in a layer-inhomogeneous system. The motion of electrons is limited to a one-dimensional potential well and a magnetic field is applied parallel to the layers. It is shown that in a static electric field an allowance even for very weak electron scattering alters greatly the expressions for the transport coefficients found in the collisionless approximation. Calculations carried out using the quantum transport theory formulas give the final results in the zeroth approximation in respect of the scattering potential, but the quadratic correction diverges. The magntoconductivity tensor is found in the approximation of the classical transport equation. It is shown that the Hall coefficient of such a system is not governed by the carrier density and that the cyclotron resonance line width depends on the magnetic field even in the absence of quantum effects.

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1. INTRODUCTION

Quantum theory of irreversible processes gives formally exact expressions for the transport coefficients of macroscopic systems. However, actual calculations can be carried out using these expressions only if we introduce additional simplifying assumptions, whose number and extent are governed by the specific features of each separate problem. In a nonquantum situation (where there is no quantization of the energy spectrum of quasiparticles, and when electric and magnetic fields are sufficiently weak), the quantum theory reproduces the results obtained from the Boltzmann transport equation and only modifies the range of their validity. Examples are the theory of the electrical conductivity developed by Edwards,[1] the formula of Titeica for the magnetoconductivity,[2] and the theory of cyclotron resonance given by Kubo, Hasegawa, and Hashitsume.[3] In all these cases the scattering is assumed to be (in some sense) weak and, consequently, the transport coefficients found in this way differ little from their values in the collisionless approximation. For example, the Hall component of the conductivity tensor in a magnetic field subject to the condition $\omega_H\tau \gg 1$ (ω_H is the cyclotron frequency and τ is the relaxation time representing the scattering processes) is completely independent of τ in its principal order:

$$\sigma_H = Nec/H, \tag{1}$$

where N is the electron density and H is the magnetic field.

Allowance for the scattering gives rise to small corrections of the order of $(\omega_H\tau)^{-2}$. The diagonal elements of σ, transverse relative to the magnetic field, are proportional to τ^{-1}, i.e., they also differ little from their collisionless value (which is zero). The longitudinal component σ_{zz} is infinite in the absence of scattering and if allowance is made for the scattering, this component is proportional to τ. However, the dynamic conductivity at the frequency of an external field ω is given by the following expression in the quantum and classical theories:

$$\sigma_{zz} = Ne^2/m(i\omega+\tau^{-1}). \tag{2}$$

If the scattering is weak in the sense that $\omega\tau \gg 1$, allowance for the scattering has little effect on the value of $\sigma_{zz}(\omega)$.

We can see that in all the examples considered so far the transport coefficients found using the quantum theory in the collisionless approximation are in a sense stable when allowance is made for weak scattering. Moreover, the results of the quantum and classical calculations are identical.

However, all these results apply to spatially homogeneous systems. We shall consider the problem of the magnetoconductivity tensor $\sigma_{ij}(H)$ for an electron gas in a stratified inhomogeneous system in which the motion of electrons along the y axis is limited by the potential

$$U(y) = {}^1/_2 m\Omega^2 y^2. \tag{3}$$

A magnetic field is directed along the z axis, i.e., it is parallel to the layers (Fig. 1). From the purely methodological point of view, this problem is interesting because the formally exact quantum transport theory is not inapplicable in practice. In making this statement we have in mind that calculations can be completed (in the density matrix method or using the Kubo formulas) only in the weak scattering approximation. Then, the zeroth approximation is the tensor $\sigma_{ij}(H)$, calculated without allowance for collisions. It is undoubtedly desirable that allowance for the scattering should give rise to small corrections to the zeroth approximation. The main distinguishing feature of our problem is that allowance for even very weak scattering alters considerably the form of the magnetoconductivity tensor. The stability of the collisionless transport coefficients mentioned above does not apply to the

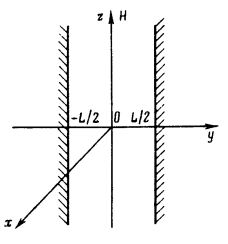

FIG. 1.

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spatially inhomogeneous case considered here and the results of the quantum and classical treatments do not agree.

In addition to investigating the methodological aspect of the problem, we shall use the simplest model to calculate the transport characteristics of the system in question and to show that

1) the Hall coefficient is not in this case governed by the mobile carrier density;

2) a magnetic field does not occur explicitly in the static values of σ_{ij} although the parameter $\omega_H\tau$ is not assumed to be large or small;

3) in the classical regime for the H-independent scattering mechanism the cyclotron resonance line width depends on the magnetic field, in contrast to the case of a homogeneous system.

This model of a parabolic layer may be realized experimentally. An example is a semiconductor film with completely ionized shallow donors placed between two metal electrodes (two Schottky barriers connected in opposition). A suitable selection of the work functions and potentials applied to the film and metals can ensure electron depletion in the film. The excess positive charge of density N_+ creates a parabolic potential (3) where $\Omega^2 = 4\pi e^2 N_+/\varepsilon m$ and ε is the permittivity of the semiconductor (Fig. 1).

Another example are electrons entrained in the field of a high-power acoustic wave in piezoelectric semiconductor.[4] In the simplest case of a standing wave when the amplitude of the piezoelectric potential is much greater than the thermal energy, electrons concentrate near the minima of the wave field where the approximation (3) applies. It is important to note that for attainable ultrasonic frequencies (10^8 sec^{-1}) the relaxation of the momentum of the entrained particles occurs earlier than a significant change in the piezoacoustic wave field.

An important opportunity for investigating transport effects in a layer-inhomogeneous systems is provided by the methods of ion implantation in semiconductors. It is now possible to create "subsurface" (i.e., separated from the surface) conducting layers, for example, a p-type layer in n-type silicon (see the results of Romanov *et al.*[5]). Holes move in a potential trough and if the carrier density is low they occupy a small parabolic part of the potential relief. The technique of measurement of the transport characteristics of such systems is analogous to the technique for investigating metal–insulator–semiconductor structures. Finally, electrons in magnetic surface layers in Schottky barriers also form a conducting inhomogeneous layer separated from the bulk of the semiconductor by a space-charge region which is practically free of electrons (exhaustion region).

2. TRANSPORT COEFFICIENTS IN THE COLLISIONLESS APPROXIMATION

We shall give the expressions for the elements of the electrical conductivity tensor σ_{ij} obtained from the quantum theory ignoring electron scattering. They are derived by the well-known method using the equation describing the evolution of the density matrix and the response of a system to a weak external field $\mathbf{F}e^{(s+i\omega)t}$, $s \to +0$. For a spatially inhomogeneous system the quantities σ_{ij} are understood to be the coefficients relating the linear spatial-average current $\int j d^3r$ with the components of the external field $\mathbf{F}$:

$$\sigma_{xy}=\frac{Nec}{H}\frac{\omega_H^2}{\tilde\omega^2-\omega^2},\quad \sigma_{xx}=\frac{Ne^2}{i\omega m}\frac{\omega^2-\Omega^2}{\omega^2-\tilde\omega^2},$$
$$\sigma_{yy}=\frac{Ne^2}{i\omega m}\frac{\omega^2}{\omega^2-\tilde\omega^2},\quad \sigma_{zz}=\frac{Ne^2}{i\omega m}. \tag{4}$$

Here, N is the average bulk density of electrons and $\tilde\omega \equiv \Omega^2+\omega_H^2$.

In the limit $\omega \to 0$, the Hall component tends to a constant value

$$\sigma_{xy}=Nec\omega_H^2/H\tilde\omega^2, \tag{5}$$

σ_{yy} tends to zero, and σ_{xx} is described by

$$\sigma_{xx}\approx Ne^2\Omega^2/im\omega\tilde\omega^2. \tag{6}$$

Comparing this expression with the dynamic polarizability of free electrons we can see that the difference is the mass renormalization: $m \to m\tilde\omega^2/\Omega^2$.

$$\alpha(\omega)=\sigma/i\omega=-Ne^2/m\omega^2, \tag{7}$$

3. METHOD OF THE BOLTZMANN TRANSPORT EQUATION

We shall now calculate the magnetoconductivity tensor of a parabolic layer using the classical transport equation. We shall allow for the carrier scattering in the simplest model: an electron experiences instantaneous elastic collisions and the kernel of the collision integral is independent of the kinetic energy of a particle.

The equilibrium distribution function f_0 depends only on the combination $E = p^2/2m + U(y)$. The nonequilibrium correction $g(\mathbf{p}, y)e^{i\omega t}$ is given by

$$i\omega g+\frac{p_y}{m}\frac{\partial g}{\partial y}+\frac{\partial g}{\partial \mathbf{p}}[\mathbf{p}\omega_H]-m\Omega^2 y\frac{\partial g}{\partial p_y}+\frac{e\mathbf{p}\mathbf{F}}{m}$$
$$+\sum_{p'}[g(\mathbf{p},y)-g(\mathbf{p}',y)]W_{pp'}=0. \tag{8}$$

The arrival and departure terms in the collision integral refer to the same value of y because of the local nature of an elementary scattering event. We shall seek the solution of Eq. (8) in the form

$$g(\mathbf{p},y)=(Ay+\mathbf{B}\mathbf{p})\frac{df_0}{dE}. \tag{9}$$

The collision integral is then equal to $\nu\mathbf{B}\cdot\mathbf{p}$, where

$$\nu \equiv \tau^{-1}=\sum_{p'}W_{pp'}[1-\cos(\hat{pp}')]$$

The coefficients A and B_j are given by

$$A=\frac{e\Omega^2}{i\omega\Delta(\omega)}[\omega_H F_x-(i\omega+\nu)F_y],$$
$$B_j=-\frac{e}{m\Delta(\omega)}\left[\left(i\omega+\nu+\frac{\Omega^2}{i\omega}\right)+\omega_H F_y\right], \tag{10}$$

$$B_y=\frac{e}{m\Delta(\omega)}[\omega_H F_x-(i\omega+\nu)F_y],\quad B_z=-\frac{e}{m}\frac{F_z}{i\omega+\nu};$$
$$\Delta(\omega)=\omega_H^2+(i\omega+\nu)(i\omega+\nu+\Omega^2/i\omega). \tag{11}$$

Using Eqs. (9)–(11), we can readily calculate the current density

$$\mathbf{j}=e\int\frac{g\mathbf{p}}{m}d^3p\,dy$$

and find the tensor $\sigma_{ij}(\omega)$. In the limit of a high frequency of the external field, $\omega\tau\gg1$, the classical values of σ_{ij} calculated in this way are identical with the quantum expressions (4). However, the situation changes greatly in the low-frequency limit, $\omega\tau\ll1$, and particularly in the case of a static field. For $\omega=0$, we obtain from Eqs. (10) and (11):

$$A=eF_x\omega_H\tau-eF_y,\quad B_x=-e\tau F_x/m,\quad B_y=0,\quad B_z=-e\tau F_z/m. \tag{12}$$

The tensor σ_{ij}, corresponding to the solution (12), is

$$\sigma=\frac{Ne^2\tau}{m}\begin{pmatrix}1&0&0\\0&0&0\\0&0&1\end{pmatrix}, \tag{13}$$

i.e., it describes a two-dimensional free particle with an isotropic mass m.

It follows that the classical and quantum results for the static magnetoconductivity tensor are very different. The Hall components σ found from the transport equation vanish, whereas the quantum calculation gives the final value $Nec\omega_H^2/H\tilde{\omega}^2$. Clearly, this is because the scattering of electrons is not allowed for in the quantum calculation and this leads directly to the statement made in the Introduction: in the problem under consideration (in contrast to the homogeneous case) allowance for even weak scattering ($\omega_H\tau\gg1$) alters greatly the Hall component σ_{xy}. We can show that the corrections to σ_{xy} due to the scattering [in the model of point impurities $v_0\delta(\mathbf{r})$] give rise, in the first order of v_0, to the usual renormalization of the chemical potential and the quadratic contribution diverges in proportion to $\Omega^2v_0^2/is$. This divergence is due to the fact that the diagonal matrix element of the operator x, which is the velocity component, does not vanish for an inhomogeneous system.[1)]

The scattering alters greatly the diagonal component σ_{xx}. This is clear from the fact that Eqs. (12) and (13) include the "bare" m and not the renormalized mass $m\tilde{\omega}^2/\Omega^2$.

It should be pointed out that the tensor (13) is completely independent of the magnetic field although no assumptions have been made about the value of ω_H. Thus, in this situation the transverse magnetoresistivity vanishes. This is not surprising because in our model all carriers have the same relaxation time and then there is no magnetoresistivity in the homogeneous problem. However, it is interesting that if τ is a function of the total energy $E=p^2/2m+U$, then—as can easily be demonstrated—the solution of the transport equation is still given by Eqs. (10) and (11) or Eq. (12) [for $\omega=0$]. Consequently, there is no magnetoresistivity even in this case although different electrons have different relaxation times. Physically this is quite clear: each carrier undergoes elastic collisions and, therefore, it is characterized by its own value of τ; for a given τ, electron drifts along the y axis until it reaches a point y_0 at which a balance is attained between the electric force eF_y, the Lorentz force $e\omega_H\tau Fx$ and the force $m\Omega^2y_0$, due to the potential (1):

$$y_0=\frac{e}{m\Omega^2}(F_y-\omega_H\tau F_x). \tag{14}$$

The solution (9) describes the shift of the equilibrium distribution in the momentum and coordinate spaces. The average value of the coordinate y,

$$\int y(f_0+g)\,dy\,d^3p,$$

is, as expected, equal to y_0 of Eq. (14), where τ is replaced with $\bar{\tau}$ defined by

$$\bar{\tau}=\int f_0(E)\tau(E)\,dy\,d^3p/f_0\,dy\,d^3p.$$

Thus, the transport coefficients of our inhomogeneous system depend much more critically on the scattering of particles than in the spatially homogeneous case. The reason for this can be found easily by analyzing the classical equations of motion in which the electron scattering is modeled by the "friction force" $\mathbf{F}_{tr}=m\mathbf{u}/\tau$ ($\mathbf{u}$ is the drift velocity). Such an analysis is given by one of the present authors and Popov.[4,12)] It is found that the characteristic equation for natural frequencies of the system has the form $\Delta(\omega)=0$ with $\Delta(\omega)$ from Eq. (11). There appears a new (compared with the homogeneous problem) relaxation parameter $\tau_0=\tau\tilde{\omega}^2/\Omega^2$, which is an additional root of the characteristic equation. The solutions obtained from the quantum formulas in the static case correspond to the initial stage of the motion of the particle when $t\ll\tau_0$. Since in the homogeneous problem we have $\Omega=0$, $\tau_0=\infty$, these solutions describe steady-state motion (in the limit $t\to\infty$) and, consequently, apply to the macroscopic kinetics of the system. In the inhomogeneous case they relax to the solutions (12) and (13) with a decrement τ_0. Naturally, the static transport equation leads directly to Eqs. (12) and (13), corresponding to the steady-state electron motion.

4. CYCLOTRON RESONANCE

The absorption of microwave electromagnetic radiation is described by the real part of $\sigma(\omega)$. For a field polarized in the x direction, the absorption coefficient is obtained from Eqs. (10) and (11):

$$q_{xx}=\frac{Ne^2\nu}{m}\frac{(\omega^2-\Omega^2)^2+\omega^2(\omega_H^2+\nu^2)}{\nu^2(2\omega^2-\Omega^2)^2+\omega^2(\tilde{\omega}^2+\nu^2-\omega^2)^2}. \tag{15}$$

A graph of the function $q_{xx}(\omega)$ in the case $\tilde{\omega}\gg\nu$ is plotted in Fig. 2. The half-maximum at zero frequency is of width τ_0^{-1} and corresponds to the absorption by free carriers: the law is ω^{-2} in the range $\omega\tau_0\gg1$. In the homogeneous case this maximum is absent for the microwave polarization perpendicular to the magnetic field. The second maximum of the $q_{xx}(\omega)$ curve corresponds to the usual cyclotron resonance frequency-shifted to the point $\omega_c=(\tilde{\omega}^2+\nu^2)^{1/2}$. It is important to note that the width of

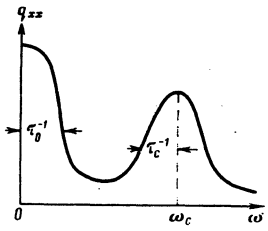

FIG. 2.

this resonance τ_c^{-1} includes an explicit dependence on the magnetic field. We find from Eq. (15) that

$$\tau_c^{-1}=\tau^{-1}(1-\Omega^2/2\bar{\omega}^2). \tag{16}$$

The dependence of the line width on H in the absence of quantum effects is a characteristic feature of the inhomogeneous problem under discussion. In the case of polarization at right-angles to the layer the absorption coefficient is

$$q_{yy}=\frac{Ne^2\nu}{m}\frac{\omega^2(\omega_H^2+\nu^2+\omega^2)}{\nu^2(\Omega^2-2\omega^2)^2+\omega^2(\bar{\omega}^2+\nu^2-\omega^2)^2}. \tag{17}$$

The curve q_{yy} has only one maximum at $\omega=\omega_c$ of the same width τ_c^{-1}. The difference from the homogeneous case is manifested also by the fact that in the limit $\omega\to 0$ the quantity q_{yy} tends to zero as ω^2 and does not reach the constant value $Ne^2\nu/m\omega_H^2$.

These features of the high-frequency conductivity of a parabolic layer may be observed when microwave radiation is absorbed by electrons populating magnetic surface levels in Schottky barriers.

5. HALL EFFECT

Vanishing of the nondiagonal components of the tensor (13) denotes formally the absence of the Hall emf in the system under discussion. However, we can see that the Hall emf ξ_H does not vanish exactly for the structure in Fig. 1 but it is small compared with the emf developed in an equivalent (in respect of free carriers) homogeneous film: $\mathcal{E}_H \ll \mathcal{E}_H^0$. The ratio $\mathcal{E}_H/\mathcal{E}_H^0$ is governed by a parameter whose smallness is the condition of validity of all the theory given above. The point is that up to now we have understood $\mathbf{F}$ to be an external (transverse) electric field. However, the solution obtained describes a shift of the equilibrium distribution along the y axis; this gives rise to a field $\tilde{F}y$ induced by distortion of the coordinate distribution of electrons. Obviously, the Hall emf is governed by this field $\tilde{F}y$:

$$\mathcal{E}_H=\int_{-\infty}^{\infty}\tilde{F}_y\,dy.$$

The field $\tilde{F}y$ is found by solving the Poisson equation in which the charge density is governed by the nonequilibrium correction to the distribution function:

$$\frac{d\tilde{F}_y}{dy}=\frac{4\pi e}{\varepsilon}\int g(\mathbf{p},y)\,d^3\mathbf{p}=\frac{4\pi e}{\varepsilon}\int Ay\frac{df_0}{dE}d^3\mathbf{p}. \tag{18}$$

Elementary calculations give

$$\tilde{F}_y=\frac{4\pi e^2}{\varepsilon}\frac{\omega_H\tau F_x-F_y}{m\Omega^2}N_0(y), \tag{19}$$

where $N_0(y)=\int f_0 d^3\mathbf{p}$ is the equilibrium local concentration of the particles. In measurements of the Hall effect the transverse field Fy vanishes. Then, the Hall emf is given by

$$\mathcal{E}_H=\frac{4\pi e^2}{\varepsilon m\Omega^2}\omega_H\tau N_s F_x; \tag{20}$$

Here, $N_s=\int N_0(y)dy$ is the density of electrons per unit surface area of the layer. In the case of a homogeneous film of thickness L the Hall emf is $\mathcal{E}_H^0=\omega\tau F_x L$ and, therefore,

$$\mathcal{E}_H/\mathcal{E}_H^0=4\pi e^2N_s/\varepsilon m\Omega^2L=\omega_p^2/\Omega^2, \tag{21}$$

where ω_p is the plasma frequency corresponding to the average electron density.

Clearly, in solving the transport equation the electron self-field can be ignored if $\omega_p^2\ll\Omega^2$. This is the parameter governing the range of validity of our treatment. The ratio ω_p^2/Ω^2 can also be written in the form N/N_+, where N is the average electron density and N_+ is the density of the positive background. We can then easily see that the Hall coefficient of the investigated system is not governed by the number of carriers but is given by

$$R=1/N_+ec. \tag{22}$$

The usual Hall mobility $\mu_H=\sigma R$ is meaningless because it is no longer dependent on the number of mobile carriers. These points have to be allowed for in the interpretation of the results of measurements of the transport characteristics of inhomogeneous layers.

6. POSSIBLE GENERALIZATIONS OF THE PROBLEM

Analytic solutions of the quantum and classical problems (transport equation and equations of motion) can be obtained only for the model of a parabolic layer (3) and a constant relaxation time. Formally, the solution remains the same also in the case when τ depends on the total energy $p^2/2m+U(y)$, but such a situation is not physically realistic. Generalizations of our model are possible in two directions.

First of all, we may consider the case of an arbitrary potential $U(y)$. Such a generalization also lifts the restriction $\omega_p^2\ll\Omega^2$ because if the number of electrons is not small compared with N_+, the screening effects result in a considerable nonparabolicity of the potential of the layer. In this case the problem requires a self-consistent solution of the transport and Poisson equations. However, we can easily show that the reasoning leading to Eq. (14) still applies to a nonparabolic well and, instead of Eq. (14), we now have

$$(dU/dy)_{y_0}=e(F_y(y_0)+F_y-\omega_H\tau F_x). \tag{23}$$

Here $F_{x,y}$ are again transverse homogeneous fields and $\tilde{F}_y$ is the self-consistent component due to the shift of the particle distribution. Hence, it follows that the steady-state motion of carriers under the action of a static external field remains effectively two-dimensional, the conductivity tensor has the form (13), and it does not depend explicitly on the magnetic field. In

the case of weak "seed" fields F_x and F_y the Hall emf can be found as follows. The average field $\langle \tilde{F}_y \rangle$, due to a shift of the coordinate distribution of electrons by an amount y_0, can be expressed in terms of the average electron density N using the obvious equality:

$$\langle \tilde{F}_y \rangle = -4\pi e y_0 N/\varepsilon. \tag{24}$$

For small values of F_x and F_y, the quantities $\langle \tilde{F}_y \rangle$ and y_0 are also small. Therefore, we can replace $(dU/dy)_{y_0}$ with $U''(0)y_0$ and the Poisson equation for the unperturbed distribution gives

$$U''(0) = \frac{4\pi e^2}{\varepsilon}(N_+ - N(0)),$$

where $N(0)$ is the density of electrons at $y=0$ (e is the electron charge!). Then it follows from Eqs. (23) and (24)

$$\langle \tilde{F}_y \rangle = \frac{N}{N+N_+-N(0)}(\omega_H \tau F_x - F_y), \tag{25}$$

so that the Hall coefficient is described by

$$ecR = (N - N(0) + N_+)^{-1} \tag{26}$$

[for $N_+ \gg N$, $N(0)$, Eq. (26) naturally reduces to Eq. (22)].

The second direction in which we can generalize the above results is associated with allowance for the dependence of τ on the kinetic energy of carriers. The resultant difficulties are similar to those encountered in the homogeneous problem for anisotropic scattering. The point is that Eq. (8) is the transport equation in a homogeneous four-dimensional space (p_x, p_y, p_z, y) for a particle with the quadratic dispersion law $p^2/2m + m\Omega^2 y^2/2$ [this is exactly why the case $\tau = \tau(E)$ is easily solved]. If τ depends only on p^2, then—as in a three-dimensional anisotropic problem—we shall seek the nonequilibrium correction to the distribution function in the form of a series in terms of the spherical harmonics,

$$g = \Phi(\mathbf{p}, y)\frac{df_0}{dE}, \quad \Phi = C + C_i p_i + {}^1/_2 C_{ik} p_i p_k + {}^1/_3 C_{ikl} p_i p_k p_l + \ldots \tag{27}$$

The tensors $C(y, p^2)$ are irreducible and symmetric in respect of all the indices. The collision operator acting on n-th term of the series (27) gives

$$\begin{aligned} &\hat{S}(C_{ik\ldots} p_i p_k \ldots) = -\tau_n^{-1}(w) C_{ik\ldots} p_i p_k \ldots, \\ &\tau_n^{-1}(w) = \sum_{p'} W_{pp'}[1 - P_n(\cos\theta)]; \quad w = p^2/2m, \quad \theta = (\widehat{\mathbf{pp}'}), \end{aligned} \tag{28}$$

where P_n is a Legendre polynomial of order n.

Next, averaging Eq. (8) with the weight 1, p_i, $p_i p_k$, etc. over the angles in the p space, we obtain a chain of equations for the spherical harmonics $C(y, p^2)$. We shall confine ourselves to the case $\omega = 0$. Averaging of Eq. (8) with a unit weight gives a closed equation for C_y:

$${}^2/_3 w C_y' - m\Omega^2 y (C_y + {}^2/_3 w \dot{C}_y) = 0. \tag{29}$$

Here the prime represents differentiation with respect to y, the dot differentiation with respect to w. The absence of a static current along the y axis means that the only permissible solution of Eq. (29) is $C_y(y, w) \equiv 0$.

We shall find the condition of validity of Eq. (13) in the case of $W_{pp'}$ depending on w. We shall do this by terminating a chain of equations retaining tensors $C(y, w)$ of rank up to two and requiring that the third term of the series in Eq. (27) is small compared with the second. Omitting the intermediate steps, we shall give the final form of this condition:

$$\begin{aligned} &(\bar{w}\dot{\tau}_1/\tau_1)\Omega\tau_2 \ll 1 \text{ for } \omega_H\tau_2 \ll 1, \\ &(\bar{w}\dot{\tau}_1/\tau_1)\Omega/\omega_H \ll 1 \text{ for } \omega_H\tau_2 \gg 1, \end{aligned} \tag{30}$$

where $\bar{w}$ is of the order of temperature of Fermi energy, depending on the degree of degeneracy of the electron gas. For the usual power-law dependence of τ_1 on the carrier energy, the factor in parentheses in Eq. (30) is of the order of unity. If Eq. (30) is satisfied (and, moreover, we still have $\Omega \gg \omega_p$), the magnetoconductivity tensor is given by Eq. (13), where τ should be replaced with $\langle \tau_1 \rangle$:

$$\langle \tau_1 \rangle = \int \tau_1(w) f_0(w + U(y)) w^{3/2} dw\, dy \left[\int f_0(w + U(y)) w^{3/2} dw\, dy \right]^{-1}. \tag{31}$$

We shall conclude by considering the question of the limiting transition to the homogeneous case $U(y) \equiv 0$. Equations (5), (10), and (11) which govern the dynamic conductivity, permit such a transition ($\Omega \to 0$) directly. Consequently, the cyclotron resonance results (see Sec. 4) reduce for $\Omega \to 0$ to the formulas for the homogeneous problem. In particular, for $\tau_c \to \tau$, the line width ceases to depend on the magnetic field. A special feature is the case of a static external field. The tensor (13) does not contain Ω explicitly so that this going to the limit is meaningless. As explained above, this is because Eq. (13) corresponds to the motion of the system at a time $t \gg \tau\bar{\omega}^2/\Omega^2$.

In Eqs. (14) and (19) the transition $\Omega \to 0$ is formally forbidden. However, if F_y in these equations represents the y component of the total field (including the self-consistent field), then in the limit $\Omega \to 0$, we have to take $F_y = \omega_H \tau F_x$. However, this is exactly the correct expression for the Hall field in a homogeneous system. However, the observed Hall coefficient is given by Eq. (26) in which the transition to the homogeneous case is quite evident: we have to substitute $N_+ = N(0)$, which corresponds to the absence of space charges in the homogeneous case [in Eq. (22) there is no such limit because this equation is valid for $\omega_p^2 \ll \Omega^2$].

The authors are grateful to F. F. Sokolov for valuable comments and discussions.

[1)] We readily see that in this situation it is meaningless to sum any sequence in the series obtained in the perturbation theory (for example, the most diverging diagrams). Since $\sigma_{xy} \neq 0$ in the zeroth approximation and the results should be $\sigma_{xy} = 0$, it is clear that nothing should be ignored in deriving this result and it is necessary to sum all the diagrams, which is impractical.

[2)] We take this opportunity to correct the error in Ref. 4 on p. 537. The transverse magnetoresistivity disappears if τ depends on the total rather than on the kinetic energy of

electrons (this is discussed later in the present paper).

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Dependence of the effects of spatial dispersion in a crystal on the exciton-damping constant

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We calculate the spectral dependences of the optical constants of two normal waves due to the spatial dispersion of ϵ in the region of the lowest excitonic state of a CdS crystal at different values of the damping constant γ. The obtained curves are used for a quantitative estimate of γ in real crystals. Two branches of the refractive index within the limits of the absorption band are measured by a direct interference method for a CdS crystal whose γ is close to the "critical" value.

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INTRODUCTION

Pekar formulated theoretically in 1957[1] the main premises of crystal optics in the region of the exciton absorption bands, when spatial dispersion of the dielectric constant $\varepsilon(\omega,\mathbf{k})$ becomes significant. In contrast to classical optics, which takes no account of the dependence of ε on the wave vector $\mathbf{k}$ of the light wave, the new theory predicts the propagation in the crystal of two normal waves having the same frequency and polarization, but different velocities, i.e., different refractive indices.

One of the most important experimental proofs of the validity of the spatial-dispersion theory was the impossibility of describing the optical properties of the CdS crystal at 4.2 K, in light transmitted through the target and reflected from it, by means of a single diffraction index n and a single absorption coefficient $\varkappa$, i.e., by a dielectric constant of the form $\varepsilon(\omega)=[n(\omega)+i\varkappa(\omega)]^2$. Thus, it was shown[2] that the spectral dependence of the reflection coefficient $R(\omega)$ calculated from the measured dispersion[3] and absorption[4] curves differs strongly from that measured in experiment. At the same time, these results can be very well reconciled by using the formulas of the spatial dispersion theory.[5] It became recently[6,7] possible to measure the phase $\varphi(\omega)$ of the reflected light in the region of the exiton band A of the CdS crystal. These results also confirmed the impossibility of describing the behavior of $\varphi(\omega)$ at 4.2 K by the formulas of classical optics and the need for resorting to the theory of spatial dispersion to explain the experimental data.

Another no less important confirmation of the essential role played by spatial dispersion in CdSe and CdS crystals at 4.2 K was the observation of interference of normal waves on the short-wave side of the exciton absorption band[8] and the reconstruction, from the obtained interference pattern, of their dispersion relations $E_1(\mathbf{k})$ and $E_2(\mathbf{k})$. It has thus been convincingly shown that the spatial dispersion exerts a significant influence on the optical properties of crystals in the region of exciton absorption bands at temperatures 4.2 K and below. It is known at the same time that at high temperatures the formulas of classical optics are well satisfied, i.e., the spatial-dispersion effects become insignificant.

The temperature dependence of the spatial-dispersion effects in CdS crystals was investigated experimentally by Voigt[9] and by Brodin *et al.*[10] With increasing temperature, a sharp increase of the area under absorption curve in the region of the exciton band A [9] and a characteristic change of the birefringence picture[10] were observed. To explain the results of[10], theoretical calculations performed by Davydov and Myasnikov[11] for anthracene crystals were used. But since exciton parameters of CdS and anthracene differ greatly, only qualitative agreement between theory and experiment, and a qualitative description of the general tendency of the variation of the optical characteristic of the substance with increasing termperature, could be expected.

We report here calculations that make it possible to follow the gradual change in various characteristics of both normal waves with increasing damping constant γ of the excitons in CdS crystals. The obtained data are used for a quantitative estimate of γ in various crystals investigated in different experimental conditions. Two

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