

Thus in this work there has been observed an interaction of two magnons of different frequencies, consisting of fusion of them into a phonon. An estimate has been made of the value of the spin-lattice relaxation and of the superheating of the spin system in parametric excitation of the waves.

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<sup>1</sup>V. V. Kveder, B. Ya. Kotyuzhanskiĭ, and L. A. Prozorova, *Zh. Eksp. Teor. Fiz.* **63**, 2205 (1972) [*Sov. Phys. JETP*

**36**, 1165 (1973)].

<sup>2</sup>B. Ya. Kotyuzhanskiĭ and L. A. Prozorova, *Zh. Eksp. Teor. Fiz.* **65**, 2470 (1973) [*Sov. Phys. JETP* **38**, 1233 (1974)].

<sup>3</sup>V. I. Ozhogin, *Zh. Eksp. Teor. Fiz.* **58**, 2079 (1970) [*Sov. Phys. JETP* **31**, 1121 (1970)].

<sup>4</sup>L. A. Prozorova and A. I. Smirnov, *Zh. Eksp. Teor. Fiz.* **67**, 1952 (1974) [*Sov. Phys. JETP* **40**, 970 (1974)].

<sup>5</sup>G. A. Melkov, *Fiz. Tverd. Tela (Leningrad)* **17**, 1728 (1975) [*Sov. Phys. Solid State* **17**, 1123 (1975)].

<sup>6</sup>V. V. Kveder and L. A. Prozorova, *Pis'ma Zh. Eksp. Teor. Fiz.* **19**, 683 (1974) [*JETP Lett.* **19**, 353 (1974)].

<sup>7</sup>V. A. Kolganov, V. S. L'vov, and M. I. Shirokov, *Pis'ma Zh. Eksp. Teor. Fiz.* **19**, 680 (1974) [*JETP Lett.* **19**, 351 (1974)].

<sup>8</sup>A. I. Akhiezer, V. G. Bar'yakhtar, and M. I. Kaganov, *Usp. Fiz. Nauk* **72**, 3 (1960) [*Sov. Phys. Usp.* **3**, 661 (1961)].

<sup>9</sup>M. H. Seavey, *Phys. Rev. Lett.* **23**, 132 (1969).

<sup>10</sup>V. S. Lutovinov, *Fiz. Tverd. Tela (Leningrad)* **20**, in press (1978) [*Sov. Phys. Solid State* **20**, in press (1978)].

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## Interaction between vortices and the surface of a type II superconductor and the field of a vortex in a cavity

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The response of a hollow, thin-walled Pb+30 at.% In superconducting cylinder in the mixed state to an external weak alternating magnetic field is investigated experimentally. It is found that vortices located near the inner surface of the cylinder produce a magnetic field in the cavity of the cylinder. In response to an external alternating field, the vortices adjacent to the outer surface of the cylinder execute reversible oscillations until their amplitude reaches a certain critical value, which is proportional to the period of the vortex lattice. This state corresponds to the flow of a critical current along the outer surface of the cylinder. The vortex oscillation amplitude in this case is of the order of several angstroms for vortices adjacent to the inner surface of the cylinder. The interaction between the vortices and the surface of the superconductor depends on the modulus of rigidity  $k$ , in accord with Eq. (3). The proportionality coefficient between the oscillation amplitudes for the outer and inner vortices is proportional to the external magnetic field.

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### 1. INTRODUCTION

The purpose of the present work is twofold. First, we want to investigate experimentally the interaction of vortices with the surface of a superconductor; second, we wish to find the magnetic field created in a macroscopic cavity by a vortex located near this cavity. The second is by no means trivial. Actually, it is well known<sup>[1]</sup> that a vortex located near the open surface of a superconductor does not create a magnetic field outside the superconductor. This is illustrated in Fig. 1a, where a vortex is shown, whose core is located near ( $x_0 < \lambda$ ) the plane surface of a superconductor. If the superconductor occupies the halfspace  $x > 0$  then the magnetic field is equal to zero in the region  $x < 0$ . Furthermore,

the total magnetic flux which is created in the superconductor by such a vortex is not equal to the flux quantum  $\Phi_0$  but is equal to<sup>[1]</sup>

$$\Phi_c = \Phi_0(1 - e^{-x_0/\lambda}),$$

where  $x_0$  is the coordinate of the core of the vortex,  $\lambda$  is the penetration depth of the weak magnetic field. If now we close the open surface of the superconductor at a distance from the vortex that is large in comparison with  $\lambda$  (Fig. 1b), the picture changes radically. A superconducting current immediately passes along the inner surface of the resulting macroscopic cavity, and a magnetic field induced by the vortex develops inside the cavity. This follows both from the result of the calcula-

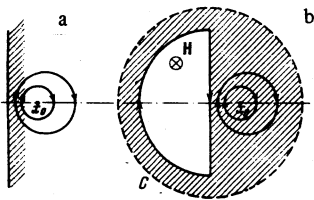


FIG. 1. a) Vortex does not create a magnetic field outside the singly-connected superconductor; b) vortex creates a magnetic field in the cavity.

tion of the particular case of a vortex near a cylindrical cavity,<sup>[2]</sup> and from general considerations. Actually, the total magnetic flux linked with the contour  $C$  in Fig. 1b should be quantized if the superconducting current is equal to zero at any point of the contour  $C$ . But the magnetic flux created by the vortex in the superconductor, as has already been remarked above, will be less than the flux quantum. Therefore a magnetic flux should develop inside the cavity making the total flux of the vortex up to one quantum.

The investigation was carried out on a long hollow, thin-walled cylinder of a type-II superconductor. A stationary external magnetic field, parallel to the axis of the cylinder, created a mixed state in it. Weak modulation of this external field at acoustic frequency caused small radial oscillations of the vortices in the cylinder. The oscillations of the vortices in turn led to field oscillations induced by the vortices in the cavity of the cylinder. The experiment consisted in the measurement of the dependence of the alternating component of the magnetic field in the cavity of the cylinder on the amplitude of the external modulating field at various values of the stationary magnetic field.

In Sec. 2, we give: 1) a derivation of the force of interaction of the vortex lattice with the plane surface of the superconductor; 2) a theoretical analysis of the dependence of the displacement of the vortices adjacent to the inner and outer surfaces of the hollow cylinder on a small change in the external magnetic field; 3) a derivation of the change in the field in the cavity of the cylinder produced by the shift in the vortex structure.

Section 3 contains a description of the experimental arrangement, the method of preparation of samples, and the experimental results, while Sec. 4 is devoted to a discussion of the results.

## 2. THEORY

### 2.1. Interaction of the vortex lattice with a plane surface of a superconductor

The interaction of the vortices with the surface of a superconductor was considered theoretically in Refs. 1, 3 and 4. It is appropriate here to carry out what we think is a very simple and, at the same time, sufficiently rigorous derivation of the expression for the strength of this interaction, applicable to the case considered.

We consider a semi-infinite type-II superconductor ( $\kappa = \lambda/\xi \gg 1$ ) occupying the halfspace  $x > 0$ . The external field  $H_0$  is parallel to the  $z$  axis and  $H_{c1} \ll H_0 \ll H_{c2}$ .

It is clear that the magnetization current

$$I_M = c|M| = \frac{c}{4\pi} (H_0 - B)$$

flows only near the surface in a layer  $\sim \lambda$ , and the current density will be

$$j_M = (I_M/\lambda) e^{-x/\lambda}.$$

We consider the case in which  $a \sim x_0 \sim \lambda$ , where  $a$  is the period of the vortex lattice,  $x_0$  is the coordinate of the first vortex series. Then the force acting on one surface vortex is equal to

$$f_1 = \frac{1}{c} j_M(x_0) \Phi_0 = \frac{1}{4\pi} \frac{H_0 - B}{\lambda} \Phi_0 e^{-x_0/\lambda}.$$

We neglect the forces acting on the more deeply located vortices. We then immediately obtain the pressure acting on the vortex lattice and balanced by the elastic forces:

$$f = \frac{f_1}{a} = \frac{|M|}{\lambda} (\Phi_0 B)^{1/2} e^{-x_0/\lambda}.$$

Here we make use of the equality

$$M = (B - H_0)/4\pi, \quad a = (\Phi_0/B)^{1/2},$$

where  $B$  is the magnetic induction in the sample.

A shift of the entire vortex lattice by  $\delta x_0$  leads to a change in the pressure on its left surface by an amount

$$\delta f_{\text{left}} = -|M| \lambda^{-2} (\Phi_0 B)^{1/2} \delta x_0. \quad (1)$$

Here we shall assume that  $x_0 \leq \lambda$  and therefore  $e^{-x_0/\lambda}$  can be set equal to unity. If we now assume that at some large distance there also exists a flat right-hand edge of the vortex lattice, and that it is also displaced a distance  $\delta x_0$ , then the change in the pressure on the right surface of the vortex lattice will be

$$\delta f_{\text{right}} = |M| \lambda^{-2} (\Phi_0 B)^{1/2} \delta x_0.$$

It then follows that a restoring force acts on the vortex lattice when it is rigidly displaced without deformation. This force is equal (per unit surface area) to

$$\delta f = -2|M| \lambda^{-2} (\Phi_0 B)^{1/2} \delta x_0.$$

The mean density of the restoring force is equal to

$$F = -\frac{2|M|}{\lambda^2 d} (\Phi_0 B)^{1/2} \delta x_0,$$

where  $d$  is the thickness of the superconducting plate in the mixed state.

Now, let a transport current  $I_T$  flow in a plate of thickness  $d$ , i. e., let a current  $I_T/2$  flow along each of the edges. Acting on the vortex line nearest the surface, this current creates a pressure on the vortex lattice

$$f_T = I_T \Phi_0 / c \lambda a = (I_T / c \lambda) (\Phi_0 B)^{1/2}.$$

Equating this pressure to the restoring force  $\delta f$ , we ob-

tain the connection between the transport current  $I_T$  and the displacement of the vortex lattice  $\delta x_0$ . Taking the critical displacement

$$\delta x_{cr} \sim a = (\Phi_0/B)^{1/2},$$

as in Ref. 3, we obtain the following value for the critical current of the plate:

$$I_c = 2c|M|\sqrt{\Phi_0}/\lambda\sqrt{B}.$$

The mean density of the critical current will then be equal to

$$j_c = 2c|M|\sqrt{\Phi_0}/\lambda d\sqrt{B}. \quad (2)$$

A formula similar to Eq. (2) was first obtained in Ref. 4, and then derived in Ref. 3 for the case  $a \ll \lambda$ . We now see that this formula is valid also for the case  $a \sim \lambda$ .

It is important for our purpose to establish the fact that, in accord with Eq. (1), the surface of the superconductor acts on the vortex lattice as a spring with stiffness

$$k = |M|(\Phi_0 B)^{1/2}/\lambda^2. \quad (3)$$

## 2.2. Elasticity of a vortex lattice having the shape a hollow cylinder

We now consider the following problem. An infinite hollow cylinder of a type-II superconductor is given ( $\kappa \gg 1$ ) the inner radius of which is equal to  $r_1$  and the outer to  $r_2$ . The stationary external magnetic field  $H_0$  is identical outside and inside the cylinder and parallel to its axis,  $H_{c1} \ll H_0 \ll H_{c2}$ . If we superimpose from outside an additional magnetic field  $b$  that is small in comparison with  $H_0$ , then this small field produces along the outer surface of the cylinder an additional Meissner current that creates a pressure  $P$  on the outer surface of the vortex lattice. It is necessary to find the response of the vortex lattice to this external influence. In other words, we want to find the dependence of the displacements of the vortex lattice near the outer and inner surfaces of the cylinder on the additional external field  $b \ll H_0$ . We shall be interested only in the linear response of the vortex system to  $b$ .

Let the displacements of the inner and outer surfaces of the vortex lattice be equal to  $\delta x_1$  and  $\delta x_2$ , respectively. After these displacements have been carried out, the pressure on the inner and outer surfaces of the lattice will be

$$P_1 = -k\delta x_1, \quad P_2 = P + k\delta x_2,$$

where  $k$  is defined by Eq. (3).

The positive sign corresponds to a displacement away from the center of the cylinder. We regard the vortex lattice itself as an isotropic (in the plane perpendicular to the axis of the cylinder) elastic medium, the elastic

moduli of which are equal to [5]

$$c_{11} \approx B^2/4\pi, \quad c_{66} \sim H_{cm}^2(1 - H_0/H_{c2})^2, \quad (4)$$

where  $H_{cm}$  is the critical thermodynamic magnetic field and  $c_{11} \gg c_{66}$ . Using the latter inequality and the usual elasticity theory, it is not difficult to obtain

$$\delta x_1 = -P \frac{r_2}{r_2^2 - r_1^2} \frac{c_{11} + c_{12}}{k^2}, \quad \delta x_2 = -\frac{P}{k}.$$

Now, assuming that the cylinder is thin-walled, i. e.,  $r_2 - r_1 = d \ll r_1$ , and taking it into account that  $c_{11} \approx c_{12}$ , we have

$$\delta x_1 = -Pc_{11}/dk^2, \quad \delta x_2 = -P/k. \quad (5)$$

## 2.3. Change produced in the field in the cavity of a cylinder by a displacement of the vortex structure

We first find that magnetic field  $H^{(1)}$  which creates a single vortex inside the cylindrical cavity (radius equal to  $r_1$ ). This vortex is located at a distance  $x$  from the cavity. In the relative units of the Ginzburg-Landau theory ( $\lambda = 1$ ,  $\sqrt{2}H_{cm} = 1$ ), this field is equal, according to Ref. 2, to

$$H^{(1)} = \frac{1}{\kappa r_1} \frac{K_0(r_1 + x)}{K_1(r) + 1/2rK_0(r)}, \quad (6)$$

where  $K_{0,1}$  are Hankel functions of imaginary argument. Assuming that  $r_1 \gg 1$  and  $x \ll r_1$ , we obtain

$$H^{(1)} = 2e^{-x/\kappa r_1^2}.$$

The change of this field  $\delta H^{(1)}$  produced by the displacement of the vortex  $\delta x_1$ , is equal to

$$\delta H^{(1)} = -2e^{-x/\kappa r_1^2} \delta x_1/\kappa r_1^2.$$

It follows from this formula that a contribution to the field  $\delta H^{(1)}$  will be made only by those vortices which are located at distances less than or of the order of the penetration depth  $\lambda$  from the inner surface of the cylinder.

From the very beginning, we have assumed that  $x_0 \sim a \sim \lambda$ . Therefore, the total change in the field inside the cavity,  $\delta H$ , due to a displacement  $\delta x_1$  of the inner vortices, will be equal to

$$\delta H = (2\pi r_1/a) \delta H^{(1)}$$

or, transforming to the ordinary Gaussian units and omitting the exponential, as in the derivation of Eq. (1), we have, finally,

$$\delta H = (2/r_1\lambda)(\Phi_0 B)^{1/2} \delta x_1. \quad (7)$$

If we now assume that the vortex structure in a hollow cylinder oscillates with amplitude  $\delta x_1$ , then an emf proportional to  $\delta H$  will be induced in a detecting coil placed in the cylinder. Thus, analyzing the signal of the de-

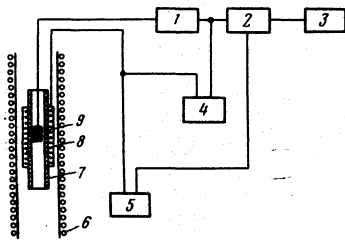


FIG. 2. Block diagram of the arrangements: 1—amplifier; 2—synchronous detector; 3—digital voltmeter; 4—oscilloscope; 5—low frequency generator; 6—solenoid; 7—sample; 8—driving coil; 9—detecting coil.

detecting coil, we can study the displacement of the vortex  $\delta x_1$ .

### 3. EXPERIMENTAL SETUP, SAMPLES AND EXPERIMENTAL RESULTS

As samples, we used cylinders of the alloy Pb + 30 at. % In. They were prepared in the following way. The Pb + In melt, quenched in oil, was subjected to the homogenizing annealing in a vacuum of  $10^{-5}$  mm Hg at a temperature of 240 °C for a period of 15 hr. Hollow cylinders were prepared from the ingots in this fashion. Immediately after preparation, these cylinders had a mirror-smooth surface. The dimensions of the cylinders were the following: length 6 cm, inner radius  $r_2 = 4.2$  mm, thickness of the wall  $d = 0.67$  mm. The cylinders thus obtained were subjected to vacuum annealing ( $10^{-5}$  mm Hg) at a temperature of 120–130 °C for 3 hr to remove internal stresses. X-ray microanalysis confirmed the homogeneity of the chemical composition of the material of the cylinder.

Since the Pb-In alloys form a solid solution at the concentrations chosen, there could be no segregations of another phase in our sample. According to Ref. 6, pinning on the grain boundaries is not observed experimentally in Pb-In alloys with an indium content of more than 15%. Therefore, our samples could be considered as sufficiently homogeneous without noticeable volume pinning.

We now give the known<sup>[7]</sup> data for our alloy:

$$T_c = 7.0 \text{ K}, H_{c2} = 3700 \text{ Oe}, H_{c1} = 145 \text{ Oe}, H_{cm} = 505 \text{ Oe}, \kappa = 5.2.$$

We then easily find  $\lambda = 1.5 \times 10^{-5}$  cm. All the data correspond to a temperature of 4.2 K.

The experimental setup is sketched in Fig. 2. The

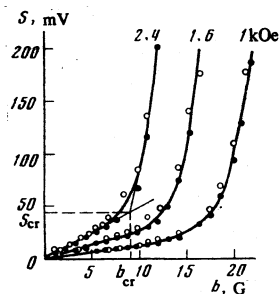


FIG. 3. Dependence of the signal  $S$  of the detecting coil on the effective value of the field of the driving coil for two identically prepared samples (the black and white circles here and below in Figs. 4–6).

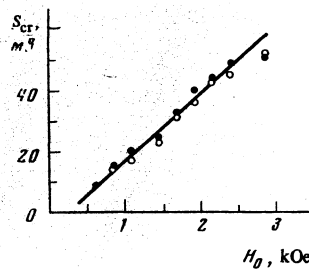


FIG. 4. Dependence of the critical signal  $S_{cr}$  on the external field  $H_0$ .

cylindrical sample was placed inside a superconducting solenoid capable of creating a maximum magnetic field of 30 kOe. The central part of the sample was encircled by a driving coil (height 2 cm), which was connected to an audio frequency oscillator (Fig. 2). The research was carried out to a frequency of 550 Hz. The detecting coil was placed inside the cylindrical sample. The audio frequency voltage generated in this coil was amplified by a narrow band amplifier U2-6 and detected by a synchronous detector V2-9. This was the directly measured signal  $S$ . An oscilloscope (see Fig. 2) enabled us to make a comparison of the phase of the signal  $S$  and the phase of the voltage on the driving coil. In each measurement of the signal  $S$ , fine adjustment of the phase of the reference signal of the synchronous detector V2-9 at maximum indication was made. All the measurements were made at a temperature of 4.2 K.

The experiment consisted in obtaining the dependence of  $S$  on the effective value of the field of the driving coil  $b$  at a given value of the stationary magnetic field created by the superconducting solenoid. Figure 3 shows typical  $S(b)$  curves. As is seen from Fig. 3, the presence of a linear portion and the  $S(b)$  plot with a subsequent rapid increase in the signal, beginning with some field  $b_{cr}$ , is common to all this family of curves. The method of determination of the field  $b_{cr}$  is shown in Fig. 3:  $b_{cr}$  is the abscissa of the point of intersection of two linear extrapolations,  $S_{cr}$  is the ordinate of this point.

The dependence of  $S_{cr}$  on the external stationary magnetic field is shown in Fig. 4. It is seen that this dependence is well approximated by the linear law  $S_{cr} \propto H_0$ . The investigation of the phase difference between  $S$  and  $b$  with the help of an oscilloscope showed that the phase difference on the linear portions is equal to  $\pi/2$ , at  $b > b_{cr}$  the phase shift sharply changed.

Finally, we note that preliminary experiments were carried out by us on a study of the signal  $S$  from the detecting coil, located on the outer surface of the cylinder being studied. In this case,  $S_{cr}$  turned out to be a quantity independent of the field  $H_0$ .

### 4. DISCUSSION OF THE RESULTS

1. It follows from the shape of the curves  $S(b)$  (Fig. 3) that the dependence of the field inside the cylinder on the external alternating field has more or less extended linear portions, which give way abruptly at some value  $b_{cr}$  to a steep increase in the  $S(b)$  dependence. It is natural to assume that at  $b < b_c$  the vortex structure executes reversible displacements, while at  $b = b_{cr}$  an irreversible

flow of vortex lines begins, i. e., a critical current flows along the outer surface and is determined by the pinning of the vortices on the external surface of the cylinder.

This assumption is confirmed by a study of the phase shift between the signal  $S$  and the field  $b$ . If the vortices are displaced elastically, and "follow" the field  $b$ , then the change in the internal field  $\delta H$  and  $b$  should be in phase, and the signal  $S \propto d(\delta H)/dt$  and  $b$  should be shifted in phase by  $\pi/2$ . This is also observed experimentally.

The sharp change in this phase relation at  $b > b_{cr}$  is also easy to understand. Now the velocity of viscous flow of the vortex  $d(\delta x_1)/dt$  is proportional to and in phase with the field  $b$ .

2. As has already been noted, a preliminary experiment has been carried out on observation of the signal of the detecting coil placed in the outer surface of the sample. In this case,  $S_2$  will be determined by the amplitude of the displacement  $\delta x_2$  of the vortices adjacent to the outer surface of the cylinder:

$$S_2 \propto \delta \Phi_2 = \frac{2\pi r_2}{\lambda} (\Phi_0 B)^{1/2} \delta x_2, \quad (8)$$

where  $\delta \Phi_2$  is the change produced in the magnetic flux in the detecting coil by the displacement of the vortex in the sample.

Since the experiment showed that the critical signal  $S_{cr2}$  does not depend on the magnetic field  $H_0 \approx B$ , we can, by using Eq. (8), find the dependence of the critical displacement of the outer vortices on the external field  $H_0$ :

$$\delta x_{2cr} \propto H_0^{-1/2}.$$

Such a dependence agrees well with the well known<sup>[1,3,4]</sup> assumption that the critical displacement  $\delta x_{2cr}$  should be of the order  $a \approx (\Phi_0/H_0)^{1/2}$ . On the other hand, numerical estimate with the aid of Eq. (8) would give a change in  $\delta x_{2cr}$  from 80 to 111 Å for the range of fields  $H_0$  employed. This is approximately an order of magnitude smaller than the period of the vortex structure.

3. We now discuss the result of the investigation of the field dependence of the critical signal  $S_{cr1}$  on the detecting coil located inside the investigated cylinder. Since  $S_{cr1} \propto H_0$  (see Fig. 4), then it follows from Eq. (7) that

$$\delta x_{1cr} \propto H_0^{1/2}.$$

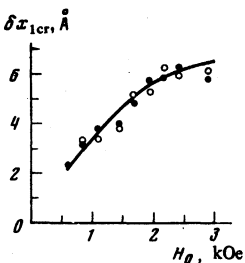


FIG. 5. Dependence of the critical displacement  $\delta x_{1cr}$  of the external vortices on the external field  $H_0$ .

The dependence of  $\delta x_{cr1}$  on  $H_0$  calculated from this formula is shown in Fig. 5. It is seen that the elastic reversible displacements of the internal vortices (adjacent to the inner surface of the cylinder) can take place only in the range of several Angstroms, which is smaller by several orders of magnitude than the period of the vortex lattice. Thus, we come to the conclusion that the vortex structure is not displaced as a single unit in our experiments, but that the outer vortices oscillate with an amplitude that is two orders of magnitude larger than those of the internal vortices.

Finally, we note that the results are not in quantitative agreement with theory in the following respects. Using the assumptions made above, we have obtained the result that  $\delta x_{2cr} \propto H_0^{-1/2}$ ,  $\delta x_{1cr} \propto H_0^{1/2}$ . Consequently, it follows from experiment that

$$\delta x_{1cr}/\delta x_{2cr} \propto H_0.$$

On the other hand, it follows from the formulas (5) and (3) that

$$\frac{\delta x_1}{\delta x_2} = \frac{c_{11}}{kd} \sim \frac{B^{1/2}}{|M|}. \quad (9)$$

Thus, experiment shows that the ratio  $\delta x_1/\delta x_2$  increases with increase in the field  $H_0$ . The theoretical result also yields an increase in this ratio with increase in the field, but the dependence on the field is quantitatively stronger here.

4. We now investigate the dependence on the field  $H_0$  of the critical amplitude of the field  $b_{cr}$  produced by the driving coil. Upon imposition of the field  $b_{cr}$ , the current

$$I = cb/4\pi,$$

flows along the outer surface of the sample. This current, as was shown in Sec. 2.1, creates a pressure on the vortex lattice

$$P = \frac{I}{c\lambda} (\Phi_0 B)^{1/2} = \frac{b}{4\pi\lambda} (\Phi_0 B)^{1/2}.$$

According to (5) and (3), the critical field  $b_{cr}$  can now be written in the form

$$b_{2cr} = (4\pi|M|/\lambda) \delta x_{2cr}. \quad (10)$$

The magnetic moment  $M$  can be represented with logarithmic accuracy in the form<sup>[6]</sup>

$$4\pi M = H_{c1} \frac{\ln(a/\xi)}{\ln \kappa}$$

or

$$4\pi M = \frac{H_{c1}}{2 \ln \kappa} \ln \left( \frac{H_{c2}}{H_0} \right).$$

Using this expression and formulas (8) and (10), we find

$$b_{cr} = K \frac{\ln(H_{c2}/H_0)}{\sqrt{B}}, \quad (11)$$

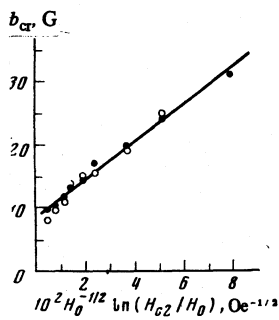


FIG. 6. Experimentally observed dependence of  $b_{cr}$  on  $H_0$ .

$$K = \frac{H_{c1} \delta \Phi_{2cr}}{4\pi r_2 \sqrt{\Phi_0} \ln \kappa} \quad (12)$$

The experimental dependence of  $b_{cr}$  on  $H_0$  is shown in Fig. 6.

Using the experimentally obtained value  $\delta \Phi_{2cr} = 2.37 \times 10^{-3} \text{ G-cm}^2$ , we obtain from Eq. (12):

$$K_{theor} = 88 \text{ G}^{3/2}.$$

The experimental value  $K_{exp}$  can be obtained from Fig. 6:

$$K_{exp} = 318 \text{ G}^{3/2}.$$

Recognizing that Eq. (12) does not contain any adjustable parameters, such agreement must be regarded as satisfactory.

5. We connect the critical amplitude of the driving coil with the appearance of the critical current  $I_c$  on the outer surface of the cylinder. This current leads to the critical amplitude of oscillations of the external vortices  $\delta x_2$ . An additional confirmation of this is the direct experiment on the measurement of the critical current in foil of the same alloy, subjected to the same temperature treatment. The external magnetic field was parallel to the plane of the foil and perpendicular to the current. The values of the critical currents, obtained in direct measurement of  $j_c$  and calculated from

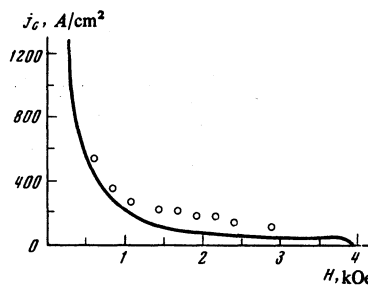


FIG. 7. Dependence of the critical current density  $j_c$  on the external field  $H$ . The points are the critical current density calculated from the values of  $b_{cr}$ ; the solid curve is the dependence of  $j_c(H)$ , measured directly.

$b_{cr}$ , are found in excellent agreement (see Fig. 7).

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<sup>1</sup>V. V. Schmidt and G. S. Mkrtychyan, Usp. Fiz. Nauk 112, 459 (1974) [Sov. Phys. Uspekhi 17, 353 (1975)].

<sup>2</sup>G. S. Mkrtychyan and V. V. Schmidt, Zh. Eksp. Teor. Fiz. 61, 367 (1971) [Sov. Phys. JETP 34, 195 (1972)].

<sup>3</sup>V. V. Schmidt, Zh. Eksp. Teor. Fiz. 61, 398 (1971) [Sov. Phys. JETP 34, 211 (1972)].

<sup>4</sup>A. M. Campbell, J. E. Evetts, and D. Dew-Hughes, Phil. Mag. 18, 313 (1968).

<sup>5</sup>R. Labusch, Phys. Stat. Sol. 32, 439 (1969).

<sup>6</sup>L. Ya. Vinnikov, O. V. Zharikov, Ch. V. Konetskii and S. I. Moskvina, 19th All-Union Conf. on Low Temperature Physics, Report of papers, Minsk, 1976, p. 423.

<sup>7</sup>J. E. Evetts and J. M. A. Wade, J. Phys. Chem. Sol 31, 973 (1970).

<sup>8</sup>P. de Gennes, Superconductivity of Metals and Alloys, Benjamin, 1965.

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