Contribution to the theory of light scattering in quantum crystals

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Correlators of the fluctuations of the lattice-deformation field and the defecton distribution functions are obtained for a quantum crystal of the Fermi type. It is shown that in the case of Rayleigh scattering of light in solid He^3, observation of collective excitations, which are coupled oscillations of the lattice sites and of the defecton density, may turn out to be realistic.

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1. INTRODUCTION

It is known that quantum crystals are characterized by a large overlap of the wave functions of the neighboring atoms. Therefore a description of such crystals in the language of small deviations of the atoms from the equilibrium positions in the crystal-lattice sites is insufficient. The delocalization of the atoms in crystals of helium isotopes (and their solutions) leads to a number of pure quantum effects, the most pronounced of which is quantum diffusion. This effect, first predicted by Andreev and Lifshitz,[1] has by now been well investigated experimentally (see the review[2]).

Andreev and Lifshitz[1] have called attention also to the fact that if the atoms are not localized in definite sites, the requirement that the number of sites be equal to the number of atoms is not obligatory. As a result, gapless excitations of a new type–zero-point defectons–can exist in quantum crystals. A crystal containing such excitations should reveal simultaneously properties characteristic of a solid and of a liquid. In particular, as noted in [1], in a Fermi-type crystal with zero-point defectons there can exist, besides the usual acoustic oscillations, also collective excitations of the zero-sound type, accompanied by oscillations of the crystal density when the lattice sites are immobile. A theory that describes such excitations was developed by Dzyaloshinskiĭ, Kondratenko, and Levchenkov.[3]

The most convenient and direct method of investigating the high-frequency spectrum of the excitations of condensed bodies is the scattering in them of light or slow neutrons. If the body temperature and the frequency of the natural oscillations are such that the weak-damping condition is satisfied, then sharp peaks are produced in the spectrum of the particles inelastically scattered at a given angle, and are due to emission and absorption of collective excitations.

A theory of Rayleigh scattering of light in a quantum liquid of the Fermi type (liquid He^3) was developed by Abrikosov and Khalatnikov.[4] For scattering and reflection of slow neutrons from the surface of a Fermi liquid, an analogous problem was solved by A. Akhiezer, I. Akhiezer, and Pomeranchuk.[5]

The purpose of the present paper is the study of particle scattering in a quantum crystal of the Fermi type (solid He^3). The main content of the paper is the calculation of the density-fluctuation correlator, in terms of which the differential extinction coefficient of the light, the differential scattering coefficient, and the neutron reflection coefficient are expressed with the aid of known relations (see, e.g.,[4,5]).

2. FLUCTUATIONS OF STABILITY IN A QUANTUM CRYSTAL

To calculate the fluctuation correlators we use for a quantum crystal of the Fermi type the phenomenological model proposed in[4]. In this model the state of the crystal is characterized by a lattice deformation field $\mathbf{u}(\mathbf{r},t)$ and by the quasiparticle distribution function $n(\mathbf{p},\mathbf{r},t)$. The change produced in the thermodynamic potential of the crystal by long-wave distortions $u_i(\mathbf{r},t)$ of the crystal lattice and by the perturbation $\delta n(\mathbf{p},\mathbf{r},t)$ of the single-particle density matrix is given according to[3] by the formula

$$\delta\Omega=\int d^3r\left\{\frac{1}{2}\rho_{ik}\frac{\partial u_i(\mathbf{r},t)}{\partial t}\frac{\partial u_k(\mathbf{r},t)}{\partial t}+\frac{1}{2}\lambda_{iklm}\frac{\partial u_k}{\partial r_i}\frac{\partial u_m}{\partial r_l}+\mathrm{Sp}_\sigma\int\frac{d^3p}{(2\pi)^3}[\varepsilon_0(\mathbf{p})-\mu+\varphi(\mathbf{p},\mathbf{r},t)]\delta n(\mathbf{p},\mathbf{r},t)+\frac{1}{2}\mathrm{Sp}_{\sigma\sigma'}\int\frac{d^3pd^3p'}{(2\pi)^6}f(\mathbf{p},\mathbf{p}')\delta n(\mathbf{p},\mathbf{r},t)\delta n(\mathbf{p}',\mathbf{r},t)\right\}, \quad (1)$$

where

$$\varphi(\mathbf{p},\mathbf{r},t)=\xi_i(\mathbf{p})\frac{\partial u_i(\mathbf{r},t)}{\partial t}+\zeta_{ik}(\mathbf{p})\frac{\partial u_k(\mathbf{r},t)}{\partial r},\quad (2)$$

$\epsilon_0(\mathbf{p})$ and $f(\mathbf{p},\mathbf{p}')$ are the unperturbed energy and the Landau correlation function, μ is the chemical potential, and $\xi_i(\mathbf{p})$ and $\zeta_{ik}(\mathbf{p})$ are parameters that describe the connection between the perturbations of the lattice and of the quasiparticle distribution function.

The corresponding change in the single-particle energy is[3]

$$\delta\varepsilon(\mathbf{p},\mathbf{r},t)=\varphi(\mathbf{p},\mathbf{r},t)+\mathrm{Sp}_{\sigma'}\int\frac{d^3p'}{(2\pi)^3}f(\mathbf{p},\mathbf{p}')\delta n(\mathbf{p}',\mathbf{r},t).\quad (3)$$

To calculate the correlators of the fluctuations it is convenient to change over from the variables δn and $\mathbf{u}$ to new variables for which $\delta\Omega$ is a quadratic form. To this end, we represent the perturbation of the single-particle density matrix in the form

$$\delta n=\frac{\partial n_0}{\partial\varepsilon_0}\Delta+\frac{1}{2}\frac{\partial^2 n_0}{\partial\varepsilon_0^2}\Delta^2+\ldots,\quad (4)$$

where n_0 is the Fermi distribution function,

$$\Delta(\mathbf{p},\mathbf{r},t)=\nu(\mathbf{p},\mathbf{r},t)+\Phi(\mathbf{p},\mathbf{r},t),\quad (5)$$

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and $\Phi(\mathbf{p}, \mathbf{r}, t)$ satisfies the integral equation

$$\Phi(\mathbf{p},\mathbf{r},t)-\mathrm{Sp}_{\sigma'}\int\frac{d^3p'}{(2\pi)^3}\frac{\partial n_0'}{\partial\varepsilon_0'}f_{pp'}\Phi(\mathbf{p}',\mathbf{r},t)=\varphi(\mathbf{p},\mathbf{r},t) \tag{6}$$

(the prime pertains here and below to the argument $\mathbf{p}$).

Substituting (4) and (5) in (1) and using (6), we obtain

$$\delta\Omega=\frac{1}{2}\int d^3r\left\{\rho_{ik}\frac{\partial u_i}{\partial t}\frac{\partial u_k}{\partial t}+\lambda_{iklm}\frac{\partial u_k}{\partial r_i}\frac{\partial u_m}{\partial r_l}+\mathrm{Sp}_{\sigma}\int\frac{d^3p}{(2\pi)^3}\frac{\partial n_0}{\partial\varepsilon_0}(\Phi^2-\nu^2)-\mathrm{Sp}_{\sigma\sigma'}\int\frac{d^3p\,d^3p'}{(2\pi)^6}\frac{\partial n_0}{\partial\varepsilon_0}\frac{\partial n_0'}{\partial\varepsilon_0'}f_{pp'}(\Phi\Phi'-\nu\nu')\right\}. \tag{7}$$

Taking into account (2) and the symmetry properties of the coefficients $\xi_i(\mathbf{p})$ and $\zeta_{ik}(\mathbf{p})$,[3] we have

$$\delta\Omega=\frac{1}{2}\int d^3r\left\{P_{ik}\frac{\partial u_i}{\partial t}\frac{\partial u_k}{\partial t}+\Lambda_{iklm}\frac{\partial u_k}{\partial r_i}\frac{\partial u_m}{\partial r_l}-\mathrm{Sp}_{\sigma}\int\frac{d^3p}{(2\pi)^3}\frac{\partial n_0}{\partial\varepsilon_0}\nu^2+\mathrm{Sp}_{\sigma\sigma'}\int\frac{d^3p\,d^3p'}{(2\pi)^6}\frac{\partial n_0}{\partial\varepsilon_0}\frac{\partial n_0'}{\partial\varepsilon_0'}f_{pp'}\nu\nu'\right\}, \tag{8}$$

where

$$P_{ik}=\rho_{ik}+\mathrm{Sp}_{\sigma}\int\frac{d^3p}{(2\pi)^3}\frac{\partial n_0}{\partial\varepsilon_0}\Xi_i(\mathbf{p})\Xi_k(\mathbf{p})-\mathrm{Sp}_{\sigma\sigma'}\int\frac{d^3p\,d^3p'}{(2\pi)^6}\frac{\partial n_0}{\partial\varepsilon_0}\frac{\partial n_0'}{\partial\varepsilon_0'}f_{pp'}\Xi_i(\mathbf{p})\Xi_k(\mathbf{p}'), \tag{9}$$

$$\Lambda_{iklm}=\lambda_{iklm}+\mathrm{Sp}_{\sigma}\int\frac{d^3p}{(2\pi)^3}\frac{\partial n_0}{\partial\varepsilon_0}Z_{ik}(\mathbf{p})Z_{lm}(\mathbf{p})-\mathrm{Sp}_{\sigma\sigma'}\int\frac{d^3p\,d^3p'}{(2\pi)^6}\frac{\partial n_0}{\partial\varepsilon_0}\frac{\partial n_0'}{\partial\varepsilon_0'}f_{pp'}Z_{ik}(\mathbf{p})Z_{lm}(\mathbf{p}'). \tag{10}$$

The functions $\Xi_i(\mathbf{p})$ and $Z_{ik}(\mathbf{p})$ are solutions of the integral equations

$$\Xi_i(\mathbf{p})-\mathrm{Sp}_{\sigma'}\int\frac{d^3p'}{(2\pi)^3}\frac{\partial n_0'}{\partial\varepsilon_0'}f_{pp'}\Xi_i(\mathbf{p}')=\xi_i(\mathbf{p}), \tag{11}$$

$$Z_{ik}(\mathbf{p})-\mathrm{Sp}_{\sigma'}\int\frac{d^3p'}{(2\pi)^3}\frac{\partial n_0'}{\partial\varepsilon_0'}f_{pp'}Z_{ik}(\mathbf{p}')=\zeta_{ik}(\mathbf{p}). \tag{12}$$

The variable ν introduced by us characterizes the non-equilibrium character of the quasiparticle distribution function. At $\nu=0$ the change of the distribution function n is due entirely to the change of the quasiparticle energy $\Phi(\mathbf{p}, \mathbf{r}, t)$ in the lattice-deformation field, which is partially screened because of the interaction between the particles.

The stability of the quantum crystal calls for the integral quadratic form (8) to be positive-definite at arbitrary values of the variables $\nu(\mathbf{p}, \mathbf{r}, t)$, $\partial u_i(\mathbf{r}, t)/\partial t$ and $\partial u_i(\mathbf{r}, t)/\partial r_k$. Besides the stability conditions obtained by Pomeranchuk[5] for a Fermi liquid, the stability of a quantum crystal requires also that the tensors P_{ik} and Λ_{iklm} be positive definite. This imposes additional limitations on the functions $\xi_i(\mathbf{p})$, $\zeta_{ik}(\mathbf{p})$ and $f(\mathbf{p}, \mathbf{p}')$. Whereas the Pomeranchuk conditions imposed on the function $f(\mathbf{p}, \mathbf{p}')$ correspond to the requirement that the Fermi surface be stable, the conditions that the tensors P_{ik} and Λ_{iklm} be positive-definite correspond to the requirement that the crystal lattice be stable.

To obtain the fluctuation correlators we use the known method used in Fermi-liquid theory.[4,5] This method implies the introduction of sources (random forces) into the equations of motion of the perturbations. The solutions of the equations must next be expressed in terms of the random forces, and the correlators of the fluctuations of the physical quantities in terms of the correlators of the fluctuations of the random forces. These correlators are determined in accordance with the general theory of fluctuations.

The equations of motion of the perturbations of the quasiparticle distribution in a crystal lattice, in the presence of random forces, take according to[3] the form (we confine ourselves hereafter to the case $\xi_i=0$)

$$\frac{\partial\delta n}{\partial t}+v_i\frac{\partial}{\partial r_i}\left\{\delta n-\frac{\partial n_0}{\partial\varepsilon_0}\left[\mathrm{Sp}_{\sigma'}\int\frac{d^3p'}{(2\pi)^3}f_{pp'}\delta n'+\zeta_{lm}\frac{\partial u_m}{\partial r_l}\right]\right\}=\frac{\partial n_0}{\partial\varepsilon_0}(I_0+y_0), \tag{13}$$

$$\rho_{ik}\frac{\partial^2 u_k}{\partial t^2}-\lambda_{iklm}\frac{\partial^2 u_m}{\partial r_k\partial r_l}-\mathrm{Sp}_{\sigma}\int\frac{d^3p}{(2\pi)^3}\zeta_{ki}\frac{\partial\delta n}{\partial r_k}=\rho_{ik}(I_k+y_k), \tag{14}$$

where $I_0\partial n_0/\partial\epsilon_0$ and $\rho_{ik}I_k$ are the collision integrals for the kinetic equation and the Hamilton equation for the lattice; y_0 and y_i are random forces (all the perturbations are considered near the Fermi surface).

Substituting (4) in (13) and (14) and taking (5), (6), (10), and (12) into account we obtain for the variables $\nu(\mathbf{p}, \mathbf{r}, t)$ and $\mathbf{u}(\mathbf{r}, t)$ the equations

$$\frac{\partial\nu}{\partial t}+Z_{ik}\frac{\partial^2 u_k}{\partial t\,\partial r_i}+v_i\frac{\partial}{\partial r_i}\left\{\nu-\mathrm{Sp}_{\sigma'}\int\frac{d^3p'}{(2\pi)^3}\frac{\partial n_0'}{\partial\varepsilon_0'}f_{pp'}\nu'\right\}=I_0+y_0, \tag{15}$$

$$\rho_{ik}\frac{\partial^2 u_k}{\partial t^2}-\Lambda_{iklm}\frac{\partial^2 u_m}{\partial r_k\partial r_l}-\mathrm{Sp}_{\sigma}\int\frac{d^3p}{(2\pi)^3}\frac{\partial n_0}{\partial\varepsilon_0}\zeta_{ki}\frac{\partial\nu}{\partial r_k}=\rho_{ik}(I_k+y_k). \tag{16}$$

Following the general method of fluctuation theory, we must now determine the time derivative of the thermodynamic potential and represent it in the form

$$\dot\Omega=\int d^3r\sum_j\dot x^{(j)}(\mathbf{r},t)X^{(j)}(\mathbf{r},t), \tag{17}$$

where $\dot x^{(j)}$ are the generalized thermodynamic velocities and $X^{(j)}$ are the generalized thermodynamic forces corresponding to them.

Differentiating (8) with respect to time, using the equations of motion (15) and (16), and recognizing that, since the system is closed, there is no energy flux through its boundary, we obtain

$$\dot\Omega=\int d^3r\left\{\frac{\partial u_i}{\partial t}\rho_{ik}(I_k+y_k)-\mathrm{Sp}_{\sigma}\int\frac{d^3p}{(2\pi)^3}\frac{\partial n_0}{\partial\varepsilon_0}\nu(I_0+y_0)+\mathrm{Sp}_{\sigma\sigma'}\int\frac{d^3p\,d^3p'}{(2\pi)^6}\frac{\partial n_0}{\partial\varepsilon_0}\frac{\partial n_0'}{\partial\varepsilon_0'}f_{pp'}\nu'(I_0'+y_0')\right\}. \tag{18}$$

We note that the equations of motion for the spin-dependent part of the perturbation of the distribution function can be regarded, within the framework of the theory considered here, as unrelated to the lattice deformation. We shall henceforth confine ourselves to spin-independent perturbations. In addition, we shall assume for simplicity that the Fermi surface is spherical[1)] and choose the collision integrals in the simplest form that satisfies the conservation laws:

$$I_0\{\nu\}=-\frac{1}{\tau_0}\left\{\nu-\nu^0-\sum_{m=-1}^{1}\nu^{1m}P_1^m(\cos\theta)e^{im\varphi}\right\}, \tag{19}$$

$$I_1\{u\}=-\frac{1}{\tau_1}\frac{\partial u_i}{\partial t}, \tag{20}$$

where P_l^m are associated Legendre polynomials, θ and φ the azimuthal and polar angles in momentum space, and τ_0 and τ_1 are the relaxation times of the distribution function and of the lattice deformations.

Expanding $\nu(\mathbf{p},\mathbf{r},t), f(\mathbf{p},\mathbf{p}')$ and $y_0(\mathbf{p},\mathbf{r},t)$ in spherical harmonics near the Fermi surface and comparing (17) with (18), we see that the generalized thermodynamic velocities and forces can be chosen in the form

$$\dot{x}_0^{lm}=\begin{cases} y_0^{lm}, & l<2 \\ -\nu^{lm}/\tau_0+y_0^{lm}, & l\geqslant 2\end{cases}, \quad -l\leqslant m\leqslant l,$$
$$\dot{x}_i=-\frac{1}{\tau_1}\frac{\partial u_i}{\partial t}+y_i; \tag{21}$$

$$X_0^{lm}=\frac{p_F^2}{\pi^2 v_F}\left(1+\frac{F_l}{2l+1}\right)\frac{1}{(2l+1)}\frac{(l+|m|)!}{(l-|m|)!}\nu^{l,-m},$$
$$X_i=\rho_{ik}\frac{\partial u_k}{\partial t}, \tag{22}$$

where $F_l=p_F^2 f_l/\pi^2 v_F$, and p_F and v_F are the limiting Fermi momentum and Fermi velocity.

In accordance with fluctuation theory, the coefficients of the linear relation between the thermodynamic forces and velocities determines the random-force correlators averaged over the fluctuations. Establishing this relation in accord with (21)-(24), we obtain for the Fourier components of the random-force correlators

$$\langle y_0^{lm}y_0^{l'm'}\rangle_{q\omega}=0 \quad \text{for} \quad l=0,\ 1,$$
$$\langle y_0^{lm}y_0^{l'm'}\rangle_{q\omega}=2\omega(N_\omega+1)\delta_{ll'}\cdot\delta_{m,-m'}\pi^2 v_F(2l+1)/\tau_0 p_F^2\left(1+\frac{F_l}{2l+1}\right), \quad l\geqslant 2;$$
$$\langle y_i y_k\rangle_{q\omega}=2\omega(N_\omega+1)\rho_{ik}^{-1}\tau_1^{-1}, \tag{23}$$

where N_ω is the Planck distribution function.

3. CORRELATORS OF THE FLUCTUATIONS OF THE PHYSICAL QUANTITIES

According to Dzyaloshinskiĭ *et al.*[3] the change of the particle-number density in a quantum crystal is due to the change in the quasiparticle-number density with changing lattice density:

$$\delta N(\mathbf{r},t)=\mathrm{Sp}_\sigma\int\frac{d^3n}{(2\pi)}\delta n(\mathbf{p},\mathbf{r},t)+\eta_{ik}\frac{\partial u_k(\mathbf{r},t)}{\partial r_i}$$

where η_{ik} is a symmetric tensor of second rank (for more details on the properties of η_{ik} see[3]). Substituting here the expression (4) and taking (5) and (6) into account, we obtain

$$\delta N=N_{ik}\frac{\partial u_k}{\partial r_i}+\mathrm{Sp}_\sigma\int\frac{d^3p}{(2\pi)^3}\frac{\partial n_0}{\partial\varepsilon_0}\nu, \tag{24}$$

$$N_{ik}=\eta_{ik}+\mathrm{Sp}_\sigma\int\frac{d^3p}{(2\pi)^3}\frac{\partial n_0}{\partial\varepsilon_0}Z_{ik}(\mathbf{p}). \tag{25}$$

The second term in (24), in accordance with the meaning of ν (see above), is the change produced in the quasiparticle density by the nonequilibrium character of the distribution function. We introduce henceforth the symbol $d(\mathbf{r},t)$ for this term.

We shall consider hereafter only the bcc phase of He^3, for which the quantum effects are the most substantial. In the case of a cubic crystal, the tensor structure of $\rho_{ik}, \zeta_{ik}, \eta_{ik}, \lambda_{iklm}$ can be substantially simplified

$$\rho_{ik}=\rho\delta_{ik}, \quad \zeta_{ik}=\zeta\delta_{ik}, \quad \eta_{ik}=\eta\delta_{ik},$$
$$\lambda_{iklm}=(\lambda_l-2\lambda_t)\delta_{ik}\delta_{lm}+\lambda_t(\delta_{im}\delta_{kl}+\delta_{il}\delta_{km}). \tag{26}$$

We now express with the aid of (15) and (16) the quantities $d(\mathbf{r},t)$ and $\mathbf{u}(\mathbf{r},t)$ in terms of the random forces. Since this is a rather complicated matter in the general case, we confine ourselves to the zeroth harmonics in the expansions of $f(\mathbf{p},\mathbf{p}')$ and $\zeta(\mathbf{p})$ in spherical functions on the Fermi surface. Leaving out the sequence of cumbersome manipulations, we obtain for the Fourier component $d(\mathbf{q},\omega)$

$$d(\mathbf{q},\omega)=\frac{i}{\Delta}\left\{\frac{1}{qv_F}\int\frac{d^3p}{(2\pi)^3}\frac{\partial n_0}{\partial\varepsilon_0}hy_0-\rho Q_{ij}q_j y_i\right\}, \tag{27}$$

where

$$\Delta=\Delta_g+\zeta_0 Q_{ij}q_i q_j,$$
$$\Delta_g=1+F_0(A_1-gA_2)+\frac{i}{qv_F\tau_0}(A_0-gA_1),$$
$$Q_{ij}=\frac{\omega}{qv_F}\gamma\zeta_0\frac{A_0-gA_1}{1+F_0}D_{ij},$$
$$g=\frac{3iA_1}{3iA_2+qv_F\tau_0},$$
$$D_{ij}=\left[\rho\left(\omega^2+i\frac{\omega}{\tau_1}\right)\delta_{ij}-\Lambda_{iklj}q_k q_l\right]^{-1}, \tag{28}$$
$$h=(1-g\cos\theta)\left[\cos\theta-\frac{\omega}{qv_F}\left(1+\frac{i}{\omega\tau_0}\right)\right]^{-1},$$
$$A_n=\frac{1}{2}\int_{-1}^{1}dx\,x^n\left[x-\frac{\omega}{qv_F}\left(1+\frac{i}{\omega\tau_0}\right)\right]^{-1};$$

$\gamma=p_F^2/\pi^2 v_F$ is the defection state density at the Fermi level.

The Fourier component of the lattice-deformation field produced by the random forces is given by

$$u_j=-\tilde{D}_{ij}\left[\rho y_j+\frac{1}{qv_F}\frac{\zeta_0}{\Delta_g}q_i\int\frac{d^3p}{(2\pi)^3}\frac{\partial n_0}{\partial\varepsilon_0}hy_0\right], \tag{29}$$

where $\tilde{D}_{ij}$ is obtained from D_{ij} by the substitution $\lambda_l\to\tilde{\lambda}_l$:

$$\tilde{\lambda}_l=\lambda_l-\frac{\omega}{qv_F}\frac{\gamma\zeta_0^2}{\Delta_g}\frac{A_0-gA_1}{1+F_0}. \tag{30}$$

The singularities of $d(\mathbf{q},\omega)$ and $\mathbf{u}(\mathbf{q},\omega)$ yield the spectra of the natural oscillations of the quantum crystal.

The correlators of the fluctuations of $d(\mathbf{q},\omega)$ and $\mathbf{u}(\mathbf{q},\omega)$ can now be expressed with the aid of relations (27) and (29) in terms of the random-force correlators (23). Omitting the cumbersome manipulations, we present the final expressions for $\langle d^2\rangle, \langle u_i u_j\rangle$ and $\langle du_i\rangle$:

$$\langle d^2\rangle_{q\omega}=\frac{2\omega(N_\omega+1)}{|\Delta|^2}\left\{\frac{\gamma\Sigma}{(qv_F)^2\tau_0}+\frac{\rho}{\tau_1}Q_{ij}Q_{ik}^*q_j q_k\right\}, \tag{31}$$

$$\langle u_i u_j\rangle_{q\omega}=2\omega(N_\omega+1)\tilde{D}_{ki}\tilde{D}_{lj}^*\left\{\frac{\rho}{\tau_1}\delta_{kl}+\frac{\gamma\zeta_0^2\Sigma q_k q_l}{(qv_F)^2\tau_0|\Delta_g|^2}\right\}, \tag{32}$$

$$\langle du_i\rangle_{q\omega}=2i\omega(N_\omega+1)\tilde{D}_{ji}^*\left\{\frac{\rho}{\tau_1}q_k Q_{jk}-\frac{\gamma\zeta_0\Sigma q_j}{(qv_F)^2\tau_0\Delta\Delta_g^*}\right\}; \tag{33}$$

here

$$\Sigma=\int\frac{do}{4\pi}|h|^2-\left|\int\frac{do}{4\pi}h\right|^2-3\left|\int\frac{do}{4\pi}h\cos\theta\right|^2.$$

The particle-density fluctuation correlator is expressed, according to (24) in terms of the correlators (31)-(33):

$$\langle\delta N^2\rangle_{q\omega}=N_0^2q_iq_j\langle u_iu_j\rangle_{q\omega}+\langle d^2\rangle_{q\omega}-iN_0q_i(\langle du_i\rangle_{q\omega}-\langle du_i\rangle_{q\omega}^*), \quad (34)$$

where

$$N_0=\eta-\gamma\zeta_0/(1+F_0).$$

When (26) is taken into account, the dispersion equation for the natural oscillations breaks up into equations for the longitudinal and transverse waves:

$$\delta=\Delta_l\Delta_g+\frac{\omega}{qv_F}q^2\gamma\zeta_0^2\frac{A_0-gA_1}{1+F_0}=0, \quad (35)$$

$$\Delta_t=\rho\left(\omega^2+i\frac{\omega}{\tau_1}\right)-\lambda_tq^2=0, \quad (36)$$

where Δ_l is defined by

$$\Delta_l=\rho\left(\omega^2+i\frac{\omega}{\tau_1}\right)-\lambda_lq^2+q^2\frac{\gamma\zeta_0^2}{1+F_0}. \quad (37)$$

We see that only the longitudinal lattice vibrations are coupled with the oscillations of the quasiparticle distribution function. This coupling, which is general is not small, vanishes in the limit at $\zeta_0=0$. The relation (35) then breaks up into two equations, $\Delta_l=0$ and $\Delta_g=0$, which coincide with the dispersion equations for longitudinal lattice sound in an ordinary crystal and for zero sound in a Fermi liquid.

Under the assumptions made, the expression for the correlator of the transverse lattice fluctuations has, according to (32), the same form as in an ordinary crystal:

$$\langle u_i^{(t)}u_j^{(t)}\rangle_{q\omega}=2\omega(N_\omega+1)\frac{\rho}{\tau_1}\frac{1}{|\Delta_t|^2}\left(\delta_{ij}-\frac{q_iq_j}{q^2}\right). \quad (38)$$

The fluctuation correlators $\langle d^2\rangle$, $\langle d\mathbf{u}\rangle$, $\langle u_i^{(l)}u_j^{(l)}\rangle$ and $\langle\delta N^2\rangle$ are proportional to $|\delta|^{-2}$ and have poles only for longitudinal coupled oscillations. We point out that in the general case of strong coupling between to oscillation modes described by (35), the pole residues corresponding to the two modes are of the same order of magnitude for the lattice-deformation fluctuation correlator and for the quasiparticle-density fluctuation correlator. Therefore in the case of strong coupling the distinction between defect-density waves and lattice sound is only arbitrary.

According to (24), the particle-density fluctuation correlator can be represented in the form

$$\langle\delta N^2\rangle_{q\omega}=\frac{2\omega(N_\omega+1)}{|\delta|^2}\left\{\frac{\gamma|\Delta_l-N_0\zeta_0q^2|^2}{(qv_F)^2\tau_0}\Sigma+\frac{\rho}{\tau_1}q^2\left|N_0\Delta_g+\frac{\omega}{qv_F}\gamma\zeta_0\frac{A_0-gA_1}{1+F_0}\right|^2\right\}. \quad (39)$$

It is easily seen that at $\zeta_0=0$ the density correlator breaks up into two terms. The first is proportional to $|\Delta_g|^{-2}$ and describes the fluctuations of the distribution function. The second is proportional to $|\Delta_l|^{-2}$ and corresponds to the lattice fluctuations. The expressions for these terms coincide with the known expression for the density-fluctuation correlator in a Fermi liquid[4,5] and in an ordinary crystal (see, e.g.,[6]).

Weakly damped natural oscillations exist at $\omega>qv_F$ when there is no Landau damping. In the limit $F_0\gg1$ the dispersion equation yields two oscillation modes with linear dispersion law and with velocities given by

$$s_{l,d}^2=\frac{1}{2}\left(\frac{\lambda_l}{\rho}+\frac{1}{3}v_F^2F_0\right)\pm\left[\frac{1}{4}\left(\frac{\lambda_l}{\rho}-\frac{1}{3}v_F^2F_0\right)^2+\frac{1}{3}\gamma\zeta_0^2\frac{v_F^2}{\rho}\right]^{1/2}. \quad (40)$$

The inequality $s_{l,d}^2>0$ is satisfied if the coupling of the quasiparticles with the lattice is not too strong,

$$f_0>\zeta_0^2/\lambda_l, \quad (41)$$

and is one of the conditions for the stability of a quantum crystal.

The condition $F_0\gg1$ can obtain if the defect density is large enough. In the opposite limiting case of low defecton concentration (at low values of the state density γ), the longitudinal oscillations of the lattice and of the defect density are weakly coupled; the phonon phase velocity is close to its value $s_l=(\lambda_l/\rho)^{1/2}$ in an ordinary crystal.

The second oscillation mode in the case of low defecton concentration can propagate of the inequality (41) is satisfied. Its velocity is close to the Fermi velocity and is equal to

$$s_d=v_F\left\{1+2\exp\left[\frac{2}{\gamma}\left(\frac{\zeta_0^2}{\lambda_l}-f_0\right)^{-1}\right]\right\}. \quad (42)$$

The density-fluctuation correlator, when damping is neglected, has δ-like maxima at the natural-oscillation frequencies and is determined in the case of low defecton concentration by the expression

$$\langle\delta N^2\rangle_{l\omega}=2\pi|N_\omega+1|\left\{q^2\frac{N_0^2}{\rho}\delta(\omega^2-s_l^2q^2)+\omega^2\frac{\gamma}{F_0^2}\left(\frac{s_d^2}{v_F^2}-1\right)\delta(\omega^2-s_d^2q^2)\right\}. \quad (43)$$

4. LIGHT SCATTERING IN SOLID He^3

The differential light-extinction coefficient is connected with the density-fluctuation correlator by the relation

$$dh=\frac{\omega_0^4}{16\pi^2c^4}\left(\frac{\partial D}{\partial N}\right)^2(1+\cos^2\vartheta)\langle\delta N^2\rangle_{q\omega}\frac{do}{4\pi}, \quad (44)$$

where D is the permittivity, N is the density of the crystal atoms, ω and $\mathbf{q}$ are the changes of the frequency and momentum of the scattered light, ϑ is the scattering angle, and ω_0 is the frequency of the scattered light. In the limit of weak coupling between the quasiparticle density and the lattice vibrations, the ratio of the extinction coefficients at these vibrations (h_d and h_l, respectively) is of the order of

$$\frac{h_d}{h_l}\sim\frac{M}{M^*}\left(\frac{N_d}{N_l}\right)^{1/3}\exp\left\{-\left(\frac{N_l}{N_d}\right)^{1/3}\right\}, \quad (45)$$

where M is the helium-atom mass, M^* is the quasiparticle effective mass, N_d is the quasiparticle density, and N_l is the lattice-site density.

The presence of a degenerate defecton liquid leads to the appearance of additional satellites in the scattered-light spectrum. Four satellites corresponding to the absorption and emission of two collective-excitation modes, are separated from the fundamental line by a distance

$$\omega = \pm 2 \frac{s_{t,d}}{c} \omega_0 \sin\frac{\vartheta}{2}. \quad (46)$$

To estimate the extinction coefficient and the frequency shift of the scattered light we note first of all that all the formulas obtained by us are valid in a region of temperatures that are much lower than the quasiparticle Fermi energies. At low temperatures the presence in the crystal of delocalized fermions should lead, besides the high-frequency effects considered above, to a contribution linear in temperature to the heat capacity. On the basis of the experimental data for liquid He^3 (see, e.g.,[7]), one can expect the heat capacity to become linear at a temperature lower by one order than the quasiparticle Fermi energy. At $T > 0.1\epsilon_F \sim 0.1$ K the Fermi excitations in liquid He^3, in view of the strong interaction with one another, are not well-defined quasiparticles and the temperature dependence of the heat capacity is determined principally by the phonon contribution.

For solid He^3, the Fermi energy of the quasiparticles is apparently low in comparison with the Fermi energy in the liquid phase. Accordingly, the linear temperature dependence shifts into the region of much lower temperatures. The presently available measurements of the heat capacity can therefore not yield data on the number of quasiparticles in solid He^3.

An appreciable delocalization of the defects is possible only in an ideal quantum crystal in which there is magnetic order besides the spatial ordering of the atom. Recently Andreev, Marchenko, and Meĭerovich[8] have advanced a hypothesis, based on magnetic measurements,[9] that zero-point vacancies at a concentration 6×10^{-3} exist in magnetic fields exceeding 0.2 T. Assuming that the quasiparticle effective mass is of the order of the mass of the He^3 atom, Andreev *et al.*[8] obtained for the Fermi energy the value $\epsilon_F \sim 0.1$ K.

The coefficient of light extinction on the zero-point oscillations of the vacancy density in solid He^3, at $M^* \sim M$ and $N_d \sim 6\times10^{-3}N_\lambda$, amounts according to (45) to 0.1% of the coefficient of light extinction on ordinary sound waves. The relative change of the light frequency is here lower by one order of magnitude than the corresponding value for scattering by phonons, and is within the present experimental capabilities.

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[1)]This assumption can be justified only for the bcc phase of He^3.

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Measurement of internal friction in solid He^4

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The internal friction in crystalline He^4 (molar volumes 19.2–20.95 cm^3/mol) was investigated at ~15 kHz. The temperature dependence of the logarithmic damping decrement down to 0.55 K and the dependence of the damping on the oscillation amplitude were measured. It is concluded on the basis of the results that the principal internal-friction mechanism is due to dislocations. The temperature dependences of the damping were reduced by the theory of Granato and Lucke; a number of dislocation parameters are obtained.

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Measurement of internal friction is an effective means of studying defects in solids. As shown in[1,2] in crystalline helium the quantum character of the point defects (vacancies) leads to a peculiar dependence of the internal friction on the temperature and frequency. The logarithmic damping decrement Δ_{vac} when the crystal is inhomogeneously deformed is described according to Meĭerovich[1] by the relation

$$\Delta_{vac} \sim N(T)\frac{\tau/\omega}{1+\tau^2\omega^2},$$

where $N(T)$ is the equilibrium vacancy concentration and varies exponentially with temperature, $N(T) \propto \exp(-\epsilon_0/T)$, $\tau \propto T^{-9}$ is the relaxation time, and ω is the cyclic fre-

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