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Abnormal states of nuclear matter and π condensation

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It is shown within the framework of relativistic field models of the πN interaction that the instability of nuclear matter to π condensation becomes stronger when the Fermi velocity tends to the relativistic limit. The results agree with Migdal's theory and point to the need for taking π condensation into account in the Lee and Wick model for abnormal states of atomic nuclei.

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1. INTRODUCTION

There have been discussions in recent years of the possible existence, at nuclear density, of an energy barrier whose surmounting (e.g., in collisions of heavy nuclei) may be the next step towards a genuine ground state of a system of N nucleons. Thus, for example, calculation of the dependence of the nuclear energy on the effective mass M^* of the nucleon, carried out in the σ model by Lee and Wick[1] (see also [2]) points to the existence of a local energy minimum at $M^* \approx 0$. An increase in the density of nuclear matter may make this minimum absolute[2]—the nucleus may go over via a relativistic phase transition into an abnormal state with $M^* \approx 0$.

On the other hand, Migdal's theory of π condensation[3] (see also later papers by Migdal and co-workers[4]) predicts, at a certain density, the onset of an inhomogeneous classical pion field in the ground state of nuclear matter.

The possibility of π condensation was not considered by Lee and Wick in connection with the problem of abnormal states. The formation of a π condensate in nuclear matter was investigated later, within the framework of the σ model of strong interactions, by Dashen, Campbell, and Manassah.[5] The nonrelativistic approximation used by them does not explain, however, the role of π condensation in the model of abnormal states with $M^* \approx 0$.

We have investigated the stability of nuclear matter to the appearance in it of a classical pion field, using the relativistic quasiclassical approach employed by Lee and Wick. In this approach the solution of the problem is similar to finding the energy $\epsilon = -(1/2)\chi H^2$ of an electron gas in an external magnetic field (χ is the magnetic susceptibility). It is known that the susceptibility of an electron gas is positive, but since the electromagnetic-interaction constant is small, the susceptibility is small compared with unity. Therefore the decrease of the energy of a metal in an external magnetic field is small compared with the self-energy $H^2/8\pi$ of the field.

The situation changes in theories with strong constants. In particular, the gain in the energy of nucleons situated in a classical pion field can exceed the self-energy of the field and may favor the formation of the π condensate.

Starting with Dirac's equation, we find the energy of a relativistic nucleon in an inhomogeneous classical ($\langle\pi\rangle \neq 0$) pion field of small amplitude. We construct next

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the single-particle density matrix and calculate the change in the energy of a relativistic gas of nucleons in the field of the π condensate. Comparison of this change with the self-energy of the classical pion field shows that the stability of incompressible nuclear matter relative to π condensation is due to the fact that the parameter

$$\zeta(v_F)=1-\frac{g^2}{6\pi^2}\left\{v_F-\frac{2}{3}v_F^3+\ln\frac{1+v_F}{1-v_F}\right\}$$

is positive (g is the πN-coupling and v_F is the top Fermi velocity), and is lost at $v_F=v_c\approx 0.11$.

When nuclear matter is considered within the framework of the field approach, it is necessary to give preference to models that take into account the approximate chiral invariance of the strong interaction of nonstrange particles. The simplest model of this type is the σ model of strong interactions with spontaneous breaking of the $SU(2)\otimes SU(2)$ chiral symmetry. Proceeding to the investigation of the σ model, we shall show that, depending on the relation between the constants of the model, an increase in the density of the nuclear matter can lead to phase transitions of various types, one of which was considered by Lee and Wick.[1] In the absence of a π condensate, the increase of the density is accompanied in accord with the conclusions of Lee and Wick by a decrease of the effective mass of the nucleon. The corresponding increase of the Fermi velocity leads, however, to a strong enhancement of the instability of the nucleon gas to π condensation, and this instability must be taken into account when calculating the ground-state energy. Consideration of abnormal states of nuclear matter with $M^*\approx 0$ and $\langle\pi\rangle=0$ is therefore inconsistent.

In the last section of this article we discuss the influence of short-range nucleon-nucleon interaction on the parameters that describe the instability of nuclear matter. Allowance for the effect of screening by the interaction on the classical pion field is apparently incapable of changing the conclusion drawn for a nucleon gas, but increases the critical value of the Fermi velocity to $v_c\approx 0.28$.

2. QUASICLASSICAL RELATIVISTIC MODEL OF π CONDENSATION

The relativistic Lagrangian that describes the interaction of nucleons with the pseudoscalar pion field is given by

$$\mathscr{L}={}^1/_2(\partial_\mu\pi)^2-{}^1/_2\mu_\pi^2\pi^2+\bar\psi\{i\gamma_\mu\partial_\mu-M-ig\gamma_5\tau\pi\}\psi, \tag{1}$$

where μ_π and M are the masses of the pion and nucleon, and g is the πN-coupling constant.

We consider a model of an incompressible nucleon liquid, in which the short-range forces between the nucleons lead to independence of the nuclear-matter density of the long-wave structure of the ground state. In the absence of π condensate, the nucleon-gas energy is

$$\mathscr{E}_0=\mathrm{Sp}_\sigma{}^\tau\int\frac{d^3x d^3p}{(2\pi)^4}\varepsilon_p n_p^{(0)}, \tag{2}$$

where $\epsilon_p+(p^2+M^2)^{1/2}$ and $n_p^{(0)}$ is the single-particle density matrix. In isotopically symmetrical nucleon matter, $n_p^{(0)}$ coincides with the Fermi distribution function n_F. In neutron matter we have

$$n_p^{(0)}={}^1/_2(1-\tau_3)n_F. \tag{3}$$

The presence of π condensate corresponds to the appearance, in the system, of a pion field whose mean value over the ground state differs from zero, $\langle\pi\rangle=\pi(x)$. The corresponding change of the nucleon energy is

$$\delta\mathscr{E}_N=\mathrm{Sp}_\sigma{}^\tau\int\frac{d^3xd^3p}{(2\pi)^3}(h_pn_p-\varepsilon_pn_p^{(0)}), \tag{4}$$

where h_p and n_p are the single-particle energy and the single-particle density matrix in the π-condensate field.

To avoid monitoring the constancy of the local density of the nucleons, we calculate their thermodynamical potential $\Omega=\mathscr{E}_N-\mu N$, where N is the total number of particles:

$$N=\mathrm{Sp}_\sigma{}^\tau\int\frac{d^3xd^3p}{(2\pi)^3}n_p, \tag{5}$$

and μ is the chemical potential determined from the equation

$$\mathrm{Sp}_\sigma{}^\tau\int\frac{d^3p}{(2\pi)^3}(n_p-n_p^{(0)})=0. \tag{6}$$

The instability of nuclear matter to the appearance of the π condensate sets in when the gain $\delta\mathscr{E}_N$ of the nucleon energy in the field $\pi(\mathbf{x})$ exceeds the self-energy of this field

$$\mathscr{E}_\pi=\int d^3x\left\{\frac{1}{2}\left(\frac{\partial\pi}{\partial\mathbf{x}}\right)^2+\frac{1}{2}\mu_\pi^2\pi^2\right\}. \tag{7}$$

To study the stability it suffices to consider fields $\pi(\mathbf{x})$ of small amplitude. In this case the change of the Fermi density matrix can be expanded in powers of the perturbations of the single-particle energy

$$n_p-n_p^{(0)}=\frac{\partial n_p^{(0)}}{\partial\varepsilon_p}(h_p-\varepsilon_p-\delta\mu)+\frac{1}{2}\frac{\partial^2n_p^{(0)}}{\partial\varepsilon_p^2}(h_p-\varepsilon_p-\delta\mu)^2+\ldots, \tag{8}$$

where $\delta\mu$ is the change of the chemical potential and, by virtue of the matrix structure of the perturbation, is of second order of smallness in the amplitude $\pi(\mathbf{x})$. Using the property $\partial n_F/\partial\epsilon_p=-\delta(\epsilon_p-\mu_0)$ (μ_0 is the Fermi energy) of the Fermi distribution function, it is easily shown that the change of the thermodynamic potential of the nucleons in the π-condensate field is given by, accurate to terms quadratic in $\pi(\mathbf{x})$,

$$\delta\Omega_N=\mathrm{Sp}_\sigma{}^\tau\int\frac{d^3x\,d^3p}{(2\pi)^3}\left\{n_p^{(0)}(h_p-\varepsilon_p)+\frac{1}{2}\frac{\partial n_p^{(0)}}{\partial\varepsilon_p}(h_p-\varepsilon_p)^2\right\}. \tag{9}$$

The problem thus reduces to a determination of the single-particle energy h_p.

Energy of relativistic nucleon in a classical pion field

The Dirac equation for a nucleon in a classical pion field is

$$i\frac{\partial\psi}{\partial t}=\hat{H}\psi,\quad \hat{H}=\hat{\alpha}\mathbf{p}+\hat{\beta}M+ig\hat{\beta}\gamma_5\boldsymbol{\tau\pi}(\mathbf{x}) \tag{10}$$

(we use the standard representation of the Dirac matrices).

We consider the quasiclassical case, when the characteristic inhomogeneity dimension q^{-1} of the field $\boldsymbol{\pi}(\mathbf{x})$ is large in comparison with the uncertainty p^{-1} of the nucleon position. In the calculation of the quasiclassical energy of the nucleon h_p in the field $\boldsymbol{\pi}(\mathbf{x})$ it is convenient to change over to the Foldy-Wouthuysen (FW) representation. In this representation the operators of the physical quantities have simple classical analogs (see, e.g., [5]). In particular, the operator of the particle average position is the usual coordinate operator $\hat{\mathbf{x}}=i\partial/\partial\mathbf{p}$.

Changing over in (10) to the momentum representation and carrying out the FW transformation with the aid of the unitary operator

$$U=(\hat{\beta}(\hat{\alpha}\mathbf{p})+M+\varepsilon_p)/[2\varepsilon_p(M+\varepsilon_p)]^{1/2}, \tag{11}$$

we obtain

$$\hat{H}_{\rm FW}=U\hat{H}U^+=\hat{\beta}\varepsilon_p+ig\hat{\beta}\gamma_5\boldsymbol{\tau\pi}(\mathbf{x}_{\rm FW}), \tag{12}$$

where $\mathbf{x}_{\rm FW}$ is the coordinate operator in the FW representation:

$$\hat{\mathbf{x}}_{\rm FW}\equiv U\hat{\mathbf{x}}U^+=\hat{\mathbf{x}}+\hat{\boldsymbol{\delta}},\quad \hat{\mathbf{x}}=i\frac{\partial}{\partial\mathbf{p}}, \tag{13}$$

$$\hat{\boldsymbol{\delta}}=\gamma_5\frac{[\hat{\alpha}\times\mathbf{p}]}{2\varepsilon_p(\varepsilon_p+M)}+\frac{i}{2\varepsilon_p}\left\{\hat{\alpha}\hat{\beta}+\frac{\hat{\beta}(\hat{\alpha}\mathbf{p})\mathbf{p}}{\varepsilon_p(\varepsilon_p+M)}\right\}. \tag{14}$$

The transition from the FW representation to the quasiclassical Hamiltonian corresponds to replacement of the operator of the average particle position $\hat{\mathbf{x}}$ by the coordinate $\mathbf{x}$. Introducing the pseudoscalar singlet[1)] $\pi(\hat{\mathbf{x}}_{\rm FW})$ we have in the case of weak inhomogeneity of the classical pion field

$$\Pi(\mathbf{x}_{\rm FW})=\Pi(\mathbf{x})+\mathbf{q}\delta\Pi'(\mathbf{qx})+{}^1/_2(\mathbf{q}\delta)^2\Pi''(\mathbf{qx}). \tag{15}$$

When this expansion is taken into account the Hamiltonian (12) takes the form

$$H_{\rm FW}=\begin{pmatrix}\varepsilon_p+g\delta_1\Pi' & -ig[\Pi+\delta_2\Pi'+{}^1/_2(\delta_1^2+\delta_2^2)\Pi'']\\ ig[\Pi+\delta_2\Pi'+{}^1/_2(\delta_1^2+\delta_2^2)\Pi''] & -\varepsilon_p+g\delta_1\Pi'\end{pmatrix}, \tag{16}$$

where

$$\delta_1=\frac{\sigma}{2\varepsilon_p}\left\{\mathbf{q}-\mathbf{p}\frac{(\mathbf{pq})}{\varepsilon_p(\varepsilon_p+M)}\right\},\quad \delta_2=-\frac{\sigma}{2\varepsilon_p}\frac{[\mathbf{p}\times\mathbf{q}]^2}{(\varepsilon_p+M)}. \tag{17}$$

The spectrum of the particles and antiparticles in a weakly inhomogeneous pion field is determined by the equation for the eigenvalues h_p of the matrix (16)

$$(h_p-g\delta_1\Pi')^2=\varepsilon_p^2+g^2[\Pi+\delta_2\Pi'+{}^1/_2(\delta_1^2+\delta_2^2)\Pi'']. \tag{18}$$

Assuming the field amplitude π small, we obtain for the particle spectrum, accurate to terms quadratic in Π/ϵ_p and q/p,

$$h_p=\varepsilon_p+\frac{g}{2\varepsilon_p}\Pi'\left[\sigma\mathbf{q}-\frac{(\sigma\mathbf{p})(\mathbf{pq})}{\varepsilon_p(\varepsilon_p+M)}\right]+\frac{g^2}{2\varepsilon_p}\left\{\Pi^2+(\Pi')^2\frac{[\mathbf{p}\times\mathbf{q}]^2}{(2\varepsilon_p)^2(\varepsilon_p+M)^2}\right.$$

$$\left.-\Pi\Pi'\frac{\sigma[\mathbf{p}\times\mathbf{q}]}{\varepsilon_p(\varepsilon_p+M)}+\Pi\Pi''\frac{1}{(2\varepsilon_p)^2}\left[\mathbf{q}^2-\frac{(\mathbf{pq})^2}{\varepsilon_p^2}+\frac{[\mathbf{p}\times\mathbf{q}]^2}{(\varepsilon_p+M)^2}\right]\right\}. \tag{19}$$

In the nonrelativistic limit ($p\ll M$) the energy increment goes over into the well-known expression for the relativistic πN-interaction Hamiltonian

$$\delta\varepsilon_p=\frac{g}{2M}\sigma\frac{\partial(\boldsymbol{\pi\tau})}{\partial\mathbf{x}}+\frac{g^2}{2M}\boldsymbol{\pi}^2. \tag{20}$$

π condensation in a relativistic nucleon gas

Substituting (9) in (19) and recognizing that for perturbations bounded in space we have

$$\int d^3x\{(\Pi')^2+\Pi\Pi''\}=0, \tag{21}$$

we obtain for the total change of the thermodynamic potential $\delta\Omega=\delta\Omega_N+\delta\Omega_\pi$ of isotopically symmetrical matter, after simple calculations,

$$\delta\Omega=\int d^3x\left\{\frac{1}{2}\zeta(v_F)\left(\frac{\partial\boldsymbol{\pi}}{\partial\mathbf{x}}\right)^2+\frac{1}{2}\mu_\pi^{*2}\boldsymbol{\pi}^2\right\}, \tag{22}$$

where

$$\zeta(v_F)=1-\frac{g^2}{6\pi^2}\left\{v_F-\frac{2}{3}v_F^3+\ln\frac{1+v_F}{1-v_F}\right\}, \tag{23}$$

$$\mu_\pi^{*2}=\mu_\pi^2+\frac{g^2}{\pi^2}\frac{p_F^2}{v_F}\left\{1-\frac{1-v_F^2}{2v_F}\ln\frac{1+v_F}{1-v_F}\right\}, \tag{24}$$

$v_F=p_F/\epsilon(p_F)$ is the Fermi velocity of the nucleons, $p_F=({}^3/_2\pi^2n)^{1/3}$ is the limiting Fermi momentum, and n is the nucleon density.

In neutron matter, $\zeta-1$ goes over into $(\zeta-1)/2$, and the Fermi momentum is equal to $(3\pi_2 n)^{1/3}$.

In the nonrelativistic limit ($v_F\ll 1$) the expression for ζ coincides with that obtained by Migdal, Markin, and Mishustin.[4] We note that when ζ is expanded in powers of v_F relativistic corrections appear only in the fifth order:

$$\zeta(v_F)=1-\frac{g^2}{2\pi^2}\left\{v_F+\frac{2}{15}v_F^5+\ldots\right\}. \tag{25}$$

Therefore the nonrelativistic formula of[4] is in fact valid up to $v_F\approx 0.6$.

The renormalization of the pion mass (24) has a simple physical meaning and is connected with the change of the effective mass of the nucleon in a classical pion field. Indeed, the nucleon energy, in the limit of a constant field, takes in accordance with the Dirac equation the form

$$\varepsilon_p^*=(\mathbf{p}^2+M^2+g^2\boldsymbol{\pi}^2)^{1/2}\approx\varepsilon_p\left(1+\frac{g^2\boldsymbol{\pi}^2}{2\varepsilon_p^2}\right). \tag{26}$$

By calculating from (2) the energy increment due to the change of the nucleon mass and proportional to $\boldsymbol{\pi}^2$, we obtain for the pion effective mass the expression (24).

Proceeding to a discussion of the relativistic formula, we note that the relativism condition $p_F\gtrsim M$ can be satisfied both as a result of the high density of the nucleons (it is more reasonable to refer this case to neutron matter) and as a result of the low effectiveness of the nucleon mass.

The function $\zeta(v_F)$ for $g^2/4\pi = 14.6$ is plotted in Fig. 1. The reversal of the sign of ζ at $v_F = v_c \approx 0.11$ points to instability to π condensation at $v_F > v_c$. For nuclear matter we have $v_F \approx 0.3$. We note, however, that when no account is taken of the short-range nucleon-nucleon interaction, a quantitative estimate of the lower limit of the instability can not be correct in our approach. A more accurate estimate will be made at the end of the article.

Owing to the finite pion mass, instability at a given $\zeta < 0$ develops only at pion-field gradients exceeding a certain critical value

$$q_c^2 \sim \mu_\pi^{*2}\zeta^{-1}. \tag{27}$$

The validity of (22) is restricted by the conditions for the applicability of the quasiclassical approximation

$$q_c < p_F, M. \tag{28}$$

In the limit of a low effective nucleon mass, the effective pion mass tends to a constant limit $\mu_\pi^{*2} = \mu_\pi^2 + g^2 p_F^2/\pi^2$ that depends only on the nucleon density. If the density increases at a fixed nucleon effective mass, the increase of μ_π^* follows a faster law than for $\zeta(v_F)$, and inequality (28) does not hold.

In connection with the foregoing, we call attention to the fact that the renormalization of the masses of the nucleon and the pion in nucleon matter depends substantially on how the bare masses are introduced into the theory. The successes of current algebra and of PCAC in the description of hadron scattering and decays (see, e.g., [7]) point to an approximate chiral invariance of the strong interaction, at least for non-strange particles. In chiral theories, the pion is a Goldstone particle, and its mass is introduced on account of "soft" violation of the chiral invariance. One can therefore expect in such theories (as we shall show below) that in nucleon matter, in a phase with spontaneously violated chiral invariance, the pion, remaining a Goldstone particle, can have a low effective mass even if renormalization is taken into account. The simplest model of SU(2) ⊗ SU(2) chiral-invariant pion-nucleon interaction is the σ model of strong interactions,[8] which we now proceed to consider.

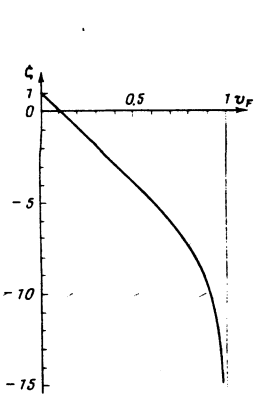

FIG. 1.

3. π CONDENSATION IN THE σ MODEL OF STRONG INTERACTIONS

The σ-model Lagrangian is

$$\mathscr{L} = \frac{1}{2}\left(\frac{\partial\sigma}{\partial x_\mu}\right)^2 + \frac{1}{2}\left(\frac{\partial\pi}{\partial x_\mu}\right)^2 - \frac{\lambda^2}{4}\left[\sigma^2+\pi^2-\left(\frac{\mu}{\lambda}\right)^2\right]^2 + \bar\psi\{i\gamma_\mu\partial_\mu - g(\sigma+i\gamma_5\tau\pi)\}\psi + \mathscr{L}_b, \tag{29}$$

where σ is a scalar field; μ and λ are parameters that can be connected with the physical masses of the particles; $\mathscr{L}_b$ is the part of the Lagrangian that violates explicitly the chiral invariance. In the case of exact SU(2) ⊗ SU(2) symmetry of the Lagrangian ($\mathscr{L}_b = 0$), the minimum of the energy of the system of interacting fields are realized in a state with spontaneously broken symmetry of vacuum, when

$$\langle\sigma^2+\pi^2\rangle = \mu^2/\lambda^2. \tag{30}$$

Chiral rotation makes it possible to choose a gauge in which $\langle\pi\rangle = 0$, $\langle\sigma\rangle = \sigma_0 = \mu/\lambda$. In this case the pions are massless Goldstone particles and the masses of the nucleon and scalar σ meson are equal to

$$m_\sigma^2 = 3\lambda^2\sigma_0^2 - \mu^2, \quad M = g\sigma_0. \tag{31}$$

Relativistic phase transition

In the absence of a π condensate the energy of a system of nucleons interacting with a homogeneous field σ is determined in the quasiclassical approximation by the expression

$$\mathscr{E} = \frac{\lambda^2}{4}\left[\sigma^2 - \left(\frac{\mu}{\lambda}\right)^2\right]^2 + \int^{p_F} \frac{4d^3p}{(2\pi)^3}(\mathbf{p}^2+g^2\sigma^2)^{1/2}. \tag{32}$$

Families of $\mathscr{E}(\sigma)$ curves for different values of the nucleon density $n = 2p_F^3/3\pi^2$ are shown in Fig. 2.[9] A relation of the type shown in Fig. 2a is obtained at not too large a σ-meson mass, $m_\sigma \gtrsim gM \sim 13$ GeV. If $m_\sigma \ll gM$, then at a nucleon density

$$n_c^{\rm I} = \frac{1}{3\cdot 6^{1/2} g^2} m_\sigma^2 M \tag{33}$$

a first-order phase transition occurs in the system (curve a of Fig. 3) into a state with restored chiral symmetry. The average scalar field changes jumpwise

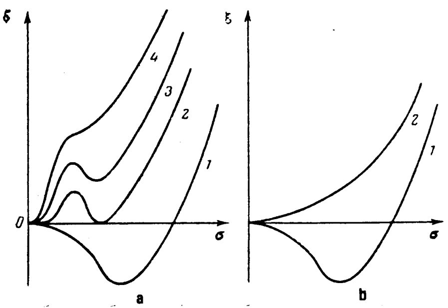

FIG. 2. a) Plot of $\mathscr{E}(\sigma)$ $n = n_c^{\rm I}$, for $m_\sigma \lesssim gM$ in the nucleon density intervals: 1) $n < n_c^{\rm I}$, 2) $n_c^{\rm I}$, 3) $n_c^{\rm I} < n < n_m$, 4) $n_m < n$. b) Plot of $\mathscr{E}(\sigma)$ for $m_\sigma \gg gM$ in the nucleon-density intervals: 1) $n < n_c^{\rm II}$, 2) $n_c^{\rm II} < n$.

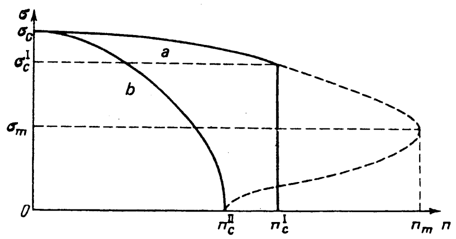

FIG. 3. Plots of the average scalar field vs. nucleon density: a) $m_\sigma \lesssim gM$, b) $m_\sigma \gg gM$.

from the value

$$\sigma_c^{\mathrm{I}}=(^3/_2)^{1/2}M/g \tag{34}$$

to zero. For a nucleon density in the interval $n_c^{\mathrm{I}} < n < n_m$, where

$$n_m=\sqrt{2}n_c^{\mathrm{I}},\quad \sigma_m=\sigma_c^{\mathrm{I}}/\sqrt{2}. \tag{35}$$

The state of matter with broken symmetry, according to Fig. 2a, is metastable.

In the limit of a very large σ-meson mass $m_\sigma \gg gM$, the average scalar field decreases smoothly with increasing n (Fig. 3, curve b), and vanishes at a nucleon density

$$n_c^{\mathrm{II}}=(1/3\sqrt{2})\pi(m_\sigma/g)^3 \tag{36}$$

(second-order phase transition).

We shall show now that in a phase with spontaneously broken symmetry the pion remains massless even with allowance for renormalization. Indeed, according to (26) and (29), the effective mass of the pion in nucleon matter (defined as the coefficient of $\frac{1}{2}\pi^2$ in the total energy of the system) is

$$\mu_\pi^{*2}=\lambda^2\left[\sigma^2-\left(\frac{\mu}{\lambda}\right)^2\right]+g^2\int\frac{4d^3p}{(2\pi)^3}(\mathbf{p}^2+g^2\sigma^2)^{-1/2}. \tag{37}$$

Comparing this expression with formula (32), we find

$$\mu_\pi^{*2}=\frac{1}{\sigma}\frac{\partial\mathscr{E}}{\partial\sigma}. \tag{38}$$

Therefore $\mu_\pi^*=0$ for the extramal energy in the phase with broken symmetry ($\sigma\neq 0$). As a result of the phase transition the π and σ mesons combine to form a mass-degenerate chiral multiplet.

Abnormal states and π condensation in the σ model

If account is taken in the Lagrangian (29) of terms that violate chiral invariance, the dependence of the pion effective mass on the nucleon density is sensitive to the manner of this violation. We consider two standard forms of non-invariant terms:

$$\mathscr{L}_b^{(1)}=-{}^1/_2\mu_\pi^2\pi^2,\quad \mathscr{L}_b^{(2)}=C_\pi\sigma, \tag{39}$$

where $C_\pi=\mu_\pi^2\sigma_0$ and in the second type of violation σ_0 is determined from the equation

$$C_\pi=\sigma_0(\lambda^2\sigma_0^2-\mu^2). \tag{40}$$

In the absence of a π condensate the presence of $\mathscr{L}_b^{(1)}$ does not change the formulas obtained above. In this case $\mu_\pi^*=\mu_\pi$. When chiral invariance is violated by the term $\mathscr{L}_b^{(2)}$, the expression of the energy of the nucleons interacting with the field σ goes over into

$$\mathscr{E}^{(2)}=\mathscr{E}(\sigma)-C_\pi\sigma. \tag{41}$$

In this case a calculation of μ_π^* similar to the foregoing one yields

$$\mu_\pi^{*2}=C_\pi/\sigma. \tag{42}$$

The behavior of $\mathscr{E}^{(2)}(\sigma)$ at different values of the nucleon density was investigated by Lee and Wick in connection with the problem of abnormal states of nuclear matter.[1]

When account is taken of a classical weakly inhomogeneous pion field of small amplitude, the energy of the nucleon matter at a given density takes, in accord with the conclusions of the preceding section, for the different types of chiral-invariance violation, the form

$$\mathscr{E}_t^{(1)}=\mathscr{E}(\sigma)+\frac{1}{2}\zeta(v_F)\left(\frac{\partial\pi}{\partial x}\right)^2+\frac{1}{2}\mu_\pi^2\pi^2, \tag{43}$$

$$\mathscr{E}_t^{(2)}=\mathscr{E}(\sigma)-C_\pi\sigma+\frac{1}{2}\zeta(v_F)\left(\frac{\partial\pi}{\partial x}\right)^2+\frac{1}{2}\frac{C_\pi}{\sigma}\pi^2, \tag{44}$$

where $\zeta(v_F)$ is determined by formula (25), and the Fermi velocity is

$$v_F=(^3/_2\pi^2n)^{1/3}[(^3/_2\pi^2n)^{2/3}+g^2\sigma^2]^{-1/2}. \tag{45}$$

In the preceding section it was shown that even in the nonrelativistic limit we have $\zeta(p_F/M)<0$ at nuclear density. In the σ model the effective nucleon masses $M^*=g\sigma$ is always smaller than the vacuum value $M=g\sigma_0$ (see Fig. 3). Therefore if the condition $\zeta<0$ is satisfied in the simplest model of πN interaction (see above) it is all the more valid, according to formulas (25) and (45), in the σ model.

Since ζ is negative at $v_F > v_c$ (Fig. 1), it follows that the minima on the plots of $\mathscr{E}_t^{(1)}=\mathscr{E}(\sigma)$ and $\mathscr{E}_t^{(2)}(\sigma)$ (Fig. 2) are saddle points in (σ, π) space, and the system is unstable to formation of π condensate.

If chiral invariance is violated by the term $\mathscr{L}_b^{(1)}$, the condition (28) that the π condensate be weakly inhomogeneous is well satisfied at relativistic values of v_F. In the second case, satisfaction of this condition depends on the relation between the constants of the model.

In concluding this section we emphasize that inasmuch as the instability of the nuclear matter arises in the nonrelativistic region of values of v_F and becomes stronger with increasing Fermi velocity, consideration of the ground state of relativistic nuclear matter without allowance for π condensation can not be consistent.

EFFECT OF NUCLEON-INTERACTION ON THE PARAMETERS THAT CHARACTERIZE THE INSTABILITY

the classical pion field, which takes place when account is taken of the short-range interaction between the nucleons. For a weakly inhomogeneous π condensate this can be done within the framework of the quasiclassical method used by us with the aid of the Fermi-liquid theory.[10]

When account is taken of the short-range *NN* interaction, the thermodynamic potential of the nucleons in the π-condensate field is determined by the formula

$$\delta\Omega=\int d^3x\left\{\mathrm{Sp}_\sigma^\tau\int\frac{d^3p}{(2\pi)^3}\widetilde{\delta\varepsilon}_p n_p^{(0)}+\mathrm{Sp}_\sigma^\tau\int\frac{d^3p}{(2\pi)^3}(\varepsilon_p+\delta\varepsilon_p-\mu)\,\delta n_p+\frac{1}{2}\mathrm{Sp}_{\sigma\sigma'}^{\tau\tau'}\int\frac{d^3p\,d^3p'}{(2\pi)^6}F_{pp'}\,\delta n_p\,\delta n_{p'}\right\},\tag{46}$$

where $\delta\epsilon_p = h_p - \epsilon_p$, and δn_p is the change of the single-particle density matrix of the interacting nucleons:

$$\delta n_p=\frac{\partial n_p^{(0)}}{\partial\varepsilon_p}(\widetilde{\delta\varepsilon}_p-\delta\mu)+\frac{1}{2}\frac{\partial^2 n_p^{(0)}}{\partial\varepsilon_p^2}(\widetilde{\delta\varepsilon}_p-\delta\mu)^2+\ldots,\tag{47}$$

$\widetilde{\delta\epsilon}_p$ satisfies the integral equation

$$\widetilde{\delta\varepsilon}_p=\delta\varepsilon_p+\mathrm{Sp}_\sigma^\tau\int\frac{d^3p}{(2\pi)^3}F_{pp'}\frac{\partial n_{p'}^{(0)}}{\partial\varepsilon_{p'}}\widetilde{\delta\varepsilon}_{p'},\tag{48}$$

and $F_{pp'}$ is the amplitude of the zero-angle nucleon scattering.

The function $F_{pp'}$ can be expanded in the invariant amplitudes and its form in the nonrelativistic limit for isotopically symmetric matter is[10]

$$F_{pp'}=\frac{\pi^2}{2Mv_F}\{f_{pp'}+\varphi_{pp'}\boldsymbol{\tau}\boldsymbol{\tau}'+\xi_{pp'}\boldsymbol{\sigma}\boldsymbol{\sigma}'+\eta_{pp'}(\boldsymbol{\sigma}\boldsymbol{\sigma}')(\boldsymbol{\tau}\boldsymbol{\tau}')\}.\tag{49}$$

In this case Eq. (48) is readily solved:

$$\widetilde{\delta\varepsilon}_p=(1+\eta_0)^{-1}\frac{g}{2M^*}\boldsymbol{\sigma}\frac{\partial(\boldsymbol{\tau}\boldsymbol{\pi})}{\partial\mathbf{x}}+(1+f_0)^{-1}\frac{g^2}{2M^*}\boldsymbol{\pi}^2,\tag{50}$$

where η_0 and f_0 are the zeroth harmonics of the expansions of the functions $\eta_{pp'}$ and $f_{pp'}$ in Legendre polynomials.

Substitution of (50) in (46) leads to the following expression for the instability parameter ζ in the nonrelativistic limit:

$$\zeta=1-\frac{g^2}{2\pi^2}\frac{v_F}{1+\eta_0}.\tag{51}$$

(We note that the inequality $1+\eta_0>0$ is the condition for the stability of the nucleon liquid relative to perturbations of the Fermi surface in the absence of the pion field.) The constant η_0 in vacuum is known from experiments on the scattering of nonrelativistic nucleons, $\eta_0\approx 0.8$.[10] For nuclear matter, the value $\eta_0\approx 1.6$ can be determined from the experimental data on the magnetic moments of spherical nuclei.[11] The Fermi velocity v_c at which ζ reverses sign, is $v_c\approx 0.28$ at $\eta_0 = 1.6$ and $g^2/4\pi = 14.6$, and is very close to the Fermi velocity of the nucleons at normal nuclear density.

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1)To simplify the notation, we omit hereafter the isotopic indices.

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