(function) of the noise $\partial N_{k'}/\partial k'$.

At $\partial N_{k'}/\partial k' < 0$, we have $\beta_k < 0$ and damping of the wave takes place; at $\partial N_{k'}/\partial k' > 0$, the sign is opposite: $\beta_k < 0$ and the wave is amplified.

In particular, for an equilibrium spectrum we obtain $N_{k'} = T\omega_{k'}^{-1}$, and $\partial N_{k'}/\partial k' < 0$ for all cases in which $\mathbf{k}'$ increases with increase in $\mathbf{k}'$, so that the equilibrium noise absorbs the wave in spite of the fact that its energy spectrum turns out to be increasing in correspondence with the Rayleigh–Jeans law.

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Possibility of decreasing the electron heat flux from open traps

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The possibility is considered of decreasing the electron heat flux from open traps when the emerging plasma stream is strongly expanded in the expansion nozzle. Allowance for weak collisions in an almost collisionless plasma, when the mean free path is much larger than the characteristic dimension R of the expansion nozzle, leads to the appearance of electrons that are trapped between the exit slit of a trap with mirror-configuration magnetic field and a wall with electrostatic potential φ_0. An analysis of the equations shows that in the case of an unlimited emissivity of the wall the blocking potential is connected with the degree of plasma expansion by a relation from which it follows that a relatively small expansion is sufficient to decrease substantially the electron heat flux. This estimate of the heat loss is an upper bound, since no account is taken of the possibility of turbulence development in the expansion nozzle.

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1. INTRODUCTION

One of the main problems in the development of thermonuclear reaction based on open traps (of the mirror anti-mirror, or baseball type) is the fact that the plasma can be rapidly cooled in the trap as a result of the runaway electrons whose flux can exceed that of the ions by $(M/m)^{1/2}$ times. In fact, the time of energy exchange between the electrons and the ions

$$\tau_{ei}^{\varepsilon} \sim 5\cdot 10^{13}\,(T[\text{eV}]/10^4)^{3/2}/n$$

is smaller for thermonuclear temperatures $T \sim 10^4$ eV than the Lawson time, so that the energy lifetime is determined by the cooling of the electrons.

In the analysis of open traps it is usually assumed that the emissivity of the walls is either low or can be substantially decreased by using special blocking grids,[1] so that the electron heat flux decreases to the ion value because of the appearance of an ambipolar potential between the trap and the wall. In all cases, however, it is desirable to expand the plasma stream emerging from the trap, for the purpose of reducing the heating of the wall, to solve the problem of directly recuperating the plasma energy by conversion to electric energy, etc. When such an expanding nozzle is used, the physical picture of the plasma flow becomes unique. It will be shown below that expansion of the plasma stream leads even on its own to a considerable lowering of the electron heat flux from the trap, to a value on the order of the ion flux.

2. QUALITATIVE APPROACH

We assume throughout that the electron emissivity of the wall is large. This takes place either when it is impossible to use blocking grids and the heating causes evaporation of the wall materials, or else through the use of special means of producing a plasma and ensuring its stability. In these cases there is a cold plasma with unlimited emissivity near the wall.

In the collisionless case without expansion, the electronic heat flux was calculated a number of times (see, e.g.,[2,3]). After a time $t_1 \sim R/V_0$ (R is the distance between the trap and the wall and V_0 is the average velocity of ion outflow of the trap), the electric potential φ_0 of the trap relative to the wall decreases from a value $\varphi_0 \sim (T_e/e)\ln(R/r_d)$[2] (Debye radius $r_d \ll R$) to $\varphi_0 \sim T_e/e$[3] because of the production of a flow of cold electrons from the wall, and the electron heat flux becomes larger than the ion one by $(M/m)^{1/2}$ times. In the presence of expansion, or if we disregard completely the collisions in the expansion nozzle, the picture remains unchanged, since the ion and electron densities depend in like manner on the radius: $n_e, n_i \sim (r_0/r)^p$ (r_0 is the width of the stream prior to expansion, and $p=1$ or 2 for two- or three-dimensional expansion, respectively). Therefore the steady-state potential is constant throughout (equal to φ_0) and only near the wall does it experience a jump $\Delta\varphi=\varphi_0\sim T_e/e$ at a distance r_d.

Allowance for the weak collisions inside the expansion nozzle leads to qualitative changes of the character of the plasma outflow. We assume that the particle mean free path in the expansion tube is $\lambda \gg R$. Then after a time $t\gg\tau_e$ (the Coulomb collision time of the electrons) electrons accumulate in the expansion nozzle and are trapped between the magnetic mirror and the electric potential barrier of the wall. Thus, at $t>\tau_e$ the expansion nozzle contains four types of electron: a) untrapped, which leave the trap and strike the wall directly, overcoming the potential barrier; b) returning electrons, reflected by the barrier and returning directly to the trap, c) emitted by the walls, d) trapped. We shall henceforth consider for simplicity the case $R/\lambda \ll (r_0/R)^p$, when the collisions are so rare that in first-order approximation we can neglect the capture of the returning and emitted electrons in one pass through the expansion nozzle.

The exchange between the subsystems of the trapped and untrapped particles is weak (relative to the parameter $R/\lambda\ll 1$). This situation is similar to that considered in[4,5], when the subsystems are weakly coupled but nevertheless allowance for the weak collisions influences strongly the general properties of the system (for example, the influence on the diffusion in the neoclassical theory[4]), in our case the value of φ_0.

If we assume that the potential φ_0 is large enough, so that $\exp(-e\varphi_0/T_e)\ll 1$, then the electrons in the region $e\varphi>T_e$ can be assumed to have a Boltzmann distribution

$$n_e\approx n_0\exp[-e(\varphi_0-\varphi)/T_e], \tag{1}$$

where n_0 is the density prior to the expansion. Then, neglecting the density of the emitted electrons (this is obviously valid at large φ), the quasineutrality condition

$$n_e=n_i=n_0(r_0/r)^p(1+2e(\varphi_0-\varphi)/MV_0^2)^{-1/2} \tag{2}$$

yields

$$e(\varphi_0-\varphi)\sim pT_e\ln(r/r_0). \tag{3}$$

We see that the potential barrier for the electrons is in fact large at $r\gg r_0$, thus justifying the assumption concerning the value of φ_0. Of course, φ_0 can not exceed the ambipolar potential, but this upper bound can be obtained only in the region of small ($e\varphi<T_e$) values of the potential, where formula (1) is not valid. From qualitative considerations, the form of the electron distribution function f_e in this region is difficult to determine. It can only be assumed that in phase space, in view of the continuity of f_e on the boundary between the trapped and untrapped electrons, the characteristic form of f_e turns out to be

$$f_e\sim n_0\exp\left[-\frac{e(\varphi_0-\varphi)}{T}-\frac{mv^2}{2T_e}\right]\approx n_0e^{-e\varphi_0/T_e} \tag{4}$$

($mv^2/2\leq e\varphi<T_e$ for trapped and returning electrons). It follows from (4) that in the region $e\varphi<T_e$ we have

$$n_e\sim n_0e^{-e\varphi_0/T_e}\left(\frac{2e\varphi}{m}\right)^{3/2}. \tag{5}$$

We see therefore that $\varphi(r)$ will differ strongly from a logarithmic function. However, to find the actual form of this dependence, as well as the behavior of $\varphi_0(R)$, we must find the form of f_e from the rigorous kinetic problem.

The appropriate analysis is contained in Secs. 3 and 4 of this paper, and the main result is given by formula (26).

3. FORMULATION OF THE KINETIC PROBLEM

The principal small parameter is the ratio $r_0/R\ll 1$.

Assuming that the plasma confinement time in the trap is much longer than the time of establishment of dynamic equilibrium between the trapped and untrapped electrons, we can regard the electron distribution function as stationary. To simplify the problem we neglect the drift of the plasma due to the curvature of the magnetic force lines, as well as the thermal spread of the ion velocities, and assume that at $r=r_0$ the ion flux is monochromatic and is directed radially: $v_r=V_0 \approx(2T_i/M)^{1/2}$ (allowance for the ion temperature influences weakly the final result).

For the electrons in the drift approximation, the following kinetic equation is valid[6]

$$v_r\frac{\partial f_e}{\partial r}+\frac{e}{m}\frac{d\varphi}{dr}\frac{1}{v}\left(v_r\frac{\partial f_e}{\partial v}-v_\perp\frac{\partial f_e}{v\,\partial\theta}\right)-\frac{1}{2}\frac{v_\perp}{r^p}\frac{\partial f_e}{\partial\theta}=\mathrm{St}(f_e), \tag{6}$$

where $v_r=v\cos\theta$, $v_\perp=v\sin\theta$.

Since in our case $R/\lambda\ll 1$, we can neglect the collisions in first-order approximation and write down directly the solution of the kinematic equation in the form of a function of the integrals of the motion[7]: $f_e=f(E,\mu)$, where $E=mv^2/2-e\varphi$ is the energy and $\mu=mv_\perp^2r^p/2$ is the adiabatic invariant. The distribution function of the untrapped electrons can then be found by starting from the specified boundary conditions at $r=r_0$ (untrapped and returning electrons) and at $r=R$ (electrons emitted by the wall). As to the trapped electrons, to determine their distribution function we must take the weak collisions into account.

We use for this purpose the method described in[8]. We represent the solution in the form

$$f_e=f(E,\mu)+f_1(E,\mu,r). \tag{7}$$

We then arrive at the following expression for f_1:

$$f_1=\int\frac{dr}{v}\,\mathrm{St}(f)+\mathrm{const}, \tag{8}$$

where the integration is along the trajectory of the motion of the trapped particle. From the condition that f_1 be single valued after going repeatedly around the closed trajectory, we get an equation for f:

$$\oint \frac{dr}{v_r} \mathrm{St}(f) = 0. \tag{9}$$

An analogous procedure for finding the distribution function of trapped particles was used also in[4,5]. Equation (9) together with the conditions that follow from the continuity on the boundaries of the region of the trapped particles determines completely their distribution function.

The region of the trapped particles is determined by the inequalities

$$mv_r^2/2 = E + e\varphi - \mu r^{-p}|_{r=r_0,\,R} < 0, \tag{10}$$

i.e.,

$$R^p E < \mu, \quad r_0^p (E + e\varphi_0) < \mu.$$

The equation $v_r^2(r_0) = 0$ defines the surface that separates the regions of the trapped and returning electrons in (E, μ) space, and it follows therefore from the continuity of f that at $r_0^p(E + e\varphi_0) = \mu$ we have

$$f = f_M = n_{e0} \left(\frac{m}{2\pi T_e}\right)^{3/2} \exp\left(-\frac{e\varphi_0}{T_e} - \frac{E}{T_e}\right). \tag{11}$$

The equation $v_r^2(R) = 0$ defines the boundary between the region of trapped particles and the empty region of the (E, μ) space at $R \ll \lambda$. A particle landing in this region should proceed unopposed to the wall. Therefore, just as in[9], we can assume that at $R^p E = \mu$ we have

$$f = 0. \tag{12}$$

At a distance of several Debye radii from the wall, the potential satisfies the quasineutrality equation

$$\int f\, d\mathbf{v} + n_{\mathrm{untr}} + n_{\mathrm{ret}} + n_{\mathrm{em}} = n_i, \tag{13}$$

where n_{untr}, n_{ret}, n_{em} and n_i are respectively the densities of the untrapped, returning, and emitted electrons and of the ions:

$$n_i = n_0 \left(\frac{r_0}{r}\right)^p \left(1 + \frac{2e(\varphi_0 - \varphi)}{MV_0^2}\right)^{-1/2},$$
$$n_{\mathrm{em}} = n_{\mathrm{em}0} \left(\frac{r_0}{r}\right)^p \left(\frac{\varphi_0}{\varphi}\right)^{1/2}, \tag{14}$$
$$n_{\mathrm{untr}} + n_{\mathrm{ret}} = \int f_M(E)\,\theta(Z(\varphi_0))\,\theta(Z(\varphi)) \frac{\pi}{2r^p} \frac{dE\, d\mu}{Z^{1/2}(\varphi)},$$

where $Z(\varphi) = E + e\varphi - \mu r^{-p}$.

It is convenient to express the collision term in (9) in the form[10]

$$\oint \frac{dr}{v_r} \left(\frac{\partial^2 f}{\partial v_\beta \partial v_\gamma} \frac{\partial^2 g}{\partial v_\beta \partial v_\gamma} - 2n_i \frac{\partial f}{\partial v_\beta} \frac{v_\beta - V_{\beta i}}{|\mathbf{v} - \mathbf{V}_i|^3} + 8\pi f^2 \right) = 0, \tag{15}$$

where

$$g(\mathbf{v}) = \int f(\mathbf{v}')\,|\mathbf{v} - \mathbf{v}'|\,d\mathbf{v}' + n_i |\mathbf{v} - \mathbf{V}_i| + n_{\mathrm{em}} |\mathbf{v} - \mathbf{V}_{\mathrm{em}}|.$$

$f(\mathbf{v}')$ takes account also of the contribution of the returning and untrapped electrons

$$\mathbf{V}_i = V_0 [1 + k^2(u_0 - u)]^{1/2} \mathbf{n}, \quad \mathbf{V}_{\mathrm{em}} = -n_{\mathrm{em}0} (2T_e/m)^{1/2} u^{1/2} \mathbf{n}, \quad \mathbf{n} = \mathbf{r}/r_0.$$

We have used here the notation

$$k^2 = 2T_e/MV_0^2 \approx T_e/T_i, \quad u_0 = e\varphi_0/T_e, \quad u = e\varphi/T_e.$$

Equations (9)–(15) form a closed system that describes the motion of the particles and the electric field at $r_0 < r < R$.

4. EQUATIONS IN THE REGION $r \gg r_0$

We consider the main region of the expansion nozzle, in which $R > r \gg r_0$. For the densities of the untrapped and returning electrons we get

$$n_{\mathrm{untr}} + n_{\mathrm{ret}} \approx {}^1\!/_4 n_{e0} \exp(u - u_0) \left(\frac{r_0}{r}\right)^p (1 + 2u_0 - 2u).$$

Changing over to dimensionless variables

$$v/[w(2T_e/m)^{1/2}] = \tilde{v} \to v, \quad f/[n_{e0} e^{-u_0} (m/2\pi T_e)^{3/2}] \to f,$$

where w is a certain constant, we rewrite the system (13)–(15) in the form

$$\int f\, d\mathbf{v} = (R/r)^p C\{1 - q[1 + k^2(u_0 - u)]^{1/2} [{}^1\!/_4 n_{e0} e^{-u_0 + u} (1 + 2u_0 - 2u) + [e^{-u_0}(1 + u_0)/2\pi^{1/2} - \gamma/qk]/u^{1/2}]\}, \tag{16}$$

$$\int \frac{dr}{v_r} \left[\frac{\partial^2 f}{\partial v_\beta \partial v_\gamma} \frac{\partial^2 g}{\partial v_\beta \partial v_\gamma} - 2\left(\frac{R}{r}\right)^p C \frac{\partial f}{\partial v_\beta} \frac{v_\beta - V_{i\beta}}{|\mathbf{v} - \mathbf{V}_i|^3} + 8\pi f^2 \right] = 0, \tag{17}$$

where

$$g(\mathbf{v}) = \int f(\mathbf{v}')\,|\mathbf{v} - \mathbf{v}'|\,d\mathbf{v}' + (R/r)^p C\{|\mathbf{v} - \mathbf{V}_i| + q[1 + k^2(u_0 - u)]^{1/2} [(1 + u_0) e^{-u_0}/2\pi^{1/2} - \gamma/qk]\, |\mathbf{v} - u^{1/2}\mathbf{n}/w|/u^{1/2}\},$$
$$C = (r_0/R)^p e^{u_0} \pi^{3/2} / (qw^3 [1 + k^2(u_0 - u)]^{1/2}),$$
$$V_i = \gamma [1 + k^2(u_0 - u)]^{1/2} / \pi^{1/2} kw,$$
$$\gamma = (m/M)^{1/2}, \quad q = n_{e0}/n_0.$$

The value of n_{e0} is determined from the condition of quasineutrality at the point r_0, which yields $q \approx 1$ at $u_0 > 1$. The boundary conditions at $r \gg r_0$ simplify and take at $v^2 - u/w^2 = v_\perp^2 (r/R)^p$ the form

$$f = 0, \tag{18a}$$

and at $v_\perp^2 = 0$

$$f = \exp(u - v^2 w^2). \tag{18b}$$

In the analysis of the system (16)–(18) we make the following assumptions:

1. At a sufficiently large ratio R/r_0 the resultant value of u_0 is so large that $\exp(-u_0) \ll 1$.

2. The potential jump at the wall is $u_w \ll 1$, so that there exists in the expansion nozzle a region in which $u \ll 1$. These assumptions will be verified later. Then in the region $u \ll 1$, where it is possible to neglect the untrapped and returning electrons, the quasineutrality condition takes the form

$$\int f\, d\mathbf{v} = (R/r)^p C \left\{ 1 - \frac{q(1 + u_0)}{2\pi^{1/2}} \frac{(1 + u_0 k^2)^{1/2}}{u^{1/2}} e^{-u_0} \left[1 - \frac{2\pi^{1/2}\gamma}{qk(1 + u_0)} e^{u_0} \right] \right\}. \tag{19}$$

We see that (19) has no solution $u=0$. Therefore when the wall is approached there must exist a region with $n_i \neq n_e$, where it is necessary to solve the Poisson equation. Since the approximation $r_d=0$ is used, this means that the solution $u(r)$ of Eq. (19) should have a singularity at the point $r=R$. The necessary condition for the singularity of the solution is that all the terms of Eq. (19) are of the same order of magnitude at $r=R$, since neglect of any term leads to a solution that is regular at the point R. This leads to an expression for the potential jump at the wall

$$u_W=a(1+u_0)^2(1+u_0k^2)e^{-2u_0}\left[1-\frac{2\pi^{1/2}\gamma e^{u_0}}{qk(1+u_0)}\right]^2, \tag{20}$$

where a is a constant of the order of unity. The smallness of u_w follows from the large value of u_0.

We put now

$$w=(1+u_0)(1+u_0k^2)^{1/2}e^{-u_0}\left[1-\frac{2\pi^{1/2}\gamma e^{u_0}}{qk(1+u_0)}\right] \tag{21}$$

and change over to the new variables $r/R \to r$ and $u/w^2 \to u$. Then

$$C=\left(\frac{r_0}{R}\right)^p e^{u_0}/\left\{q(1+u_0)^3(1+u_0k^2)^2\left[1-\frac{2\pi^{1/2}\gamma e^{u_0}}{qk(1+u_0)}\right]^3\right\} \tag{22}$$

and the system takes the form

$$\int f\,d\mathbf{v}=C(1-q'/u^{1/2})/r^p, \tag{23}$$

$$\oint\frac{dr}{v_r}\left[\frac{\partial^2 f}{\partial v_\beta\,\partial v_\gamma}\frac{\partial^2 g}{\partial v_\beta\,\partial v_\gamma}-2C\frac{\partial f}{\partial v_\gamma}\frac{v_\beta-V_{i\beta}}{r^p|\mathbf{v}-\mathbf{V}_i|^3}+8\pi f^2\right]=0, \tag{24}$$

where

$$q'=q/2\pi^{1/2},$$

$$g(\mathbf{v})=\int f(\mathbf{v}')|\mathbf{v}-\mathbf{v}'|d\mathbf{v}'+C(|\mathbf{v}-\mathbf{V}_i|+q'|\mathbf{v}+\mathbf{n}u^{1/2}|/u^{1/2})r^{-p}.$$

The boundary conditions at $v^2-u=v_\perp^2 r^p$ are

$$f=0, \tag{25a}$$

and at $v_\perp$

$$f=1. \tag{25b}$$

In (23) and (24) we have assumed for simplicity that φ_0 is not too high, so that we can neglect the term $2\pi^{1/2}\gamma e^{u_0}/qk(1+u_0)$ compared with unity. At the same time, $V_i \ll v$. We see that we have obtained a system of equations, (23)–(25), which contains not even a single parameter ($q' \approx 0.3$) and having an eigenvalue C. We can therefore assume that $C \sim 1$. In fact, if $C \gg 1$ or $C \ll 1$ in (24), then we can either cancel it or neglect it. Then the solution of (24) is a function f such that $f \sim 1$ in the region of untrapped particles. This yields $\int f\,d\mathbf{v} \sim u^{3/2}$, but then at $r=1$ ($u(1) \sim 1$) it follows from the condition for the existence of the singularity that $C \sim 1$. The obtained contradiction proves in fact that $C \sim 1$. Obviously, all the conclusions remain in force also at large φ_0, when $V_i > v$.

We thus obtain the universal expression

$$\left(\frac{R}{r_0}\right)^p=Ce^{u_0}\left\{(1+u_0)^3(1+k^2u_0)^2\left[1-\frac{\gamma}{q'k}\frac{e^{u_0}}{1+u_0}\right]^3\right\}^{-1} \tag{26}$$

(C is a constant on the order of unity), which makes it possible to determine from the dimension of the expansion nozzle the value of the blocking potential. In the determination of u_0 the constant turns out to be under a logarithm sign, so that its actual value is immaterial. From (26) it is seen that at sufficiently large R/r_0 we obtain $\exp(-u_0) \ll 1$, but then the assumptions 1 and 2 of the present section are satisfied.

Thus, the behavior of $\varphi(r)$ has the following character. With increasing r, the decrease of the potential from the value φ_0 is initially logarithmic, until a value $e\varphi_0 \sim T_e$ is reached. After this, in that region of the expansion nozzle, where the contribution of the emitted electrons can still be neglected, it follows from (24) that $\int f\,d\mathbf{v} \sim u^{3/2}$, i.e., $u(r) \sim r^{-2p/3}$ at $e\varphi_0 < T_e$. In the principal volume of the expansion nozzle the contribution of the emitted electrons must of course be taken into account, therefore the potential variation is faster than given by a power law and becomes singular at $r=R$ (i.e., it leads, as indicated above, to appearance of a potential jump at the wall).

5. HEAT FLUX FROM THE TRAP

The total flux of kinetic energy Q from the trap consists of the thermal fluxes q_i, q_e, and q_{em} of the ions and of the untrapped electrons and those emitted by the wall:

$$Q=q_e+q_i-q_{em}.$$

The escape of trapped electrons at $R/\lambda \ll 1$ can be neglected. In the calculation of q_i we assume for simplicity that the ion distribution function is Maxwellian at $v_r 0$. At $r=r_0$ we have

$$\begin{aligned}q_e&=\int f_M(E)\theta(E+e\varphi_0-\mu r_0^p)\theta(E-\mu R^{-p})\pi(E+e\varphi_0)\,dE\,d\mu/2r_0^p\\&=T_e[(2T_e/\pi m)^{1/2}n_{e0}(1+u_0/2)\exp(-u_0)+u_0 j_{untr}],\\ q_i&=T_i(2T_i/\pi M)^{1/2}n_0,\quad q_{em}=n_{em0}\left(\frac{2e\varphi_0}{m}\right)^{1/2}e\varphi_0,\end{aligned} \tag{27}$$

where n_{e0} is determined from the condition $j_e=j_i-j_{untr}$; j_i, j_e, and j_{untr} are respectively the fluxes of the ions and the emitted and untrapped electrons. From this we get

$$n_{em0}\left(\frac{2e\varphi_0}{m}\right)^{1/2}=j_u-\left(\frac{2T_i}{\pi M}\right)^{1/2}\frac{n_0}{2}.$$

The heat flux is equal to

$$Q=n_0T_i\left(\frac{2T_i}{\pi M}\right)^{1/2}\left(1+\frac{1}{2}u_0\right)\left[\left(1+\frac{1}{2}u_0\frac{T_e}{T_i}\right)\Big/\left(1+\frac{1}{2}u_0\right)+q\left(\frac{T_e}{T_i}\right)^{3/2}\exp(-u_0)/\gamma\right].$$

At $T_e=T_i=T$ this formula takes the simpler form

$$Q(T)=n_0T\left(\frac{2T}{\pi M}\right)^{1/2}\left(1+\frac{1}{2}u_0\right)\left[1+\left(\frac{M}{m}\right)^{1/2}e^{-u_0}\right], \tag{28}$$

since $q \approx 1$.

It is seen that at $u_0 \approx 0.5$ the heat flux exceeds the ion flux by $(M/m)^{1/2}$, and at $u_0 = 4$ we have $Q = 6q_i$, i.e., the use of the expansion nozzle decreases the heat flux from the trap by approximately one order of magnitude.

6. DISCUSSION OF RESULTS

The results are valid if the particle mean free path is $\lambda \gg R$. The additional restriction $R/\lambda \ll (r_0/R)^p$ indicated in the Introduction is merely a simplification, since it obviates the need of considering the structure of the boundary layer between the trapped and returning electrons. Obviously, even if all the returning electrons become trapped after one pass through the expansion tube (i.e., $R/\lambda > (r_0/R)^p$), then at $R \ll \lambda$ the boundary conditions (25) do not change radically. The quantity entering here is the mean free path in the region $u \ll 1$, where the trapped electrons make the decisive contribution to the electron density. The electron density coincides with the ion density: $n_e \approx n_0(r_0/R)^p/(1+u_0k^2)^{1/2}$. The characteristic energy of the trapped electrons is determined by the value of u_w:

$$\varepsilon_e \sim T_e(1+u_0)^2 \frac{1+u_0k^2}{\exp(2u_0)}.$$

Then the Coulomb mean free path in this region is

$$\lambda \sim \lambda_0 \frac{n_0}{n_e}\left(\frac{\varepsilon_e}{T_e}\right)^2 \sim \lambda_0(1+u_0)(1+u_0k^2)^{1/2}, \qquad (29)$$

where λ_0 is the mean free path in the trap. Consequently, it follows from $\lambda_0 > R$ that $\lambda \gg R$, and therefore the criterion for the applicability of the results (26) can be taken to be the condition $\lambda_0 > R$.

The model considered in the present paper is laminar in its character, i.e., it does not take into account the possible development of two-stream instabilities[11,12] or instabilities connected with the anisotropy of the distribution function of the trapped electrons. It must be noted, however, that all turbulent processes introduce additional friction for the electron flow and will contribute to further decrease of the heat flux from the trap.[11,12] Therefore the heat flux (28) calculated with the aid of (26) is an upper bound of the expected heat loss.

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