Eksp. Teor. Fiz. 62, 1606 (1972).
[18] I. D. Novikov, Astron. Zh. 38, 564 (1961) [Sov. Astron. 5, 423 (1961)].
[19] G. Lemaître, Ann. Soc. Sci., Bruxelles I A 53, 51 (1933).
[20] M. Simpson and R. Penrose, Int. J. Theor. Phys. 7, 183 (1973).
[21] E. M. Lifshitz and I. M. Khalatnikov, Usp. Fiz. Nauk 80, 391 (1963) [Sov. Phys. Usp. 6, 495 (1964)].
[22] Ya. B. Zel'dovich and I. D. Novikov, Stroenie i Évolyutsiya Vselennoĭ (Structure and Evolution of the Universe), Nauka, Moscow (1975).
[23] I. D. Novikov, Zh. Eksp. Teor. Fiz. 71, 393 (1976) [Sov. Phys. JETP 44, 207 (1976)].

Translated by R. M. Durham

Nuclear excitation during positron annihilation in the *K* shell of heavy atoms

D. P. Grechukhin and A. A. Soldatov

I. V. Kurchatov Institute of Atomic Energy, Academy of Sciences of the USSR, Moscow
(Submitted 1 June 1977)
Zh. Eksp. Teor. Fiz. 74, 13–22 (January 1978)

The excitation of a nucleus by a positron beam during the annihilation of the positrons with atomic electrons is discussed. Results are reported of a calculation of the excitation cross section for ^{115}In and ^{235}U. The calculations were performed in the transition current and charge scheme, using wave functions obtained by the relativistic Hartree-Fock-Slater method through a numerical integration of the Dirac equation. The Weisskopf single-particle nuclear transition matrix elements were used for ^{115}In to estimate the cross section for the nuclear $E1$ transition induced by a monochromatic positron beam. The cross section at resonance is found to be $\sigma_{res}(E1)\sim 10^{-25}$ cm^2. A similar calculation for ^{235}U yielded $\sigma_{res}(E1)\sim 5\times 10^{-26}$ cm^2. More accurate cross sections have been obtained for particular levels on the basis of existing experimental data on the nuclear-level spectrum.

PACS numbers: 34.90.+q, 23.20.Js

INTRODUCTION

The annihilation of positrons with atomic electrons is one of the possible processes during the scattering of a positron beam by atoms. The annihilation process can be accompanied by the emission of one or more photons, or by the excitation of the nucleus. In this paper we report the results of an analysis of the cross section for nuclear excitation by a positron beam during the annihilation of positrons with atomic electrons.

If an atom intercepts a positron beam of energy E_+ and energy spread ΔE, annihilation between a positron and an atomic electron with quantum numbers nlj (n is the principal quantum number and j and l are the resultant and orbital angular momenta of the electron) may be accompanied by resonant excitation of nuclear states with energies in the interval ΔE around $E_f = E_+ + E_{nlj}$ (E_+ and E_{nlj} are, respectively, the total relativistic energies of the positron and the electron). It follows that, if a sufficiently narrow positron beam is available (ΔE less than the separation between the nuclear levels), one can scan the nuclear spectrum by varying the energy of the incident positrons. One would expect that the main contribution to the cross section would be that due to K-shell electrons. Our calculations have shown that, for the L_I shell, the cross section is already smaller than the K-shell cross section by roughly an order of magnitude. We shall therefore confine our attention to K-shell electrons, i.e., we shall assume that only nuclear levels with energies $E_f = E_+ + E_{1s}$ are excited.

The above problem has already been treated theoretically by Present and Chen,[1] and the nuclear excitation cross section for positron annihilation was calculated in the Born approximation without taking into account the finite width of the K-shell hole. A more accurate theoretical analysis has become necessary following the work of Mukoyama and Shimizu,[2] who reported an experimental attempt to determine the cross section for the positron excitation of ^{115}In, and who concluded that the cross section was greater by two orders of magnitude as compared with the theoretical prediction.[1]

In this paper, the nuclear excitation cross section will be calculated within the framework of the well-known transition current and charge scheme.[3] The wave functions of the electron and incident positron will be taken to be the solution of the Dirac equation for the average atomic field deduced by the Hartree-Fock-Slater method.

The cross sections have been calculated for ^{115}In and ^{235}U. The choice of ^{115}In was dictated by the fact that the first estimates of the cross sections were made[2] for this nucleus, whereas ^{235}U is an example of a heavy fissile nucleus for which it is interesting to investigate excitation during positron annihilation as a possible way of investigating nuclear fission in the subbarrier region. In both cases (^{115}In and ^{235}U), there are modern experimental data on the nuclear excitation spectrum at energies E_f below ~1.5 MeV. In this energy range, the excitation cross section can be calculated for individual levels by using experimental reduced nuclear transition probabilities or some accepted model. For example, the Nilsson model was used to identify the ^{235}U levels. In the region of nuclear excitations where there are no

 0038-5646/78/4701-0006$02.40

nuclear structure data, the excitation cross section can be estimated by calculating the cross sections obtained with the Weisskopf single-particle nuclear matrix elements for ML and EL multipoles. Estimates of this kind, in the case of the $E1$ nuclear transition, lead to values of about 10^{-25} cm^2 for ^{115}In and $\sim 5\times 10^{-26}$ cm^2 for ^{235}U at excitation energies $E_f \gtrsim 1.5$ MeV in the case of the resonance cross section (i.e., at the point of strict resonance for a monochromatic positron beam).

1. CROSS SECTION FOR NUCLEAR EXCITATION DURING THE ANNIHILATION BETWEEN A POSITRON AND AN *nlj* ATOMIC ELECTRON

1. The states of the atomic electron and the incident positron will be described within the framework of the relativistic variant of the Hartree-Fock-Slater method, i.e., we shall assume that all the electrons move in the same atomic field with a centrally symmetric potential which is obtained by replacing the exchange interaction in the Hartree-Fock method by an approximate local potential. Accordingly, the electron-positron field operators will be written in the form of an expansion over the eigenfunctions of the Dirac equation with this mean atomic field. The continuous-spectrum functions will be taken to be such that the asymptotic behavior of the positron wave function $\Psi^{(+)}_{p\mu}$ at infinity can be described by a plane wave plus an outgoing spherical wave (p is the wave vector and μ the polarization of the positron). According to the transition current and charge scheme,[3] the effective matrix element for the interaction between the electron-positron field and the nuclear field can be written in the form

$$V_{21}=\int d\mathbf{R}\,d\mathbf{r}\left[-\rho_e(\mathbf{R})\rho_n(\mathbf{r})+\frac{1}{c^2}\mathbf{j}_e(\mathbf{R})\mathbf{j}_n(\mathbf{r})\right]\frac{\exp\{ik|\mathbf{R}-\mathbf{r}|\}}{|\mathbf{R}-\mathbf{r}|}, \qquad (1)$$

where $\mathbf{j}_n(\mathbf{r})$ and $\rho_n(\mathbf{r})$ are the matrix elements of the current and charge operators for the $1\to 2$ nuclear transitions; $k=(E_2-E_1)/\hbar c$; $\rho_e = e\Psi^+(-E_+)\Psi_{nljm}$, $\mathbf{j}_e = ec\Psi^+(-E_+)\times\alpha\Psi_{nljm}$; $\Psi(-E_+)$ is the solution of the Dirac equation for $E=-E_+$, corresponding to the wave function of the incident positron with energy E_+, wave vector $\mathbf{p}$, and spin component μ; Ψ_{nljm} is the wave function of the atomic electron,

$$\alpha=\begin{pmatrix}0 & \sigma\\ \sigma & 0\end{pmatrix},$$

is the Dirac matrix, σ are the Pauli matrices, and e is the elementary charge ($e>0$).

The nuclear excitation process during the annihilation of the positron with the atomic electron has a resonant character, so that it is necessary to take into account the decay of the final state of the system whose width Γ consists of the nuclear level width Γ_N and the width $\Gamma(nlj)$ of the hole in the atomic shell. The cross section averaged over the polarizations of the incident-positron spin and the components of the angular momenta of the initial nuclear and the nlj atomic states, summed over the final states of the system (i.e., the cross section per electron in the nlj shell) can be written in the form

$$\sigma(nlj)=\sum_{I_2\Pi_2} a_0^{N(L)}\int dE_+ S(E_+)D(E_2-E_1-E_{nlj}-E_+)\cdot\frac{2I_2+1}{2I_1+1}|\langle I_2E_2\|\Lambda L\|I_1E_1\rangle|^2\xi(\Lambda L,(nlj),E_+), \qquad (2)$$

where the resonance factor is given by

$$D(x)=\frac{1}{2\pi}\frac{I_0\Gamma}{x^2+(\Gamma/2)^2}, \qquad I_0=\frac{me^4}{\hbar^2}\approx 27\text{ eV}; \qquad (3)$$

$a_0=\hbar^2/me^2\approx 0.529\times 10^{-8}$ cm,

$$N(L)=\begin{cases}-2 \text{ for the } ED \text{ monopole}\\ -2 \text{ for the } EL \text{ and } ML \text{ multipoles } (L\neq 0);\end{cases}$$

ΛL are, respectively, the type and multipolarity of the $1\to 2$ transition, and $S(E)$ is the energy distribution of the incident positron beam which is normalized so that

$$\int_0^\infty dE\,S(E)=1. \qquad (4)$$

The nuclear matrix element can be expressed in terms of the E_0 monopole and the EL and ML $(L\neq 0)$ multipoles, as follows:

$$e\langle I_2\|E0\|I_1\rangle=\left\langle I_2M_2\left|\int d\mathbf{r}\,r^2Y_{00}(\mathbf{r})\hat{\rho}_N(\mathbf{r})\right|I_1M_1\right\rangle \quad (I_1=I_2), \qquad (5)$$

$$e\langle I_2\|EL\|I_1\rangle(LI_1MM_1|I_2M_2)=\left\langle I_2M_2\left|\int d\mathbf{r}\,r^LY_{LM}(\mathbf{r})\hat{\rho}_N(\mathbf{r})\right|I_1M_1\right\rangle, \qquad (6)$$

$$e\langle I_2\|ML\|I_1\rangle(LI_1MM_1|I_2M_2)=\left\langle I_2M_2\left|\frac{1}{c}\int d\mathbf{r}\,r^L\mathbf{Y}_{LLM}(\mathbf{r})\hat{\mathbf{j}}_N(\mathbf{r})\right|I_1M_1\right\rangle, \qquad (7)$$

where $(LI_1MM_1|I_2M_2)$ is the Clebsch-Gordan coefficient, $Y_{LM}(r)$ is the spherical harmonic, and $\mathbf{Y}_{LLM}(r)$ is a spherical harmonic vector.[3] The matrix elements in (6) and (7) are related to the reduced nuclear transition probability[4] as follows:

$$e^2\frac{2I_2+1}{2I_1+1}|\langle I_2\|EL\|I_1\rangle|^2=B(EL;I_1\to I_2),$$
$$e^2\frac{2I_2+1}{2I_1+1}\frac{L}{L+1}|\langle I_2\|ML\|I_1\rangle|^2=B(ML;I_1\to I_2). \qquad (8)$$

The summation in (2) is performed over all the nuclear levels $|E_2I_2\Pi_2\rangle$ that fall into the energy interval determined by the spread of the positron beam $S(E_1)$ and the resonance factor $D(E_2-E_1-E_{nlj}-E_+)$.

The dependence of the positron annihilation probabilities on the states of the atomic electron and the incident positron for given ΛL is represented by the factors $\xi(\Lambda I, nlj, E_+)$, given by the following expressions:

$$\xi(E0,(nlj)^1,E_+)=\delta_{j1/2}\frac{2\pi^3}{9}\frac{1}{(pa_0)^3}\frac{E_+-mc^2}{mc^2}\left|\left[\frac{g_{nlj}G_{jl}+F_{jl}f_{nlj}}{x^2}\right]_{x=0}\right|^2 \qquad (9)$$

$$\xi(ML,(nlj)^1,E_+)=8\pi^3\frac{1}{2j+1}\frac{1}{2L+1}\frac{(ka_0)^{2L+2}}{[(2L+1)!!]^2}\frac{E_+-mc^2}{mc^2}\frac{1}{(pa_0)^3}$$

$$\times\sum_{j_2l_2}(2j_2+1)\left|A(j_2l_2Ljl')\int_0^\infty dx\,h_L^{(1)}(ka_0x)(G_{j_2l_2}f_{nlj}+F_{j_2l_2}g_{nlj})\right|^2, \qquad (10)$$

$$\xi(EL,(nlj)^1,E_+)=8\pi^3\frac{1}{2j+1}\frac{1}{2L+1}\frac{(ka_0)^{2L+2}}{[(2L+1)!!]^2}\frac{E_+-mc^2}{mc^2}\frac{1}{(pa_0)^3}$$

$$\times\sum_{j_2l_2}(2j_2+1)\Big|B(j_2l_2Ljl)\Big[\int_0^\infty dx\,h_L^{(1)}(ka_0x)(G_{j_2l_2}g_{nlj}+F_{j_2l_2}f_{nlj})$$

$$+\int_0^\infty dx\,h_{L-1}^{(1)}(ka_0x)(F_{j_2l_2}g_{nlj}-G_{j_2l_2}f_{nlj})\Big]$$

$$-\left(\frac{L+1}{L}\right)^{1/2}A(j_2l_2Ljl)\int_0^\infty dx\,h_{L-1}^{(1)}(ka_0x)(F_{j_2l_2}g_{nlj}+G_{j_2l_2}f_{nlj})\Big|^2, \qquad (11)$$

where $l'=2j-l$, $h_L^{(1)}(z)$ is the spherical Hankel function of the first kind,

$$B(j_2l_2Ljl)=[(2L+1)(2l+1)(2j+1)]^{1/2}(Ll00|l_20)W(Llj_2{}^1/_2;l_2j),$$

$$A(j_2l_2Ljl)=\frac{j_2(j_2+1)-j(j+1)+l(l+1)-l_2(l_2+1)}{[L(L+1)]^{1/2}}B(j_2l_2Ljl), \qquad (12)$$

and $W(abcd;ef)$ is the Racah function.

The above expressions are given in atomic units for which $x=r/a_0$. The radial wave functions for the discrete spectrum (g_{nlj} is the large, and f_{nlj} the small component) are normalized so that

$$\int_0^\infty dx(g_{nlj}^2+f_{nlj}^2)=1, \qquad (13)$$

and the functions G_{jl} and F_{jl} form the solution of the radial Dirac equation for energy $E=-E_+$, normalized so that, as $x\to\infty$:

$$G_{jl}\to\sin(pa_0x-l\pi/2+\delta_{jl}), \qquad (14)$$

where δ_{jl} are the phase shifts. The component F_{jl} is determined in accordance with this normalization,[3] $\hbar cp=(E_+^2-m^2c^4)^{1/2}$, and p is the positron wave number.

2. We shall now give the expression for the cross sections in the limiting cases of a narrow (monochromatic) and a broad energy distribution of the positron beam.

a) Monochromatic positron beam: $\Gamma\gg\Delta E$. In this case, the entire sum over the nuclear spectrum reduces to a single term corresponding to the nuclear level with energy $E_2=E_1+E_{nlj}+E_+$, and the cross section is

$$\sigma(\Lambda L,(nlj)^1,E_+)$$
$$=a_0^{N(L)}\xi(\Lambda L,(nlj)^1,E_+)\frac{2I_2+1}{2I_1+1}|\langle I_2\|\Lambda L\|I_1\rangle|^2D(E_2-E_1-E_{nlj}-E_+). \qquad (15)$$

b) "Broad" positron beam: $\Gamma\ll\Delta E$. We shall assume that the nuclear spectrum is sufficiently dense, i.e., the interval ΔE contains a large number of nuclear levels. Assuming, in addition, that the nuclear matrix element and $|\langle I_2\|\Lambda L\|I_1\rangle|^2$ and $\xi(\Lambda L,(nlj)^1,E_+)$ are slowly-varying functions of energy, we can write the cross section in the following form by introducing the average nuclear matrix element for a group of nuclear states:

$$\bar\sigma((nlj)^1,E_+)$$
$$=\sum_{I_2\Pi_2}a_0^{N(L)}\xi(\Lambda L,(nlj)^1,E_+)\overline{|\langle I_2\|\Lambda L\|I_1\rangle|^2}\frac{2I_2+1}{2I_1+1}\rho_{I_2\Pi_2}(E_1+E_{nlj}+E_+), \qquad (16)$$

where $\rho_{I\Pi}(E)$ is the density of nuclear states, i.e., the number of levels with given angular momentum I, parity Π, and energy E in the interval ΔE is $\rho_{I\Pi}(E)\Delta E$.

2. CROSS SECTION FOR THE EXCITATION OF ^{115}In AND ^{235}U DURING ANNIHILATION OF A FILLED *K*-SHELL

As already noted, the main contribution to the cross section for the excitation of the nucleus during the annihilation of a positron with an atomic electron is provided by the *K*-shell electrons. Accordingly, we have calculated the factors $\xi(\Lambda L,(1s)^1,E_+)$ for *K*-shell electrons in In and U in the case of $E0$, $E1$, $E2$, and $M1$ multipoles.

The behavior of the factors $\xi(\Lambda L,(1s)^1,E_+)$ as functions of the incident-positron energy E_+ is shown in Figs. 1 and 2. Figure 3 illustrates the dependence of ξ (and, correspondingly, of the cross sections) on the atomic number Z at energies well removed from the threshold, so that the threshold effects can be neglected. We note that the law $\sigma\propto Z^3$, predicted within the framework of the Born approximation,[1] is not valid. This is so because the Born approximation is not very acceptable for $Ze^2/\hbar c\gtrsim 40/137$ in the present case.

The wave functions for the *K*-shell electron and the positron were obtained by numerical solution of the Dirac equation. The mean atomic Hartree-Fock-Slater potential was computed by a program written by Band and Trzhaskovskaya.[5]

All the numerical estimates will henceforth refer to the case of an exact resonance and a monochromatic positron beam, i.e., we shall use (15) at the resonance point $E_2=E_1+E_{1s}+E_+$:

$$\sigma_{res}(\Lambda L,(1s)^2,E_+)=2a_0^{N(L)}\xi(\Lambda L,(1s)^1,E_+)\frac{2I_2+1}{2I_1+1}|\langle I_2\|\Lambda L\|I_1\rangle|^2\frac{2}{\pi}\frac{I_0}{\Gamma}. \qquad (17)$$

When $Z\geqslant 40$ and $E_2\leqslant 7$ MeV, the main contribution to the final-state width Γ is, as a rule, provided by the width of the *K*-shell hole, $\Gamma(K)$. Using the Coulomb functions and taking into account only the radiative *L*-shell electron transitions to the *K*-shell hole, we obtain a very

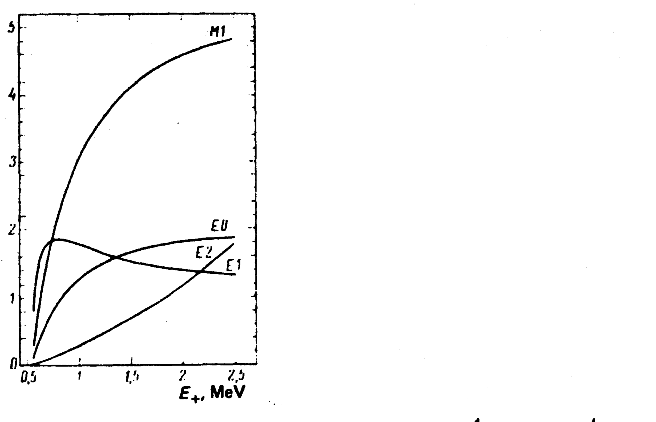

FIG. 1. Dependence of the factors $\xi[E0,(1s)^1,E_+]\times10^{-4}$, $\xi[E1,(1s)^1,E_+]$, $\xi[E2,(1s)^1,E_+]\times10^{-4}$, $\xi[M1,(1s)^1,E_+]\times10$ on the incident-position energy E_+ in the case of the indium atom.

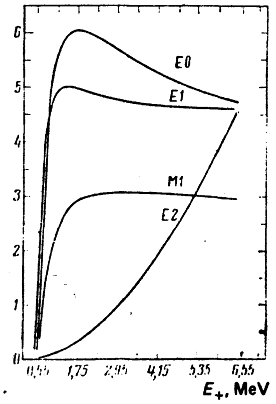

FIG. 2. Dependence of the factors $\xi[E0,\ (1s)^1,\ E_+]\times 10^{-5}$, $\xi[E1,\ (1s)^1,\ E_+]$, $\xi[E2,\ (1s)^1,\ E_+]\times 10^{-5}$, $\xi[M1,\ (1s)^1,\ E_+]$ on the incident positron energy E_+ in the case of the uranium atom.

rough estimate for $\Gamma(K)$, which is probably a little too low. The results are $\Gamma(K)=7.2$ eV and 89 eV for indium and uranium, respectively. Next, we neglect the contribution of the nuclear width Γ_N, but it must be remembered that the nuclear-level spectrum may contain states connected to the ground state by an accelerated $E1$ or $(M1)$ transition. The radiative width of such states may be comparable with $\Gamma(K)$, in which case, the contribution of Γ_N to the total width Γ must be taken into account.

a) ^{115}In. To estimate the resonance cross section σ_{res}, we use the single-particle Weisskopf nuclear matrix element.[4] According to (17), the cross sections for the excitation of the nucleus during the annihilation of positrons on a filled K shell of the indium atom are then given by (in barns):

$$\begin{aligned} &\sigma_{res}(E1,\ (1s)^2, E_+)=7.4\cdot 10^{-2}\xi(E1,\ (1s)^1, E_+),\\ &\sigma_{res}(M1,\ (1s)^2, E_+)=7.6\cdot 10^{-2}\xi(M1,\ (1s)^1, E_+),\\ &\sigma_{res}(E2,\ (1s)^2, E_+)=5.8\cdot 10^{-10}\xi(E2,\ (1s)^1, E_+),\\ &\sigma_{res}(E0,\ (1s)^2, E_+)=1.6\cdot 10^{-9}\xi(E0,\ (1s)^1, E_+). \end{aligned} \tag{18}$$

In these expressions, the matrix element $|\langle I_2\|E0\|I_1\rangle|^2$ for the $E0$ transition was replaced by $R^4/4\pi=(1.2A^{1/3})^4/4\pi$ fm^4. It follows from (18) that σ_{res} for single-particle $E1$ transitions may reach values $\sim 10^{-25}$ cm^2 when $E_+ \gtrsim 0.7$ MeV.

TABLE I. Summary of Data * on the nuclear spectrum of ^{115}In[6] and the corresponding cross sections for the resonance excitation during annihilation on a filled K shell.

E, MeV	$I\Pi$	$B(E2\uparrow)$, $e^2\cdot b^2$	$g\Gamma_0(E2)$, 10^{-4} eV	$g\Gamma_0(M1)$, 10^{-4} eV	$\sigma_{res}(E2, (1s)^2, E_+)$, 10^{-6} b	$\sigma_{res}(M1, (1s)^2, E_+)$, 10^{-6} b
0	$9/2^+$					
1.0778	$5/2^+$	0.0095–0.0227			0.626–1.49	
			2.8–4.7		1.58–2.66	
1.1325	$(11/2)^+$	0.060–0.108			7.70–13.9	
			12.9–15	60–61	11.1–12.9	4.44–4.52
1.2905	$(13/2)^+$	0.042–0.060			13.0–18.6	
			12–14	<5–6	13.0–15.1	<0.486–0.582
1.448		0.0124–0.0151			6.12–7.46	
			6.4	<4	6.12	<0.364
1.450			~5	~4–6	~4,8	~0.364–0.546
1.4625		0.0096–0.0124			4.92–6.34	
			6.7	50	6.38	4.50
1.487		0.0060–0.0087			3.24–4.70	
			3.5	>7	3.22	>0.612

¹*$\Gamma_0(\Lambda L)$ is the width of the state for the ΛL transition to the ground state, $g=(2I+1)/10$; $\sigma_{res}[\Lambda L,\ (1s)^2,\ E_+]$ is the excitation cross section.

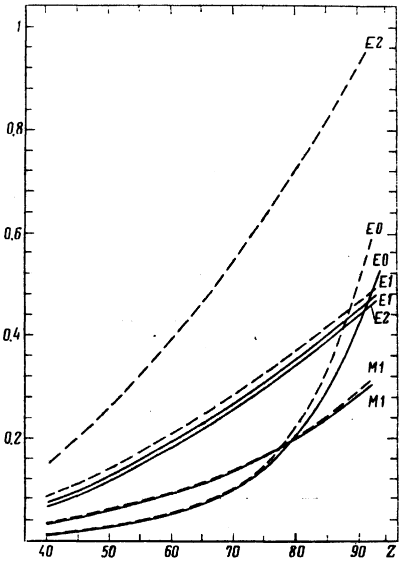

FIG. 3. Behavior of the factors $\xi[\Lambda L,\ (1s)^1,\ E_+]$ as functions of Z. Solid curves correspond to $E_+=6.55$ MeV, broken curve $-E_+=2.95$ MeV. Curves marked $E0$: $\xi[E0,\ (1s)^1,\ E_+]\times 10^{-5}$, curves marked $E1$: $\xi[E1,\ (1s)^1,\ E_+]\times 10^{-1}$, curves marked $M1$: $\xi[M1,\ (1s)^1,\ E_+]\times 10^{-1}$, broken curve $E2$: $\xi[E2,\ (1s)^1,\ E_+]\times 10^{-5}$, solid curve $E2$: $\xi[E2,\ (1s)^1,\ E_+]\times 10^{-6}$.

Existing experimental data[6] on the nuclear-level spectrum of ^{115}In enable us to calculate more accurately the excitation cross sections for some of the levels in ^{115}In in the range $E_2\approx 1-1.5$ MeV. Table I lists these experimental data and the calculated values of σ_{res} for the annihilation of positrons on the filled K-shell of In. We note that this energy interval of ^{115}In level spectrum was found not to contain levels excited by the $E1$ transition from the ground state. This is in conflict with published data.[1]

b) ^{235}U. If we use the single-particle Weisskopf matrix elements[4] for the EL and ML transitions in ^{235}U, the excitation cross sections turn out to be (in barns)

$$\begin{aligned} &\sigma_{res}(E1,\ (1s)^2, E_+)=9.6\cdot 10^{-3}\xi(E1,\ (1s)^1, E_+),\\ &\sigma_{res}(M1,\ (1s)^2, E_+)=6.1\cdot 10^{-3}\xi(M1,\ (1s)^1, E_+),\\ &\sigma_{res}(E2,\ (1s)^2, E_+)=1.2\cdot 10^{-10}\xi(E2,\ (1s)^1, E_+),\\ &\sigma_{res}(E0,\ (1s)^2, E_+)=3.4\cdot 10^{-10}\xi(E0,\ (1s)^1, E_+). \end{aligned} \tag{19}$$

Hence, it follows that, for the single-particle $E1$ transition, σ_{res} can be of the order of 5×10^{-26} cm^2.

For energies $E2<2$ MeV, the spectrum of ^{235}U has been investigated in sufficient detail by Rickey *et al.*,[7] who identified a number of ^{235}U states with Nilsson neutron orbits $|[Nn_z\Lambda]\Omega\Pi\rangle$. Thus, the ground state of ^{235}U $(I_1\Pi_1+7/2^-)$ is interpreted as the $|[743]7/2^-\rangle$ neutron orbit. Out of the entire set of identified ^{235}U levels, we selected those for which the $E1$ transition from the ground state is possible and, for these levels, we calculated the excitation cross sections with matrix elements for the $E1$ transition of the nucleus for all the neutron orbits participating in the structure of the corresponding ^{235}U states.

The data on the head levels of rotational bands reported by Rickey *et al.*[7] are as follows:

1) $I_2\Pi_2=7/2^+$, 970-keV level; structure:

$$90\%\,|[624]7/2^+\rangle,\quad 10\%\,\{|[743]7/2^-\rangle\times Q(3,0)\};$$

$Q(3,0)$ is the octupole $(K=0)$ phonon. The $9/2^+$, 1030-keV level is the next level excited by the $E1$ transition.

2) $I_2\Pi_2=5/2^+$, 1116-keV level; structure:

$$14\%\,|[622]5/2^+\rangle,\quad 10\%\,|[633]5/2^+\rangle,\quad 76\%\,\{|[633]5/2^+\rangle\times Q(2,0)\};$$

$Q(2,0)$ is the quadrupole $(K=0)$ phonon. The $7/2^+$, 1157-keV and $9/2^+$, 1212-keV levels are the next in this band that can be excited by the $E1$ transition from the ground state.

3) Proposed $I_2\Pi_2=9/2^+$, 1438-keV level, where the main contribution (~66%) to the structure of the state is provided by the orbits $|[615]9/2^+\rangle$ and $|[613]7/2^+\rangle$.

We used these data to calculate the single-particle matrix elements for the $E1$ transition in ^{235}U, using neutron wave functions for the deformed Woods-Saxon potential well, as given by Gareev *et al.*[8]

Table II lists the calculated matrix elements for the $E1$ transition $\langle I_2\|E1\|I_1\rangle$ from the initial neutron state $|[743]7/2^-\rangle$ to the levels indicated above. The effective neutron charge was assumed to be Z/A. These matrix elements were then used to calculate the cross section

TABLE II. Single-particle nuclear matrix elements $\langle I_2\|E1\|I_1\rangle$. Calculated with the wave functions reported by Gareev *et al.*[8] and the corresponding resonance cross sections for the excitation of ^{235}U during annihilation on a filled K shell.

E_2, MeV	$I_2\Pi_2$	$\vert[Nn_z\Lambda]\Omega\Pi\rangle$	$\langle I_2\|E1\|I_1\rangle$, 10^{-3} F	$B(E1\uparrow)\cdot10^{4}$, $e^2\cdot$b	$\xi(E1)$	$\sigma_{res}(E1,(1s)^2)$, $10^{-6}\cdot$b
0.970	$7/2^+$	[624] $7/2^+$	1.51	2.27	0.678	0.600
1.030	$9/2^+$	[624] $7/2^+$	−0.725	0.657	1.93	0.496
1.1162	$5/2^+$	[622] $5/2^+$	2.80	5.87	3.26	7.46
1.1162	$5/2^+$	[633] $5/2^+$	5.60	23.6	3.26	30.0
1.1573	$7/2^+$	[622] $5/2^+$	−1.32	1.74	3.66	2.48
1.1573	$7/2^+$	[633] $5/2^+$	−2.64	6.95	3.66	9.92
1.212	$9/2^+$	[622] $5/2^+$	0.420	0.221	4.05	0.350
1.212	$9/2^+$	[633] $5/2^+$	0.835	0.875	4.05	1.38
(1.438)	$(9/2^+)$	[615] $9/2^+$	−2.47	7.63	4.33	12.9
(1.438)	$(9/2^+)$	[613] $7/2^+$	1.65	3.40	4.33	5.76

$\sigma_{res}(E1,(1s)^2,E_+)$ for the excitation of ^{235}U levels by a monochromatic positron beam.

The single-particle estimate for the reduced $E1$ transition probability is $0.074\,e^2.\ b$, so that all the $E1$ transitions in ^{235}U that we have considered are retarded by a factor of between about 3×10^{-3} and 3×10^{-5}, and this explains the low values of the cross section $\sigma_{res}(E1)$. The quadrupole $E2$ transitions to the levels in the β and γ vibrational bands lie in the same energy interval $E_2\sim1-1.5$ MeV and are known to be accelerated by a factor of ~2 – 6 relative to the single particle estimate $B(E2)$. Consequently, the cross section for the excitation of $E2$ transitions in ^{235}U may be $\sigma_{res}(E2)\sim10^{-5}-10^{-6}\,bn$, which is comparable with the excitation cross section for retarded $E1$ transitions in ^{235}U.

[1]R. D. Present and S. C. Chen, Phys. Rev. 85, 447 (1952).
[2]T. Mukoyama and S. Shimizu, Phys. Rev. C 5, 95 (1972).
[3]A. I. Akhiezer and V. B. Berestetskiĭ, Kvantovaya élektrodinamika, Fizmatgiz, Moscow, 1959 (Quantum Electrodynamics, Krieger, New York, 1965).
[4]A. Bohr and B. A. Mottelson, Nuclear Structure, Benjamin, New York, 1969 (Russ. Transl., Mir, Moscow, 1971), Vol. 1.
[5]I. M. Band and M. B. Trzhaskovskaya, Preprint No. 92, Institute of Nuclear Physics, Leningrad, 1974.
[6]S. Raman and H. J. Kim, Nucl. Data Sheets 16, 195 (1975).
[7]F. A. Rickey, E. T. Turney, and H. C. Britt, Phys. Rev. C 5, 2072 (1972).
[8]F. A. Gareev, S. P. Ivanova, L. A. Malov, and V. G. Soloviev, Nucl. Phys. A 171, 134 (1971).

Translated by S. Chomet