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Spatial amplification of helical waves in the course of uniaxial deformation of Ge crystals

V. M. Bondar, V. V. Vladimirov, V. P. Doskoch, and A. I. Shchedrin

Institute of Physics, Academy of Sciences of the Ukrainian SSR, Kiev
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Measurements were made of the spatial gain of helical waves in Ge crystals compressed and stretched along the ⟨111⟩ direction. At frequencies corresponding to the maximum amplification, the value of the gain increased (decreased) in n-type samples and decreased (increased) in p-type samples as a result of elongation (compression). The results are attributed to a change in the velocity of the ambipolar drift of helical waves in the course of intervalley redistribution of electrons caused by the deformation. The calculated values of the gain are in qualitative agreement with the experimental data. Analogies with gaseous plasma are pointed out.

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1. The absolute instability of helical waves (oscillation regime) was first observed in the experiments of Ivanov and Ryvkin,[1] who discovered an instability of the current in Ge samples subjected to a sufficiently strong longitudinal magnetic field. The results of these experiments were explained by Glicksman[2] on the basis of the theory of helical instabilities developed by Kadomtsev and Nedospasov[3] for gaseous plasmas. The regime of spatial amplification of helical waves (convective instability) was first investigated by Hurwitz and McWhorter.[4] This regime appears in the case of ambipolar drift of helical perturbations in the direction of an electric field and is characteristic of semiconductor plasmas when the drift velocity can be controlled by varying the ratio of the electron and hole densities through suitable doping of a sample. The phase velocity of helical waves is equal to the velocity of ambipolar drift. An increase in this phase velocity is accompanied by a reduction in the maximum value of the gain k_{im} and an increase in the frequency corresponding to the gain maximum ($f=f_m$).[4] A considerable change in the ambipolar drift velocity results from an intervalley redistribution of electrons when the anisotropy of the electron mobility becomes important.[5] In this case, the drift occurs even when the densities of electrons and holes are equal ($n=p$). The influences of this anisotropic drift resulting from such an intervalley redistribution on the absolute instability of helical waves was studied in detail by Bondar *et al.*[5] in Ge and Si crystals. We shall show that this effect greatly alters the nature of the spatial amplification of helical waves.

A considerable intervalley redistribution of electrons in Ge occurs when a crystal is deformed along a ⟨111⟩ axis. The constant-energy surfaces of Ge near the bottom of the conduction band are four ellipsoids of revolution elongated along such ⟨111⟩ axes. Compression transfers electrons to a valley parallel to the direction of deformation and elongation removes electrons from this valley.[6] In the former case, the transverse mobility averages over all the valleys is greater than the longitudinal value ($\overline{\mu}_{e\perp}>\overline{\mu}_{e\parallel}$), because the electrons are transferred to a valley with a higher transverse mobility, whereas, in the latter case, we have $\overline{\mu}_{e\parallel}>\mu_{e\perp}$. An expression for the ambipolar mobility, which governs the drift velocity ($v_a=\mu_0 E$), has the following form in the anisotropic case[5]

$$\mu_a=\mu_h(n\bar{\mu}_{e\perp}-p\bar{\mu}_{e\parallel})/(n\bar{\mu}_{e\perp}+p\mu_h), \quad (1)$$

where μ_h is the hole mobility (a scalar quantity). Compression (elongation) of an n-type Ge crystal ($n \geq p$) should, in accordance with Eq. (1), increase (reduce) the drift velocity, whereas, in the case of a p-type sample ($p \geq n$), the reverse should occur. The principal characteristics of spatial amplification of helical waves should change accordingly. For example, in the case of n-type samples, the value of k_{im} should decrease (increase) and the frequency f_m should increase (decrease) on compression (elongation). In the case of p-type crystals, the values of k_{im} and f_m should vary in opposite ways. The results of our experiments, carried out in weak magnetic fields ($\mu_{i=e,h}H/c \ll 1$) are in agreement with these predictions. In weak magnetic fields, the mobility anisotropy is entirely due to the intervalley redistribution of electons.

2. Our measurements were carried out at room temperature on n- and p-type Ge with near-intrinsic conductivity ($\rho = 60\Omega\cdot$cm and $50\Omega\cdot$cm, respectively, $n \approx p$). Our samples were rectangular slabs of $20\times1\times1$ mm dimensions. One of the end contacts was ohmic and the other injecting. Carriers were injected electrically or optically. The distribution of the injected carriers along the sample was determined by a microwave method in the free and deformed states. This made it possible to identify the cases of weak plasma density gradients along the sample and to check the quality of the contacts. Etching ensured a low surface recombination velocity ($s \leq 50$ cm/sec). The excitation of a signal and measurements of the amplitude of a helical wave were carried out using symmetric pairs of ohmic probes deposited along the length of a sample in steps of 1.5 mm. Measurements of the gain were carried out using a small-amplitude signal so that the conditions were far from saturation.

The system employed in the probe method is shown schematically in Fig. 1. An electric field pulse (of duration 1-2 msec) was applied to probes 1-1′ simultaneously with an exciting alternating voltage produced by a signal generator and passed through a transformer, which was amplitude-calibrated to ensure constancy of the voltage when the frequency was varied in the 10–200 kHz range. The output signal from the other pairs of probes was recorded with the aid of an oscillograph.

Figure 2 shows the resultant dependences of the gain [$k_i = l^{-1}\ln(A_{33'}/A_{22'})$, where l is the distance between the 3-3′ and 2-2′ pairs of probes and A is the signal amplitude] on the frequency, obtained for different values of compressive and tensile stresses applied along the ⟨111⟩ direction coinciding with the axis of the sample. We can clearly see that, in the case of n-type Ge (Fig. 2a), a

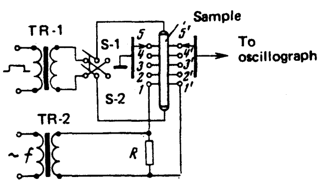

FIG. 1. Schematic diagram of the apparatus used in measurements of the spatial gain by the probe method.

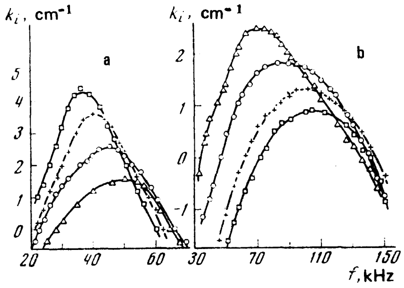

FIG. 2. Spatial gain of helical waves measured on deformation of Ge crystals along the ⟨111⟩ direction at $T=300$°K. a) Crystals of n-type Ge, $\rho=60\ \Omega\cdot$cm, $E=35$ V/cm, $H=6.5$ kOe: ○) $P=0$ kg/cm^2; △) $P=330$ kg/cm^2 (compression); +) $P=330$ kg/cm^2, □) $P=660$ kg/cm^2 (tension). b) Crystals of p-type Ge, $\rho=50\ \Omega\cdot$cm, $E=35$ V/cm, $H=5$ kOe: ○) $P=0$ kg/cm^2; △) $P=400$ kg/cm^2 (compression); +) $P=400$ kg/cm^2, □) $P=600$ kg/cm^2 (tension).

compressive stress reduces the maximum gain k_{im} and shifts the frequency f_m toward higher values. Elongation increases k_{im} and reduces f_m. Opposite dependences are obtained for p-type Ge (Fig. 2b). Thus, the results of the measurement are in agreement with the above qualitative analysis. It should be pointed out that the deformation along a symmetric direction ⟨100⟩, when there is no intervalley redistribution, causes no changes in the dependences $k_i(f)$. The bell-shaped frequency dependence of the gain (Fig. 2) essentially reflects the nature of the change in the helical-wave increment ($k_i = v_a^{-1}\,\mathrm{Im}\,\omega$) on the wave vector $k_r (k = k_r + ik_i)$. For small values of the wave vector (or frequency, because $k_r = v_a^{-1}\,\mathrm{Re}\,\omega$), the drift current causing an instability is small but, for large values of the wave vector, there is a strong longitudinal diffusion which suppresses the helical waves.

3. A calculation of the spatial gain was carried out for spherical helical waves[4] within the framework of the two-valley model. It was assumed that an electron gas consists of two ensembles in which electrons have different mobilities along and across the electric field. In the first ensemble (I) the mobilities $\mu_\parallel$ and $\mu_\perp$ correspond to the long and short axes of an ellipsoid of revolution. This ensemble is equivalent to a valley elongated along the ⟨111⟩ deformation axis. In the second ensemble (II), representing the other valleys, the mobilities $\mu_{\parallel\mathrm{eff}}$ and $\mu_{\perp\mathrm{eff}}$ are given by

$$\mu_{\parallel\mathrm{eff}} = {}^8/_9\mu_\perp + {}^1/_9\mu_\parallel, \quad \mu_{\perp\,\mathrm{eff}} = {}^4/_9\mu_\parallel + {}^5/_9\mu_\perp. \quad (2)$$

It was assumed that static electric and magnetic fields ($\mu_{i=e,h}H/c \ll 1$) were directed along the deformation axis. The calculations were carried out for a cylindrical sample with a low surface recombination velocity. Perturbations were assumed to be of the form

$$A' = A_1(r)\exp(i\omega t - im\varphi - ikz),$$

where $|m| = 1$ (a helical wave). The procedure for obtaining the dispersion relationship was the same as that given in our earlier paper.[5] This relationship was analyzed earlier in respect of the absolute instability. We shall now give the main characteristics of the spatial amplification of helical waves.

The gain and threshold values of the magnetic field and frequency are given by

$$k_i=\mathrm{Im}\,k=\frac{8}{3}\frac{D_h\bar{\mu}_{e\perp}(n+p)}{R^2\mu_a(\bar{\mu}_{e\perp}n+\mu_h p)E}\left[2\frac{\omega}{\omega_{th}}\frac{H}{H_{th}}-\left(\frac{\omega}{\omega_{th}}\right)^2-1\right], \quad (3)$$

$$H_{th}=\frac{8cD_h\bar{\mu}_{e\perp}(n+p)(\bar{\mu}_{e\perp}n+\mu_h p)}{k_a R^2 E\mu_h np(\bar{\mu}_{e\parallel}+\mu_h)(\mu^2+\bar{\mu}_{e\perp}\mu_h)}, \quad (4)$$

$$\omega_{th}=k_{th}\mu_a E, \quad k_{th}^2=\frac{8}{3R^2}\frac{\bar{\mu}_{e\perp}}{\bar{\mu}_{e\perp}+\bar{\mu}_{e\parallel}}, \quad (5)$$

where

$$\bar{\mu}_{e\parallel}=(\mu_\parallel\tilde{n}_{\rm I}+\mu_{\parallel\rm eff}\tilde{n}_{\rm II}), \quad \bar{\mu}_{e\perp}=(\mu_\perp\tilde{n}_{\rm I}+\mu_{\perp\rm eff}\tilde{n}_{\rm II}),$$
$$\mu^2=(\mu_\perp^2\tilde{n}_{\rm I}+\mu_{\perp\rm eff}^2\tilde{n}_{\rm II}). \quad (6)$$

The following notation is used above: D_h is the diffusion coefficient of holes; R is the radius of the sample; E is the electric field intensity; μ_a is defined by Eq. (1); $\tilde{n}_{\rm I,II}=n_{\rm I,II}/(n_{\rm I}+n_{\rm II})$, where $n_{\rm I,II}$ are the electron densities in the ensembles. The relationships (3)–(5) are derived on the assumption that

$$\mu_a E\gg D_h/R.$$

In the absence of anisotropy ($\bar{\mu}_{e\parallel}=\bar{\mu}_{e\perp}$), Eqs. (3)–(5) reduce to the expressions obtained earlier.[4] The above above calculations assume the relationship[7]

$$\frac{n_{\rm I}}{n_{\rm II}}=\frac{1}{3}\exp\left(\frac{4}{9}\frac{\Sigma_u P}{c_{44}kT}\right), \quad (7)$$

which represents the degree of redistribution of electrons on the application of a stress P. In Eq. (7), $\Sigma_u=18$ eV (deformation potential constant), $c_{44}=0.67\times10^{12}$ dyn/cm^2 (elastic constant), and T is the temperature of the crystal.

In analyzing the dispersion relationship, allowance was only made for the first two terms of the expansion in terms of the small imaginary argument of the Bessel functions, representing the solutions of the initial equations for the density and potential perturbations.[5] This procedure was justified in the range of low frequencies (small wave vectors) when

$$fR/\mu_a E\ll 1. \quad (8)$$

Figure 3 shows the calculated frequency dependences of k_i for the cases of compressive and tensile deformation of n- and p-type Ge samples. The agreement with the experimental data (Fig. 2) is both qualitative and quantitative. Deterioration in the quantitative agreement at high frequencies is due to the fact that the condition (8) is not satisfied accurately and this condition is used in the derivation of the analytic results. It should be pointed out that, in the case of the carrier mobility anisotropy, the regime of spatial amplification of helical waves appears even in a neutral plasma ($n=p$) because, in accordance with Eq. (1), there is ambipolar drift even in this case. Therefore, in strong magnetic fields, which give rise to anisotropy of the electron mobility, the regime of spatial amplification of helical waves should also appear in the plasma of a positive column of a gaseous discharge. This regime of a helical instability, preceding the well-known absolute instability,[3,8] has hardly been investigated for gaseous plasmas.

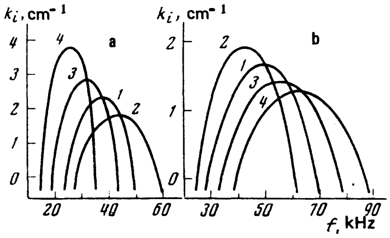

FIG. 3. Spatial gain of helical waves calculated for the deformation of Ge crystals along the ⟨111⟩ direction at 300°K ($E=25$ V/cm, $H=5$ kOe). a) Crystals of n-type Ge, $n/p=1.3$: 1) $P=0$ kg/cm^2; 2) $P=300$ kg/cm^2 (compression); 3) $P=300$ kg/cm^2; 4) $P=600$ kg/cm^2 (tension). b) Crystals of p-type Ge, $n/p=0.7$; the curves have the same meaning as in Fig. 3a.

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