Langmuir solitons

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We give in more detail than in our preliminary reports [JETP Lett. **23**, 562 (1976); **25**, 145 (1977)] the results of experiments which investigate the formation of solitons due to the self-compression of large-amplitude Langmuir waves. The experiments are performed in the regime of a magnetized collisionless plasma in contrast to the experiments by Wong *et al*., and Ikezi and coworkers [A. Y. Wong and B. H. Quon, Phys. Rev. Lett. **34**, 1499 (1975); H. Ikezi, K. Nishikawa, and K. Mima, J. Phys. Soc. Japan **37**, 766 (1974); H. Ikezi, R.P.H. Chang, and R. A. Stern, Phys. Rev. Lett. **36**, 1047(1976)] in which there was no magnetic field and the plasma was collisional. The build-up of Langmuir waves in the plasma is realized in two independent ways: by an external rf electrical field and by an electron beam. The two methods for wave pumping give results which agree. We show that a quasi-one-dimensional Langmuir soliton (an equilibrium density well filled with Langmuir waves) exists, that it has the size of the order of the one expected theoretically, and that it turns out to be a relatively stable configuration. The lifetime of a free soliton (when there is no pumping) with an energy density $W/nT \approx \delta n/n \approx 5$ to 10% and a size $\Delta \approx 20\ r_D$ exceeds several tens of thousands of Langmuir oscillation periods and its mean free path (together with the moving plasma) exceeds 40 to 50 cm.

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INTRODUCTION

In the present paper we study experimentally the formation of quasi-stationary self-compressed large-amplitude Langmuir wavepackets (solitons) in a collisionless, magnetized plasma.

The effect of the self-compression (physical collapse) of waves belongs to a class of general non-linear effects which are characteristic for waves of very different nature in dispersive media.[1-6] According to the theory it must appear in plasma physics for various branches of oscillations, determining the dynamics and fate of plasma turbulence[1b,3,4,6-11] and turning out to have an important effect on the nature of the relaxation of beams and the heating of the plasma,[7,10] the resonance absorption of powerful microwaves in a non-uniform plasma,[11] and on the anomalous transfer processes in a plasma.[12,13] The non-linear effect of self-compression of waves is the opposite of the well known spreading out of linear wavepackets which occurs due to the dispersion of the group velocity. Under well defined conditions the process of the self-compression of waves compensates in some stage the dispersive spreading out; in that case a quasi-stationary self-compressed wavepacket is formed—a soliton.[1,4-9,12,15,16]

It is clear from the literature cited here that a large number of theoretical papers has been devoted to the study of the dynamics of the formation of Langmuir solitons. However, the authors of these papers do not always apply a consistent terminology or identical numerical estimates. This fact stimulated us to preface the present paper with a short introduction of a survey nature (§1), basically with the aim of making the terminology and the qualitative considerations referring to Langmuir solitons and those instabilities which lead to their formation more precise. The material given in §1 is necessary for us for the discussion of the experimental data.

§1. BASIC DEFINITIONS

1) It is well known that the spreading out of a linear (i.e., small-amplitude) wavepacket of length[1)] L is determined by the way the group velocity v_g depends on the wavenumber k and that it proceeds with a speed $\partial L/\partial t = |\partial v_g/\partial k|\varkappa$, where $\varkappa \approx \pi/L \ll k$ is the range of wavenumbers which make up the packet. The characteristic time for the spreading of the wavepacket is

$$\tau \approx \pi / |\partial v_g/\partial k|\varkappa^2. \tag{1}$$

2) If the dispersing packet is non-linear (i.e., when the wavefrequency ω depends on its amplitude E_0), under well defined conditions it turns out to be unstable with respect to self-compression, i.e., with respect to division into separate wave bunches. This is called the modulational instability.[3,4,1,2] Let

$$\omega = \omega_k + \alpha E_0^2, \tag{2}$$

where ω_k is the frequency of the linear wave with wavenumber k and α is a coefficient. The condition for the modulational instability of the wave with respect to a growth in time of a spatial modulation of scale $2\pi/\varkappa$ (where $\varkappa \ll k$) then has the form[1-4]

$$\alpha \partial v_g/\partial k < 0 \quad \text{(Lighthill criterion)}, \qquad |\partial v_g/\partial k|\varkappa^2 < 4|\alpha| E_0^2. \tag{3}$$

From comparing the last condition with Eq. (1) it is clear that it has an obvious physical meaning:[1,2] The growth rate of the (non-linear) self-compression of the

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packet $\gamma \approx |\alpha| E_0^2$ exceeds the reciprocal $1/\tau$ of the time of its spreading.

3) We apply Eqs. (2) and (3) to Langmuir waves for which

$$\omega_k = \omega_p + {}^3/_2 (kr_D)^2 \omega_p.$$

If the wavelength is sufficiently long so that the condition

$$\partial\omega/\partial k \ll c_s = (T_e/M)^{1/2}, \tag{4a}$$

i.e.,

$$kr_D \ll {}^1/_3 (m/M)^{1/2} \tag{4b}$$

(where M is the ion mass) is fulfilled, the non-linear correction to the frequency equals

$$\alpha E_0^2 = -E_0^2 \omega_p / 32\pi nT;$$

here $\omega_p = (4\pi ne^2/m)^{1/2}$ is the Langmuir frequency, n the unperturbed plasma density, $T = T_e$ the electron temperature (we assume that the ion temperature $T_i \ll T_e$, in contrast to Ref. 1, where one took $T_i = T_e$), and $r_D = (T_e/4\pi ne^2)^{1/2}$ the electron Debye radius. In that case

$$\omega = \omega_p + \frac{3}{2}(kr_D)^2 \omega_p - \frac{W}{2nT}\omega_p, \tag{5}$$

where $W = E_0^2/16\pi$ is the electrical energy density of the wave.

It is clear from (2) and (5) that the non-linear correction to the frequency of the Langmuir waves is negative. This correction is connected with the ponderomotive force which expels plasma from a region with increased electrical field strength of the oscillations.[1,2,5] The volume density of that force is

$$F = -\frac{ne^2}{4m\omega^2}\nabla E_0^2,$$

i.e., for Langmuir waves $F = -\nabla W$. This force causes the occurrence of density "wells" in the plasma with a depth δn which is determined by equilibrium between the force F and the excess thermal pressure of the plasma; when condition (4) holds

$$\delta n/n = W/nT. \tag{6}$$

The formation of a density well causes a decrease in the local Langmuir frequency:

$$\frac{\delta\omega}{\omega} = -\frac{1}{2}\frac{\delta n}{n} = -\frac{1}{2}\frac{W}{nT},$$

and this leads to Eq. (5).

Therefore, for Langmuir waves

$$\alpha < 0, \quad \partial v_r/\partial k = \partial^2\omega/\partial k^2 = 3r_D^2\omega_p > 0,$$

i.e., the Lighthill criterion (3) is satisfied. The second condition (3) means

$$W/nT > {}^3/_2 (\varkappa r_D)^2 \tag{7}$$

and determines the energy threshold for the modulational instability considered here:

$$(W/nT)_c = {}^3/_2 (\varkappa r_D)^2 \ll {}^3/_2 (kr_D)^2. \tag{7'}$$

The physical meaning of the modulational instability consists in the fact that if there occurs randomly a density well in the plasma, the self-compression effect leads to an increase in the wave energy density in the well which in turn leads to a deepening of the well and to a further increase in the wave energy density etc..

4) Let there be included in the Langmuir wavepacket of size $L \approx \pi/\varkappa$ a sufficiently large field energy, which satisfies condition (7) for the modulational instability. The packet then starts to collapse and its energy (when there is no external pumping) will be conserved: WL = const. i.e., the energy density will increase as $W \propto 1/L$ as L decreases. Since the dispersive correction to the frequency of the oscillations, $\frac{3}{2}(\varkappa r_D)^2$ will in that case increase in proportion to L^{-2} (i.e., more steeply than W) for some L^* Eq. (7′) will be satisfied rather than inequality (7) and the collapse process will come to a halt. In other words, while for $L > L^*$ non-linearity (collapse of the packet) dominates, for $L < L^*$ dispersion (spreading of the packet) dominates; when $L = L^*$ the non-linearity and dispersion effects exactly balance. In the equilibrium state a self-compressed wavepacket is formed with an envelope in the form of a soliton; this is the so-called Langmuir envelope soliton.

One sees easily that this equilibrium state is possible only in the one-dimensional case. Indeed, in the two-dimensional case $W \propto L^{-2}$, in the three-dimensional case $W \propto L^{-3}$ and as the dispersive correction to the frequency is as before $\propto L^{-2}$, the non-linearity is not balanced by dispersion and the packet collapses.[4,9]

The electrical field of a Langmuir soliton (see Refs. 4, 6) has the form[7]

$$E(z,t) = \frac{E_0}{\mathrm{ch}[k_0(z-ut)]}\exp\{i(kz-\omega t)\}, \tag{8}$$

where the oscillation frequency is

$$\omega = \omega_p + {}^3/_2 r_D^2 (k^2 - k_0^2), \tag{9}$$

the wavenumber is

$$k_0 = r_D^{-1}(W/6nT)^{1/2} \tag{10}$$

and the propagation speed is

$$u = \partial\omega/\partial k = 3\omega_p r_D^2 k < c_s. \tag{11}$$

The density well depth is

$$\delta n/n = W/nT, \tag{9'}$$

and the width of the soliton (at a level $1/e$ of its amplitude)

$$\Delta = \frac{2}{k_0} = r_D\left(\frac{24nT}{W}\right)^{1/2} = r_D\left(\frac{24n}{\delta n}\right)^{1/2}. \tag{10'}$$

The envelope solition considered in which

$$k_0 < k, \quad \omega > \omega_p, \quad \frac{W}{nT} = \frac{\delta n}{n} \approx (\varkappa r_D)^2 \ll (k r_D)^2, \tag{12}$$

is a non-spreading Langmuir wavepacket with self-consistent phases and amplitudes. Owing to its large extension ($\varkappa^{-1} \gg r_D$) and low oscillation intensity (12) such a soliton, although it is of some interest as a matter of principle, is not the best example of that situation which corresponds to the concept of a physical wave collapse.[4]

4) Of incomparably more interest than the envelope soliton is the large amplitude Langmuir soliton which is formed in the regime

$$W/nT > (k r_D)^2 \tag{12'}$$

and which has a size less than the initial wavelength of the oscillations $2\pi/k$. This soliton is also characterized by the properties (8) to (10′). However, according to (12′) now the non-linear lowering of the frequency of the oscillations (due to the formation of a very deep density well) already exceeds the dispersive correction to the frequency (the third term on the right-hand side of (5) has a larger absolute magnitude than the second one) and $\omega < \omega_p$, i.e., the frequency of the oscillations in the soliton is less than the Langmuir frequency in the surrounding plasma. Due to this the Langmuir oscillations turn out to be trapped in the density well (caviton[4]) as in a resonator. (In an envelope soliton with a relatively small amplitude (12), $\omega > \omega_p$ and the field of the oscillations is not locked in.)

The size of a large amplitude soliton is less than the initial wavelength: $k_0 > k$. The "non-linear" dispersion is thus already given by the magnitude of k_0 and the speed of the soliton is, of course, no longer connected through Eq. (11) with the wavenumber of that initial wave from which the soliton is formed. In the new situation the soliton speed u is a free parameter which is subject only to the condition $u < c_s$, and Eq. (11) can be considered to be merely the definition of that quantity k which for a given magnitude of u must be substituted into Eq. (9) to obtain the exact value of the frequency of the oscillations.

6) The large amplitude Langmuir soliton which occurs in the regime (12′) is, like an envelope soliton, formed due to the instability of a wavepacket with respect to self-compression.[4] This instability has essentially the same physical meaning as the earlier considered modulational instability of a quasi-monochromatic wave which forms an envelope soliton, but it has also its own peculiar features. It is called the aperiodic[13] or (in the non-Russian literature) the oscillating two-stream instability (OTSI).[17] (The modulational instability is correspondingly denoted by MI.) We consider the conditions for the occurrence of the aperiodic instability.[13,17] Let a Langmuir wave with frequency $\omega_k = \omega_p + \frac{3}{2}(k r_D)^2 \omega_p$ be in an external hf electrical field of frequency ω_0, energy density W and spatial period $L \gg 1/k$ (quasi-uniform field). The wave amplitude will then grow aperiodically with time (with a growth rate of the order of the ion-sound frequency) provided

$$\omega \leqslant \omega_k, \tag{13}$$

$$W/nT > (\omega_k - \omega_0)/\omega_p. \tag{14}$$

We now apply these relations to a determination of the threshold for the aperiodic instability of a free Langmuir wave (with no external field) with respect to a (low-frequency) "modulation" with wavenumber $\varkappa > k$. As such a "modulation" increases the dispersive correction to the Langmuir wave frequency by $\frac{3}{2}(\varkappa r_D)^2 \omega_p$ the threshold of the aperiodic instability of the wave considered is according to (14)

$$W/nT \approx {}^3/_2 (\varkappa r_D)^2 > (k r_D)^2. \tag{15}$$

The strength of this inequality is determined by the relation between $\varkappa$ and k. Thus, if the modulational instability described above is not aperiodic, it develops for $\varkappa < k$ and has a comparatively low threshold (7′), and the aperiodic instability considered occurs for $\varkappa > k$ and is characterized by a relatively high threshold (15).

In this case the instability of the originating wave of wavenumber $\varkappa$ turns out to be modulated by the initial wave (of longer wavelength, $k < \varkappa$) and further undergoes self-compression[4] leading (in the one-dimensional geometry) to the formation of Langmuir solitons.[4,6,15,17b] This distance between the solitons which appear depends on several "subtleties" and may be uneven.

For sufficiently large amplitudes of the oscillations one can consider both instabilities as variants (branches) of the "modulational decay":[2,4,18] when $\varkappa < k$ the MI occurs, and when $\varkappa > k$ the OTSI. As $k \to 0$ the two branches merge. The difference between them is also of no great importance in the case when one does not consider a quasi-monochromatic wave, but a "gas" of Langmuir plasmons with a spread in wavenumbers $\Delta k \approx k$ as in that case the modulational instability also has the threshold (15). In other words, when $\Delta k \approx k$ the modulational instability develops for a large intensity of oscillations and it is thus possible to form also a large amplitude Langmuir soliton, (12′).

7) The soliton amplitude $E_0 \sim k_0$, according to (10), is determined by the pumping conditions and limited by Landau damping when k_0 is so large that the size of the soliton is commensurable with the electron Debye radius. When there is no magnetic field a one-dimensional soliton collapses fast in the transverse directions.[4]

8) It is interesting to compare the behavior of a Langmuir soliton with that of the linear Langmuir wavepacket for the same size and propagation speed ($\lesssim c_s$). Let, in agreement with experimental data (see §5) this size be $\Delta \equiv 2/k_0 \approx 20\, r_D \approx 1$ cm. In a hydrogen plasma of density $n \approx 3 \times 10^9$ cm^{-3} and temperature $T_e \approx 10$ eV a linear packet would according to Eq. (1) have a "mean free path" $l \lesssim \tau c_s$ (until it has spread by a factor two) which is not larger than the initial size Δ. On the other hand, a soliton of size of the order of 20 r_D (corresponding to a well depth $\delta n/n \approx 5\%$) must propagate without spreading.

9) Under well defined conditions a one-dimensional

density well, formed by a strong external hf field in the plasma, contains in it not one, but many solitons.[11,19] Such a configuration with a size much larger than the width (10′) of a separate soliton is called a caviton. Solitons are sometimes called spikons.[6,8]

10) We consider the condition for the modulational instability of Langmuir waves excited by an electron beam in a plasma. The wavenumber of these waves is determined by the well known Cherenkov condition: $k\approx \omega_p/u$ where u is the velocity of the electron beam. From this condition and condition (4) for the existence of the instability we get the threshold for the energy W_1 of the beam electrons:

$$mu^2/2T_e = W_1/T_e \geq {}^9/_2 M/m,$$

where M is the ion mass in the plasma and T_e the temperature of the plasma electrons. For molecular hydrogen this means: $W_1 \geq 200$ keV; for heavier ions one needs already relativistic energies for the beam electrons. Hence, in order that the Langmuir waves produced by an electron beam of measured energy may undergo the modulational instability, their wavenumbers must be relatively considerably decreased, for instance, through induced scattering by ions or as the result of decays involving the emission of ion sound. And, indeed, experiments[20] and calculations[15,16] show that the start of the modulational and aperiodic instabilities is preceded by the stage in which Langmuir waves decay into Langmuir waves (with smaller wavenumbers) and sound waves (decay of the type $l \to l' + s$[2]).

§2. STATEMENT OF THE PROBLEM

One can conveniently split the experimental studies of Langmuir solitons which have so far been performed into two groups.

Experiments of the first group[19-22,27] have two principal features. One of these features consists in the fact that in these experiments one applies that plasma regime which we call collisional: the scattering time τ_e for a plasma electron relative to the loss of momentum is much shorter than the "flight time" of ion sound along the system ($\tau_s = L/c_s$). In such a regime the Langmuir soliton (the speed of which is less than c_s, as we have already mentioned) moves during a time τ_e a distance much shorter than the length of the plasma L. After the "pumping" source (external hf field[21] or electron beam[20,22]) is switched off, the soliton which is formed hardly shifts its position during the time of the collisional damping of the waves ($\sim\tau_e$). Under such experimental conditions the problem of the stability of the soliton (or caviton) remains unsolved. The second peculiarity of the first group of experiments is connected with the absence of a magnetic field ($H=0$) and does not enable us to check the results of the existing theory of the stability of a Langmuir soliton in a magnetized plasma.

In experiments of the second group, to which so far only our experiments,[24] belong a collisionless regime ($\tau_e \gg \tau_s$) of a strongly magnetized plasma ($\omega_H \gg \omega_p$) is applied. The present paper, which differs from the short notes[24] by an appreciably more detailed and fuller exposition of the results and their discussion, is devoted to a description of these experiments.

The experiments described here are performed on a moving plasma in order to have the possibility to register the solitons—fixed or non-moving relative to the plasma.[7] The aim of the experiments consists in the observation, by producing Langmuir waves of sufficiently large amplitude which exceeds the threshold of their instability with respect to self-compression, the spatial "collapse" of the waves and to study the properties of the wave bunches which are formed: either (stable) solitons, or (unstable) spikons.

Before describing the experimental set-up we make the following remark about the principle. The fact is that when one studies the proposed space-time evolution of Langmuir waves in a magnetic field it is necessary to take into consideration the real dispersion function $\omega(k)$ of the electron waves in a plasma column of limited radius (size a transverse to the magnetic field). In a magnetized cold plasma ($\omega_H > \omega_p$, $T_e \to 0$) this function is given by the formula

$$\omega(k_\parallel) = \omega_p \cos\theta = \omega_p \frac{k_\parallel}{(k_\parallel^2 + k_\perp^2)^{1/2}}, \tag{16}$$

where $k_\parallel$ and $k_\perp$ are the wavenumbers of the oscillations along and at right angles to the plasma filament; for the most important (first) mode of the oscillations $k_\perp \approx 1/a$. Waves with frequencies $\omega \ll \omega_p$ are well described by the cold plasma approximation; this branch of the electron oscillations is called the Trivelpiece-Gould mode.[25] The wave dispersion near the Langmuir frequency is, of course, no longer described by the cold plasma approximation and has the usual form:

$$\omega_k \approx \omega_p + {}^3/_2 (k_\parallel r_D)^2 \omega_p. \tag{16′}$$

One sees easily that the properties of the waves described by the branches (16) and (16′) must be different in principle from the point of view of the possibility of the self-compression effect. Indeed, the Lighthill criterion (3) for the modulational instability is satisfied in the case (16′) (as we noted already above) while it is not satisfied in the case (16) where $\partial^2\omega/\partial k_\parallel^2 < 0$; in the latter case solitons cannot occur according to the theory. One must thus choose the conditions for the experiment on the formation of Langmuir solitons in such a way that one "falls on" the (concave upwards) curve (16′).

§3. EXPERIMENTAL ARRANGEMENT

The experimental arrangement (Fig. 1; for details see Ref. 24) includes the following elements: a solenoid of length 250 cm producing an approximately uniform magnetic field, in the experiments $H = 2 \times 10^3$ Oe; a vacuum volume evacuated by the diffusion pump 1; a discharge chamber 2 in which a plasma is produced by an electron beam from the gun 3 which passes through the gas (hydrogen) which is present in the chamber; the

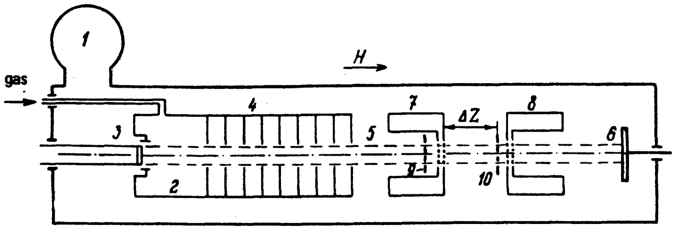

FIG. 1. Diagram of the experimental apparatus (see text for symbols).

set of diaphragms which form the so-called gas trapping line 4 of length ~80 cm in which the plasma, moving along the magnetic field (with velocity $\sim c_s$), is decontaminated from neutral gas for a time which is sufficient for the experiment;[24] the plasma filament 5 of diameter 3 to 4 cm; the beam collector 6; a working volume of length $L \gtrsim 50$ cm in which there are two quarter-wave resonators: a master oscillator 7 and a diagnostic one 8 (the working spaces of the resonators are covered by grids with a transparency of ~95%); two rf generators powering these resonators; a pair of "modulating" grids 9; a radio-frequency probe 10 and a P5-20 receiver to register the Langmuir oscillations; three pulse power sources: two (independent) ones to control the operation of the electron gun and one to control the pulse valve which regulates the supply of gas to the discharge chamber.

We performed two series of experiments with this arrangement which were done under the same conditions except for the characteristics which were connected with the actual mode of pumping the Langmuir waves: an external rf field[24a] or an electron beam.[24b] In all cases the plasma was produced in the discharge chamber, which was filled in pulses by hydrogen, by a pulsed (10 to 20 μs) beam of electrons with energies 0.5 to 2 keV and a current strength of tens to hundreds milliamps. The plasma "flowed" from the discharge chamber along the magnetic field and arrived ~50 μs after the switching-off of the beam at the working volume where it continued to propagate with (approximately) the sound velocity $c_s \approx 2 \times 10^6$ cm/s. The gas pressure in the working volume (in the regime when the plasma was flowing) was $\sim 3 \times 10^{-6}$ mm Hg (and remained such for ~10 μs after the plasma had passed through). The temperature of the electrons (estimated using the Langmuir probe) $T_e \approx 10$ eV, a typical plasma density $n \approx 3 \times 10^9$ cm^{-3}, $r_D \approx 3 \times 10^{-2}$ cm.

In the experiments with beam excitation of the oscillations we applied not only the first ("basic") beam, but also a second "pumping beam"; it was produced by the same electron gun 3 and had independently regulated parameters: electron energy (~80 to 150 eV), current strength (several milliamp), pulse length, and the moment it was switched on relative to the first beam.

The plasma with the above parameters was magnetized ($\omega_H \gg \omega_p$) and collisionless:

$$\tau_e = [n_0\langle\sigma v\rangle_0 + n\langle\sigma v\rangle_e]^{-1} \approx 2\cdot 10^{-4} \text{ s},$$
$$\tau_s = L/c_s \approx 0{,}25\cdot 10^{-4} \text{ s}, \tag{17}$$

i.e.,

$$\tau_e \gg \tau_s. \tag{18}$$

Here n_0 is the neutral hydrogen density in the working volume, v the velocity of the plasma electrons, σ_0 and σ_e the cross sections for the scattering of plasma electrons by hydrogen molecules and by the charged plasma particles ($\sigma_0 \approx 1 \times 10^{-16}$ cm^2,[26a] $\sigma_e \approx 6 \times 10^{-13} \times T_e^{-2}$(eV) $\times$ cm^2 [26b]). We note that in the experiments of Refs. 20 to 22, 27 the gas (argon) pressure was $p \gtrsim 1 \times 10^{-4}$ mm Hg, i.e., about two orders of magnitude higher than in our experiments, while the sound velocity was less (due to the larger ion mass); as a result a condition which is the opposite of (18) was satisfied, i.e., the plasma regime was essentially collisional.

§4. DIAGNOSTICS

We used a resonator method to measure the plasma density and its time-dependence; the essence of this method as applied in the present paper consists of the following. When the plasma passes through the capacitance gap of the coaxial quarter-wave resonator its resonance frequency changes corresponding to the change in the permittivity of the medium in the gap; in that mode of oscillations which is used for plasma diagnostics in our experiments (see below) the permittivity of that part of the gap which is filled by plasma is given by the equation

$$\varepsilon = 1 - \frac{4\pi e^2}{m\omega^2} n, \tag{19}$$

where ω is the frequency of the pumping external field resonator ($\omega > \omega_p$). If the resonator is pumped by an external generator when there is no plasma at some "vacuum" resonance frequency, when there is a plasma present, the resonator is detuned: its frequency increases. The amplitude of the field oscillations in the resonator which is measured by means of a diagnostic loop which is in the position of the maximum magnetic field component then decreases (the signal detected with the loop is supplied on the storage S1-42 oscillograph). Varying the generator frequency and observing the oscillograms of the signal from the loop (when the plasma passes through the diagnostic resonator) we can thus easily determine the time-dependence of the plasma density. In particular, we can in this way observe in the plasma the density well connected with the Langmuir soliton.

In our experiments the characteristic ("vacuum") frequency of the diagnostic resonator was 720 MHz. It differed very considerably from the eignefrequency of the pumping resonator (which was 500 MHz) so that there was no coupling whatever between the two resonators which might lead to an effect from one on the other.

To elucidate what we have said we turn to Fig. 2 in which we show (schematically) as a function of time the change in the resonance frequency ω_{res} of the diagnostic resonator (from ω_{min} to ω_{max}) when a plasma column with a region of lowered density passes through its gap

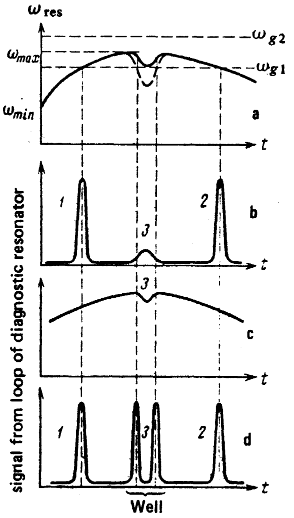

FIG. 2. a) Time-dependence of the eigenfrequency of the diagnostic resonator when a plasma column with a region of lowered density δn passes through; b) to d) time-dependence of the signal from the loop of the diagnostic resonator. 1: Leading front of the moving plasma; 2: trailing front of the plasma; 3: region of lowered density.

(ω_{min} corresponds to the absence of the plasma, ω_{max} to the presence of the plasma with maximum density). If the generator frequency ω_g lies within the limits $\omega_{min} \leq \omega_g = \omega_{g1} \leq \omega_{max}$, the signal from the detector loop of the resonator looks as shown in Fig. 2b: in the points $\omega = \omega_{g1}$ (resonance) the signal has a maximum and the region of lowered density corresponds to an increase in the signal from the loop. If, on the other hand, $\omega_g = \omega_{g2} > \omega_{max}$ the region of lowered density corresponds to a decrease in the signal from the loop–see Fig. 2c. The sensitivy of the method depends on the geometry of the system and the Q of the resonator; in our experiments one could reliably fix wells with relative depths of 2 to 3%.

In the experiments described below we applied widely the following device: for methodological purposes (see below) we produced in the plasma artifically an extended region of lowered density–a "well." The registration of such a well proceeded in the same way as for smaller wells. For instance, for $\omega_{min} \leq \omega_g \leq \omega_{max}$ and for a deep well (dashed curve in Fig. 2a) a typical oscillogram of the signal from the detector loop would correspond to Fig. 2d; for another adjustment of the generator frequency relative to the eigenfrequency of the resonator or for a shallower well the oscillogram might have the form of Figs. 2b, c.

The Q of the resonator was 250 (the width of the resonance curve was 5 MHz).

2. To indicate the relative intensity of the electrical field of the Langmuir oscillations we applied an rf probe, the signal from which we supplied by means of a coaxial cable to the 75 Ohm input of a selective tunable P5-10 receiver; the detected signal from the receiver was taken out onto a storage S1-42 oscillograph. The working surface of the probe was achieved in the form of a grid with a 4 cm diameter and a 2×2 mm mesh of tungsten wire of 0.1 mm diameter. The probe was put into the plasma flow in such a way that its plane was at right angles to the velocity of the flow–see Fig. 1.

We did not perform in the present work absolute measurements of the electrical field of the Langmuir waves. One can in principle manage without them as in the soliton the quantity $E_0^2/16\pi$ is determined by the depth of the density well $\delta n/n$–see Eq. (9′). In Refs. 19, 20, where $H=0$ a method of probing with a transverse electron beam was applied to measure the absolute magnitude of the electrical field strength. Under the conditions of the present paper it is impossible to apply that method because of the presence of a strong magnetic field.

§5. EXPERIMENTAL PROCEDURE AND EXPERIMENTAL DATA

We performed two series of experiments in the present work: in the first of those we applied an rf electrical field from an external source to pump the large amplitude Langmuir waves[24a] and in the second we applied an electron beam.[24b]

1. Generation of Langmuir solitons by an external rf field

The pumping of Langmuir waves by an rf field was accomplished by means of a quarter-wave pumping resonator (7 in Fig. 1). The pumping resonator was in principle constructed in the same way as the diagnostic one, but it had a considerably lower eigenfrequency. It was excited by a pulse generator of decimeter waves working on a frequency of 495 MHz; the density of the plasma was then chosen such that the frequency f_E of the field was sufficiently close to the electron Langmuir frequency. The excitation of the resonator was realized by means of a loop which was placed in the antinode of the magnetic field component in the resonator. The excited electrical field strength was a maximum in the working gap of the resonator and had a direction parallel to the external constant magnetic field **H**. The length of the generator pulse was 1 μs, and the pumping power several watts.

In order that the oscillations at the pumping frequency could be trapped in the plasma we chose the initial plasma density corresponding to the condition $f_E \lesssim f_p$, where $f_p = \omega_p/2\pi$. We then observed that the pumping of the oscillations proceeded more efficiently if the density of charged particles in the pumping resonator gap and near it was somewhat lower than in the surrounding plasma, and is scanned in time. The density well was therefore produced in the plasma somewhat earlier than the pumping pulse–by means of a field pulse of 5 μs length supplied on a pair of grids positioned directly in front of the working gap of the pumping resonator and causing an appreciable charge separation in the gap between the grids (during 5 μs). As the plasma was moving, after it passed the grid 6 there appeared in it a density well of length $\sim c_s \cdot 5 \times 10^{-6} \approx 10$ cm, which grad-

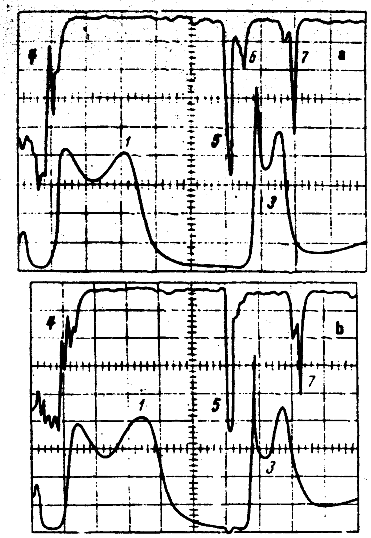

FIG. 3. Oscillogram from the rf probe (upper curve) and from the diagnostic resonator (lower curve); $\Delta Z = 15$ cm (see Fig. 1), time base: 10 μs per division. The designation of the peaks is explained in the text.

ually increased along the working volume. The pumping of rf oscillations in the plasma proceeded after that as the leading front of the well passed through the working gap of the pumping resonator. Therefore, during the pumping time the (in the resonator gap) local Langmuir frequency is scanned in time from a magnitude $f_{p1} > f_B$ to a mganitude $f_{p2} < f_B$; we shall see from the results obtained that the excited Langmuir oscillation turned out to be trapped between the leading and the trailing front of the well. Furthermore the configuration obtained—a gradually spreading density well and localized in it a Langmuir wavepacket—passed past the rf probe (10 in Fig. 1) and the diagnostic resonator 8; the latter were mechanically clamped together (the distance between them was 3 cm) and during the time of the experiment could be shifted along the axis of the apparatus.

We turn to a description of the experimental data.

The most general result of this series of experiments is characterized by the oscillograms of Fig. 3a of which the lower one (trace deflection upwards) is the signal from the diagnostic resonator (plasma density indicator) and the upper one the signal from the Langmuir oscillation receiver (trace deflection downwards). The diagnostic resonator works in the regime of Fig. 2d. The pulse 1 is the leading front of the moving plasma with a density exceeding 3×10^9 cm^{-3}, pulse 2 the trailing front of the plasma—there was no place for it in the figure, the pulse 3 the density well produced by the pulse 5. On the upper oscillogram the pulse 4 are the oscillations ($f \approx 500$ MHz) at the end of the pulse of the primary beam which produces the plasma; 5 is the pulse of the pumping resonator at a frequency $f_B = 495$ MHz which coincides with the pulse of the "well generator", 6 and 7 are pulses (packets) of Langmuir oscillations at a frequency of 492 MHz which are registered by the rf probe at a distance $\Delta Z = 15$ cm from the place of the pumping. It is clear that the wavepackets 6 and 7 occur at the leading and trailing fronts of the density well.

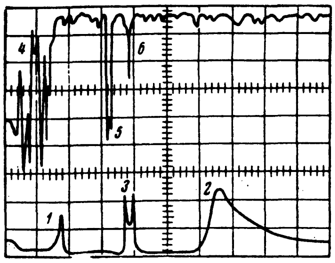

FIG. 4. The same as in Fig. 3, but for $\Delta Z = 20$ cm; time base: 25 μs per division.

It is fundamentally important that the wave bunch 7 (trailing edge of the well) is observed after 20 μs after the pumping pulse when the bunch 6, connected with the leading front of the well is no longer there; [24a] in other words, the bunches 6 and 7 are independent of one another. To complement what we have said we give Fig. 3b, in the conditions of which the wave bunch 7 exists only at the trailing front of the well. Under other conditions the (only) wave bunch is observed either at the leading front of the well, or in its center (Fig. 4).

The wave bunches observed in Figs. 3,4 are thus independent solitary Langmuir wavepackets—solitons which move freely in the plasma. The soliton corresponding to the pulse 7 in Fig. 3 lives in the plasma not less than 10^4 periods of the Langmuir oscillations after the pumping is switched off, traversing during that time tens of centimeters. Its width (length multiplied by the ion sound velocity) is not more than 0.7 cm, i.e., not more than 20 r_D. This width is five to six times less than the diameter of the plasma filament. Packet 6 moves approximately five times faster than packet 7 and has a width of 3 to 4 cm which is comparable to the diameter of the plasma filament.

The observed free soliton is localized in the plasma and propagates with it along the axis of the apparatus with a velocity of not more than 10^6 cm/s. For the indicated dimensions such a soliton must have a corresponding density well with a depth of several percents. The observation of the well in the experiments discussed here was made very difficult due to the large density gradients at those fronts of the artificial well at which the solitons are usually localized. This fact is particularly clearly demonstrated in Figs. 5a, b where on the background of an exceptionally sharp drop in density at the leading edge of the well it is impossible to distinguish the well corresponding to the soliton.

The presence of density wells which are self-consistent with the field of the solitary Langmuir waves is demonstrated in the second series of experiments—with an electron beam (§5.2).

The character of the evolution of the solitons when they propagate along the apparatus can be seen from Fig. 5. Case a corresponds to a distance of observation from the pumping resonator of $\Delta Z = 5$ cm and case b to a distance of 10 cm. It is clear that when one goes away from the pumping resonator the wave bunch localized at the trailing front of the well grows and is observed proportionally later. From the set of available results one can conclude that the characteristic length for the

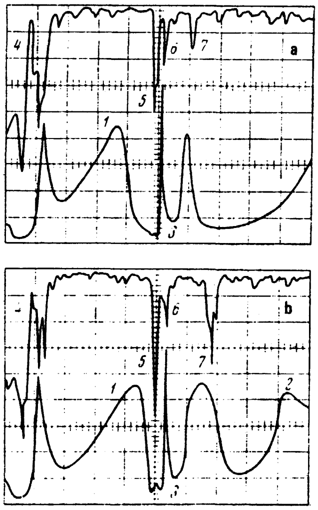

FIG. 5. The same as in Fig. 3, but a) for $\Delta Z = 5$ cm, b) for $\Delta Z = 10$ cm. Time base: 10 μs perdivision.

formation of a soliton from Langmuir waves is of the order of 10 cm while the characteristic time for the soliton formation is of the order of several thousand Langmuir periods.

Sometimes more than one soliton is localized at the fronts of the well.

We note that the experiments described here are the first experiments on the production of Langmuir solitons in a magnetized plasma; the above cited works by other authors were performed when there was no magnetic field present. There exists in a magnetized plasma on the Trivelpiece-Gould branch an electron soliton which is not a Langmuir soliton. The mechanism for the formation of this soliton, studied in Refs. 28, is a different one, it is related to the mechanism for the formation of an ion-sound soliton and is not connected with the modulational (or aperiodic) instability of a wavepacket with respect to self-compression.

In connection with the class of problems considered here we must refer also to Ref. 14.

2. Generation of Langmuir solitons by an electron beam

The electron beam used to pump Langmuir waves in the plasma had a pulse length which approximately coincided with the length of the plasma pulse. In this series of experiments the pumping resonator stood as before along the path of the plasma flow although the pumping generator was disconnected; the role of this resonator was reduced to self-modulation of the beam at a fixed frequency $f = 500$ MHz ($\approx f_p$) which made it easier to observe solitons with the P5-20 receiver which was tuned to that frequency. This resonator fixed the start of the working volume.

The basic experimental fact observed in this series of measurements consists of the following. When the current strength of the pumping beam exceeds some threshold (~3 mA for an energy of the beam electrons of ~100 eV) the Langmuir oscillations excited by the beam in the plasma acquire the character of relatively rare, but strong electrical field pulses at a frequency of 500 MHz ($\approx f_p$). At the same time there arises a sharply pronounced modulation in the plasma density: at the positions where the wave bunches are localized density wells appear in which the electron density turns out to be lowered at the maximum by 5 to 10% as compared to the density of the surrounding plasma. This is shown in Fig. 6 where the upper oscillogram (displacement of the line downwards) is the envelope of the electrical field of the Langmuir waves (at a frequency $f = 500$ MHz $\approx f_p$) and the lower one (displacement of the line upwards) is the oscillogram of the diagnostic resonator[2)] (see Ref. 24b for the calculated time-behavior of the plasma density).

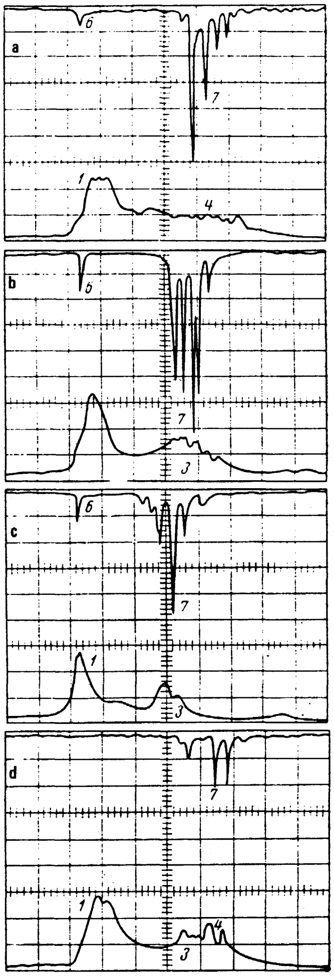

FIG. 6. Upper line: signal from the rf probe; lower line: signal from the diagnostic resonator; 3: density "well"; 4: density "well"; 7: pulse of the electrical field of the Langmuir waves; a), b), d) $\Delta Z = 30$ cm, c) $\Delta Z = 10$ cm; time base: 10 μs per division.

The width of the wave bunches which are localized in the plasma density wells turns out to be the smaller the larger their amplitude (or the depth of the well). When $\delta n/n \approx 0.05$ to 0.1 this width does certainly not exceed 1 cm, i.e., 25 to 30 times r_D and these estimates are upper bounds as they include the resolving time (1 μs) of the (P5-20) receiver of the rf oscillations.

As in the experiments with an external hf field (section 5.1) the generation of the wave bunches turns out to be more efficient if an artificial density well is produced in the plasma; possibly the latter is a resonator in which waves are built up to a larger amplitude than when it is not present and are on the whole stopped relative to the plasma (see below for a discussion of the experimental data). This well is the "marker" used to measure the motion of the plasma. The well is present in Figs. 6b,c,d, but not in Fig. 6a.

The picture of the Langmuir bunches and the density wells connected with them is changed appreciably when the diagnostic apparatus is shifted along the working volume. While at relatively large distances ΔZ from the start of the working volume the development of the plasma density modulation is observed and the number of Langmuir bunches is relatively large (Fig. 6a,b, where $\Delta Z = 30$ cm), at small distances ($\Delta Z = 10$ cm) the density modulation develops appreciably more weakly and only single wave bunches are observed–Fig. 6c. This means that the characteristic length for the evolution of wave bunches is of the order of 10 cm, i.e., the characteristic time is ~10 μs (several thousand Langmuir periods)–approximately the same as in the experiments with the pumping of the wave by an external rf field.

The measurements show that the wave bunches localized in the plasma density wells move together with the artificial well, i.e., they are to a first approximation fixed relative to the plasma.

We show in Fig. 7a the delay τ in the arrival time of the signals from the hf probe and the diagnostic resonator as function of the shift of these devices along the working volume. It is clear that the wave bunches move freely in the apparatus for at least 50 cm, being retarded along that path by ~40 μs. In other words, the life time of the field bunches is at least 2×10^4 periods of the electron plasma oscillations. Bunches observed at larger distances from the start of the working volume usually turn out to be larger in absolute magnitude and narrower than at shorter distances. We show in Fig. 7b the amplitude $A_\sim$ of the electrical field in a bunch (in relative units) as function of the distance travelled by it.

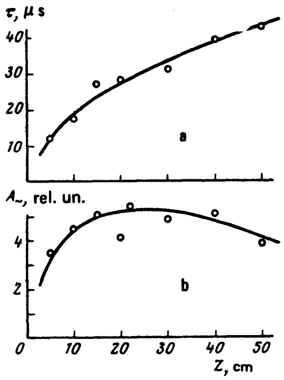

FIG. 7. a) Delay in the time of arrival of signals from the rf probe and the diagnostic resonator as function of the shift of these devices along the working volume. b) Amplitude of the electrical field in the bunch as function of the distance travelled by it ($Z = \Delta Z$).

According to all indications the Langmuir wave bunches studied which are localized in the plasma density wells can be identified as Langmuir solitons. Under the conditions of the experiments described here the solitons turn out to be approximately fixed relative to the plasma.

We make one important remark referring to the relations between the Langmuir wave bunches and the plasma density wells. The fact is that in the conditions of Fig. 6 there is in general a good spatial correspondence between the field bunches and the density wells: the field bunches are trapped in the wells. The experiments show that such a picture is statistically the most probable one. However, in a number of cases a different situation is observed: together with the "filled" wells there are also "empty" wells; for instance well 5 (indicated by an arrow) in Fig. 8 is not filled with Langmuir waves. It is possible that this situation reflects the stage of strong interactions between the solitons which are already formed and the continuing "pumping." In that stage for well-defined phase relations the Langmuir oscillations trapped in the wells may be repressed by the pumping, after that appear again, etc.; the density wells may in that case exist for some time without their Langmuir filling.[15,23,29]

We indicate yet another curious detail which is clearly visible on the oscillograms of Fig. 6: when there is no artificial density well one observes on the leading front of the moving plasma column a Langmuir soliton (pulse 6) with which is connected a self-consistent well (the latter is clearly detected on the concave side of that section of the oscillogram which corresponds to the increase in density—see also §4).

We have so far not yet studied the problem of the frequency spectrum of the oscillations with sufficiently quantitative accuracy.

§6. DISCUSSION OF THE EXPERIMENTAL DATA

The Langmuir wave bunches observed in the experiments described (§5) cannot be considered to be linear wavepackets and are essentially non-linear formations. Indeed, as was shown sub 8 in §1 a linear packet of the observed dimensions (~25 $r_D \approx 1$ cm) would be spread

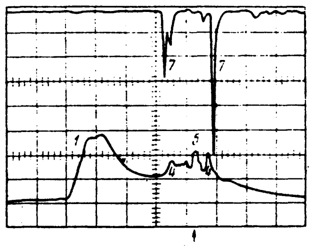

FIG. 8. The same as in Fig. 8, but with $\Delta Z = 33$ cm, time base: 10 μs per division; 4: density well filled with Langmuir oscillations, 5: "empty" well.

out already over a distance of its own initial width whereas the observed bunches travel without noticeable spreading over tenfold larger distances. In the present work we have thus observed the formation of solitons as the result of self-compression of non-linear Langmuir waves; this is the first time this was done in a magnetized collisionless plasma. The results of the experiments on the pumping of Langmuir waves by two different methods were close to one another.

The self-compression of waves has the character of an instability. The time for the development of this instability (several microseconds) is completely sufficient from the point of view of the theory for the formation of solitons: for $\delta n/n = W/nT \approx 10^{-2}$ the expected growth rate for the modulational or aperiodic instabilities is of the order of $10^7 s^{-1}$ (see §1, sub 2, 6). The dimensions of the observed self-compressed wavepackets (not exceeding 25 to 30 times r_D) is close to the theoretical dimensions of Langmuir solitons (§1, sub 4). Indeed, for the observed depth of the density well $\delta n/n = 5\%$ a soliton should, according to Eq. (10′) of §1 have a width $\Delta \approx 22\ r_D$ whereas experimentally we found $\Delta \lesssim 25\ r_D$. The solitons turn out to be relatively stable configurations: their "lifetime" is not less than 30 to 40 μs (tens of thousand Langmuir oscillation periods) and their mean free path (together with the moving plasma) is not less than several tens of centimeters.

One can make the following estimates with regards to the wavelengths of the oscillations from which the solitons are formed. In the case of beam pumping of the Langmuir waves their wavenumber is given by the Cerenkov relation:

$$k \approx \frac{\omega_p}{u} \approx \frac{3\cdot 10^9}{6\cdot 10^8} \approx 5\,\text{cm}^{-1}. \qquad (20)$$

Hence the wavelength $2\pi/k \approx 1.3$ cm and $kr_D \approx 1/6$. This gives

$$kr_D \gg {}^1/_3 (m/M)^{1/2} \approx 1/180, \qquad (21)$$

in clear contradiction to the necessary condition (4) for the modulational instability (§1) which, hence, cannot appear under such conditions. However, the aperiodic instability (§1) does also not develop for waves determined by Eq. (20) as the wavenumbers of the observed solitons $k_0 \approx 2/\Delta \approx 3\ \text{cm}^{-1}$, i.e.,

$$k_0 \ll k = \omega_p/u, \qquad (22)$$

whereas as the result of the development of the aperiodic instability waves must occur with wavelengths shorter than the initial one. And as both instabilities mentioned are considered by us to be the causes for the formation of the observed solitons we obtain something like a contradiction.

In our opinion, the solution of this contradiction is connected with two facts. Firstly, if condition (21) is satisfied, decay processes (§1) of the type $l \to l' + s$ (and the induced scattering of waves by ions[3,4]) occur and cause the wavenumbers of the Langmuir waves to diminish. The Langmuir "condensate" which is then formed

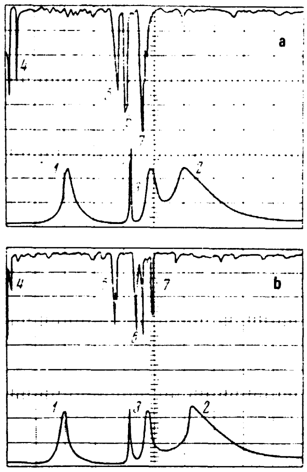

FIG. 9. Upper trace: signal from the rf probe; lower trace signal from the diagnostic resonator. a) 1, 2: leading and trailing fronts of the moving plasma; 3: density well; 4: termination of the discharge pulse producing the plasma; 5: voltage pulse, producing the density well 3; 6: wave bunch at the leading front of the well; 7: wave bunches at the trailing front of the well. b) 6, 7: wave bunches in the middle of the well and its trailing front. $\Delta Z = 20$ cm, current strength of the "pumping beam" ~4 mAmp, time base: 25 μs per division.

has wavelengths which are appreciably longer than the initial ones. Estimates show that during the time observed in the experiment (~10 $\mu s \approx 5 \times 10^3$ Langmuir periods) decay processes succeed to play an important role.[2] Secondly, in our experiments it is most probable that solitons are formed from standing waves which are built up in the density well, as in a resonator. Decay processes which lengthen waves as compared to the estimate (20) also facilitate the formation of standing waves. However, standing waves (which have zero group velocity) satisfy the relation from which (when applied to travelling waves) we obtained condition (4) for the modulational instability. We note, incidentally, that the theoretical interpretation[15] of the experimental results of Ref. 20 is also connected with the formation of standing waves.

We are thus forced to assume that the Langmuir waves from which the solitons are formed have a large spread in wavenumbers: $\Delta k \approx k$. As we noted in §1 (sub 6), in that case the difference between the modulational and the aperiodic instabilities is eroded. In our experiments both instabilities considered might thus be responsible for the formation of solitons.

In conclusion we note once again the difference between the experiments performed in this work and those carried out in Refs. 20 to 22 which are connected with the collisionless regime of the plasma. It is additionally illustrated by the oscillograms of Fig. 9 obtained in the regime with a short duration of the pumping beam (~3 μs in contrast to all other data of the present section obtained with a beam of duration ~200 μs). It is clear that solitons are observed with a large delay from the moment the beam is switched off: at a distance of 20 cm from the start of the working volume it reaches 35 μs, and at larger distances one must (according to Fig. 7a) expect even longer delays. If the plasma regime were collisional, we could not observe a free motion of the solitons (in the absence of the beam): in Refs. 20 to 22, and 27 the solitons vanished practically instantaneously after the switching off of the "pumping" due to the collisional damping of the Langmuir waves.

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[1)] In the present paper we consider only one-dimensional wave-packets.

[2)] In all oscillograms of Sec. 5.2 the pulses of the upper line are delayed relative to those of the lower line by ~1.5 μs and they should therefore be shifted to the left. This fact is caused by the inertia of the *P*5–20 receiver which causes also an apparent broadening of the observed Langmuir pulses by approximately 1 μs.

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