Photon absorption in Coulomb collisions in a degenerate plasma in a quantizing magnetic field

G. G. Pavlov, A. N. Panov, and D. G. Yakovlev

A. F. Ioffe Physicotechnical Institute, USSR Academy of Sciences
(Submitted 4 October 1977)
Zh. Eksp. Teor. Fiz. **74**, 1356–1365 (April 1978)

We obtain the dissipative part of the high-frequency conductivity, which describes the absorption of electromagnetic waves in electron-ion collisions in a degenerate plasma in a quantizing magnetic field $\hbar\omega_B > \epsilon_F$ (ω_B is the cyclotron frequency and ϵ_F is the Fermi energy). We investigate the dependence of the longitudinal and transverse effective collision frequencies $\nu_\parallel$ and $\nu_\perp$, in terms of which the conductivity tensor is expressed, on the magnetic field B and on the photon frequency ω. A comparison is made with the case of a nondegenerate plasma in a quantizing magnetic field and a degenerate plasma at $B=0$. Account is taken of the effect of low temperatures $T \ll \epsilon_F$ on the shape of the absorption curves. It is shown that besides the usual cyclotron resonances at $\omega = s\omega_B$ ($s = 1,2, \ldots$) the frequency dependence $\nu_{\parallel,\perp}(\omega)$ contains sharp peaks at $\hbar\omega = \epsilon_F$, the positions of which can be used to measure the Fermi energy in degenerate semiconductors. If $\hbar\omega < \delta_F$, similar absorption singularities appear at frequencies $\hbar\omega = \epsilon_F - n\hbar\omega_B$, where $n = 0, 1,2, \ldots, N$ and $N = [\epsilon_F/\hbar\omega_B]$. It follows in particular from the results that the radiant thermal conductivity of a degenerate plasma (cores of white dwarfs, surface layers of neutron stars) decrease in proportion to B^{-1} at $\hbar\omega_B > \epsilon_F$.

PACS numbers: 52.25.Ps, 52.20.Fs, 72.30.+q, 95.30.Qd

1. The emission and absorption of photons in electron–ion collisions are among the main interactions between radiation and a magnetized plasma. For a nondegenerate plasma, these processes were considered in detail in [1]. It is of interest to consider the analogous problem for a degenerate electron plasma. This problem is of importance in the study of the absorption of electromagnetic waves in degenerate semiconductors (absorption by free carriers as they are scattered by charged impurities), as well as in astrophysics for the study of processes in white dwarfs and neutron stars, which have, as recently established, tremendous magnetic fields. Greatest interest attaches to the case of quantizing magnetic fields $B \gtrsim 3.10^{-7} N_e^{2/3}$ G(n_e is the electron density in cm^{-3}), when the distance between the Landau levels $\hbar\omega_B = \hbar eB/mc$ is larger than or of the order of the Fermi energy ϵ_F. For a degenerate semiconductor with $N_e \approx 10^{18}$ cm^{-3}, a field $B \gtrsim 30$ kG is quantizing. For the surface layers of neutron stars and the central regions of white dwarfs ($N_e \sim 10^{28} - 10^{30}$ cm^{-3}) fields $B \gtrsim (1-30)\cdot 10^{12}$ G are quantizing. These values of the field intensities for the surfaces of neutron stars are at present universally accepted (see e.g., [2]). The possible existence of such fields in the cores of white dwarfs does not contradict the observed surface fields $\sim 10^7 - 10^8$ G and is confirmed by certain model calculations.[3]

The absorption of electromagnetic waves by a fully degenerate plasma in a quantizing magnetic field was considered by Silin and Uryupin.[4] They, however, have made a number of errors. In particular, as will be shown below, their results are qualitatively incorrect for the case of practical importance $\hbar\omega < \epsilon_F$(ω is the frequency of the absorbed radiation). In addition, certain important limiting cases were left unstudied in [4], no comparison of the results with the case $B=0$ was made, and the comparison with the case of a nondegenerate plasma in a quantizing field is not quite satisfactory, in view of errors in a number of the deductions.

In this paper we calculate the high frequency conductivity, which describes the absorption of electromagnetic waves in a nondegenerate plasma at $\hbar\omega_B > \epsilon_F$ for different values of $\hbar\omega/\epsilon_F$ and carry out a comparison with the case of a degenerate plasma at $B=0$ and a nondegenerate plasma in a quantizing magnetic field. Account is taken of the effect of low but finite ($T \ll \epsilon_F$) temperatures on the shape of the absorption curves. An important feature of the results is the presence at $\hbar\omega_B > \epsilon_F$ of an absorption peak at the frequency $\hbar\omega = \epsilon_F$ (besides the usual peaks at the cyclotron harmonics $\omega = s\omega_B$). This uncovers, in particular, an additional possibility of measuring the Fermi energy in degenerate semiconductors. It is indicated that in a weaker magnetic field, $\hbar\omega_B < \epsilon_F$, the absorption curves will contain series of similar singularities at the frequencies $\hbar\omega = \epsilon_F - n\hbar\omega_B$, where $n = 0,1,2,\ldots,N, N = [\epsilon_F/\hbar\omega_B]$. The obtained conductivity values can also be used to calculate the coefficients of absorption, emission, and radiant thermal conductivity, which are needed for the construction of models of neutron stars and white dwarfs with strong magnetic fields.

2. The plasma dissipative processes that determine the absorption of the electromagnetic waves are described by the Hermitian part of the conductivity tensor $\sigma'_{\alpha\beta}$. The components of this tensor are determined by the effective frequencies $\nu_\parallel$ and $\nu_\perp$ of the electron–ion collisions along and across the magnetic field (see, e.g., Silin's monograph[5]). These components take the simplest form in cyclic coordinates $\alpha, \beta = 0 \pm 1$ with z axis along the magnetic field:

$$\sigma_{\alpha\beta}' = \frac{\omega_p^2}{4\pi} \frac{\nu^{(\alpha)}}{(\omega - \alpha\omega_B)^2} \delta_{\alpha\beta}, \quad \nu^{(0)} \equiv \nu_\parallel, \quad \nu^{(+1)} = \nu^{(-1)} \equiv \nu_\perp, \tag{1}$$

where ω_p is the Langmuir frequency. Formula (1) is valid if spatial dispersion is neglected for frequencies much higher than $\nu_{\parallel,\perp}$ and the ion cyclotron frequency, as well as those not too close to the first cyclotron resonance ($|\omega - \omega_B| \gg \nu_{\parallel,\perp}$).

 0038-5646/78/040710-05$02.40

The effective collision frequencies $\nu_{\parallel,\perp}$ in a magnetic field, for an arbitrary electron distribution function $f_{n\sigma}(p_z)$ ($n=0, 1, 2, \ldots$ is the number of the Landau level, p_z and $\sigma = \pm 1/2$ are the projections of the momentum and of the electron spin on the magnetic-field direction) and for an arbitrary potential $U(r)$ of the interaction of the electron with the scatterer are given in the Born approximation by formula (20) of[1]. By integrating in this formula with respect to p_z at a fixed longitudinal-momentum transfer $q_z = p_{z'} - p_z$, we rewrite it in the form

$$\left\{ \begin{matrix} \nu_\parallel \\ \nu_\perp \end{matrix} \right\} = \frac{N_i m^2 \omega_B^2}{4\pi\hbar^3\omega} \sum_{n',n=0}^{\infty} \sum_{\sigma=\pm 1/2} \frac{n!}{n'!} \int_0^\infty \frac{dw}{w} \times \int_0^\infty du e^{-u} u^{n'-n} [L_n^{n'-n}(u)]^2 |U_q|^2 \cdot \left\{ \begin{matrix} 2w \\ u \end{matrix} \right\} [f_{n\sigma}(p_+) - f_{n\sigma}(p_-)],$$
$$p_\pm = \left(\frac{m\hbar\omega_B}{2\omega} \right)^{1/2} \left[w + n' - n \mp \frac{\omega}{\omega_B} \right]. \qquad (2)$$

Here N_i is the concentration of the scatterers, w and u are the squares of the longitudinal a_z and transverse $q_\perp = (q_x^2 + q_y^2)^{1/2}$ momentum transfers in units of $2m\hbar\omega_B$. U_q is the Fourier transform of the potential $U(r)$ and $L_n^{n'-n}$ is a Laguerre polynomial. The term containing $f_{n\sigma}(p_-)$ takes into account the contribution of the inverse transitions (stimulated emission).

We calculate first $\nu_{\parallel,\perp}$ for a fully degenerate ($T=0$) plasma for electron collisions with ions of charge Ze in a quantizing magnetic field $\hbar\omega_B > \epsilon_F$. In that case the only populated Landau level is [1)] $n=0$, $\sigma = 1/2$ and

$$f_{n\sigma}(p_z) = \delta_{n0}\delta_{\sigma,-1/2}(2p_F)^{-1}\theta(p_F - |p_z|), \quad p_F = (2m\varepsilon_F)^{1/2},$$
$$\varepsilon_F = \frac{2\pi^4\hbar^4 N_e^2}{m^3\omega_B^2} = \varepsilon_0 \left(\frac{4\varepsilon_0}{3\hbar\omega_B} \right)^2, \quad U_q = -\frac{2\pi Ze^2\hbar}{m\omega_B(u+w+a)}, \qquad (3)$$

where ϵ_F and ϵ_0 are the Fermi energies in the quantizing magnetic field and at $B=0$, N_e is the electron density, and the quantity $a = a(u, w) \sim \hbar/m\omega_B \rho_D^2$ takes into account the screening of the Coulomb potential ($\rho_D \sim p_F/m\omega_p$ is the Debye radius). We note that the necessary condition for the applicability of the here-employed Born approximation for the Coulomb potential is $\epsilon_F \gg Z^2 m e^4 \hbar^{-2}$. It is easy to show that both at $B=0$ and in a quantizing magnetic field this condition is satisfied at least for an ideal ($Ze(N_e/Z)^{1/3} \ll \epsilon_F$) degenerate electron gas.

Substituting (3) in (2) we obtain

$$\nu_{\parallel,\perp} = \sum_{s=0}^{\infty} \nu_{\parallel,\perp}^{(s)}, \quad \nu_\parallel^{(s)} = \frac{\nu_*}{x} \int dw W_s(w),$$
$$\nu_\perp^{(s)} = \frac{(s+1)\nu_*}{2x} \int \frac{dw}{w} W_{s+1}(w), \qquad (4)$$
$$x = \frac{\hbar\omega}{\varepsilon_F}, \quad \nu_* = \frac{\pi N_i Z^2 e^4}{\varepsilon_F p_F}, \quad W_s(w) = \frac{1}{s!} \int_0^\infty \frac{u^s e^{-u} du}{(u+w+a)^2}.$$

The region of integration with respect to w in (4) is determined from the condition that the difference of the distribution functions in (2) not be equal to zero. At $\hbar(\omega - s\omega_B) < \epsilon_F$ both functions $f_{n\sigma}(p_\pm) = 0$ and $\nu_{\parallel,\perp}^{(s)} = 0$. At $\hbar\omega > \epsilon_F$, the function $f_{n\sigma}(p_-) = 0$ and the integration is over the interval $w_1 < w < w_4$. At $s=0$ and $\hbar\omega < \epsilon_F$, both distribution functions differ from zero, and the integration region consists of two intervals: $w_1 < w < w_2$ and $w_3 < w < w_4$. Here

$$w_{1,4} = [1 \mp (1 + x - sb)^{1/2}]^2/b, \quad w_{2,3} = [1 \mp (1-x)^{1/2}]^2/b, \quad b \equiv \hbar\omega_B/\varepsilon_F. \qquad (5)$$

A formula corresponding to (4) (with an extra factor 2) was obtained in[4], but the limits of integration with respect to w at $x < 1$ were incorrectly determined—the integral was taken over the interval $w_1 < w < w_4$, just as at $x > 1$, and this, will be shown to lead to substantial errors.

3. We proceed to the investigation of the dependence of $\nu_{\parallel,\perp}$ on the emission frequency and on the magnetic field. We assume, as in[4], that $\omega \gg \omega_p$. Then at $x \ll 1$ the ratio $a/w_1 \lesssim \omega_p^2/\omega^2$, i.e., the effect of the Debye screening on the photon absorption is immaterial, in contrast to the conclusion in[4]. At $s > 1$ for $\nu_\parallel^{(s)}$ and at $s > 0$ for $\nu_\perp^{(s)}$ the characteristic values in (4) are $u \gtrsim 1$. Since the condition for the applicability of the Born approximation (see above) means that $a \ll 1$, it can be neglected compared with u. In the expressions for $\nu_\parallel^{(1)}$ and $\nu_{\parallel,\perp}^{(0)}$, the Debye screening can certainly be neglected in the case of practical interest $\epsilon_F > \hbar\omega_p$, which in fact the case we shall consider. Then (see e. e.g.,[6])

$$W_s(w) = -\frac{(-w)^{s-1}}{s!} \left[(s+w)e^w \operatorname{Ei}(-w) + 1 - \sum_{k=0}^{s-2} \frac{k!(s-k-1)}{(-w)^{k+1}} \right], \qquad (6)$$

where Ei is the integral exponential function. Expressions (4) for $\nu_{\parallel,\perp}^{(s)}$ can now be written in the form:

$$\nu_\parallel^{(0)} = \frac{\nu_*}{x} [\Phi_0(w_1) - \Phi_0(w_2) + \Phi_0(w_3) - \Phi_0(w_4)],$$
$$\nu_\perp^{(0)} = \frac{\nu_*}{2x} [\psi_0(w_1) - \psi_0(w_2) + \psi_0(w_3) - \psi_0(w_4)] \qquad (7)$$

at $x < 1$ and $s=0$, and

$$\nu_\parallel^{(s)} = \frac{\nu_*}{x} [\Phi_s(w_1) - \Phi_s(w_4)], \quad \nu_\perp^{(s)} = \frac{\nu_*}{2x} [\psi_s(w_1) - \psi_s(w_4)] \qquad (8)$$

at $x > 1$ for $s=0$ and $x > sb - 1$ for $s > 0$. In (7) and (8)

$$\Phi_s(w) = \frac{1}{s!} \int_0^\infty du \frac{e^{-u}u^s}{u+w} = \frac{(-w)^s}{s!} \left[-e^w \operatorname{Ei}(-w) + \sum_{k=1} \frac{(k-1)!}{(-w)^k} \right],$$
$$\psi_s(w) = \frac{1}{s!} \int_0^\infty du e^{-u} u^{s-1} \ln \frac{u+w}{w} - \Phi_s(w), \qquad (9)$$
$$\psi_{s>0}(w) = \sum_{k=0}^{s-1} \frac{\Phi_k(w)}{k+1} - \Phi_s(w).$$

Using relations (4)–(9), we easily obtain simple formulas for $\nu_{\parallel,\perp}$ in different limiting cases. At $x \ll 1$ and $b > 1$ we have $w_{2,4} - w_{1,3} \ll w_{1,3}$

$$\nu_\parallel^0 = 2\nu_* \left[1 - \frac{2}{b} \Phi_0\left(\frac{4}{b}\right) \right],$$
$$\nu_\perp^{(0)} = \frac{\nu_*}{2} \left[\ln \frac{4b}{x^2} - C - 2 + \left(1 + \frac{4}{b}\right) \Phi_0\left(\frac{4}{b}\right) \right]. \qquad (10)$$

At $x \lesssim 1$ and $b \gg 4$ we have $w_4 \ll 1$,

$$\nu_\parallel^{(0)} = \frac{4\nu_*}{x} \ln \frac{1+(1+x)^{1/2}}{1+(1-x)^{1/2}}, \quad \nu_\perp^{(0)} = \frac{2\nu_*}{x} \left(\ln \frac{b}{x} - C - 1 \right) \ln \frac{1+(1+x)^{1/2}}{1+(1-x)^{1/2}} \qquad (11)$$

and analogously at $1 < x \ll b$ and $b \gg 4$

$$\nu_\parallel^{(0)} = \frac{2\nu_*}{x}\ln\frac{(1+x)^{1/2}+1}{(1+x)^{1/2}-1}, \quad \nu_\perp^{(0)} = \frac{\nu_*}{x}\left(\ln\frac{b}{x}-C-1\right)\ln\frac{(1+x)^{1/2}+1}{(1+x)^{1/2}-1} \tag{12}$$

From (10) and (11) at $x \ll 1$ and $b \gg 4$ we have

$$\nu_\parallel^{(0)} = 2\nu_*, \quad \nu_\perp^{(0)} = \nu_*\left(\ln\frac{b}{x}-C-1\right). \tag{13}$$

At $1+x-sb \ll sb$, $sb \gg 4$, and $s>0$ we have $w_4 \ll s$,

$$\nu_\parallel^{(1)} = \frac{4\nu_*}{bx}\Big[(1+x-b)^{1/2}\ln\frac{b}{|x-b|}-\frac{2+x-b}{2}\times\ln\frac{(1+x-b)^{1/2}+1}{|(1+x-b)^{1/2}-1|}-C(1+x-b)^{1/2}\Big],$$
$$\nu_\parallel^{(s>1)} = \frac{4\nu_*(1+x-sb)^{1/2}}{bxs(s-1)}, \quad \nu_\perp^{(s)} = \frac{\nu_*}{xs}\ln\frac{(1+x-sb)^{1/2}+1}{|(1+x-sb)^{1/2}-1|}. \tag{14}$$

From (14) at $|x-sb| \ll 1$ and $sb \gg 4$ we have

$$\nu_\parallel^{(1)} = \frac{4\nu_*}{b^2}\left(\ln\frac{b}{4}-C\right), \quad \nu_\parallel^{(s>1)} = \frac{4\nu_*}{b^2s^2(s-1)},$$
$$\nu_\perp^{(s>0)} = \frac{\nu_*}{bs^2}\ln\frac{4}{|x-sb|}; \tag{15}$$

at $x-sb+1 \ll 1$ and $sb \gg 4$ we have

$$\nu_\parallel^{(1)} = \frac{4\nu_*}{b(b-1)}(1+x-b)^{1/2}(\ln b-C-1), \quad \nu_\parallel^{(s>1)} = \frac{4\nu_*(1+x-sb)^{1/2}}{b(sb-1)s(s-1)},$$
$$\nu_\perp^{(s>0)} = \frac{2\nu_*}{s(sb-1)}(1+x-sb)^{1/2}; \tag{16}$$

at $x-sb \gg 1$ and $b>1$ we have $w_4-w_1 \ll w_4$ and

$$\nu_\parallel^{(s)} = \frac{4\nu_*(x-sb)^{1/2}}{bx}W_s\left(\frac{x-sb}{b}\right), \quad \nu_\perp^{(s)} = \frac{2\nu_*(s+1)}{x(x-sb)^{1/2}}W_{s+1}\left(\frac{x-sb}{b}\right). \tag{17}$$

From (17) at $1 \ll x-sb \ll (s+1)b$ we have

$$\nu_\parallel^{(0)} = \frac{4\nu_*}{x^{1/2}}, \quad \nu_\parallel^{(1)} = \frac{4\nu_*}{bx}(x-b)^{1/2}\left(\ln\frac{b}{x-b}-C-1\right),$$
$$\nu_\parallel^{(s>1)} = \frac{4\nu_*(x-sb)^{1/2}}{bxs(s-1)}, \tag{18}$$
$$\nu_\perp^{(0)} = \frac{2\nu_*}{x^{1/2}}\left(\ln\frac{b}{x}-C-1\right), \quad \nu_\perp^{(s>0)} = \frac{2\nu_*}{sx(x-sb)^{1/2}};$$

at $x-sb \gg (s+1)b$

$$\nu_\parallel^{(s)} = \frac{4\nu_* b}{x(x+sb)^{1/2}}, \quad \nu_\perp^{(s)} = \frac{2\nu_* b^2(s+1)}{x(x-sb)^{3/2}}; \tag{19}$$

at $x-sb \gg 1$ and $s \gg 1$

$$\nu_\parallel^{(s)} = 4\nu_*(x-sb)^{1/2}bx^{-3}, \quad \nu_\perp^{(s)} = 2\nu_* b^2 s x^{-3}(x-sb)^{-1/2}. \tag{20}$$

In these formulas, $C=0.577$ is the Euler constant. At not too small values of b formulas (10)–(20) enable us to calculate the values of $\nu_{\parallel,\perp}$ for practically all values of the frequency x. The limiting cases (13), (15), (18), and (19) were considered in[4]. The formula for $\nu_\parallel^{(0)}$ obtained there for the case (18) contains an extra factor 8 (compared with our formula), and the remaining formulas of the case (18) and of the cases (15) and (19) contain an extra factor 2. For the case (13) the formulas of[4] take (in our notation) the form

$$\nu_\parallel^{(0)} = \frac{8\nu_*}{x}\ln\frac{4(ab+4)}{4ab+x^2} \approx \frac{16\nu_*}{x}\ln\frac{4}{x},$$
$$\nu_\perp^{(0)} = \frac{8\nu_*}{x}\left[\frac{1}{4}\ln\frac{4}{x}+\ln\frac{4(ab+4)}{4ab+x^2}\right]\ln\frac{1}{a} \approx \frac{18\nu_*}{x}\ln\frac{4}{x}\ln\frac{1}{a}. \tag{13a}$$

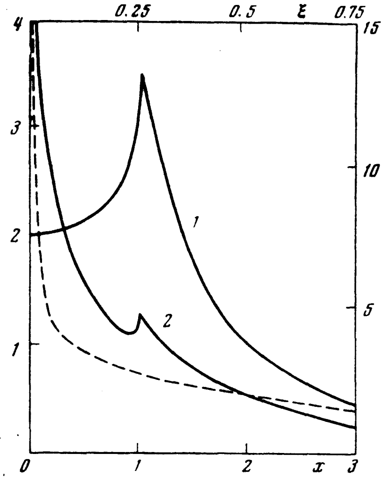

FIG. 1. Plots of $\nu_\parallel$ and $\nu_\perp$ (curves 1 and 2) in a degenerate plasma ($T=0$) against the radiation frequency at $b=\hbar\omega_B/\epsilon_F=10$, $x=\hbar\omega/\epsilon_F<3$. The lower and upper horizontal axes designate the frequency, referred to the Fermi energy, in a magnetic field (x) and without a field (ξ). The left-hand vertical axis represents the collision frequencies referred to ν_*, and the right-hand vertical scale shows them referred to ν_0, see (4) and (21). Dashed line--collision frequency (12) at $B=0$.

The quantities in (13a), first depend on the emission frequency, on the magnetic field, and on the Debye screening parameter in an entirely different manner than our Eq. (13). Second, they quantitatively exceed (13) by a factor $\gg 1$. The main cause of this discrepancy is that no account was taken in[4] of the contribution of the stimulated emission at $x<1$ (see above). The limiting cases (10), (11), (12), (14), (16), (17), and 20) were not considered in[4].

4. An analysis of the obtained frequency dependences $\nu_{\parallel,\perp}(x)$ (they are shown in Figs. 1 and 2 for $b=10$) is best carried out by comparing them with the corresponding dependences for the case of a degenerate plasma at $B=0$ and a nondegenerate plasma in a quantizing magnetic field.[1] The expression for the effective collision frequency ν at $B=0$ can be easily obtained if the known expression (see e.g., [7]) for the coefficient of inverse bremsstrahlung of an electron with given energy is averaged with the Fermi distribution function. In the Born approximation we obtain by this method:

$$\nu = 2\nu_0\int_0^\infty\frac{d\varepsilon}{\hbar\omega}\ln\frac{(\varepsilon+\hbar\omega)^{1/2}+\varepsilon^{1/2}}{(\varepsilon+\hbar\omega)^{1/2}-\varepsilon^{1/2}}\left[\left(e^{(\varepsilon-\mu)/T}+1\right)^{-1}-\left(e^{(\varepsilon-\mu+\hbar\omega)/T}+1\right)^{-1}\right], \tag{21}$$

where $\nu_0=\nu_*(B=0)=\pi Z^2e^4/\epsilon_0 p_0$, $\epsilon_0=\mu(T=0)$, and μ is

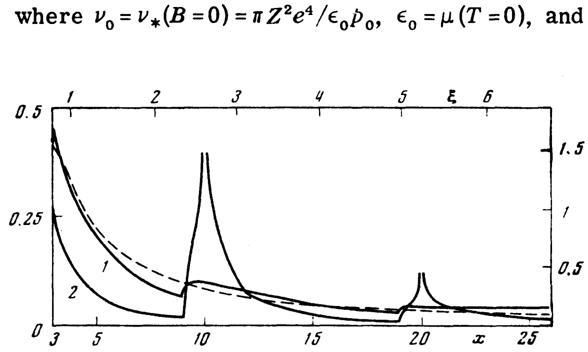

FIG. 2. The same as in Fig. 1, at $x>3$.

the chemical potential of the electron gas at the temperature T. At $T=0$

$$\nu=\frac{2\nu_0}{\xi}\left[2\ln\frac{1+(1+\xi)^{1/2}}{1+(1-\xi)^{1/2}}+\xi\ln\frac{1+(1+\xi)^{1/2}}{1-(1-\xi)^{1/2}}-(1+\xi)^{1/2}+(1-\xi)^{1/2}\right]$$
$$\text{at}\quad \xi\equiv\hbar\omega/\varepsilon_0\leqslant 1, \tag{22}$$

$$\nu=\frac{2\nu_0}{\xi}\left[\left(1+\frac{\xi}{2}\right)\ln\frac{(1+\xi)^{1/2}+1}{(1+\xi)^{1/2}-1}-(1+\xi)^{1/2}\right]\quad\text{at}\qquad \xi\geqslant 1,$$

and in the limiting cases

$$\nu(\xi\ll 1)=2\nu_0\ln(4/\xi),\quad \nu(\xi\gg 1)={}^8/_3\nu_0\xi^{-1/2}. \tag{23}$$

The curve calculated from (22), $\nu=\nu(\xi)$, is shown by the dashed lines in Figs. 1 and 2. We note the presence of a kink (a discontinuity in the derivative $\nu'(\xi)$) on the curve at the point $\xi=1$: $\nu'(1-0)-\nu'(1+0)\approx 0.15\nu(1)$. This kink is due to the steep Fermi distribution at $T=0$ and, as follows from (21), becomes smoothed out at $T\neq 0$. The first formula of (23) was obtained (without the factor 4 under the logarithm sign) by Chandrasekhar.[8] The expression for $\nu(\xi\gg 1)$ is independent of ϵ_0 and it is easy to show with the aid of (21) that it is valid at any degree of electron degeneracy, provided that the electron characteristic energy $\epsilon\ll\hbar\omega$.

For the analysis of the collision frequencies in a quantizing magnetic field it is convenient to use the general formula (2). If the electron energy is $\epsilon\ll\hbar\omega_B$ and if $\omega\ll\omega_B$, then only the term $n=n'=0$ and $\sigma=1/2$, in which the characteristic values are $w\ll 1$, is significant in this formula. It is then easy to obtain the expressions

$$\nu_\perp=\frac{\nu_\parallel}{2}\left(\ln\frac{\omega_B}{\omega}-C-1\right),$$
$$\nu_\parallel=\frac{2\nu_*}{x}\int_0^\infty\frac{d\varepsilon}{[\varepsilon(\varepsilon+\hbar\omega)]^{1/2}}\left[(e^{(\varepsilon-\mu)/T}+1)^{-1}-(e^{(\varepsilon-\mu+\hbar\omega)/T}+1)^{-1}\right], \tag{24}$$

which are valid for any ratio μ/T (the quantities ν_* and x contain here, as before, the value (3) $\epsilon_F=\mu(T=0)$). It is seen that in a quantizing magnetic field at $\omega\ll\omega_B$ the ratio $\nu_\parallel/\nu_\perp$ does not depend on the degree of degeneracy, whereas the quantities $\nu_{\parallel,\perp}$ themselves depend on it substantially. For a nondegenerate plasma ($2\mu=-T\ln(\pi T/4\epsilon_F)$) we obtain directly from (24) the known relation (see[1])

$$\nu_\parallel=\frac{8\pi N_iZ^2e^4}{\hbar\omega(2\pi mT)^{1/2}}\,\mathrm{sh}\frac{\hbar\omega}{2T}K_0\left(\frac{\hbar\omega}{2T}\right),$$
$$\nu_\parallel(\hbar\omega\ll T)=2\left(\frac{2\pi}{m}\right)^{1/2}\frac{N_iZ^2e^4}{T^{3/2}}\left(\ln\frac{4T}{\hbar\omega}-C\right), \tag{25}$$

from which it is clear that the quantities $\nu_{\parallel,\perp}$ contain an additional (compared with the degenerate case) large logarithm $\ln(T/\hbar\omega)$ at $T\gg\hbar\omega$ and have no peaks at $\omega<\omega_B$.

Comparison of (25) with (10)–(13) shows that in a degenerate plasma the magnetic field alters the effective collision frequencies more strongly than in a nondegenerate one. Whereas in a nondegenerate plasma the magnitude field changes the collision frequencies by not more than a large logarithmic factor (the Coulomb logarithm changes), in a degenerate plasma we have besides the logarithmic change also $\nu_{\parallel,\perp}\propto B^3$ (due to the decrease of the Fermi energy in the quantizing field). This in turn causes, in a sufficiently strong field ($\omega_B\gg\omega$), the absorption coefficients $k_\parallel\propto\nu_\parallel\omega^{-2}$ and $k_\perp\propto\nu_\perp\omega_B^{-2}$ for photons with polarization along and across the field in a degenerate plasma (without allowance for the logarithmic dependence on B) to increase: $k_\parallel\propto B^3$ and $k_\perp\propto B$, whereas in a nondegenerate plasma we have $k_\parallel\propto B^0$ and $k_\perp\propto B^{-2}$. As a result, for example, the radiant thermal conductivity of a degenerate plasma in a quantizing field decreases in proportion to B^{-1}, while in a degenerate plasma it increases in proportion to B^2.[9] Next, in a degenerate plasma the kind of the $\nu(\xi)$ curve at the point $=1$ at $B=0$ is replaced at $\nu_{\parallel,\perp}(x)$ by peaks of the curves $\nu_{\parallel,\perp}(x)$ at the point $x=1$. In addition, whereas in a nondegenerate plasma in the case $\hbar\omega<T$ $\nu_\parallel$ increases by 1.5 times and $\nu_\perp$ increases by a factor $0.75\ln(\omega_B/\omega)$ on going from a magnetic field to a quantizing field, such a transition in a degenerate plasma causes $2\nu_0\ln(4\epsilon_0/\hbar\omega)$ to be replaced by $2\nu_*$ for $\nu_\parallel$ and by $\nu_*\ln\omega_B/\omega$ for $\nu_\perp$.

The most interesting feature of the functions $\nu_{\parallel,\perp}(\omega)$ in a degenerate plasma in a quantizing field is the presence of absorption peaks at $\hbar\omega_B>\epsilon_F$ (Fig. 1), which were not noted in[4]. In contrast to the ordinary peaks at the cyclotron harmonics $x=sb$ (see below), which are much more pronounced on the $\nu_\perp(\omega)$ curve, this peak is no less noticeable on the $\nu_\parallel(\omega)$ curve than on the $\nu_\perp(\omega)$ curve. So long as $\hbar\omega>\epsilon_F$, any electron with energy $0<\epsilon<\epsilon_F$ can absorb a photon. As soon as $\hbar\omega$ becomes smaller than ϵ_F, the electrons with energy $0<\epsilon<\hbar\omega$ cease to participate in the absorption, owing to the turning-on of the reverse-transition channel (stimulated emission). In a quantizing magnetic field, owing to the "one-dimensionality" of the electron motion, the density of the number of electronic states in the region of small ϵ is large (proportional to $\epsilon^{-1/2}$). Therefore the exclusion of these states from the absorption process greatly weakens the latter. At $B=0$ the density of the number of the electronic states is proportional to $\epsilon^{1/2}$ and the exclusion of states of low energy with decreasing ω leads only to a jumplike decrease of the slope of the absorption curve at the point $\hbar\omega=\epsilon_0$. According to (11) and (12), at $T=0$ the $\nu_{\parallel,\perp}(x)$ have a discontinuity of the derivative at the maximum of the peak ($x=1$), with $\nu'_{\parallel,\perp}(1-0)=\infty$. Using (24), we can easily find that at $T\ll\epsilon_F$ the temperature corrections to the functions $\nu_{\parallel,\perp}(x)$ obtained in Sec. 3 are substantial precisely in the region $|x-1|\lesssim T/\epsilon_F$ near the peak. At $T\neq 0$ the discontinuity of the derivatives of the $\nu_{\parallel,\perp}(x)$ curves at the peak maximum disappears; the peaks become ever smoother with increasing T and their height decreases. In particular, the point $\hbar\omega=\mu$ we have near the maximum of the peak

$$\nu_\parallel=2\nu_*[2\ln(1+2^{1/2})-\alpha(T/\varepsilon_F)^{1/2}],\quad \alpha=\pi^{1/2}|\zeta(0.5)|(2^{1/2}-1)\approx 1.06, \tag{26}$$

where $\zeta(z)$ is the Riemann zeta-function. For example, at $T=0.1\epsilon_F$, the height of the peak is $\nu_\parallel/2\nu_*=1.43$ instead of the value 1.76 at $T=0$, but the peak remains fully discernible and can be used for an experimental determination of the Fermi energy. It is easy to show with the aid of (24) that at small T we have $\nu'_{\parallel,\perp}(x=\mu/\epsilon_F)$

>0. Since at such values of T the chemical potential is $\mu \approx \epsilon_F + \pi^2 T^2/12\epsilon_F$, when T is increased the peak shifts slightly towards higher frequencies.

We note that in a weaker magnetic field $\hbar\omega_B < \epsilon_F$ the Landau levels $n=0, 1, \ldots, N$ (N is the integer part of ratio $\epsilon_F/\hbar\omega_B$) are populated and the density of the initial states of the electron has root singularities at $\epsilon = n\hbar\omega_B$. Therefore on passing through the point $\hbar\omega = \epsilon_F - n\hbar\omega_B$ with decreasing frequency, the turning-on of the channel of reverse transitions to the n-th level decreases strongly the number of electrons that participate in the phonon absorption and causes consequently a sharp decrease in the absorption (by an amount proportional to $(1-nb-x)^{1/2}$). As a result, the previously considered peak at $\hbar\omega = \epsilon_F$ is replaced by a series of sharp kinks on the absorption curves, at the frequencies $\hbar\omega = \epsilon_F - n\hbar\omega_B$ at $n=0, 1, 2, \ldots, N$. To the left of these kinks the derivatives $\nu_{\parallel,\perp}(x) \propto (1-nb-x)^{-1/2}$ tend to infinity as $x \to 1-nb-0$, and on the right they can be either negative (in which case a peak is produced) or positive. It is obvious that the appearance of these singularities is not connected with the concrete form of the potential $U(r)$. For the particular case of absorption by electrons with emission of an optical phonon (cyclotron-phonon absorption in semiconductors), the possibility of appearance of analogous singularities at the frequencies $\hbar\omega = \epsilon_F - n\hbar\omega_B + \hbar\omega_0$ (ω_0 is the frequency of the optical phonon) was noted in [10,11].

At $x \ll 1$ and $T \ll \epsilon_F$ there are present in expressions (24) for $\nu_{\parallel,\perp}$ terms that contain the large logarithm $\ln x^{-1}$. These terms are due to the appearance of a logarthmic divergence of the integral with respect to ϵ in (24) at the point $\epsilon=0$ as $\hbar\omega \to 0$, and can be separated by integrating by parts. In a degenerate plasma, however, these terms (which are proportional to $e^{-\mu/T} \ln x^{-1}$) are insignificant. Their role increases as the degeneracy is lifted and at $\mu \sim T$ and $x \ll 1$ they become decisive. In particular, they are responsible for the presence of the large logarithm $\ln(T/\hbar\omega)$ in (25).

When the values $\hbar\omega = s\hbar\omega_B - \epsilon_F$ are reached with increasing emission frequency, transitions to the s-th Landau level begin to contribute to the absorption, and this causes the absorption to grow. In contrast to the nondegenerate case, [1] the turning-on of the new channels in a degenerate plasma is abrupt [see (16)]. The behavior of the region of the cyclotron resonances $\omega = s\omega_B$ is qualitatively the same as in a nondegenerate plasma: $\nu_\parallel$ has near these points peaks of limited height, which attenuate rapidly with increasing s; $\nu_\perp$ has at these points logarithmic singularities connected with the root singularities in the density of the number of final states of the electron. There are a number of mechanisms that broaden the cyclotron resonances and annihilate the divergences. Their action is the same as in a nondegenerate plasma (see, e.g., [5,1]).

At $x \gg b$ the main contribution to the collision frequencies (4) is made by the term with $s \gg 1$. The sum over s can then be replaced by an integral in which it suffices to substitute the values of $\nu_{\parallel,\perp}^{(s)}$ taken from (20). As a result the quantities

$$\nu_{\parallel,\perp} \approx \int_0^{x/b} ds\, \nu_{\parallel,\perp}^{(s)} = \frac{8}{3}\nu_* x^{-3/2} \qquad (27)$$

are independent of B and ϵ_F and coincide with the collision frequency (23) $\nu(\xi \gg 1)$ at $B=0$. The character of the approach of the $\nu_{\parallel,\perp}(x)$ curves to $\nu(x)$ with increasing x is seen from Fig. 2.

The author thanks L. I. Korovin for calling their attention to the works of Levinson and Bakanas. [10,11]

[1] G. G. Pavlov and A. N. Panov, Zh. Eksp. Teor. Fiz. 71, 572 (1976) [Sov. Phys. JETP 44, 300 (1976)].
[2] F. Dyson and D. ter Haar, Neutron Stars and Pulsars (Russ. transl.), Mir, 1973).
[3] J. P. Ostriker, transl. in: Belye karliki (White Dwarfs), Mir, 1975, p. 236.
[4] V. P. Silin and S. A. Uryupin, in: Kratk. Soobshch. Fiz. No. 4, 30 (1977).
[5] V. P. Silin, Vvedenie v kineticheskuyu teoriyu gazov (Introduction to the Kinetic Theory of Gases), Nauka, 1971, Chap. 10.
[6] G. G. Pavlov and D. G. Yakovlev, Zh. Eksp. Teor. Fiz. 70, 753 (1976) [Sov. Phys. JETP 43, 389 (1976)].
[7] D. A. Frank-Kamenetskiĭ, Fizicheskie protsessy vnutri zvezd (Physical Processes Inside Stars), Fizmatgiz, 1959, ch. 6.
[8] S. Chandrasekhar, Proc. R. Soc. London Ser. A 133, 241 (1931).
[9] G. G. Pavlov and D. G. Yakovlev, Astrofizika 13, 173 (1977).
[10] I. B. Levinson and R. K. Bakanas, Pis'ma Zh. Eksp. Teor. Fiz. 11, 141 (1970) [JETP Lett. 11, 87 (1970)].
[11] R. K. Bakanas, Fiz. Tverd. Tela (Leningrad) 12, 3408 (1970) [Sov. Phys. Solid State 12, 2769 (1971)].

Translated by J. G. Adashko