should be fulfilled.

For the above-indicated values of the parameters a second localized magnon level is possible only when $T_c < 20$ K. Hence, we can, in particular, conclude that the donor states of large radius that arise upon the doping of EuO ($T_c = 69$ K) with gadolinium can lead to the appearance of only one localized magnon on each donor atom.

The author is grateful to D. I. Khomskiĭ for valuable comments.

[1)]This Hamiltonian should be interpreted not as a true Hamiltonian, but as an equivalent one, since its eigenvalues give the correct spectrum, but the probability density of a definite spin configuration is, generally speaking, given not by the square of the modulus of its eigenfunction, but by a more complicated expression.

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Spin waves in amorphous and finely divided ferromagnets with allowance for dipole-dipole interaction

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Complex dispersion relations are obtained, in the long-wave approximation, for spin waves in a ferromagnet with parameters that fluctuate randomly in space (for the corresponding results without allowance for dipole-dipole interaction, see V. A. Ignatchenko and R. S. Iskhakov, [Sov. Phys. JETP **45**, 526 (1977)]). When the exchange constant α fluctuates, allowance for dipole-dipole interaction shifts the break in the modified dispersion law toward longer waves and leads to a change in the damping law for long spin waves: when k is less than a certain critical value k_c, the damping $\omega'' \sim k^7$; for $k > k_c$, we get $\omega'' \sim k^5$. When the axis of magnetic anisotropy fluctuates, allowance for dipole-dipole interaction modifies both channels of interaction of the random inhomogeneity function $\rho(\mathbf{r})$ with the spin wave $\mathbf{m}(\mathbf{r}, t)$ and leads to the appearance of a new channel of interaction of $\mathbf{m}$ with ρ via the stochastic magnetostatic fields produced by the stochastic magnetic structure. The dispersion law now contains a characteristic wave number for dipole-dipole interaction, $k_M = (4\pi/\alpha)^{1/2}$.

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INTRODUCTION

In an earlier paper,[1] we calculated the modification of the dispersion relation and of the damping for long ($ka \ll 1$, where a is the lattice parameter) spin waves in a medium with either isotropic or anisotropic inhomogeneities, having an arbitrary correlation radius r_0. The phenomenological theory of spin waves developed in Ref. 1 is correct both for finely divided and for amorphous ferromagnets. Because there is at present no systematic theory of amorphous magnetism, the correlation radius r_0 of the fluctuations of the corresponding parameter (the exchange parameter or the amount and direction of the anisotropy) cannot so far be calculated theoretically and occurs in the theory as a phenomenological constant. Therefore particular interest attaches to results that predict from what experimental observations r_0 can be determined. In particular, a possible basis for determination of r_0 might be the experimental observation of the characteristic break in the dispersion curve $\omega(k^2)$ at $k = \frac{1}{2} r_0$, which was obtained in Ref. 1 for the case of fluctuations of the exchange constant α.

When there are spatial fluctuations of the axis of magnetic anisotropy (this phenomenon may be characteristic of certain classes of amorphous magnets), there must occur in the material a stochastic static magnetic structure whose correlation properties are determined by a magnetic-field-dependent correlation radius $r_H = (\alpha M/H)^{1/2}$. This leads to the result that the inhomogeneities interact with the spin waves by two paths: directly, and through the stochastic magnetic structure. In the dispersion relation both characteristic radii, r_0 and r_H, occur.

Both in Ref. 1 and in all works known to us on this topic, dipole-dipole interaction was neglected. Allowance for this interaction is the purpose of the present paper.[1)] Formally, the problem reduces to supplementing the effective magnetic field of Ref. 1 with a field $\mathbf{H}_m(\mathbf{r}, t)$ determined by the equations of magnetostatics (we neglect effects of propagation of electro-

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magnetic waves):

$$\operatorname{div}(\mathbf{H}_m+4\pi\mathbf{M})=0,\quad \operatorname{rot}\mathbf{H}_m=0.$$

Inclusion of long-range forces greatly complicates the integrands of the terms in the perturbation-theory series. Whereas in Ref. 1 all the integrals were taken exactly, here we must restrict ourselves to various approximate estimates. But in a comparison with experiment in the radio-frequency range, all that is of interest is results obtained with allowance for dipole-dipole interaction, since its influence in many cases is decisive.

We consider below how the dipole-dipole interaction modifies the results obtained in Ref. 1: the position and amount of the "break" on the dispersion curve, the amount of damping, the dispersion of the stochastic magnetic structure, the FMR frequency. All the notation of this paper corresponds to the notation of Ref. 1, with the exception of $r_H=(\alpha M/H)^{1/2}$. In Ref. 1, we called this quantity the exchange-correlation radius and denoted it by r_α. Although this name reflects the physical origin of r_H, it is not altogether accurate and in a number of situations can lead to confusion; for example, in fluctuation of the exchange parameter α the true correlation radius of the exchange fluctuations is the phenomenological parameter r_0. Therefore in the present paper we shall call the quantity r_H the radius of interaction of the magnetization with the magnetic field. This name and notation are especially suitable in calculation of the dipole-dipole interaction, which, as is shown below, is characterized by a radius of dipole-dipole interaction $r_M=(\alpha/4\pi)^{1/2}$, corresponding to the value of r_H when $H=4\pi M$.

1. INHOMOGENEITY OF THE EXCHANGE CONSTANT

When the exchange constant is inhomogeneous, the magnetization and the effective magnetic field can be represented in the form

$$\mathbf{M}(\mathbf{r},t)=\mathbf{M}+\mathbf{m}(\mathbf{r},t),\quad \mathbf{H}^e(\mathbf{r},t)=\mathbf{H}+\mathbf{h}^e(\mathbf{r},t) \tag{1.1}$$

and the linearized Landau-Lifshitz equation takes the form

$$\dot{\mathbf{m}}=-g[\mathbf{M},\ ((\alpha+\rho\Delta\alpha)\nabla^2\mathbf{m}+\Delta\alpha(\nabla\rho)(\nabla\mathbf{m})+\mathbf{h}_m)]-g[\mathbf{m}\times\mathbf{H}], \tag{1.2}$$

where $\mathbf{h}_m(\mathbf{r},t)$ is the alternating dipole-dipole field. On expanding (1.2) in plane waves, we get the following system of integral equations:

$$\begin{gathered}\left(\frac{i\omega}{g}+4\pi M\frac{k_xk_y}{k^2}\right)m_x+\left(H+\alpha Mk^2+4\pi M\frac{k_y^2}{k^2}\right)m_y=G_y,\\ \left(H+\alpha Mk^2+4\pi M\frac{k_x^2}{k^2}\right)m_x+\left(-\frac{i\omega}{g}+4\pi M\frac{k_xk_y}{k^2}\right)m_y=G_x,\end{gathered} \tag{1.3}$$

where G_i are interaction terms of a spin wave with inhomogeneities:

$$G_i=-\Delta\alpha M\int m_i(\mathbf{k})\rho(\mathbf{k}-\mathbf{k}_1)\mathbf{k}\mathbf{k}_1\,d\mathbf{k}_1. \tag{1.4}$$

We express m_x and m_y from (1.3) in terms of G_x and G_y and substitute these "solutions" in (1.4). Then on averaging the system (1.3) in the same approximation as in Ref. 1, we get the dispersion relation, which, after carrying out of the integration over the azimuthal angle φ_1, takes the form

$$\begin{gathered}\frac{\omega}{g}=[(H+\alpha Mk^2)(H+\alpha Mk^2+4\pi M\sin^2\theta)]^{1/2}-\frac{1}{2}\alpha k^2M\left(\frac{\Delta\alpha}{\alpha}\right)^2\\ \times\left\{J_1+J_2+(J_1-J_2)\frac{H+\alpha Mk^2+2\pi M\sin^2\theta}{[(H+\alpha Mk^2)(H+\alpha Mk^2+4\pi M\sin^2\theta)]^{1/2}}\right\}.\end{gathered} \tag{1.5}$$

Here the integrals J_1 and J_2 are determined by the expressions

$$\begin{gathered}J_{1,2}=\frac{2}{\pi}\alpha Mk_0\\ \times\int_0^\infty\int_0^\pi\frac{(k_0^2+k^2+k_1^2-2kk_1\cos\theta\cos\theta_1)k_1^4\sin\theta_1\,dk_1\,d\theta_1}{[(k_0^2+k^2+k_1^2-2kk_1\cos\theta\cos\theta_1)^2-(2kk_1\sin\theta\sin\theta_1)^2]^{3/2}}\\ \times\frac{[\omega/g\pm(H+\alpha Mk_1^2+2\pi M\sin^2\theta_1)]}{[(\omega/g)^2-(H+\alpha Mk_1^2)(H+\alpha Mk_1^2+4\pi M\sin^2\theta_1)]}.\end{gathered} \tag{1.6}$$

The expression (1.5) with $\Delta\alpha=0$ is an exact dispersion relation for long spin waves in a homogeneous medium with allowance for dipole-dipole interaction. The angle value $\theta=0$ corresponds to waves propagated along the direction of the equilibrium state of the magnetization $\mathbf{M}$. For such waves, in a homogeneous medium, the dipole-dipole interaction term drops out of the dispersion relation, and it takes its simplest form.

The correction to the dispersion relation necessitated by the inhomogeneity of the medium also depends on the direction of propagation of the spin waves. But even for a wave with $\theta=0$, the correction depends on a dipole-dipole interaction term; for in the modified dispersion relation, for a wave with $\theta=0$, a contribution is made by waves with arbitrary θ_1:

$$\omega=\omega_0+\alpha gMk^2[1-(\Delta\alpha/\alpha)^2J(k)], \tag{1.7}$$

$$\begin{gathered}J(k)=\frac{2k_0}{\pi}\int_0^\infty\int_{-1}^{+1}\frac{k_1^4\,dk_1\,dx}{(k_0^2+k^2+k_1^2-2k_1kx)^2}\\ \times\frac{[2k_H^2+k^2+k_1^2+{}^1/_2k_M^2(1-x^2)]}{\{(k^2+k_H^2)^2-(k_1^2+k_H^2)[k_1^2+k_H^2+k_M^2(1-x^2)]\}},\end{gathered} \tag{1.8}$$

where $x=\cos\theta_1$, $k_H=(H/\alpha M)^{1/2}=r_H^{-1}$ is a characteristic wave number for interaction of the magnetization with the magnetic field, and $k_M=(4\pi/\alpha)^{1/2}=r_M^{-1}$ is a characteristic wave number for dipole-dipole interaction. Because of the fundamental importance of the case $\theta=0$ (it is realized experimentally in observation of linear spin-wave resonance in thin films), we shall analyze it in greater detail.

The improper integral over dk_1 has been calculated exactly by the method of residues. The result obtained was a cumbersome complex expression $J=J'+iJ''$ in which the integration over dx could be carried out only in various limiting cases. Analysis showed that the dispersion curve in an inhomogeneous medium has, qualitatively, the form shown in Fig. 1 (solid curve). Both for small and for large k, the dispersion law is quadratic in k, but with different coefficients in front of k^2. The nonquadratic terms are important only in the vicinity of a certain critical value $k=k_u$, where the dispersion curve $\omega(k^2)$ experiences a "break": a transition from one asymptote (Curve 1 in Fig. 1) to the other (Curve 2 in the same figure). The first asymp-

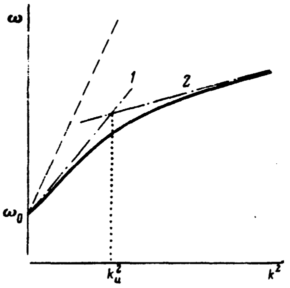

FIG. 1. Dispersion relation for spin waves with fluctuation of the exchange constant (solid curve) and in an ideal crystal (dashed line). The dash-dot straight lines 1 and 2 are asymptotes to the solid curve.

tote corresponds to the value of J' when $k=0$; in this case the integration over dx is carried out under the condition $k_H \ll k_M$. The second asymptote corresponds to expansion of J' at large k up to terms of order k^{-2}; in this case, the integration could be carried out only for the case $k_H \ll k_M \ll k_0/2$:

$$J' \approx 1 - k_M^2/2(k_0+k_M)^2, \quad k=0,$$
$$J' \approx \frac{5}{4} - \left[\left(\frac{k_0}{2}\right)^2 - k_M^2\right] \Big/ 4k^2, \quad k \gg \frac{k_0}{2}. \tag{1.9}$$

On substituting these expressions in (1.7), we obtain the equations of the straight lines 1 and 2 of Fig. 1. The intersection of these two asymptotes determines a characteristic value k_u of the wave number k_u, corresponding to the "break" on the dispersion curve:

$$k_u^2 = \left[\left(\frac{k_0}{2}\right)^2 - k_M^2\right] \Big/ \left[1 + \frac{2k_M^2}{(k_0+k_M)^2}\right]. \tag{1.10}$$

When $k_M \to 0$, $k_u \to k_0/2$ in agreement with the results of Ref. 1. Inclusion of dipole-dipole interactions shifts k_u toward the longer-wave region.

Analysis of the integral J'', which is responsible for the damping of spin waves, showed that there is a critical value k_c of the wave number, in the vicinity of which there occurs a change of the law of behavior of the damping:

$$k_c^2 = k_H(k_H^2 + k_M^2)^{1/2} - k_H^2. \tag{1.11}$$

An analytic calculation of J'' was possible for $k \ll k_c$ and for $k \gg k_c, k_M$ (in the latter case, the expression of Ref. 1 is obtained):

$$J'' \approx -\begin{cases} k^5/k_0^3 k_M^2, & k \ll k_c \\ 2k^3/k_0(k_0^2+4k^2), & k \gg k_c, k_M \end{cases} \tag{1.12}$$

On substituting these expressions in (1.7) we see that the imaginary correction ω'' to the frequency $\sim k^7$ for k small and $\sim k^5$ for k large (but smaller than the characteristic wave number k_0 for inhomogeneity of the exchange parameter; for $k \gg k_0$, we have $\omega'' \sim k^3$).

Thus allowance for dipole-dipole interaction led to the appearance of a region $k < k_c$ (when $k_M \to 0$, $k_c \to 0$, and this region disappears) in which the damping decreases extremely rapidly with decrease of k.

The physical meaning of this result is as follows. The damping that we get here is caused by linear scattering of a spin wave by inhomogeneities; in such scattering, the frequency of the spin wave is conserved, and only its momentum can change. Without allowance for dipole-dipole interaction, the dispersion law is isotropic; in the scattering, the value of $\omega = \alpha g M(k_H^2 + k^2)$ must be conserved, and its conservation is insured by equality of the modulus k for the incident and scattered waves; the scattering angles θ and φ may be arbitrary. With allowance for dipole-dipole interaction, the dispersion law becomes nonisotropic: in the scattering, the quantity whose value must be conserved is now

$$\omega = \alpha g M(k_H^2 + k^2)^{1/2}(k_H^2 + k^2 + k_M^2 \sin^2\theta)^{1/2}.$$

The modulus of k is now not conserved on scattering. The angle φ, as before, may be arbitrary. But all values of the angle θ are not always possible (Fig. 2). If $\omega > \omega_1 = \alpha g M k_H(k_H^2 + k_M^2)^{1/2}$, then scattering is possible over the whole range of angles θ. The value $\omega = \omega_1$ corresponds, on the branch $\theta = 0$, to the wave-number value $k = k_c$ determined by the relation (1.11). When $\omega < \omega_1$ ($k < k_c$), scattering is possible only within a solid-angle space whose bisectors coincide with the directions $\theta = 0$ and $\theta = \pi$; on approach of ω to ω_0, these solid angles decrease, shrinking to the directions $\theta = 0$ (scattering forward) and $\theta = \pi$ (scattering backward, reflection). Thus when $k < k_c$, with decrease of k there is a decrease of the number of states into which a spin wave can be transformed upon scattering by an inhomogeneity. This decrease of the number of states corresponds to a change from the law $\omega'' \sim k^5$, valid for $k > k_c$, to the law $\omega'' \sim k^7$ for $k < k_c$.

2. INHOMOGENEITY OF THE MAGNETIC ANISOTROPY

With spatial inhomogeneity of the orientation of the axis of magnetic anisotropy, the representation (1.1) is incorrect,[1] and the linearization with respect to a dynamic variable must be carried out with a representation of $\mathbf{M}(\mathbf{r}, t)$ and $\mathbf{H}^e(\mathbf{r}, t)$ in which the ground state is a function of the spatial coordinates:

$$\mathbf{M}(\mathbf{r}, t) = \mathbf{M}(\mathbf{r}) + \mathbf{m}(\mathbf{r}, t), \quad \mathbf{H}^e(\mathbf{r}, t) = \mathbf{H}^e(\mathbf{r}) + \mathbf{h}^e(\mathbf{r}, t). \tag{2.1}$$

On substituting these expressions in the Landau-Lifshitz equation, we obtain two systems of equations: a static, for the ground state $\mathbf{M}(\mathbf{r})$, and a dynamic, for the elementary excitations $\mathbf{m}(\mathbf{r}, t)$ that are propagated against the background of the inhomogeneous ground state.

The static system, in the same approximation as in Ref. 1 but with allowance for dipole-dipole interaction, has the following solution for the Fourier components:

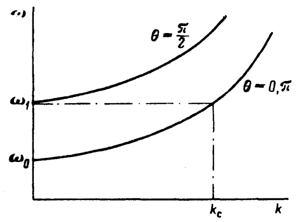

FIG. 2.

$$M_x(\mathbf{k})=\frac{\beta M}{\alpha^2}\frac{\rho_{xz}(k_H^2+k^2+k_M^2k_y^2/k^2)+\rho_{yz}k_M^2k_xk_y/k^2}{(k_H^2+k^2)(k_H^2+k^2+k_M^2\sin^2\theta)}$$
$$M_y(\mathbf{k})=\frac{\beta M}{\alpha^2}\frac{\rho_{yz}(k_H^2+k^2+k_M^2k_x^2/k^2)+\rho_{xz}k_M^2k_xk_y/k^2}{(k_H^2+k^2)(k_H^2+k^2+k_M^2\sin^2\theta)} \quad (2.2)$$

where ρ_{ij} are the Fourier components of the random functions $\rho_{ij}(r)$ that describe the fluctuations of the anisotropy axis.[1]

The mathematical expectation of the random functions M_i is zero, but the spectral density of M_x and of M_y is the same and, after integration over the azimuthal angle φ, has the form

$$S_M(k,\theta)=\pi\left(\frac{\beta M}{\alpha}\right)^2[(k_H^2+k^2)^{-2}+(k_H^2+k^2+k_M^2\sin^2\theta)^{-2}]S(k), \quad (2.3)$$

where $S(k)$ is the spectral density of the functions ρ_{ij}[1]:

$$S(k)=Dk_0/\pi^2(k_0^2+k^2)^2. \quad (2.4)$$

When $k_M\to 0$, the expression (2.3) reduces to the corresponding expression of Ref. 1. It is evident that allowance for dipole-dipole interaction leads to a breakdown of the symmetry of the spectral density: for each direction, at a definite angle θ, the variation of S_M with k is now different.

There are at present electron-optical observations of stochastic magnetic structure ("magnetization ripple") in thin, finely divided magnetic films (for appropriate references, see Ref. 1). Arrangement of similar experiments on amorphous magnets is highly desirable. What is measured in such experiments is certain one-dimensional (integral) components of the complete function $S_M(k_x,k_y,k_z)$ and the dispersion D_M of the stochastic structure, which is determined by the integral of the expression (2.3) over all values of θ and k. We have calculated the dispersion D_M approximately: the logarithm that occurs after integration of (2.3) over θ was expanded as a power series in $k_M^2/(k_M^2+k_H^2+k^2)$, and then the integration over dk was carried out. For estimation of D_M, we can restrict ourselves to the first term of the series:

$$D_M=D\left(\frac{\beta M}{2\alpha}\right)^2\left[\frac{2}{k_H(k_0+k_H)^3}+\frac{1}{k_B(k_0+k_B)^3}+\frac{2k_0+k_H+k_B}{(k_H+k_B)(k_0+k_H)^2(k_0+k_M)^2}\right], \quad (2.5)$$

where

$$k_B^2=k_H^2+k_M^2=(H+4\pi M)/\alpha M.$$

When $k_M\to 0$, (2.5) reduces to the corresponding expression of Ref. 1. Analysis of formula (2.5) shows that inclusion of the dipole-dipole interaction decreases the dispersion D_M; that is, this interaction, as was to be expected, exerts an ordering influence on the stochastic magnetic structure. The limits of applicability of the expression (2.5) are the same as of the corresponding expression in Ref. 1: $8D_M\ll M^2$, which imposes a bound on k_H (i.e., on the value of the magnetic field) from below.

We turn not to consideration of a dynamic system of equations. In obtaining a linearized dynamic system, the following approximate values were used for the z components of the static and dynamic magnetizations:

$$M_z\approx M-(M_x^2+M_y^2)/2M,\quad m_z\approx-(m_xM_x+m_yM_y)/M. \quad (2.6)$$

After expansion in plane waves, the dynamic system takes the form

$$\Phi_1=\beta[F_1(\mathbf{k})-G_1(\mathbf{k})]-L_1(\mathbf{k}),\quad \Phi_2=\beta[F_2(\mathbf{k})-G_2(\mathbf{k})]-L_2(\mathbf{k}). \quad (2.7)$$

Here the left sides of the equations, Φ_1 and Φ_2, are the same as the left sides of equations (1.3) of the present paper. The terms on the right side of the equations correspond to different channels of interaction of the anisotropic inhomogeneities with the spin wave (Fig. 3). The expressions for the terms F_i and G_i were given in Ref. 1; the terms F_i correspond to direct interaction of the spin wave $\mathbf{m}(\mathbf{r},t)$ with the random inhomogeneity function $\rho(\mathbf{r})$, while the terms G_i describe the interaction of the spin wave with the stochastic magnetic structure $\mathbf{M}(\mathbf{r})$ resulting from $\rho(\mathbf{r})$. Allowance for dipole-dipole interaction substantially modifies both channels of interaction and the form of the stochastic magnetic structure. Furthermore, this interaction leads also to the appearance of a new channel of interaction: there now act on the spin wave $\mathbf{m}(\mathbf{r},t)$, in addition to ρ and $\mathbf{M}$, the stochastic magnetostatic fields $\mathbf{H}_m(\mathbf{r})$ produced by the stochastic magnetic structure $\mathbf{M}(\mathbf{r})$. All three channels of interaction give contributions of the same order. The terms L_i in (2.7) correspond to the effects of dipole-dipole interaction:

$$\begin{aligned}L_1(\mathbf{k})=4\pi\int d\mathbf{k}_1\Big\{&\frac{k_{1z}}{k_1^2}m_y(\mathbf{k}-\mathbf{k}_1)[k_{1x}M_x(\mathbf{k}_1)+k_{1y}M_y(\mathbf{k}_1)]\\&-\frac{k_{1z}}{k_1^2}M_y(\mathbf{k}-\mathbf{k}_1)[k_{1x}m_x(\mathbf{k}_1)+k_{1y}m_y(\mathbf{k}_1)]\\&-\frac{k_yk_z}{k^2}[m_x(\mathbf{k}_1)M_x(\mathbf{k}-\mathbf{k}_1)+m_y(\mathbf{k}_1)M_y(\mathbf{k}-\mathbf{k}_1)]\Big\}\\-\frac{4\pi}{M}\iint d\mathbf{k}_1\,d\mathbf{k}_2\Big\{&\frac{k_{1y}}{k_1^2}[k_{1x}M_x(\mathbf{k}_1)+k_{1y}M_y(\mathbf{k}_1)][m_x(\mathbf{k}_2)M_x(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\\&+m_y(\mathbf{k}_2)M_y(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)]\\&+\frac{k_{2y}}{2k_2^2}[k_{2x}m_x(\mathbf{k}_2)+k_{2y}m_y(\mathbf{k}_2)][M_x(\mathbf{k}_1)M_x(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)+M_y(\mathbf{k}_1)M_y(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)]\\&-\frac{k_{1z}^2}{k_1^2}M_y(\mathbf{k}-\mathbf{k}_1)[m_x(\mathbf{k}_2)M_x(\mathbf{k}_1-\mathbf{k}_2)+m_y(\mathbf{k}_2)M_y(\mathbf{k}_1-\mathbf{k}_2)]\\&+\frac{k_{1z}^2}{2k_1^2}m_y(\mathbf{k}-\mathbf{k}_1)[M_x(\mathbf{k}_2)M_x(\mathbf{k}_1-\mathbf{k}_2)+M_y(\mathbf{k}_2)M_y(\mathbf{k}_1-\mathbf{k}_2)]\Big\}\end{aligned} \quad (2.8)$$

while $L_2(\mathbf{k})$ is obtained from $L_1(\mathbf{k})$ by replacement of x by y and of y by x. The components $M_i(\mathbf{k})$ that occur here are determined by the expressions (2.2).

On expressing m_x and m_y from the left sides of the system (2.7), substituting them in the right sides, and

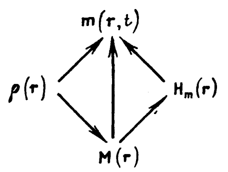

FIG. 3. Scheme of interactions of a spin wave $\mathbf{m}(\mathbf{r},t)$ with the random function of the inhomogeneities of the anisotropy axis, $\rho(\mathbf{r})$.

averaging (2.7), we get the dispersion relation in the form

$$\omega/g=\alpha M(k_H^2+k^2)^{1/2}(k_H^2+k^2+k_M^2\sin^2\theta)^{1/2}+\beta^2R(k, k_0, k_H, k_M), \qquad (2.9)$$

where β^2R, the effective magnetic field resulting from inhomogeneity of the axis of magnetic anisotropy, is determined by a cumbersome expression containing integrals over $d\mathbf{k}_1$ and $d\mathbf{k}_2$. We succeeded in evaluating these integrals (in the same approximations as for the calculation of the dispersion (2.5) of the magnetic structure) only for the uniform oscillation $k=0$, $\theta=0$, describing the FMR frequency. The damping caused by linear scattering in this case vanishes, and for the modified FMR frequency one obtains a cumbersome expression that reduces for $H\gg 4\pi M$ to the corresponding expression of Ref. 1, while in the case $H\ll 4\pi M$ it takes the following form:

$$\omega\approx\omega_0-4\pi gM\beta^2D[(M/H)^2-{}^1/_2(\alpha k_0^2)^{-3/2}(M/H)^{1/2}]. \qquad (2.10)$$

Without allowance for dipole-dipole interaction, an expression for the FMR frequency in the small-magnetic-field range can be obtained by expanding formula (3.6) of Ref. 1 as a series:

$$\omega\approx\omega_0-(22/5)gM\beta^2D(\alpha k_0^2)^{-1/2}(H/M)^{1/2}. \qquad (2.10')$$

It is seen that the corrections to the frequency in these two cases have quite different behaviors: without allowance for dipole-dipole interaction, the correction decreases on decrease of H, vanishing when $H=0$; with allowance for dipole-dipole interaction, the principal role in this field range begins to be played by entirely different terms (resulting from the channel $\rho\to\mathbf{M}\to\mathbf{H}_m\to\mathbf{m}$ of Fig. 3), and the correction increases on decrease of H. As has already been stated, the value of H is bounded from below by the relation $8D_M\ll M^2$ obtained in the calculation of the dispersion of the stochastic magnetic structure. From the requirement that the modification of the frequency in (2.10) be small in comparison with the frequency itself there follows directly another bound to H from below, which may be more stringent. Using it, we find that (2.10) is correct in the magnetic-field interval

$$(4\pi\beta^2D)^{1/2}\ll H/M\ll 4\pi. \qquad (2.11)$$

In conclusion, we shall treat briefly the case of spatial inhomogeneity of the magnetic anisotropy constant, supposing that the direction of the axis of anisotropy is the same throughout the material and coincides with the direction of the external magnetic field $\mathbf{H}$. Then a stochastic magnetic structure does not occur, and the represenation (1.1) is correct. The dispersion relation for a wave with $\theta=0$ has the form

$$\frac{\omega}{g}=H+\beta M+\alpha Mk^2-(\Delta\beta)^2\frac{M}{\alpha k_0^2}J_0(k), \qquad (2.12)$$

where the integral $J_0(k)$ is determined by an expression that differs from the expression (1.8) in the numerator of the integrand: in $J_0(k)$, k_1^2 occurs instead of k_1^4. This integral was calculated in the same approximations as the integral (1.8). The expressions obtained are cumbersome; the principal qualitative results reduce to the following. The modification of the real part of the dispersion relation is everywhere smaller than the modification obtained without allowance for dipole-dipole interaction[1] and approaches the latter when $k\gg k_M$. The damping decreases abruptly for long waves, $k<k_c$, where k_c is determined by the expression (1.11); for $k\gg k_c$, the damping approximates the expression obtained in Ref. 1.

1)Some results of the present work have been briefly communicated in Ref. 2.

[1] V. A. Ignatchenko and R. S. Iskhakov, Zh. Eksp. Teor. Fiz. **72**, 1005 (1977) [Sov. Phys. JETP **45**, 526 (1977)].
[2] V. A. Ignatchenko and R. S. Iskhakov, Summaries of Papers, All-Union Conference on the Physics of Magnetic Phenomena, Donetsk Physicotechnical Institute, Donetsk, 1977, p. 95.

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