Surface thermoelectric effects in superconductors

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The thermoelectric effects in a circuit consisting of bulk superconductors are considered in the case when an undamped current flows in the circuit. It is shown that there arises, along with a volume thermoelectric current of normal excitations, a unique superfluid-velocity-dependent thermoelectric current in the surface region of thickness of the order of the penetration depth λ. The nature of this current corresponds to the effect predicted and considered by Aronov for the case of a thin superconducting cylinder. It turns out that, despite the surface nature of the indicated current, its contribution to the thermoelectric correction to the magnetic flux linked with the circuit is of the same order of magnitude as in the case when the thermoelectric-current is uniformly distributed over the sample cross section with a density equal to the indicated surface value. Also investigated in detail in connection with the temperature dependence of λ is the contribution to the measurable temperature-dependent magnetic flux due to the redistribution of the total flux trapped between the circuit aperture and the surface layer of the superconductor. It is shown that in some fairly typical experimental situations the contribution of the redistribution effect can be appreciably reduced, which facilitates the observation of the thermoelectric effects in its background.

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Noticeable interest is being shown at present in the investigation of thermoelectric effects in superconductors, the possibility of which was first pointed out by Ginzburg.[1] To this question has, in particular, been devoted in recent years a number of theoretical papers.[2-9] The phenomena considered in these papers are due to the fact that, as was suggested by Ginzburg, there arises in superconductors in the presence of a temperature gradient a normal current that in a bulk sample is compensated by the superconducting-condensate current on account of the Meissner effect. In this case, as has been shown,[2,3] since the condensate current is proportional to the order-parameter phase gradient, there arises an order-parameter phase difference at the boundaries of a homogeneous, isotropic sample to which a temperature difference has been applied; this phase difference can be measured with, for example, a superconducting interferometer. In its turn, there arises in the normal thermoelectric circuit consisting of two superconductors an unquantized temperature-dependent correction to the magnetic flux linked with the circuit.1) [3,4]

Similar—in their manifestations—acoustoelectric effects in superconductors (in which the "bare" normal-excitation current is due to the dragging of the electrons by acoustic waves propagating in the sample) were predicted and investigated in Refs. 2, 10, and 11. The photoelectric effect in superconductors has also been investigated.[12]

Let us note a number of papers[12-14] in which the appearance of the above-mentioned effects in sample regions corresponding to the inhomogeneity of the "bare" normal-excitation current in the longitudinal direction (in particular, in the vicinities of contacts and boundaries with dielectrics) has been theoretically studied. It has been shown that in typical experimental situations these contact phenomena cannot significantly change the unquantized correction to the magnetic flux in a closed thermoelectric circuit.

In this case the distribution of the "bare" normal-excitation current was assumed[2-14] to be homogeneous over the sample cross section. Such an assumption corresponds in a great measure to the real picture of the thermoelectric effect, while for the acoustoelectric effect it corresponds to a sound flux uniformly distributed over the sample cross section.

The thermoelectric phenomena consisting in the appearance of a temperature-dependent correction to the magnetic flux in a superconducting thermoelectric circuit have been repeatedly investigated experimentally.[13-17] At the same time, whereas in the work published in Ref. 15 a fairly good agreement with theory[3,4] was obtained, in the investigation published in Ref. 17 (in which the experiment was performed on relatively "dirty" Nb and Ta samples) the observed temperature-dependent correction to the flux exceeded the theoretical estimates[3] by several orders of magnitude. Temperature-dependent magnetic fields significantly exceeding the theoretical values were also observed in the work published in Ref. 18, which was devoted to the experimental investigation of the thermoelectric effects in anisotropic superconductors.[1]

Phenomena of a somewhat different nature have been considered by Aronov.[19] He theoretically investigated the influence on the thermoelectric effects in superconductors of the condensate motion due to the presence of an undamped current. As is well known, in the presence of a condensate current the quasiparticle energy acquires a correction, $\mathbf{p}\cdot\mathbf{v}_s$: $\tilde{\epsilon}_\mathbf{p}=\epsilon_\mathbf{p}+\mathbf{p}\cdot\mathbf{v}_s$ (v_s is the superfluid velocity). It was shown that, owing to this circumstance, there arises in the situation under consideration in the first-order approximation in ∇T a nonequilibrium correction to the quasiparticle distribution function, a correction which depends on the total energy $\tilde{\epsilon}_\mathbf{p}$ (and, on account of this, which relaxes only on the phonons). On the other hand, this correction leads to the appearance of a specific thermoelectric current j_a. The latter depends on the quantity v_s, and does not have a pronounced normal or superconducting character. Since, as a rule, the phonon-induced relaxation time, τ_{ph}, significantly exceeds the impurity-induced relaxation time, τ_i, the current $\mathbf{j}_a$ can, even at not too large values of v_s, exceed the "ordinary" normal-excitation thermoelectric current.

However, in this paper the case of the homogeneous distribution of the currents over the cross section (the superconducting samples being assumed to be thin, i.e., to be of thickness much smaller than the penetration depth λ) was also considered. The realization of such a situation in an experiment is a matter of some difficulty. And in the case of thick samples the corresponding "bare" thermoelectric current, $j_a=f(v_s)$, turns out to be localized in the surface region.

In view of this, it is, in our opinion, of interest to consider the thermoelectric effects in the current state in bulk superconductors, when the corresponding thermoelectric currents are localized near the surface.[2)]

In the situation under discussion, v_s and, consequently, the corresponding source responsible for the state of nonequilibrium are localized in a surface layer of thickness of the order of the penetration depth. Therefore, it is necessary to take into account the diffusion of the nonequilibrium quasiparticles from this region.[3)] We shall see that for the main group of particles this leads to a substantial decrease in the contribution to the effect in comparison with the homogeneous situation.[19] In the surface region, however, there exists, as a result of the presence of the correction $\mathbf{p}\cdot\mathbf{v}_s$ in the total quasiparticle energy, a group of particles whose total energy is less than the gap Δ (which determines the minimum energy of the quasiparticles in the interior of the sample). Therefore, such particles cannot leave the surface region through motion along their trajectories—they are "trapped": it is precisely these trapped particles that make the dominant contribution to $\mathbf{j}_a$. In this case it turns out that, because of the presence of a singularity in the density of states, such a separation of the group of quasiparticles with energies $\tilde{\varepsilon} < \Delta$ (which correspond to only one hemisphere of the Fermi surface) can (for a sufficiently large value of v_s) lead to a nonanalytic dependence of j_a on v_s ($j_a(v_s) \sim v_s^{3/2}$) and, in the final analysis, even to some increase in j_a as compared to the estimates given in Ref. 19.

Let us now turn to electrodynamics. In a bulk sample, the current j_a is nonzero on the surface and decreases with distance from it, and, thus, curl $\mathbf{j}_a \neq 0$; the screening condensate currents due to the presence of the $\mathbf{j}_a$ current behaves in much the same way, so that the distribution of the surface currents of thermoelectric nature has a rotational character. Because of this, there arises a magnetic flux linked with the surface layer of the sample and depends on the temperature gradient.[4)]

However, if we consider a closed thermoelectric circuit, then, since $v_s=0$ in the interior of the samples, the total trapped flux (i.e., the flux linked with a contour going through the interior of the samples) does not change. Consequently, in such a situation there should arise an experimentally-measurable unquantized correction to the flux in the aperture. We shall show that, in spite of the surface nature of the thermoelectric current $\mathbf{j}_a$, the magnitude of the effect can be of the same order as in the case when the "bare" thermoelectric current is uniformly distributed over the sample cross section with a density equal to the surface value of $\mathbf{j}_a$. It seems that this conclusion is also valid for surface effects of a different nature, that correspond to different mechanisms of generation of the nonequilibrium surface currents, in particular, for the acoustoelectric effect, which occurs during the propagation of surface acoustic waves (which generate a surface current of dragged normal excitations).

We shall also consider in detail another effect that appears in the presence of undamped condensate currents in a thermoelectric circuit. This effect was first discussed and experimentally observed by Pegrum and Guenault.[20] It consists in the following. The indicated undamped currents give rise to some magnetic flux that threads the sample's surface layer of thickness ~λ. On account of the dependence of λ, as well as of the "number of superconducting electrons," N_s, on temperature, this flux is temperature dependent. However, the total flux trapped by the superconducting circuit remains unchanged; therefore, a change in the surface flux should lead to a change in the flux in the circuit aperture. In other words, a change in the temperature of one of the junctions leads to a redistribution of the total flux between the surface layer of the sample and the aperture.[5)] This redistribution leads to the appearance of a temperature-dependent, experimentally-measurable correction to the flux in the aperture. Let us note that, compared to the "thermoelectric" flux, the indicated correction can be quite substantial even for low intensities of the undamped currents (due, for example, to the remanent "background" magnetic fields in the system).

We shall investigate the case of two different geometric configurations of the thermoelectric circuit, that correspond to a long cylinder and a thin ring. It will be shown that, whereas the thermoelectric effects weakly depend on the geometry of the experiment, the effect of the redistribution can significantly depend on the geometry factors. In particular, in the case of a thin ring (of dimensions much greater than the thickness of the conductors), if the trapped flux is due to an external magnetic field that does not change after the superconducting transition, the influence of the effect of the redistribution turns out to be substantially reduced. On the other hand, this significantly reduces the limitations, which follow from estimates made in Ref. 20, on the background magnetic fields on the investigation of the normal volume thermoelectric effects in superconductors.[1-9] On the other hand, as we shall see, with allowance for the indicated factor even for the surface thermoelectric effects, which are fundamentally connected with the presence of a trapped flux, the thermoelectric correction to the flux can be comparable in magnitude to the contribution of the redistribution effect. It seems that the present circumstances, as well as the differences in the parities of the thermoelectric effects and the effect of the redistribution with respect to ∇T and $\mathbf{v}_s$, simplify the problem of the experimental separation of these effects. We shall also consider possible explanations of the experimental data on the investigation of the thermoelectric effects in superconductors[15-18] within the scope of the allowance for the contribution of the redistribution effect (the necessity of which is pointed out in Ref. 20).

1. To begin with, let us determine the magnitude of the specific thermoelectric current, $\mathbf{j}_a$, that arises near the surface of bulk superconductors in the presence of undamped condensate currents. Here, since we assume characteristic sample dimensions much greater than the penetration depth, in computing this current we can restrict ourselves to the geometry of a superconducting half-space, assuming ∇T to be directed along the surface. We shall also assume that $\mathbf{v}_s \parallel \nabla T$, which is the most interesting situation for experiment (as is easy to see, when $\mathbf{v}_s \perp \nabla T$, the effect can arise only in higher orders in $\mathbf{v}_s$ or ∇T). In this case, to simplify the analysis, let us require that

$$\mathbf{p}\mathbf{v}_s \ll \min(\Delta, T). \tag{1}$$

We shall use the kinetic equation to compute $\mathbf{j}_a$.[10,3] Therefore, let us assume that the appropriate conditions for its applicability are fulfilled[10]: $l \gg \xi, \lambda \gg \xi$ ($\xi = \hbar v_F/\Delta$ is the coherence length and l is the mean free path of the electrons). Notice that the second condition ensures the applicability of the London equation, which we shall use below. We have noted that the group of particles localized in the surface layer as a result of the presence of the correction $\mathbf{p}\cdot\mathbf{v}_s$ in the total energy turns out to be important in the computation of $\mathbf{j}_a$. As Azbel'[21] (who has investigated in detail the energy spectrum of the particles localized near the surface) has shown, the classical description for such particles for arbitrary values of the momentum is applicable only for the not too weak magnetic fields corresponding, in the case of a type-II superconductor, to the inequality[6)]

$$\mathbf{p}\mathbf{v}_s/\Delta \gg (\xi/\lambda)^2. \tag{1a}$$

Because $\lambda/\xi \gg 1$, this inequality is consistent with (1); we shall assume it to be fulfilled.

The present conditions, which correspond to a pure type-II superconductor, are fairly rigid. However, on the one hand, the approach based on the kinetic equation provides quite a graphic physical picture. On the other hand, let us emphasize that the indicated limitations apply only to the computation of the magnitude of the current density $\mathbf{j}_a$. At the same time, the distinctive features of the electrodynamics of the effects under study are connected only with the fact that this current is localized near the surface at depths $\sim\lambda$; the specific decay law only determines a number of the order of unity. Therefore, the electrodynamic calculation (Secs. 2 and 3) and its results are valid irrespective of the numerical j_a values, which can correspond to a broader parameter region than is considered here. In particular, as has been demonstrated,[21] the quasiparticle states localized near the surface (states from which a substantial contribution to the thermoelectric effect can be expected) exist also in the region of weaker magnetic fields, as well as in type-I superconductors (although the quantity j_a itself may then differ from the quantity computed by us).

With allowance for the foregoing, we have for the quasiparticle distribution function $n_\mathbf{p}$ the equation

$$\frac{\partial\tilde{\varepsilon}_\mathbf{p}}{\partial\mathbf{p}}\frac{\partial n_\mathbf{p}}{\partial\mathbf{r}} - \frac{\partial\tilde{\varepsilon}_\mathbf{p}}{\partial\mathbf{r}}\frac{\partial n_\mathbf{p}}{\partial\mathbf{p}} = \hat{I}_i n_\mathbf{p} + \hat{I}_{ph} n_\mathbf{p}, \tag{2a}$$

where

$$\tilde{\varepsilon}_\mathbf{p} = \varepsilon_\mathbf{p} + \mathbf{p}\mathbf{v}_s, \quad \varepsilon_\mathbf{p} = (\xi_\mathbf{p}^2 + \Delta^2)^{1/2}, \quad \xi_\mathbf{p} = p^2/2m + p_z^2/2m - \mu$$

(μ is the electrochemical potential; for simplicity of computation, we shall everywhere below assume the electron spectrum to be isotropic and quadratic); $\hat{I}_i$ and $\hat{I}_{ph}$ are the operators of collision respectively with impurities and phonons. Separating the equilibrium part, $n_0(\tilde{\varepsilon}_p)$, and taking into consideration the fact that $\nabla\mu = 0$ in a superconductor, we have in the linear approximation in ∇T the equation

$$\frac{\partial\tilde{\varepsilon}_\mathbf{p}}{\partial p_x}\nabla T\frac{\tilde{\varepsilon}_\mathbf{p}}{T}\frac{\partial n_0}{\partial\tilde{\varepsilon}_\mathbf{p}} + \frac{\partial\tilde{\varepsilon}_\mathbf{p}}{\partial p_z}\frac{\partial n_\mathbf{p}^1}{\partial z} - \frac{\partial\tilde{\varepsilon}_\mathbf{p}}{\partial z}\frac{\partial n_\mathbf{p}^1}{\partial p_z} = \hat{I}_i n_\mathbf{p}^1 + \hat{I}_{ph} n_\mathbf{p}^1 \tag{2b}$$

($x \parallel \nabla T \parallel \mathbf{v}_s$; z is measured from the surface along the normal to it). Let us separate from the function $n_\mathbf{p}^1$ the part that depends on the momentum only through the total energy $\tilde{\varepsilon}_\mathbf{p}$:

$$\begin{gathered} n_\mathbf{p}^1 = \bar{n} + \{n\}^-, \\ \bar{f}(\varepsilon, z) = \frac{1}{\rho(\varepsilon, z)}\sum_\mathbf{p}\delta(\varepsilon - \tilde{\varepsilon}_\mathbf{p}(z))f_\mathbf{p}, \\ \rho(\varepsilon, z) = \sum_\mathbf{p}\delta(\varepsilon - \tilde{\varepsilon}_\mathbf{p}(z)), \quad \{f\}^- = f - \bar{f}. \end{gathered} \tag{3}$$

For the purpose of obtaining the equation for $\bar{n}$, let us average (2a) over the constant-energy surface for $\tilde{\varepsilon}$ (at fixed z) after separating the symmetric and antisymmetric parts. Since

$$\hat{I}_i f = \sum_{\mathbf{p}'} W_{\mathbf{p}\mathbf{p}'}(f_\mathbf{p} - f_{\mathbf{p}'})\left(1 + \frac{\xi_\mathbf{p}\xi_{\mathbf{p}'} - \Delta^2}{\varepsilon_\mathbf{p}\varepsilon_{\mathbf{p}'}}\right)\delta(\tilde{\varepsilon}_\mathbf{p} - \tilde{\varepsilon}_{\mathbf{p}'}), \tag{4}$$

we have $\overline{\hat{I}_i f} = 0$. With allowance for this, and introducing the operator

$$\hat{B} = \frac{\partial\tilde{\varepsilon}_\mathbf{p}}{\partial p_z}\frac{\partial}{\partial z} - \frac{\partial\tilde{\varepsilon}_\mathbf{p}}{\partial z}\frac{\partial}{\partial p_z}$$

we obtain

$$\overline{\frac{\partial\tilde{\varepsilon}_\mathbf{p}}{\partial p_x}}\nabla T\frac{\tilde{\varepsilon}_\mathbf{p}}{T}\frac{\partial n_0}{\partial\tilde{\varepsilon}_\mathbf{p}} + \overline{\hat{B}\{n\}^-} = \overline{\hat{I}_{ph}\bar{n}}, \tag{5a}$$

$$\left\{\frac{\partial\tilde{\varepsilon}_\mathbf{p}}{\partial p_x}\right\}^-\nabla T\frac{\tilde{\varepsilon}_\mathbf{p}}{T}\frac{\partial n_0}{\partial\tilde{\varepsilon}_\mathbf{p}} + \{\hat{B}\{n\}^-\}^- + \{\hat{B}\bar{n}\}^- = \hat{I}_i\{n\}^-. \tag{5b}$$

We have used the relations $\hat{B}f(\tilde{\varepsilon}_\mathbf{p}) = 0$, $\hat{I}_i f(\tilde{\varepsilon}_\mathbf{p}) = 0$, and have neglected the term $\hat{I}_{ph}n^1$ in (5b) (on the basis of the fact that $\tau_{ph} \gg \tau_i$ and $v_F\tau_{ph} \gg \lambda$) and the term $\hat{I}_{ph}\{n\}^-$ in (5a). Let us now turn to the quantity

$$\overline{\frac{\partial \tilde{\varepsilon}_p}{\partial p_x}} = \frac{1}{\rho(\varepsilon, z)} \sum_p \delta(\varepsilon - \tilde{\varepsilon}_p) \left(v_x \frac{\xi_p}{\varepsilon_p} + v_s \right).$$

Let us transform the sum over p:

$$\int d^3p = \int d\Omega \int_0^\infty p^2\, dp = m \int d\varphi \int d(\cos\vartheta) \int_{-\mu}^{\infty} d\xi_p p.$$

Here as the polar axis we have chosen the x axis, so that $\cos\vartheta = p_x/p_F$. Notice that for the trapped particles ($\tilde{\varepsilon}_p - \Delta < 0$) only values of $\cos\vartheta < 0$ are possible, the limiting $\cos\vartheta$ value for trapped particles with a given energy $\tilde{\varepsilon}_p = \epsilon$ being determined by the quantity

$$a(\varepsilon, z) = \frac{\varepsilon^2 - \Delta^2}{2\varepsilon p_F v_s}. \tag{6}$$

Let us point out that we should, in integrating over ξ_p, take into consideration the fact that $\mathbf{p}\cdot\mathbf{v} = 2(\mu + \xi)$. Finally, we obtain

$$\overline{\frac{\partial \tilde{\varepsilon}_p}{\partial p_x}} = v_s \mathscr{F}(\varepsilon, z); \quad \mathscr{F} = \begin{cases} \frac{1}{3}, & \frac{\varepsilon^2 - \Delta^2}{\varepsilon} \gg p_F v_s \\ \frac{8}{15}\left(a^2 - \frac{1}{2}a + \frac{3}{8}\right), & \varepsilon < \Delta \end{cases}. \tag{7}$$

This differs somewhat from the value used in Ref. 19, since we have taken into account the fact that the result of the d^3p integration of the functions that are odd in ξ are nonzero in first order in ξ/μ.

With the aid of simple transformations we can show that:

$$\overline{\hat{B}\{n\}^-} = \frac{1}{\rho(\varepsilon, z)} \operatorname{div} \mathbf{j}_\varepsilon = \frac{1}{\rho} \frac{\partial}{\partial z} j_\varepsilon, \quad \mathbf{j}_\varepsilon = \sum_p \delta(\varepsilon - \tilde{\varepsilon}_p) \frac{\partial \tilde{\varepsilon}_p}{\partial \mathbf{p}} \{n\}^-. \tag{8}$$

Thus, the equation for $\bar{n}$ assumes the form

$$F(\tilde{\varepsilon}_p, r) + \frac{1}{\rho} \operatorname{div}\Big|_{\tilde{\varepsilon}_p = \text{const}} \mathbf{j}_\varepsilon = \overline{\hat{I}_{ph}\bar{n}}, \tag{9}$$

where

$$F = (\mathbf{v}_s \nabla T) \mathscr{F} \frac{\tilde{\varepsilon}_p}{T} \frac{\partial n_0}{\partial \tilde{\varepsilon}_p}. \tag{9a}$$

To begin with, let us consider the situation $l \ll \lambda$, which corresponds to the most graphic physical picture. From (5b) we then obtain

$$\{n\}^- \approx \hat{I}_i^{-1} \left[\left\{ \frac{\partial \tilde{\varepsilon}_p}{\partial p_x} \right\}^- \nabla T \frac{\tilde{\varepsilon}_p}{T} \frac{\partial n_0}{\partial \tilde{\varepsilon}} + \{\hat{B}\bar{n}\}^- \right]. \tag{10}$$

The first term on the right-hand side of (10) is responsible for the "normal" volume thermoelectric effect. As is easy to see, it does not make a contribution when substituted into the expression for $\overline{\hat{B}\{n\}^-}$ because of the oddness of the operator $\hat{B}$ with respect to v_s. (Notice that the oddness of the first term with respect to ξ_p allows us to neglect the quantity $\hat{I}_{ph}\{n\}^-$ in the equation for $\bar{n}$, something which we did earlier.) Substituting the second term into (9), and taking (8) into account, we obtain the diffusion equation

$$F(\tilde{\varepsilon}_p, z) + \frac{1}{\rho(\tilde{\varepsilon}_p, z)} \frac{\partial}{\partial z}\Big|_{\tilde{\varepsilon}_p = \text{const}} \rho(\tilde{\varepsilon}_p, z) D_{zz} \frac{\partial}{\partial z}\Big|_{\tilde{\varepsilon}_p = \text{const}} \bar{n} = \overline{\hat{I}_{ph}\bar{n}}, \tag{11}$$

where

$$D_{zz} = \overline{\frac{\partial \tilde{\varepsilon}_p}{\partial p_z} \hat{I}^{-1} \frac{\partial \tilde{\varepsilon}_p}{\partial p_z}}.$$

Then, using the explicit form of the operator $\hat{I}_i$, (4), we can easily show that $D_{zz}(\tilde{\varepsilon}_p, z) \sim v_F^2 \tau_i |\xi_p|/\epsilon_p$ for all values of the total energy $\tilde{\varepsilon}_p$.

Let us, to begin with, consider the region of untrapped particles, whose characteristic energies correspond to $\tilde{\varepsilon}_p - \Delta \gg p_F v_s$. In such a case the dependence on v_s in the expressions for $\rho(\tilde{\varepsilon}_p, z)$ and $D(\tilde{\varepsilon}_p, z)$ can be neglected: $\rho \approx \rho(\epsilon_p)$ and $D \approx D(\epsilon_p)$. The diffusion equation (11) then assumes the usual form, with $F(\epsilon, z) = F_0(\epsilon) e^{-z/\lambda}$ when (7) is taken into consideration. We also assume (in accordance with the results of Ref. 19) the admissible estimate $\hat{I}_{ph}\bar{n} \sim \bar{n}/\tau_{ph}$, $\tau_{ph} \sim \Theta^2/T^3$. Solving, with allowance for this, Eq. (11) by standard methods with the use of the boundary conditions $\bar{n}(\infty) = 0$, $D\nabla\bar{n}|_{z=0} = 0$ (the second condition guarantees the vanishing of the current j_ϵ at the surface), we have

$$\bar{n} \approx \frac{\lambda L_0}{D_{zz}} F_0 e^{-z/\lambda}, \quad L_0 = (D_{zz}\tau_{ph})^{1/2}. \tag{12}$$

We have used the fact that, in reality, $L_0 \gg \lambda$ (since the applicability of the kinetic equation requires, in any case, that $l \gg \xi$). Since $\lambda L_0/D = \tau_{ph}\lambda/L_0$, the contribution of these particles to the effect turns out to be considerably less than in the homogeneous situation, which is connected with the removal of particles from the nonequilibrium region on account of diffusion.

Let us therefore turn to the group of trapped particles, which cannot leave the near-surface region. Let us, by integrating it, transform (11) into an integral equation:

$$\bar{n} = C_2 + \int_0^z dz' \frac{1}{\rho D} \left[C_1 + \int_0^{z'} \left(\frac{\bar{n}}{\tau_{ph}} - F \right) \rho\, dz'' \right]. \tag{13}$$

The boundary conditions correspond to the vanishing of the current j_ϵ on the surface at the point $z_0(\tilde{\varepsilon}_p)$ determined by the condition $\xi(\tilde{\varepsilon}_p, z_0) = 0$ (i.e., at the farthest attainable point for particles with a given energy, $\tilde{\varepsilon}_p$):

$$\rho(\varepsilon, z) D(\varepsilon, z) \nabla\bar{n}|_{0, z_0} = 0.$$

With allowance for this, we have

$$C_1 = 0, \quad \int_0^{z_0} \left(\frac{\bar{n}}{\tau_{ph}} - F \right) \rho\, dz = 0. \tag{13a}$$

Since for almost all z (with the exception of a small neighborhood of z_0), $D \gg \lambda^2/\tau_{ph}$, while with allowance for (13a) the integral over z' converges as $z' \to z_0$, we obtain from (13) the estimate

$$\bar{n}|_{\tilde{\varepsilon}<\Delta} \sim C(\tilde{\varepsilon}_p) = \tau_{ph} \langle F \rangle_z, \tag{14}$$

where $\langle \ldots \rangle_z$ denotes averaging over z within the limits

$(0, z_0)$. Thus, for the trapped particles, $\bar{n}$ is of the same order of magnitude as in the homogeneous situation. So, in going from the trapped to the untrapped particles $\bar{n}$ decreases sharply on account of the "switching on" of diffusion. Let us estimate the width of the energy region corresponding to the transition from one regime to the other. For this purpose, let us take into consideration the fact that the diffusion coefficient, D, for the untrapped particles that are close to the trapped ones turns out to be small outside the localization region for v_s. Indeed, for $\tilde{\varepsilon}_p - \Delta \ll \Delta$ and $z \gtrsim \lambda$, we have $D \sim v_F^2 \tau_1 (\epsilon_p - \Delta)^{1/2}/\Delta$; thus, the reciprocal of the effective time of removal of such a particle from the surface layer is equal to

$$\frac{1}{\tau_{eff}} \sim \left(\frac{\varepsilon_p - \Delta}{\Delta}\right)^{1/2} \frac{v_F^2 \tau_1}{\lambda^2}. \tag{15}$$

Comparing this time with τ_{ph}, we obtain the sought "transition" region width: $\tilde{\varepsilon}_p - \Delta \sim \Delta\lambda/L_0$. Notice that, with allowance for (1a) in the case when the classical description is applicable, this width is, in the most realistic situation when $\lambda^3/\xi^2 L_0 \ll 1$, significantly less than the width of the trapped-particle region in energy terms, which is $\sim p_F v_s$, and this allows us to limit ourselves to the consideration of the trapped particles. (In the opposite case such an approach would require the following additional limitation imposing a lower bound on v_s: $p_F v_s \gg \Delta\lambda/L_0$.)

Let us now turn to the case of a purer sample, for which $l \gg \lambda$. We have seen that for $l \ll \lambda$ diffusion guarantees the effective removal of the untrapped particles from the surface layer and the smallness of $\bar{n}$ in this energy region. It is clear that this result is also true in the $l \gg \lambda$ case, since the efficiency of removal increases with increasing l right up to $l \sim \lambda$ and then, as is not difficult to understand, ceases to depend on l. As to the trapped particles, for $l \ll \lambda$, the diffusion ensured the equalization of the concentration of such particles with a given energy, so that the energy distribution $\bar{n}(\tilde{\varepsilon}_p)$ did not depend on the coordinates. It is natural to expect this result, (14), to remain valid in the $l \gg \lambda$ case (in such a situation we can speak of a particle distribution over quasiclassical surface levels[21]). This can be shown more rigorously. In particular, using the solution to Eq. (5b), expressed in terms of a path integral, we can obtain $\{n\}^- \sim (\bar{n} - \langle \bar{n} \rangle_z)/\lambda$. Accordingly, from (11) with allowance for the fact that $\langle v_z | \xi_p | / \tilde{\varepsilon}_p \rangle_z \tau_{\text{ph}}/\lambda \gg 1$, we obtain the estimate (14).

Now we can find the thermoelectric current $\mathbf{j}_a$ due to the nonequilibrium correction, $\bar{n}$, to the quasiparticle distribution function; in this case, in accordance with our estimates, we can neglect the contribution of the untrapped particles, restricting ourselves to the region of energies $\tilde{\varepsilon}_p < \Delta$. As Aronov[19] has shown, the contribution of $\bar{n}$ to the current is due, generally speaking, to both the presence in $\bar{n}$ of a current part and the renormalization of N_s. Let us first estimate the direct contribution of $\bar{n}$ to the current:

$$j_a' = 2e \sum_p v_x \bar{n} = 4ep_F \int_0^{2\pi} d\varphi \int_{-1}^{0} d(\cos\vartheta)\cos\vartheta \int_{\Delta}^{\Delta - p_F v_s \cos\vartheta} d\varepsilon_p \frac{\varepsilon_p}{(\varepsilon_p^2 - \Delta^2)^{1/2}} \bar{n}(\tilde{\varepsilon}_p), \tag{16}$$

or with the use of the explicit form of $\bar{n}$ (determinable on the basis of (14), (9a), and (7)), as well as with allowance for the relation

$$2 \sum_p \frac{\partial n_0}{\partial \varepsilon_p} = \frac{1}{\mu}(N_s - N),$$

we obtain the estimate

$$j_a' \sim e\tau_{ph} \left(\frac{|v_s^0| \nabla T}{T}\right) \frac{\Delta}{p_F}(N_s - N) \frac{(p_F |v_s^0| \Delta)^{1/2}}{T} f(z). \tag{17}$$

Here

$$f(z) \leqslant C \exp(-\alpha z/\lambda), \ \alpha \sim 1, \ C \sim 1; \ v_s^0 = v_s(z=0).$$

It is not difficult to see that the obtained estimate exceeds the estimate given in Ref. 19 by the parameter $(p_F v_s/\Delta)^{-1/2} \gg 1$. The nonanalyticity in (17) is connected, on the one hand, with the presence of a singularity in the density of states and, on the other, with $\bar{n}(\tilde{\varepsilon}_p)$'s behavior in the low-energy region, which corresponds to the separation of the contribution of the trapped particles (for which $p_x < 0$, and, thus, the current part of $\bar{n}$ does not possess an additional smallness).[7)]

As to the renormalization of N_s, it can be shown (on the basis of estimates similar to those given in Ref. 19) that, on account of the smallness—in terms of phase volume—of the group of trapped particles, this renormalization and the contribution to j_a connected with it are smaller by a factor equal to the parameter $(p_F v_s \Delta)^{1/2}/T$ that the corresponding results for the homogeneous situation.[19] Thus, the contribution, computed by us, of the current part of $\bar{n}$ is the dominant one, and we set $j_a \approx j_a'$.

2. Let us now proceed to the electrodynamics of the surface thermoelectric effects. Let us, to begin with, consider the thermoelectric effects in the current state in a long thick cylinder (whose height, h, is much greater than the bore radius, R, and the wall thickness $d \gg \lambda$) composed of two superconductors (Fig. 1a). In the present geometry the field in the cylinder bore is determined only by the magnitude of the total ring current flowing along the inner surface of the cylinder, and does not depend on the distribution of this current in the surface layer. (This important circumstance, which significantly simplifies the calculation, is due to the fact that, for $h \gg R$, the field of the cylindrical current inside the corresponding cylindrical surface is uniform.)

Let an undamped current flow along the inner surface, and let the corresponding superfluid velocity be equal to v_{s0}. The condition $R \gg \lambda$ allows us to reduce the prob-

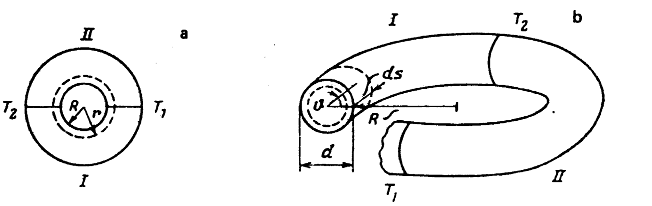

FIG. 1.

lem of the distribution of the current near the surface to a one-dimensional problem by introducing the variable $z=r-R$ and to describe the displacement along the surface by the variable s (so that $T=T(s)$). To find the corresponding "thermoelectric" correction to the magnetic flux in the bore, we should use the equations of the electrodynamics of a superconductor with allowance for the "bare" thermoelectric current. Because of the presence of a temperature gradient in the circuit, there arise both a volume current of normal excitations, j_T (determined by the first term in the expression for the distribution function $\{n\}^-$, (10)), and the specific current, j_a, predicted by Aronov and estimated by (17). As has already been noted, in not too pure superconductors the current density, j_a, near the surface can significantly exceed the quantity j_T; with allowance for (10) and (17) the ratio of these quantities is equal to

$$\sim\frac{\tau_{ph}}{\tau_i}\frac{p_F v_s (p_F v_s \Delta)^{1/2}}{T^2}.$$

For simplicity, we shall consider just this situation. (In the opposite case the major role will be played by the "volume" thermoelectric effects.[1-9])

The presence of the current $\mathbf{j}_a$ gives rise to screening condensate currents, so that the total density of the surface currents has the form

$$\mathbf{j}=eN_s\mathbf{v}_{s0}+\mathbf{j}_a+eN_s\mathbf{v}_{sa},$$

where $\mathbf{v}_{sa}$ is the corresponding correction to the superfluid velocity. Since, in reality, as we shall see, $v_{sa} \ll v_{s0}$, the problem is linear in ∇T, so that in computing the required "thermoelectric" correction to the vector potential $\mathbf{A}_a$ we can regard $\mathbf{j}_a$ as given. With allowance for the expression for the superfluid velocity,

$$\mathbf{v}_s=\frac{\hbar}{2m}\left(\nabla\chi-\frac{2e}{\hbar c}\mathbf{A}\right)=-\frac{e}{mc}\tilde{\mathbf{A}} \tag{18}$$

(χ is the phase of the order parameter), we obtain an equation for $\tilde{\mathbf{A}}_a$:

$$\text{curl curl }\tilde{\mathbf{A}}_a=-\frac{1}{\lambda^2}\tilde{\mathbf{A}}_a+\frac{4\pi}{c}\mathbf{j}_a, \tag{19}$$

with curl curl $\to -\partial^2/\partial z^2$ owing to the condition $R\gg\lambda$.

In studying the electrodynamics we shall, for simplicity, operate within the framework of a model calculation, replacing in (17) $f(z)$ by the exponential law[8)] $j_a=j_a^0 e^{-z/L_a}$, where

$$j_a^0\sim e\tau_{ph}\frac{|v_s^0|\nabla T}{T}(N_s-N)\frac{\Delta}{p_F}\frac{(p_F|v_s^0|\Delta)^{1/2}}{T}. \tag{20}$$

Such an approach is admissible, since, on the one hand, all our calculations are order-of-magnitude estimates (for which only the characteristic penetration depth of the current distribution is important, whereas the specific distribution law determines a number of the order of unity). On the other hand, the generalization of the results obtained in the computation of the electrodynamics of the effect to the case of surface effects of a different nature (e.g., the acoustoelectric effect, arising during the propagation of a surface acoustic wave) seems possible.

One of the constants determining the solution to (19) corresponds to the obvious condition $\tilde{A}\to 0$ as $z\to\infty$. Another constant is specified by the magnitude of the corresponding total current of thermoelectric nature:

$$\int_0^\infty dz\,(j_a+eN_s v_{sa})=I_a.$$

Strictly speaking,

$$I=\int_0^\infty j\,dz$$

is the current per unit length of the cylinder, i.e., the surface density of the current. However, in the case of cylindrical symmetry we shall, for simplicity, call this quantity the total current.

For convenience of the subsequent analysis, let us separate from $\tilde{A}_a$ the part

$$\tilde{A}_{a2}=-I_a\frac{4\pi}{c}\lambda e^{-z/\lambda},$$

which satisfies the homogeneous equation, and which corresponds to the condition

$$\int_0^\infty j_{a2}\,dz=-\int_0^\infty\frac{c}{4\pi\lambda^2}\tilde{A}_{a2}\,dz=I_a.$$

The quantity $A_a-A_{a2}\equiv A_{a1}$ then satisfied (19) and corresponds to zero total current (as in the case of an open sample):

$$\int_0^\infty j_{a1}\,dz=\int_0^\infty\left(-\frac{c}{4\pi\lambda^2}\tilde{A}_{a1}+j_a\right)dz=0. \tag{21}$$

We shall call the current density, j_{a1}, which does not contribute to the total current, the eddy component. Taking (21) into consideration, we obtain for $\tilde{A}_{a1}$ the expression

$$\tilde{A}_{a1}=-\left[\frac{\lambda^3 L_a}{L_a^2-\lambda^2}j_a^0\right]\frac{4\pi}{c}e^{-z/\lambda}+\frac{4\pi}{c}\frac{\lambda^2 L_a^2}{L_a^2-\lambda^2}j_a^0 e^{-z/L_a}. \tag{22}$$

The quantity I_a is determined from the following arguments. Let us find the thermoelectric correction, Φ_a, to the flux linked with the circuit:

$$\Phi_a=\oint\tilde{A}_a|_{z=0}\,ds.$$

On the other hand, in the case of a long superconducting cylinder this correction to the flux is due only to the magnitude of the total current I_a:

$$\Phi_a=\frac{1}{c}I_a\mathscr{L},\quad \mathscr{L}=2\pi^2R^2.$$

Thus, we obtain the equation

$$I_a\mathscr{L}=\oint ds\,(\tilde{A}_{a2}|_{z=0}+\tilde{A}_{a1}|_{z=0})=\oint ds\left[-\frac{4\pi}{c}\lambda I_a+\frac{4\pi}{c}j_a^0\frac{\lambda^2 L_a}{\lambda+L}\right]. \tag{23}$$

Since

$$\oint ds\,\tilde{A}_a|_{z\to\infty}\to 0,$$

the right-hand side of (23) can be interpreted as a flux linked with the near-surface region of the superconductor, and Eq. (23) can be interpreted as the invariance condition for the total flux trapped by the cylinder. As is not difficult to see from (23), the mean current density in the surface layer, which is $\sim I_a/\lambda$, turns out to be smaller by a factor $\sim\lambda/R$ than the characteristic value of the eddy component j_{a1}; in this case we can neglect on the right-hand side of (23) the term $\sim I_a$. Thus, we can assume that, in the first approximation, the distribution of the surface currents of thermoelectric nature has a purely eddy character, while the correction to the total current I_a is determined in the next order in λ/R from the requirement of invariance of the total flux. Taking account of the foregoing, we have

$$\Phi_a\approx\oint ds\frac{4\pi}{c}j_a^0\frac{\lambda^2L_a}{L_a+\lambda}. \qquad (24)$$

Or in our case, in which $L_a\sim\lambda$,

$$\Phi_a\approx\oint ds\,\lambda^2\frac{4\pi}{c}j_a^0. \qquad (25)$$

Notice that it is precisely the correction to the flux in the cylinder bore that will be recorded in an experiment with the aid of a magnetometer coupling coil located in the bore.

By comparing (25) with the results obtained in Ref. 3, we can easily convince ourselves that in the case under consideration the correction, Φ_a, to the magnetic flux is of the same order of magnitude as in the case when the "bare" thermoelectric current is uniformly distributed over the sample thickness with a density equal to the surface-current value j_a.

Since $(N-N_s)$ decreases exponentially as $T/T_c\to 0$, we shall, for simplicity, assume that for one of the conductors (II) $T\ll T_{cII}$, while for the conductor I the quantity $(T_{cI}-T_I)\sim T_I$. Therefore, only the conductor I makes a substantial contribution to the effect, and we shall drop the index I from the material constants. In that case, on the basis of (24) and with allowance for (20), we obtain after simple transformations the estimates:

$$\frac{\Phi_a}{\delta T}\sim\frac{\Phi_0}{T}\left(\frac{I_0}{\Phi_0 c}\right)^{3/2}\tau_{ph}\frac{\Delta_0}{p_F}(\xi_0^{1/2}\lambda_0^{3/2})8\pi^{5/2}F_a(T_1,T_2)$$

$$F_a=\begin{cases}\dfrac{N_s-N}{N_s}\dfrac{\Delta}{T}\left(\dfrac{\Delta}{\Delta_0}\right)^{1/2}\left(\dfrac{\lambda}{\lambda_0}\right)^{3/2}, & \delta T\ll T_c-T_1\\[2ex] \dfrac{1}{2}\dfrac{\Delta_0}{\delta T}\ln\left(\dfrac{T_c-T_1}{T_c-T_2}\right), & \dfrac{T_c-T_1}{T_c}\ll 1.\end{cases} \qquad (26)$$

Or, taking into account the fact that $\Phi\approx c^{-1}\mathcal{L}I_0$, we obtain

$$\frac{\Phi_a}{\delta T}\sim\frac{\Phi}{T}\left(\frac{\Phi}{\Phi_0}\right)^{1/2}\tau_{ph}\frac{\Delta_0}{p_F}\frac{\xi_0^{1/2}\lambda_0^{3/2}}{\mathcal{L}^{3/2}}8\pi^{5/2}F_a. \qquad (26a)$$

Here $\delta T=T_2-T_1$, Φ is the total flux trapped by the contour, Φ_0 is the flux quantum, and $\Delta_0, \xi_0, \lambda_0=\Delta, \xi, \lambda$ at $T=0$.

Let us now find the temperature-dependent correction to the flux in the bore, due to the redistribution effect. The latter, as has been noted eariler, is caused by the temperature dependence of the flux produced in the surface layer of the superconductor by an undamped current of density $j_0=(I_0/\lambda)e^{-z/\lambda}$. Let us express the magnitude of the gradient-invariant vector potential, $\tilde{A}_0=-4\pi c^{-1}\lambda^2 j_0$, corresponding to this current in terms of the total superfluid current I_0, which, in the zeroth approximation in λ/R, we can assume to be temperature independent:

$$\tilde{A}_0|_{z=0}=-\frac{4\pi}{c}\lambda(T)I_0. \qquad (27)$$

Since $\tilde{A}\to 0$ as $z\to\infty$, the circulation $(-\tilde{A}_0)$ along the inner surface of the cylinder yields the required temperature-dependent near-surface flux, which, added to the flux in the bore, constitutes the total trapped flux

$$\Phi=\frac{\Phi_0}{2\pi}\oint(\nabla\chi)_0\,ds=n\Phi_0.$$

Thus, the temperature-dependent correction to the flux in the bore is equal to

$$\Delta\Phi=\oint\tilde{A}_0|_{z=0}ds=\frac{4\pi}{c}I_0\oint ds\,\lambda(s)=\frac{4\pi}{c}\frac{I_0}{\nabla T}\int_{T_1}^{T_2}dT(\lambda_I(T)+\lambda_{II}(T)). \qquad (28)$$

Since, as a rule, in experiments we investigate the dependence on the temperature of the "hot" junction, let us separate out the corresponding contribution, subtracting from the integrand in (28) the quantity $[\lambda_I(T_1)+\lambda_{II}(T_1)]$; for $\delta T\ll T_c-T_1$ we have

$$\frac{\Delta\Phi(T_2)}{\delta T}=\frac{4\pi}{c}\frac{I_0}{\nabla T}\left[\frac{\partial}{\partial T}(\lambda_I+\lambda_{II})\right]\delta T=\Phi\left(\frac{4\pi}{\mathcal{L}}\right)\frac{\delta T}{\nabla T}\frac{\partial}{\partial T}(\lambda_I+\lambda_{II}). \qquad (29)$$

If the temperature T_I is close to T_{cI}, $(T-T_{cI})/T_{cI}\ll 1$, then the first conductor makes the dominant contribution:

$$\frac{\Delta\Phi(T_2)}{\delta T}=\frac{4\pi}{c}\frac{I_0}{\nabla T}\lambda_0\frac{T_c^{1/2}\delta T}{(T_c-T_1)^{3/2}}F_0(T_2)=\Phi\left(\frac{4\pi}{\mathcal{L}}\right)\frac{\lambda_0}{\nabla T}\frac{T_c^{1/2}\delta T}{(T_c-T_1)^{3/2}}F_0(T_2),$$

$$F_0(T_2)=(T_c-T_1)\left[\frac{(T_c-T_1)-(T_c-T_2)-2[(T_c-T_1)(T_c-T_2)]^{1/2}}{(T_2-T_1)^2}\right] \qquad (29a)$$

(the plot of F_0 is shown in Fig. 2).

Let us point out that, separating from the original superfluid current density the part that does not depend on temperature, we can ascribe the temperature-dependent part of the near-surface flux (with which the effect under consideration is connected) to the purely eddy component:

$$j_0=\frac{I_0}{\lambda_0}e^{-z/\lambda_0}+I_0\left(\frac{e^{-z/\lambda(T)}}{\lambda(T)}-\frac{e^{-z/\lambda_0}}{\lambda_0}\right). \qquad (30)$$

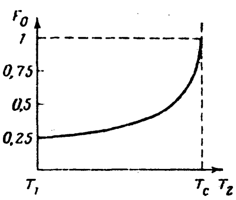

FIG. 2.

The first term (which describes the current distribution at $T=0$) is not temperature dependent, and does not make a contribution to the effect of interest to us. The second term, on the other hand, makes no contribution to the total current and, thus, has a "purely eddy" character. In its turn, the condition of invariability of the total flux (28) is guaranteed by the appearance in the next order in λ/R of a correction to the total current that gives rise to a change in the flux in the cylinder bore:

$$\Delta I_0=\Delta\Phi/\mathscr{L}. \tag{31}$$

In thermoelectric experiments, however, a more realistic situation is the one in which the circuit dimensions (~R) exceed the cross section (~d) of the conductors ($R, d \gg \lambda$). The simplest model for this case is the thermoelectric ring (Fig. 1b). In this case the field in the hole is not uniform; on the other hand, it, generally speaking, depends on the current distribution in the surface layer (contributions being made by both the inner—with respect to the circuit hole—and the outer surfaces of the conductors). In its turn, although the law $v_{s0}=v_{s0}^0 e^{-z/\lambda}$ (z is the distance from the surface) and our estimates for j_a are valid also in the present situation (on account of the fact that $\lambda \ll d$), the quantity v_{s0}^0 and, consequently, the quantity[9]

$$I_0=\int_0^\infty eN_s v_{s0}\, dz$$

and $j_a(v_s)$ are, generally speaking, different for different parts of the conductor surface (i.e., for different ϑ in Fig. 1b). (In particular, they can be different for the inner ($\vartheta \sim 0$) and outer ($\vartheta \sim \pi$) surfaces.) The electrodynamic calculation in such a situation can be carried out by standard methods, but because of the tediousness of such a calculation, it is reasonable to limit ourselves to order-of-magnitude estimates, after analyzing the difference between the present situation and the electrodynamics of a long cylinder, as described by (21)–(31). In this case, using (30) and (31), we can perform the analysis for the thermoelectric and redistribution effects in a unified scheme.

First of all, because $\lambda \ll d$, the formulas (19), (22), and (30) remain valid (the quantities I_0 and J_a^0 depending in this case on ϑ as on a parameter), so that for the eddy current component, which satisfies the condition

$$\int_0^\infty j\, dz=0,$$

we can use the obtained expressions. However, in computing (in the next order in λ/R) the total ring current

$$J=\frac{d}{2}\int d\vartheta\, I, \quad I=\int_0^\infty dz\, j,$$

which is determined from the condition of invariance of the total flux, we should also take into account the contribution of the eddy component to the flux in the hole (whereas in the case of the long cylinder this component did not produce a field in the bore).

In accordance with the Biot-Savart law, the field produced by an element, ds, of the ring at some point of the hole

$$d\mathbf{H}=\int_0^{2\pi} d\vartheta \int_0^\infty dz\, j(z,\vartheta)\frac{[d\mathbf{s}\,\mathbf{r}]}{r^3}, \quad \mathbf{r}=\mathbf{r}(z,\vartheta). \tag{32}$$

Since the eddy component corresponds, by definition, to the relation

$$\int_0^\infty dz\, j=0,$$

for $r \gg d$ we have for the field produced in the hole by this component the estimate:

$$dH \sim \tilde{j}\frac{\lambda^2 d}{r^3} ds,$$

where $\tilde{j}$ is the characteristic value of the "eddy" current density in the surface layer. Thus, as is easy to see, the field produced by the eddy component in the hole is localized near the conductors over a distance ~d and decreases like $1/r^2$ with distance from them.

Let us ascertain, in view of this, what precisely can be measured in experiment. As a rule, in the corresponding thermoelectric experiments the superconducting coupling coil is connected in series with the magnetometer in the thermoelectric circuit. It is then possible to neglect the contribution of the material of the coil to the effect under consideration (this is, in any case, correct if the smallness of ∇T is ensured in the coil, or if the T_c of the coil is sufficiently high), so that it can be assumed that the measuring element is coupled to the magnetic flux threading a section of the circuit hole far from the effective sections of the conductors. Since the flux produced in the hole by the eddy component is localized near the conductors, we measure in such a situation a quantity that is proportional to the correction to the total current, $\tilde{J}(T_2, T_1)$, of the circuit, or, more exactly, proportional to the flux, $\tilde{\Phi}$, produced by this correction in the hole: $\tilde{\Phi}=\tilde{J}L$ (L is the corresponding inductance).

Let the ring be cut by the infinite plane corresponding to the horizontal plane of symmetry (Fig. 1b), and let us find the temperature-dependent fluxes that link with this plane, which are of interest to us. Taking into account the fact that the magnetic field does not penetrate into the interior of the superconductor, we can divide in a natural fashion the part of the indicated plane threaded by the magnetic field into four regions: (a) the hole of the ring, (b) the inner surface layer, (c) the outer surface layer, and (d) the part of the plane outside the ring. On the basis of the condition of invariability of the total trapped flux (which corresponds to the regions (a) and (b)), we have

$$\tilde{J}L+\Phi_{si}+\Phi_i=0. \tag{33}$$

Here Φ_{st} is the temperature-dependent near-surface flux; because $V_s=0$ in the interior of a superconductor, this quantity is, as before, determined by the integral of the corresponding superfluid current density along a closed contour lying along the inner surface of the ring; Φ_i is the correction, produced by the purely eddy component, to the flux in the hole. To find Φ_i, let us estimate the distribution of the magnetic flux produced by the eddy component.

Let us consider a small section, ds, of the ring. Let us denote the temperature-dependent fluxes corresponding to the inner and outer surfaces of the ring by $\delta\Phi_{si}$ and $\delta\Phi_{se}$; the magnetic fluxes produced by the eddy component in the hole of the ring by $\delta\Phi_i^i$, $\delta\Phi_e^i$, and outside the ring by $\delta\Phi_e^e$, $\delta\Phi_i^e$. Here the upper i indices denote the hole of the ring, the e indices the region outside the ring, while the lower indices denote the surface near which the currents producing the flux in question are localized. Taking into account the fact that the overall sum of the magnetic fluxes linking with the infinite plane is, for any given current system (i.e., for both the inner and the outer currents), equal to zero, and neglecting (on the basis of the parameter λ/d) the fluxes produced by the inner current system in the outer surface layer and by the outer current system in the inner surface layer, we have:

$$\delta\Phi_i^e+\delta\Phi_i^i=-\delta\Phi_{si},\qquad \delta\Phi_e^i+\delta\Phi_e^e=-\delta\Phi_{se}\,. \tag{34}$$

It can be seen on the basis of (32) that, for a specified current distribution law, $j(z,\vartheta)$, a scaling law for the H-field distribution is fulfilled with respect to the current strength j. Thus, the division of the flux between the hole of the ring and the region outside the ring is determined by the geometry factors: $\delta\Phi_i^e=k_i\delta\Phi_i^i$, $\delta\Phi_e^i=k_e\delta\Phi_e^e$, where k_i and k_e are parameters that depend on the geometry of the circuit. Notice that, from the point of view of geometry, the substitutions $\delta\Phi_i^i\to\delta\Phi_e^e$ and $\delta\Phi_e^i\to\delta\Phi_i^e$ correspond in the flux calculation to the replacement of the inner region of the ring by the outer region; this replacement thus manifests itself at distances ~R from the conductors.[10] But, as has been noted, the field produced by the eddy component is localized near the conductors over distances ~d. Therefore, the difference between the quantities k_i and k_e has an order of magnitude ~d/R. Taking this into account, we obtain

$$\delta\Phi_e^e=\left(k_i+O\left(\frac{d}{R}\right)\right)\delta\Phi_e^i,\qquad \delta\Phi_i^i(1+k_i)=-\delta\Phi_{si},$$
$$\delta\Phi_e^i\left(1+\frac{d}{k_i}+O\left(\frac{d}{R}\right)\right)=-\delta\Phi_{se}. \tag{35}$$

From these relations and with allowance for (33) and (35), we obtain for the quantity of interest to us, $\delta\bar{\Phi}=\delta(\bar{J}L)$, the expression

$$-\delta\bar{\Phi}=\delta\Phi_{si}+(\delta\Phi_i^i+\delta\Phi_e^i)=\frac{k_i}{k_i+1}(\delta\Phi_{si}-\delta\Phi_{se})+O\left(\frac{d}{R}\right)\delta\Phi_{si}. \tag{35a}$$

Thus, it can be seen that, if $\delta\Phi_{si}\neq\delta\Phi_{se}$, then the magnitude of the measureable temperature-dependent flux is of the same order of magnitude as in the case of a long cylinder (for a given value of the characteristic surface density of the superfluid current I_0). In particular, for the thermoelectric effect $j_a\sim f(|v_s|)\nabla T$ and, thus, j_a has the same sign at the inner and outer surfaces of the ring. Therefore, as is easy to verify, $\delta\Phi_{si}$ and $\delta\Phi_{se}$ have different signs, so that

$$\bar{\Phi}\sim-\Phi_{si}\sim\oint\bar{A}_a|_{z=0}\,ds.$$

In this case the formulas (26) can be used to estimate the measurable quantity. Notice, however, that in the case of a ring the parameter $\mathcal{L}$, which determines the coupling of the trapped flux, Φ, to the characteristic surface density of the superfluid currents I_0 ($\mathcal{L}\sim\Phi/I_0$), depends on the relation between the trapped flux and the external magnetic field strength $\mathcal{H}$. In particular, if $\Phi\approx\int\mathcal{H}d\sigma$ (the integration is performed over the cross section of the hole) and, consequently,

$$J_0=\frac{d}{2}\int I_0\,d\vartheta\ll\frac{\Phi}{L},$$

then $\mathcal{L}\sim 2\pi^2R^2$; this situation corresponds to an antisymmetric distribution of the superfluid currents over the outer and inner surfaces of the ring. If, on the other hand, $\Phi\gg\int\mathcal{H}d\sigma$, then $\mathcal{L}\sim dL$.

In the case of the redistribution effect, as is easy to see from (30), the condition $\delta\Phi_{si}\approx\delta\Phi_{se}$ can be fulfilled, in particular, in the case of an antisymmetric distribution of the superfluid velocity over the inner and outer surfaces ($J_0\ll\Phi/L$). The latter circumstance is realized if the trapped flux is due to an external magnetic field that does not change after the superconducting transition. In such a situation the observable variation of the flux with temperature is significantly less than the variation predicted by (29)—by a factor ~d/R that represents the geometry factor. On the other hand, for $J_0\ll\Phi/L$ the picture is appreciably complicated by a strong dependence on the geometry factor. For example, in the case of a circuit of a more complex shape the geometry factor can be different for different sections of it. Therefore, since at different junction temperatures the dominant contribution to the effect can be made by different sections of the circuit, the temperature dependence of the measurable flux can also become complicated in comparison with (29). Thus, if there is tapering near the "hot" junction (Fig. 3), then for the measurable correction to the flux we can, in the case when $\delta T/(T_c-T_2)\lesssim R/d$, obtain the estimate:

$$\bar{\Phi}\sim\Phi_{si}\frac{d}{R}\left(\frac{T_c}{T_c-T_2}\right)^{1/2}\ln\left(\frac{R}{d}\frac{(T_c-T_2)}{(T_2-T_1)}\right), \tag{36}$$

so that in this case $\bar{\Phi}$ increases rapidly as $T_2\to T_c$.

3. Let us now qualitatively discuss the results obtained. We have shown that the thermoelectric effects in the current state of superconductors, considered by Aronov, can be observed in bulk superconductors too. In this case, despite the surface nature of the thermoelectric current, j_a, the magnitude of the effect is of the same order of magnitude as in the case when the "bare" thermoelectric current is uniformly distributed over the sample thickness with a density equal to the

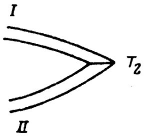

FIG. 3.

surface value j_a. It seems that this result is also valid for surface effects of a different nature, that correspond to different mechanisms of generation of nonequilibrium near-surface currents in superconductors (e.g., the acoustoelectric mechanism).

In observing the thermoelectric effect in the current state, we should also take into account in connection with the dependences $\lambda(T)$ and $N_s(T)$ the redistribution, caused by the variation of the temperature (and considered in Ref. 20), of the magnetic flux linked with the thermoelectric circuit between the hole of the circuit and the surface layer of the superconductors. This redistribution also leads to a temperature-dependent flux in the hole. With allowance for (26) and (29), the ratio of the thermoelectric correction to the flux to the correction due to the redistribution effect has, for $T_1 \sim T_c$, the order of magnitude

$$\frac{\Phi_a}{\Delta\Phi} \sim \left(\frac{\Phi}{\Phi_0}\right)^{1/2} \left(\tau_{ph}\frac{\Delta_0}{p_F}\right) \frac{\xi_0^{4/3}\lambda_0^{1/3}}{\mathscr{L}^{1/2}\bar{R}} \left(\frac{T_c-T_1}{T_c}\right)^{1/2} K^{-1} \ln\left(\frac{T_c-T_1}{T_c-T_2}\right). \quad (37)$$

(Here K is the geometry factor; we have also taken account of the fact that a temperature gradient can be established only in a section of the contour of dimension $\bar{R}$.) This quantity increases as the flux Φ increases. However, the latter is limited by the fact that the field near the superconductors must not exceed $H_{c1} \sim \Phi_0/\lambda^2$. From this it is easy to see on the basis of (37), as well as (26) and (29), that the closeness of T_1 to T_c is less preferable for the observation of a thermoelectric effect of the type considered by Aronov, and that the optimal situation corresponds to

$$N_s|_{T_1} \sim N - N_s.$$

It can be seen that, as a rule, the contribution of the redistribution effect significantly exceeds the thermoelectric contribution. An exception may be the thin-ring (i.e., $d \ll R$) situation in the case when the trapped flux is due to an external field that does not change after the superconducting transition. In this case $K \sim d/R \ll 1$. With allowance for all the foregoing, we can obtain an estimate for the ultimate ratio $\Phi_a/\Delta\Phi$ ($\Phi \sim H_{c1}R^2, N_s \sim N - N_s$):

$$\frac{\Phi_a}{\Delta\Phi} \sim \frac{\tau_{ph}}{\bar{R}} \frac{\Delta_0}{p_F} \frac{R}{d}.$$

Since d/R can be fairly small ($\sim 10^{-2}$–10^{-3}), it can be seen that, for superconductors with sufficiently low T_c, the effects under consideration can be of the same order of magnitude at $T < 2$ K ($\tau_{ph}(T) \gtrsim 10^{-7}$ sec).

It can be seen from our analysis that, to reduce the role of the redistribution effect, it is advisable to make the range of variation of the temperature narrower (i.e., to make ∇T greater) and, in any case, avoid bends in the conductors in this region (Fig. 4). In spite of the relative smallness of the contribution of the surface thermoelectric effects, its separation can be facilitated by the following circumstances. One of them is the difference in the temperature dependences of the thermoelectric contribution and the contribution of the redistribution effect (cf. (26) and (29)). Another circumstance is that the sign of the thermoelectric correction to the flux depends on the direction of ∇T. At the same time, the redistribution effect's contribution, which is determined by the local variation of the temperature, does not depend on the sign of ∇T provided the magnetic field and the geometry factor are symmetrically distributed with respect to the "hot" and "cold" junctions, in particular, if a circuit with a configuration symmetric with respect to the interchange $T_1 \rightleftharpoons T_2$ (a circuit of the type shown in Fig. 4) is located in a uniform external field. Therefore, the use of a scheme with two heaters[15] can allow the separation of the thermoelectric contribution from the contribution of the redistribution effect (cf. Ref. 20). Finally, let us note that the contribution of the redistribution effect changes sign when the sign of v_s is changed, whereas the thermoelectric contribution does not change when this is done ((20), (26)). Therefore, it is possible that the use of variable or inhomogeneous external magnetic fields will also enable us to separate the contribution of the thermoelectric effects.

Let us now discuss the question of the possible influence, indicated in Ref. 20, of the redistribution effect on experimental investigations of volume thermoelectric effects in superconductors.[1-9] Such an influence is due to the fact that, as a rule, a trapped "stray" magnetic flux exists in a thermoelectric circuit. The ratio of the contributions of these two effects to the measurable flux is determined with allowance for (29) and the results of Ref. 3 by the quantity

$$\frac{\Phi_T}{\Delta\Phi} \sim K^{-1}\left(\frac{\eta T_c}{I_0}\right) \frac{\lambda_0}{R} \frac{(T_c-T_1)^{3/2}}{T_c^{1/2}} \frac{1}{\delta T} \ln\left(\frac{T_c-T_1}{T_c-T_2}\right) \quad (T_1 \sim T_c). \quad (38)$$

Here η is the thermoelectric coefficient ($\eta \sim \alpha\sigma$, α is the thermoelectromotive force in the normal state and σ is the conductivity), I_0 is the surface density of the "stray" superfluid currents. If the trapped flux is due to the remanent magnetic field, $\mathscr{H}$, of the system, then $I_0 \sim c\mathscr{H}/4\pi$.

Notice that the quantitative estimates obtained in Ref. 20 for the contribution of the redistribution effect and its relationship with the thermoelectric contribution are in accord with our results for $K \sim 1$.[11)] An exception is thus the case of a ring with an antisymmetric distribution of the superfluid current over the inner and outer surfaces; as we have shown, for such a ring $K \sim d/R \ll 1$. Since it can be assumed that the remanent fields do not change in the course of the experiments, it is precisely such a distribution of the Meissner currents

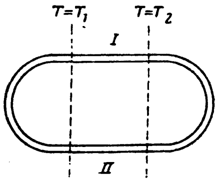

FIG. 4.

that is realized in the contour in question when $d/R \ll 1$. In our opinion, the present circumstance significantly facilitates the separation of the thermoelectric contribution (in comparison with the estimates made in Ref. 20); in this case it is important to avoid having taperings in the contour in the vicinity of the junctions, since they may not only increase the contribution of the redistribution effect, but also impart to it a "singular" temperature dependence.

We wish to point out another possibility for the appearance of "stray" currents in the contour. If in the course of raising the temperature of the "hot" junction it exceeds the T_c of one of the conductors, then there arises in the contour a normal section with a temperature gradient. The thermal e.m.f. arising in it generates a current that circulates around the contour and produces a magnetic flux that can be partially trapped during the subsequent decrease of the temperature. A rough estimate of the resulting surface density of the superfluid current yields $I_0 \sim \eta \overline{\nabla T} d$, where $\overline{\nabla T}$ is some characteristic value of the temperature gradient. Notice that for this current $K \sim 1$. In this case, if ∇T changes its direction in the cause of the experiments, but passage through T_c is accomplished each time, then the indicated current and its contribution to the redistribution effect will change their signs.

Let us now proceed to the discussion of specific experiments. As seems to us, there exist a number of circumstances indicating that it was precisely the thermoelectric effect that was observed in Zavaritskiĭ's experiments[15]: (a) the numerical estimates made within the framework of theory[3] yield, in any case, the correct order of magnitude of the observed effect, while the temperature dependence is fairly well described by the theoretical curves; (b) for the purpose of separating out the spurious effects in these experiments the sign of ∇T was changed, whereupon a change of sign of the measurable correction to the flux was observed; (c) it is significant that the observed effect depended, in accord with theory, on the sample purity—for "dirtier" samples the effect was significantly weaker (on account of the fact that $j_T \sim \eta$). At the same time, as is easy to see, for $l \gg \xi_0$ the redistribution effect does not, in general, depend on the sample purity.

The totality of these three circumstances indicates, in our opinion, a thermoelectric effect. At the same time, the remaining experimental investigations,[16-18] in which anomalously high temperature-dependent fluxes were observed, are, apparently, indeed connected with the redistribution effect. Thus, assuming the strength of the remanent field is an order of magnitude less than the upper limit given in Ref. 17, we can, on the basis of (29), obtain the correct order of magnitude of the temperature-dependent magnetic flux observed in Ref. 17. On the other hand, the temperature dependence of this flux (see Ref. 17) is, with the exception of the rapid growth in the vicinity of T_c, in fairly good agreement with (29) (Fig. 2). This growth may be connected with the presence of a tapering of the contour near the "hot" junction (cf. (36)).

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1) Let us note that the possibility of the appearance of magnetic fields in simply-connected, inhomogeneous, and anisotropic superconductors in the presence of a temperature gradient was pointed out by Ginzburg.[1]

2) Such a situation is fairly typical, since in actual experiments there exists, as a rule, a trapped magnetic flux in the thermoelectric circuit.

3) The author is grateful to Yu. M. Gal'perin, who pointed out this circumstance.

4) The appearance of a near-surface magnetic flux during the propagation of a surface acoustic wave, which gives rise to a current of dragged normal excitations, was first pointed out by Yu. M. Gal'perin.

5) Strictly speaking, what is important for the effect in question is not the presence of a temperature gradient, but the local variation of the temperature.

6) It is easy to see that (1a) follows from the condition $\hbar/\delta p \ll \lambda$, where $\delta p \equiv p_1 - p_2$ is the momentum transferred during the Andreev reflection of a particle from an inhomogeneity of the field of the velocities v_s; $p_{1,2}^2 = 2m\{[\mu - (p_x^2 + p_y^2)/2m] \pm [(\epsilon - p_x \times v_s)^2 - \Delta^2]^{1/2}\}$. The latter inequality is also given in Ref. 21. It is then easy to see that, in order of magnitude, $\hbar/\delta p$ determines the scale of the damping of the particle wave function in the classically forbidden region. In fact, the present condition implies an additional limitation on the applicability of the kinetic equation for trapped particles and should, strictly speaking, also be supplemented by the condition $\hbar/\delta p \ll l$.

7) Let us note that the estimate (17) is, in order of magnitude, also valid in the homogeneous situation involving thin conductors and not too low v_s, (1a).

8) Notice that the real function $f(z)$ could have easily been taken into account by using the Laplace transformation.

9) Since all the quantities of interest to us decrease rapidly when $z \gtrsim \lambda$, we shall, for simplicity, perform the integration over z within the limits $0, \infty$.

10) Notice that in the case of a circuit of more complex configuration the quantity R, which has the meaning of a characteristic dimension of the system near a given section of the circuit, is determined by $\min(R^*(s), a(s))$, where $R^*(s)$ and $a(s)$ are respectively the radius of curvature at the given point of the circuit and the distance to the opposite conductor.

11) Notice that the experimental study of the redistribution effect was carried out in the work published in Ref. 20 under conditions when an additional undamped current was excited in the circuit. As we have shown, in the presence in the circuit of a ring current $J_0 \sim \Phi/L$ the quantity $K \sim 1$.

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Trap charge exchange waves in compensated germanium

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We have observed experimentally, at 90 K, impedance oscillations due to excitation of trap charge-exchange waves in *n*-Ge compensated with gold. It is shown that the observed impedance singularities (shift of the oscillations to lower frequencies with increasing dc voltage and with decreasing sample length, decrease of the oscillation period with decreasing frequency, change of frequency with changing conductivity) agree with the "inverse" dispersion law $\omega^{-1} = kv\tau\tau_M$ that is characteristic of these waves.

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1. It was shown earlier[1] that in a compensated monopolar semiconductor it is possible to excite weakly damped trap charge-exchange waves. The main feature of these waves is their "inverse" dispersion law. The frequency ω and the wave vector k are connected by the relation[1]

$$\omega^{-1}=kv\tau\tau_M, \qquad (1)$$

where $v=\mu E$ is the electron drift velocity, is their lifetime, $\tau_M=\epsilon/4\pi\sigma$, and $\sigma=en\mu$ is the conductivity. In this paper we present experimental proof of the existence of these waves.

The onset of charge-exchange waves should lead to singularities in the behavior of the impedance of a crystal.[2,3] If a traveling wave-charge wave is present in the sample, a phase shift appears between the current and the voltage, and the admittance has accordingly a reactive component. The phase shift due to the wave vanishes when the sample spans an integral number of waves. The sample impedance will therefore oscillate with changing frequency of the alternating field. This reasoning is apparently valid for all waves propagating in a homogeneous medium. However, the impedance oscillations will have a different character as a function of the nature of the wave. For charge-exchange waves, the impedance singularities are due to the dispersion relation (1). It was shown[2] that the susceptance of the sample corresponds to a capacitance greatly exceeding the geometrical value. Under conditions when the conductivity is controlled by trapping on one compensated level of the impurity, the expression for the low-frequency capacitance can be approximated by

$$C=C_0\frac{\tau v}{d}\left(1-\cos\frac{d}{\omega\tau v\tau_M}\right)+C_0, \qquad (2)$$

where $C_0=\epsilon S/4\pi d$ is the geometric capacitance of the sample, S is the cross-section area, d is the sample length, and ϵ is the permittivity. Expression (2) is valid for short samples ($d\ll v\tau, v\tau_M$) and if diffusion is neglected. When $d/\omega\tau v\tau_M=2\pi m$ (m are natural numbers), i.e., precisely when the sample length is equal to an integer number of charge-exchange wavelengths, the capacitance of the sample is minimal.

Some other singularities of the impedance are also obvious consequences of the dispersion law (1). The oscillations of the frequency dependence of the capacitance should shift towards lower frequencies with increasing constant field E or with decreasing density n of the free electrons, while the maxima and the minima should come closer together with decreasing frequency. Since the charge-exchange wavelength depends on n and E, the capacitance of the sample should oscillate also if n and E vary and the frequency ω is fixed. At low frequencies the conductivity also acquires an oscillating increment

$$\frac{\Delta(\mathrm{Re}\,Y)}{X_0}=\frac{\omega\tau v\tau_M}{d}\sin\frac{d}{\omega\tau v\tau_M}. \qquad (3)$$

It is seen from (2) and (3) that the frequencies of the ReY oscillations should become higher than those of

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