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Experimental results and analysis of structural fluctuations in photosensitive Josephson junctions

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Large and small structural fluctuations in Pb–CdS–In junctions were investigated. It was found that such photosensitivity junctions were particularly suitable for investigations of this kind. The dependence of the maximum Josephson current on the magnetic field was determined and a good agreement was obtained between the experimental and theoretical results.

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1. INTRODUCTION

Influence of inhomogeneities in the barrier layer on the properties of a Josephson tunnel junction has been investigated comprehensively by Yanson.[1,2]1) Of special interest are "structural fluctuations," which are randomly distributed inhomogeneities altering greatly the properties of a contact. In particular, Yanson studied the influence of such structural fluctuations on the dependence of the critical current on the applied magnetic field. Since these fluctuations disturb the homogeneity along a Josephson junction, the observed effect was particularly noticeable for low values of the barrier energy, namely near the minimum of the dependence of the superconducting current on the magnetic field. Although the experiments described above were in agreement with the theory, some of the effects predicted by the theory have not yet been confirmed experimentally.

We investigated photosensitive semiconductor junctions[3] which made it possible, as shown below, to study more effectively the problem mentioned above. Yanson's theory was considered from a somewhat different point of view and it was developed further. Experimental dependences of the critical current on the magnetic field in the presence of fluctuations were obtained and discussed. A comparison was made with the theory and a very good agreement was obtained.

2. THEORY

We shall consider the main results of Yanson[1,2] in a somewhat modified form. The relationship between the Josephson current and the applied magnetic field is known to be given by the modulus of the Fourier component of the maximum current density:

$$I_J(\varphi)=\left|W\int\limits_{-L/2}^{L/2}J_1\exp\left(\frac{2\pi i}{L}\varphi x\right)dx\right|. \tag{1}$$

It is assumed that the barrier is in the (x,y) plane; L and W are the dimensions of the junction along the x and y axes; φ is the normalized magnetic flux $\varphi=\Phi/\Phi_0$, where Φ is the flux produced by an external magnetic field B directed along the y axis and $\Phi_0=h/2e$ is a quantum of this flux.

We shall allow for the presence of structural fluctuations by rewriting the current in the one-dimensional form:

$$J_1=J_1(x)+J_f(x)=J_1(x)+\sum_{n=1}^{\infty}\left(a_n\cos\frac{2\pi n}{L}x+b_n\sin\frac{2\pi n}{L}x\right); \tag{2}$$

here, $J_1(x)$ is the distribution on the current density in the absence of fluctuations and $J_f(x)$ is the random distribution of inhomogeneities, such that $\langle J_f(x)\rangle=0$, where the angular brackets represent averaging over the junction area. We shall assume that the correlation function of $J_f(x)$ is of the form[4]

$$\langle J_f(x_1)J_f(x_2)\rangle=\langle J_f^2\rangle\exp(-|x_1-x_2|/r); \tag{3}$$

$\langle J_f^2\rangle$ is the average value of the square of the amplitude of the fluctuations (it is assumed that $\langle J_f^2\rangle$ is constant over the whole junction) and r is the correlation radius characterizing the size of inhomogeneities.

Substituting Eq. (2) into Eq. (1), and allowing for Eq. (3), we obtain

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$$I_J(\varphi)=\left\{\mathcal{F}^2(\varphi)+I_0^2[a(r',\varphi)+b(r',\varphi)]\frac{\sin^2\pi\varphi}{\pi^2\varphi^2}\right\}^{1/2}; \quad (4)$$

here, $\mathfrak{F}$ is the Fourier transform of $J_1(x)$; $I_0=WL(2r'\langle J_f^2\rangle)^{1/2}$ and $r'=r/L$. The expressions for $a(r',\varphi)$ and $b(r',\varphi)$ are as follows:

$$\begin{aligned} a(r',\varphi)&=2\sum_{n=1}^{\infty}\left(\frac{\varphi^2}{\varphi^2-n^2}\right)^2\frac{1}{1+(2\pi nr')^2}\\ &-r'\left(1-\exp\left(-\frac{1}{r'}\right)\right)\left[2\sum_{n=1}^{\infty}\frac{\varphi^2}{\varphi^2-n^2}\frac{1}{1+(2\pi nr')^2}\right]^2,\\ b(r',\varphi)&=2\sum_{n=1}^{\infty}\left(\frac{n\varphi}{\varphi^2-n^2}\right)^2\frac{1}{1+(2\pi nr')^2}\\ &+r'\left(1-\exp\left(-\frac{1}{r'}\right)\right)\left[2\sum_{n=1}^{\infty}\frac{n\varphi}{\varphi^2-n^2}\frac{2\pi nr'}{1+(2\pi nr')^2}\right]^2. \end{aligned} \quad (5)$$

For small-scale fluctuations ($r'\ll 1$) and a weak magnetic field ($2\pi r'\varphi\ll 1$) the expressions in Eq. (5) become[5]

$$a(r',\varphi)=2\sum_{n=1}^{\infty}\left(\frac{\varphi^2}{\varphi^2-n^2}\right)^2=\frac{\pi^2\varphi^2}{2\sin^2\pi\varphi}+\frac{\pi\varphi}{2}\operatorname{ctg}\pi\varphi-1,$$

$$b(r',\varphi)=2\sum_{n=1}^{\infty}\left(\frac{n\varphi}{\varphi^2-n^2}\right)^2=\frac{\pi^2\varphi^2}{2\sin^2\pi\varphi}-\frac{\pi\varphi}{2}\operatorname{ctg}\pi\varphi,$$

so that

$$I_J(\varphi)=\left[\mathcal{F}^2(\varphi)+I_0^2\left(1-\frac{\sin^2\pi\varphi}{\pi^2\varphi^2}\right)\right]^{1/2}. \quad (6)$$

In discussing this expression it is clear that the usual dependence $I(\varphi)=|\mathfrak{F}(\varphi)|$ is modified by the presence of fluctuations. In particular, we can see that in addition to the term oscillating with the magnetic field, there is also a constant background of amplitude I_0. Moreover, in the absence of a magnetic field there is no change in the current. In general, an analysis of Eqs. (4) and (5) shows that the main result, which is the appearance of the background current, is retained but its amplitude decreases on increase of the magnetic field.

3. EXPERIMENTAL RESULTS AND DISCUSSION

In the case of small-scale fluctuations and a weak external field it is not possible to separate the average amplitude of fluctuations from the correlation radius. All the information about the fluctuations follows from Eq. (6) and is contained in I_0. In the other limiting case ($\varphi\gg 1$) a reduction in the background current makes is possible to determine independently r' and also $\langle J_f^2\rangle$.

Yanson carried out experiments on tin–tin monoxide–tin junctions. His results showed that the experimental conditions corresponded to the first case of small-scale fluctuations and a weak magnetic field. The values of the parameter $\gamma=I_0/I_J(0)$ found by Yanson[1] was 0.066.

We investigated photosensitive semiconductor junctions. Considerable structural fluctuations were expected because of the strongly inhomogeneous nature of the CdS barrier. Moreover, we assumed that the structures in question were most suitable for the verification of the theoretical approximation. We were dealing only with the Josephson current induced optically, i.e., we considered samples through which no current flowed under zero voltage in the absence of illumination. This guaranteed the absence of the background current, due to short-circuiting, in I_J considered as a function of B. Moreover, illumination made it possible to increase the Josephson current to a value allowing for careful measurements even of the secondary maxima.[2)]

Figure 1 shows the experimental dependences of the maximum value of the current on the magnetic field for a Pd-CdS-In junction (points). These results were obtained for a sample whose electrodes and barrier were formed by evaporation on a substrate made of ordinary glass; the evaporation process took place in vacuum of $(1-2)\times10^{-6}$ Torr. A CdS barrier was deposited at a rate of 15 Å/sec until a thickness of about 450 Å was reached. Undesirable effects, associated with the formation of pinholes, were prevented by placing a sample (after evaporation) for 2 h in an atmosphere of pure oxygen at room temperature and pressure of 760 Torr. The junction was cross-shaped with dimensions 0.34 $\times$0.17 mm. In darkness the normal resistance was 0.5 Ω and under zero voltage there was no current (to within 1 μA). A magnetic field, perpendicular to the long side of the junction, was produced by two Helmholtz coils (1 G ≈ 6 mA). The whole cryostat was surrounded by two coaxial screens made of a magnetic metal. The Josephson current was induced by illumination with light from an iodine-in-quartz lamp. The current maximum was measured mainly by the method of Balsamo *et al.*[7] The maximum current in zero field was $I_J(0)$ = 2.2 mA.

The results in Fig. 1a exhibited a slight increase in the constant background current, indicating that the in-

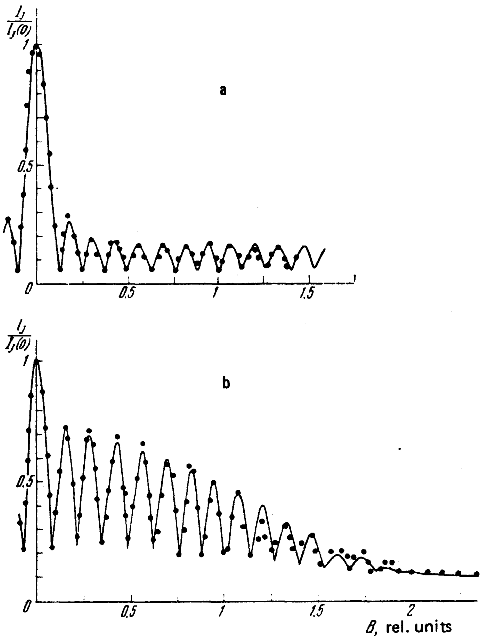

FIG. 1. Photoinduced Josephson current in a magnetic field flowing across a Pb–CdS–In junction (a: small-scale fluctuations; b: large-scale fluctuations). The points are the experimental results and the continuous curves are theoretical.

fluence of structural fluctuations could be analyzed by means of Eq. (6) (small-scale fluctuations in a weak field). The continuous curve was the theoretical dependence obtained from Eq. (6) assuming that the ratio $\gamma = I_0/I_J(0)$ had the value $\gamma = 0.06$. Moreover, a suitable step-like profile was selected for the current density $J_1(x)$ so as to allow for the interference contribution resulting from the edge effects.[8,9] 3) We assumed that some discrepancy between the theory and experiment in Yanson's work[1] could be due to the presence of such edge effects. It should be noted that our value of γ was close to that obtained by Yanson. However, in our case the current measurements were possible in relatively strong magnetic fields, which enabled us to determine also the quantitative values of r' and $\langle J_f^2 \rangle$. For this purpose we had to take into account the two-dimensional nature of the problem. This gave the same dependence of the Josephson current on the applied field as before except that now $\mathcal{F}(\varphi)$ was the Fourier transform of the integral of the current density along the y axis and

$$I_0 = WL(4a\langle J_f^2 \rangle / LW)^{1/2},$$

where a is the average area of structural inhomogeneities. The results in Fig. 1a enabled us to estimate the upper limit of the correlation radius $r' \lesssim 0.003 (r \lesssim 1\ \mu)$. In fact, only for these values of r' did Eq. (4) predict the background current in the $\gamma = 0.06$ case and, according to the experimental results, this current showed no significant reduction. Consequently, the rms amplitude of the fluctuation current represented $\sigma \gtrsim 0.7$ of the maximum current density.

The experimentally determined dependence of the maximum current on the applied magnetic field, exhibiting strong fluctuation effects, is plotted in Fig. 1b (points). In this case the sample was prepared by the same technology as before. A CdS film was deposited at a rate of 7 Å/sec until a thickness of 500 Å was obtained. The dimensions of the junction were 0.20 ×0.11 mm. In darkness the normal tunnel resistance was 22.7 Ω. Once again there was no current under zero voltage; the maximum optically induced superconducting current was 230 μA. A magnetic field was applied along the short side of the junction.

A theoretical curve (shown continuous in Fig. 1b) was obtained from Eq. (4) for the values4) $r' = 0.016$ and $\gamma = 0.25$. In this case an increase in the magnetic field reduced the background current and weakened considerably the modulation of the current, which was a consequence of the fluctuations. Knowing the adjustable parameters r' and γ, we could find the correlation radius r and the relative amplitude of the fluctuations σ. In particular, the values $r \approx 2\ \mu$ and $\sigma \approx 1$ were obtained. Such a high amplitude of the fluctuations could be attributed to the presence of a large number of oxidized pinholes, which were impermeable to the tunneling pairs. This result should not be regarded as final because of the lack of independent data on the nature of the inhomogeneities in a semiconductor film of this kind.

We thus found that the use of photosensitive junctions made it possible to investigate in detail the problem of structural fluctuations in Josephson junctions, confirming the validity of the theoretical approach. The values of the parameters used to match the experimental data to the theory made it possible to find the size of the inhomogeneities and the amplitude of the fluctuations of the current density.

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1) The authors are grateful to I. K. Yanson for information on his investigations.

2) In any case, the photoinduced Josephson current should not exceed the values at which the junction becomes larger than the Josephson length.[6]

3) It follows from an analysis of our data[9] (notation as above) that in this case $\xi = 0.01$ and $s' = 0.01$.

4) As in the preceding case, use was made of a step-like current density profile with $\xi = 0.06$ and $s' = 0.06$.

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