Landau damping in the interaction between a regular wave and noise

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The nonlinear interaction of a regular wave with noise is considered. It is shown that the interaction of the wave with low-frequency noise leads to its damping, while in the case of high-frequency noise, the wave can be amplified as a result of the transfer of energy from the noise field to the wave. If the frequency of the noise greatly exceeds the frequency of the wave, then the character of the process is determined by the interaction of the wave with the wave packets of the noise field, the projection of the group velocity of which, in the direction of propagation of the wave, is identical with the phase velocity of the considered wave. Here, as in the Landau effect, the wave can be either damped or amplified, depending on the properties of the noise spectrum.

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In the propagation of a wave in a region of a medium excited by intense noise, interaction of the wave and the noise takes place as a result of the nonlinear effects, leading to a transfer of energy from one to the other. Under certain conditions, this process leads to effects of the type of Landau damping.[1,2]

The basic contribution to this process is made by the interaction of the wave with those components of the noise field which form resonance or near-resonance triplets. This enables us to describe the interaction of the wave

$$a_{\mathbf{k}} \exp\{i\mathbf{k}\mathbf{r}-i\omega_{\mathbf{k}}t\}$$

with the noise, which can be represented in the form of a discrete set of waves

$$\sum_{\mathbf{k}'} C_{\mathbf{k}'} \exp\{i\mathbf{k}'\mathbf{r}-i\omega_{\mathbf{k}'}t\}$$

in the three-wave approximation of the theory of nonlinear waves. Here it is convenient to apply the average-field method and represent the amplitude of the considered wave of frequency $\omega_{\mathbf{k}}$ in the form of the sum of its mean value and a fluctuation increment

$$\bar{a}_{\mathbf{k}}+b_{\mathbf{k}}. \tag{1}$$

Thus the amplitude of the wave which propagates in the direction $\mathbf{k}'$ consists of three parts:

$$a_{\mathbf{k}}\delta_{\mathbf{k}\mathbf{k}'}+C_{\mathbf{k}'}+b_{\mathbf{k}'}. \tag{2}$$

Substituting this expression in the equation describing the three-wave interaction

$$\dot{a}_{\mathbf{k}}=\sum_{\mathbf{k}'} V_{\mathbf{k},\mathbf{k}',\mathbf{k}-\mathbf{k}'}a_{\mathbf{k}'}a_{\mathbf{k}-\mathbf{k}'}e^{i\Delta\omega_{\mathbf{k}}t},$$

where $V_{\mathbf{k},\mathbf{k}',\mathbf{k}-\mathbf{k}'}$ is the interaction potential, and $\Delta\omega_k$ is expressed in terms of the frequency of the interacting waves: $\Delta\omega_{\mathbf{k}}=\omega_{\mathbf{k}'}+\omega_{\mathbf{k}-\mathbf{k}'}-\omega_{\mathbf{k}}$, we obtain two equations.

One of these describes the change in the amplitude of the fluctuating contribution

$$b_{\mathbf{k}'}=\sum_{\mathbf{k}'-\mathbf{k}} V_{\mathbf{k}',\mathbf{k},\mathbf{k}'-\mathbf{k}}a_{\mathbf{k}}C_{\mathbf{k}'-\mathbf{k}}. \tag{3}$$

The summation extends over all the noise components forming a resonance triplet with the considered wave. The other equation refers to the averaged amplitude of the wave. In the approximation linear in the fluctuating contribution, it takes the form

$$\dot{\bar{a}}_{\mathbf{k}}=\sum_{\mathbf{k}'} V_{\mathbf{k},\mathbf{k}',\mathbf{k}-\mathbf{k}'}\tilde{C}_{\mathbf{k}'}\tilde{C}_{\mathbf{k}-\mathbf{k}'}e^{i\Delta\omega_{\mathbf{k}}t}. \tag{4}$$

Here $\tilde{C}_{\mathbf{k}'}=C_{\mathbf{k}'}+b_{\mathbf{k}'}$ is the modified amplitude of the noise field.

Substitution in the latter relation of the expression for the fluctuating contribution (3) leads to an equation for the mean amplitude, the solution which can be written in the form[3]

$$\bar{a}_{\mathbf{k}}=a_{0\mathbf{k}}e^{-\beta_{\mathbf{k}}t}, \tag{5}$$

where

$$\beta_{\mathbf{k}}=-\sum_{\mathbf{k}'} V_{\mathbf{k},\mathbf{k}',\mathbf{k}-\mathbf{k}'}V_{\mathbf{k}',\mathbf{k},\mathbf{k}'-\mathbf{k}}N_{\mathbf{k}'-\mathbf{k}}\delta(\Delta\omega_{\mathbf{k}}). \tag{6}$$

Here $N_{\mathbf{k}'-\mathbf{k}}=C_{\mathbf{k}'-\mathbf{k}}C^*_{\mathbf{k}'-\mathbf{k}}$ is the number of interacting noise waves and is connected with the spectral energy density of the noise by the usual relation[2]

$$N_{\mathbf{k}'}\omega_{\mathbf{k}'}=\varepsilon_{\mathbf{k}'}.$$

In this case

$$\int \varepsilon_{\mathbf{k}'}d\mathbf{k}'=\int \varepsilon_{k'}\,dk'=E,$$

where E is the energy density of the noise field,

$$\Delta\omega_{\mathbf{k}}=\omega_{\mathbf{k}'}+\omega_{\mathbf{k}-\mathbf{k}'}-\omega_{\mathbf{k}}$$

is the value of the dephasing of the interacting waves.

Formula (6) is a convenient starting point for making clear the features of the process of interaction of the wave with the noise. It is convenient here to rewrite

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this relation in a more symmetric form:

$$\beta_k = -\frac{1}{2}\sum_{k'} V_{k,k',k-k'}\delta(\Delta\omega_k)\,[V_{k',k,k'-k}N_{k'-k}-V_{k-k',k,-k'}N_{k'}]. \quad (7)$$

The obtained expression allows us to establish the fact that the process of interaction of the wave with the noise takes place differently in the case of low-frequency and high-frequency noises.

To be precise, if the noise is low-frequency, so that the relations $\omega_k>\omega_{k'}$, $\omega_k>\omega_{k-k'}$ are satisfied, then, by virtue of the relations

$$V_{k',k,k'-k}=V_{k-k',k,-k'}=-V_{k,k',k-k'}$$

we obtain

$$\beta_k=\sum_{k'}|V_{k,k',k-k'}|^2\delta(\Delta\omega_k)N_{k'-k}, \quad (8)$$

whence it follows that $\beta_k>0$ and damping of the wave takes place. On the other hand, if the wave interacts with the high-frequency noise, so that

$$\omega_{k'}>\omega_k,\quad \omega_{k-k'}>\omega_k,$$

then it is convenient to take the following into consideration in order to obtain the final result.

In the framework of the considered approximation and the random-phase condition, the regular wave interacts with each pair of noise components, forming triplets that do not interact with one another. Each of the triplets in formula (7) is taken into account twice in the summation over all the noise components forming the resonant triplets. We can therefore transform to single summation without changing the answer, carrying it out, for example, only over those values of **k'** for which $\omega_k>\omega_{k-k'}$ and simultaneously omitting the factor $\frac{1}{2}$ in front of the formula.

Recognizing that under the conditions

$$\omega_{k'}>\omega_k,\quad \omega_{k-k'}>\omega_k,\quad \omega_{k'}>\omega_{k-k'}$$

we have

$$V_{k',k,k'-k}=-V_{k-k',k-k'},\quad V_{k,k',k-k'}=-V_{k',k,k'-k}, \quad (9)$$

and for the damping coefficient, we obtain

$$\beta_k=\sum|V_{k,k',k-k'}|^2(N_{k'-k}-N_{k'})\delta(\Delta\omega_k).$$

As is seen in this case, the sign of β_k can become negative (under the condition $N_{k'-k}<N_{k'}$), which means that the energy in this case will be transferred from the noise field to the wave. Thus, the interaction of the noise can lead both to damping and to amplification of the wave, which has been pointed out in particular, by a number of authors.[4-6]

The effect of amplification of the wave by the noise is essentially a reflection of the fact of the decay of the high-frequency components relative to the low-frequency excitation in the form of the wave. The wave amplified by the energy of the noise acquires all the statistical marks of the noise components and becomes indistinguishable from its other components in the case in which the wave and the noise are waves of the same form. In this sense, the process of amplification of the wave by the noise can also be regarded as the heating of the low-frequency degrees of freedom up to the "temperature" of the noise field; here the amplitude of the wave takes on the equilibrium value

$$a_{k_0}^2=T\omega_k^{-1},$$

where T is the temperature of the field.[2] This circumstance imposes a restriction on the process of wave amplifications as a result of its interaction with the noise. If the initial energy of the wave is less than the equilibrium value, then processes of four-particle interaction turn out to be the principal ones, during the course of which, a jump first takes place in the low-frequency noise, due to the jump in the high-frequency noise components, and then the absorption of the wave in the low-frequency noise takes place.

We note further that if the frequency of the noise appreciably exceeds the frequency of the considered wave, then the relation

$$\omega_{k'}\gtrsim\omega_{k-k'}\gg\omega_k$$

follows from the conditions of synchronism (see the drawing), whence it follows that the frequencies of the noise components taking part in the interaction with the considered wave differ little from one another. This allows us to make the expansion

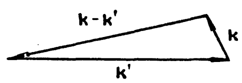
$$N_{k'-k}=N_{k'}-\frac{\partial N_{k'}}{\partial \mathbf{k}'}\mathbf{k},\quad \omega_{k'-k}=\omega_{k'}-\frac{\partial\omega_{k'}}{\partial\mathbf{k}'}\mathbf{k},$$

which leads to the following expression for β_k:

$$\beta_k=-\frac{1}{2}\sum_{k'}|V_{k,k',k-k'}|^2\left(\mathbf{k}\frac{\partial N_{k'}}{\partial\mathbf{k}'}\right)\Big|_{\omega_{k'}=\mathbf{k}\mathbf{u}} \quad (10)$$

where $\mathbf{u}=\partial\omega_k\partial/_k$ is the group velocity.

The obtained expression means that the damping of the wave takes place as a consequency of its interaction with the wave packets of the noise field (formed by two harmonics of the noise), the projection of the group



velocity of which in the direction of the wave vector **k** coincides with the phase velocity of the considered wave.

This result corresponds exactly with the well known Landau relation of the electromagnetic wave in a plasma as a result of its interaction with particles whose velocity is identical with the phase velocity of the wave. Furthermore, just as in the phenomenon considered by Landau, the character of the process turns out to be dependent on the properties of the spectral distribution (function) of the noise $\partial N_{k'}/\partial k'$.

At $\partial N_{k'}/\partial k' < 0$, we have $\beta_k < 0$ and damping of the wave takes place; at $\partial N_{k'}/\partial k' > 0$, the sign is opposite: $\beta_k < 0$ and the wave is amplified.

In particular, for an equilibrium spectrum we obtain $N_{k'} = T\omega_{k'}^{-1}$, and $\partial N_{k'}/\partial k' < 0$ for all cases in which k' increases with increase in k', so that the equilibrium noise absorbs the wave in spite of the fact that its energy spectrum turns out to be increasing in correspondence with the Rayleigh–Jeans law.

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Possibility of decreasing the electron heat flux from open traps

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The possibility is considered of decreasing the electron heat flux from open traps when the emerging plasma stream is strongly expanded in the expansion nozzle. Allowance for weak collisions in an almost collisionless plasma, when the mean free path is much larger than the characteristic dimension R of the expansion nozzle, leads to the appearance of electrons that are trapped between the exit slit of a trap with mirror-configuration magnetic field and a wall with electrostatic potential φ_0. An analysis of the equations shows that in the case of an unlimited emissivity of the wall the blocking potential is connected with the degree of plasma expansion by a relation from which it follows that a relatively small expansion is sufficient to decrease substantially the electron heat flux. This estimate of the heat loss is an upper bound, since no account is taken of the possibility of turbulence development in the expansion nozzle.

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1. INTRODUCTION

One of the main problems in the development of thermonuclear reaction based on open traps (of the mirror anti-mirror, or baseball type) is the fact that the plasma can be rapidly cooled in the trap as a result of the runaway electrons whose flux can exceed that of the ions by $(M/m)^{1/2}$ times. In fact, the time of energy exchange between the electrons and the ions

$$\tau_{ei}^{\varepsilon} \sim 5\cdot 10^{13}\,(T[\mathrm{eV}]/10^4)^{3/2}/n$$

is smaller for thermonuclear temperatures $T \sim 10^4$ eV than the Lawson time, so that the energy lifetime is determined by the cooling of the electrons.

In the analysis of open traps it is usually assumed that the emissivity of the walls is either low or can be substantially decreased by using special blocking grids,[1] so that the electron heat flux decreases to the ion value because of the appearance of an ambipolar potential between the trap and the wall. In all cases, however, it is desirable to expand the plasma stream emerging from the trap, for the purpose of reducing the heating of the wall, to solve the problem of directly recuperating the plasma energy by conversion to electric energy, etc. When such an expanding nozzle is used, the physical picture of the plasma flow becomes unique. It will be shown below that expansion of the plasma stream leads even on its own to a considerable lowering of the electron heat flux from the trap, to a value on the order of the ion flux.

2. QUALITATIVE APPROACH

We assume throughout that the electron emissivity of the wall is large. This takes place either when it is impossible to use blocking grids and the heating causes evaporation of the wall materials, or else through the use of special means of producing a plasma and ensuring its stability. In these cases there is a cold plasma with unlimited emissivity near the wall.

In the collisionless case without expansion, the electronic heat flux was calculated a number of times (see, e.g.,[2,3]). After a time $t_1 \sim R/V_0$ (R is the distance between the trap and the wall and V_0 is the average velocity of ion outflow of the trap), the electric potential φ_0 of the trap relative to the wall decreases from a value $\varphi_0 \sim (T_e/e)\ln(R/r_d)$[2] (Debye radius $r_d \ll R$) to $\varphi_0 \sim T_e/e$[3] because of the production of a flow of cold electrons from the wall, and the electron heat flux becomes larger than the ion one by $(M/m)^{1/2}$ times. In the presence

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