Concerning a bound neutron in matter

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An experiment is proposed for the detection of quasistationary neutron states in matter. It is based on the resonance dependence of the transmission coefficient of a neutron passing through a double-hump barrier. The feasibility of constructing a neutron spectrometer for neutrons with energies $\lesssim 10^{-7}$ eV is discussed.

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1. It was noted first by Kagan[1] that bound neutron states with energies of about 10^{-7} eV may exist in an irregular crystal in the presence of large multivacancies, pores, microcracks, etc. In the same paper, a single experiment was proposed for the observation of bound neutrons in matter, but has not been carried out so far for technical reasons. In the present work a new experiment for detection of a bound neutron in matter is proposed and the feasibility of constructing a neutron spectrometer for neutrons with energies $\lesssim 10^{-7}$ eV is discussed.

2. It is well known that the interaction of a low-energy neutron with a medium can be described, neglecting magnetic interaction, by the effective potential energy

$$V=2\pi\hbar^2 Nb/m,$$

where m is the neutron mass, N the number of nuclei in a unit volume, b the coherent scattering length for the scattering of a neutron by a bound nucleus. Just this interaction, whose order of magnitude is 10^{-7} eV for most substances, may lead to the formation of a bound neutron state in matter. Thus for a neutron incident perpendicularly to the plane of a target of thickness d, made of a substance with a coherent scattering length $b>0$, the effective potential energy has the shape of a rectangular barrier of height V and width d.

The proposed experiment is based on the penetration of neutrons through a multilayer target. The first and third layers of this target are made of the same substance, with a coherent scattering length $b_1>0$; their thicknesses are d_1 and d_3 respectively. The second layer, of thickness d_2, consists of a material with a coherent scattering length b_2, where $b_2<b_1$, and b_2 may even be negative. Then for a neutron incident at right angles to the target plane the effective potential energy represents the double-hump barrier shown schematically in Fig. 1.

A double-hump potential barrier contains a potential well, so that quasi-stationary neutron states may arise inside the well at certain values of its width and depth. The energy of these states can be estimated in the quasi-classical approximation

$$E_n=V_{\min}+\frac{\pi^2}{2md_2^2}\left(n+\frac{1}{2}\right)^2,\quad n=0,1,2,\ldots$$

We consider now the transmission coefficient of a neutron passing through a double-hump potential barrier. It is known that, owing to the possibility of formation of quasi-stationary states within the barrier, the transmission coefficient of a neutron incident on such a double-hump barrier depends on the neutron energy in a resonant manner. This can be shown most simply in the quasi-classical approximation. At a neutron energy $E<V_{\min}$, the transmission coefficient for a neutron passing through the barrier shown in Fig. 1 can be written in the form

$$P(E<V_{\min})=P_AP_BP_C,$$

where

$$P_{A,B,C}\approx\exp\left\{-\frac{2}{\hbar}d_i[2m(V_{A,B,C}-E)]^{1/2}\right\}$$

are the transmission coefficients of the neutron penetrating through barriers A, B, and C of respective thicknesses d_1, d_3, and d_2.

At neutron energies $V_{\min}<E<V_{\max}$ the transmission coefficient for the penetration of a double-hump barrier by a neutron takes, according to Ref. 2, the form

$$P(E)=\left\{4\cos^2\sigma\left((P_AP_B)^{-1/2}+\frac{1}{16}(P_AP_B)^{1/2}\right)^2+\frac{1}{4}\sin^2\sigma\left[\left(\frac{P_A}{P_B}\right)^{1/2}+\left(\frac{P_B}{P_A}\right)^{1/2}\right]^2\right\}^{-1}$$

$$\sigma=\frac{d_2}{\hbar}(2m(E-V_{\min}))^{1/2}.$$

It is seen from this formula that at $\sigma=\pi(n+\frac{1}{2})$, where $n=0,1,2,\ldots$, the neutron transmission coefficient reaches a maximum

$$P_{\max}=4P_AP_B/(P_A+P_B)^2,$$

and for $\sigma=\pi n$ a minimum

$$P_{\min}=P_AP_B/4.$$

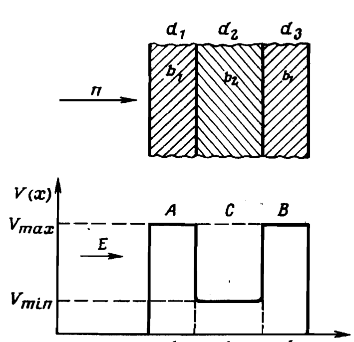

FIG. 1. Schematic representation of the effective potential energy of the interaction of a neutron with a three-layer target.

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The transmission coefficient shows the most pronounced resonant character when $P_A = P_B$. In this case $P_{max} = 1$ and $P_{min} = P_A^2/4$.

In the quasi-classical approximation one can find not only the energy of the n-th resonance in the transmission coefficient

$$E_n = V_{min} + \frac{\pi^2}{2md^2}\left(n + \frac{1}{2}\right)^2$$

but also its width $\Gamma_n = \hbar\omega(P_A + P_B)/4$, the energy interval between resonances $\Delta E = E_{n+1} - E_n = \hbar\omega$, and the energy dependence of the transmission coefficient near the resonance

$$P(E \sim E_n) = \Gamma_n^2/(\Gamma_n^2 + (E - E_n)^2),$$

where

$$\omega = \frac{\pi}{md_2}(2m(E - V_{min}))^{1/2}.$$

3. Experiments aimed at observation of quasi-stationary neutron states in matter can be conducted with ultracold neutrons in two variants. In the first variant the existence of quasi-stationary neutron states can be inferred by measuring the difference of the number of neutrons passing through a multilayer and a single-layer target. For this purpose a stream of neutrons is passed through a multilayer target of thickness $d_1 + d_2 + d_3$ and with layer coherent scattering lengths b_1, b_2, and b_3, and the number of transmitted neutrons is measured. The experiment is repeated, but this time the neutrons are passed through a single-layer target of thickness $d_1 + d_3$ and a coherent scattering length b_1. As a result of these measurements we obtain different counts, owing to the resonant dependence of the transmission coefficient of the multilayer target on the neutron energy. We will evaluate this difference for the case where $P_A = P_B$, the three-layer target consists of beryllium and aluminum, and the single-layer target is of beryllium. In this case $V_A = V_B = 2.4\times10^{-7}$ eV, $V_C = 0.55\times10^{-7}$ eV; the thicknesses of the layers are taken to be $d_1 = d_3 = 250$ Å and $d_2 = 800$ Å. Then, for a uniform flux of neutrons with an energy between zero and 1.72×10^{-7} eV, the integral penetrability will be 1.4% of the total flux for the multilayer and 0.05% for the single-layer target.

Figure 2 shows the dependence of the neutron transmission coefficient on the neutron energy for a multilayer and a single-layer target, obtained in the quasi-classical approximation. For a rectangular double-hump barrier the neutron transmission coefficient can be calculated without using the quasi-classical approximation, by solving the Schrödinger equation numerically. It is seen from Fig. 3 that the results of this calculation are in good agreement with the results of the quasi-classical calculation.

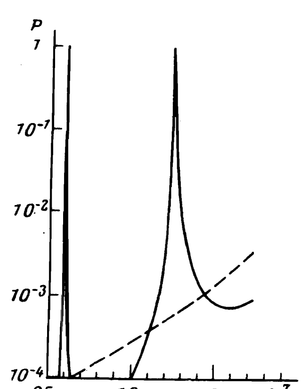

FIG. 2. Transmission coefficient of neutrons passing through a rectangular double-hump potential barrier vs the incident-neutron energy (eV) in the quasi-classical approximation. Barrier parameters: $V_A = V_B = 2.4\times10^{-7}$ eV, $V_C = 0.55\times10^{-7}$ eV, $d_1 = d_3 = 250$ Å, $d_2 = 800$ Å.

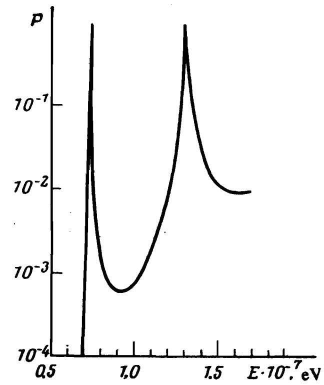

FIG. 3. Transmission coefficient of neutrons passing through a rectangular double-hump potential barrier vs the incident-neutron energy (eV) (numerical solution of the Schrödinger equation). Parameters the same as in Fig. 2.

In the second variant of the experiment, a target with covered end faces must be employed. Then, using a pulsed source of ultracold neutrons, one can search for delayed neutrons. The delay time is determined by the lifetime of the quasi-stationary state

$$T \sim \frac{4}{\omega(P_A + P_B)}.$$

The picture of the interaction of neutrons with a medium, as described above, will be distorted by radiative capture and inelastic scattering of the neutrons by phonons. Since the principal characteristic time is the lifetime T of the quasi-stationary states, it is required that $T < T_{n\gamma}$ and $T < T_{nn'}$. Naturally, this imposes certain conditions both on the choice of material of the layers and on the temperature of the target. By introducing a complex double-hump barrier we can hope to be able to describe the neutron transmission coefficient with allowance for the possible radiative capture and inelastic scattering of the neutrons by the target nuclei. This is done in the simplest manner by assuming that the imaginary part of the potential is constant and different from zero only in the region between the barriers.

Then in the quasi-classical approximation the phase σ will have both a real part θ and an imaginary part η. The expressions for the transmission coefficient P, reflection coefficient T, and absorption coefficient R will take the form

$$P = \{(4A^2\cos^2\theta + {}^1/_4B^2\sin^2\theta)\,\mathrm{ch}^2\,\eta + (4A^2\sin^2\theta + {}^1/_4B^2\cos^2\theta)\,\mathrm{sh}^2\eta + 2AB\,\mathrm{ch}\,\eta\,\mathrm{sh}\,\eta\}^{-1},$$

$$T = \{(4C^2\cos^2\theta + {}^1/_4D^2\sin^2\theta)\,\mathrm{ch}^2\,\eta + (4C^2\sin^2\theta + {}^1/_4D^2\cos^2\theta)\,\mathrm{sh}^2\,\eta - 2CD\,\mathrm{ch}\,\eta\,\mathrm{sh}\,\eta\}P,$$

$$R = 1 - T - P,$$

where

$$A = \frac{16 + P_AP_B}{16(P_AP_B)^{1/2}}, \quad B = \frac{P_A + P_B}{(P_AP_B)^{1/2}},$$

$$C=\frac{16-P_AP_B}{16(P_AP_B)^{1/2}},\quad D=\frac{P_B-P_A}{(P_AP_B)^{1/2}}.$$

In these experiments θ coincides with σ, $\eta=W_0\partial\theta/\partial E$, and W_0 is the imaginary part of the potential.

4. The proposed experiment for the detection of quasi-stationary neutron states in matter has, apart from its scientific interest, also a practical significance. A large volume of work with ultracold neutrons is planned at present,[3] and the problem arises of measuring energy spectra of neutrons with energies $\lesssim 10^{-7}$ eV. It is seen from Figs. 2 and 3 that one can use for this purpose the three-layer target described above, which transmits selectively neutrons with definite energies. Placing such a target in the path of the neutrons toward a detector, we record neutrons of definite energies. A set of calibrated three-layer targets made of various materials, with different or even variable thickness, will make possible measurements of energy spectra in the neutron energy range $\lesssim 10^{-7}$ eV. We note that the principle of operation of the proposed neutron spectrometer is analogous to that of the widely known Fabry-Pérot optical interferometer.[4]

One can hope that a neutron spectrometer utilizing the wave properties of the neutron will have better characteristics with respect to the resolution $\Delta E=\hbar\omega(P_A+P_B)/4$ and small dimensions and will prove more convenient in operation than a gravitation neutron spectrometer.

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Theory of the temperature dependence of the muon precession frequency shift for anisotropic muonium

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Equations have been obtained for the polarization density matrix for muonium for the case of anisotropic hyperfine interaction which may occur in impurity hydrogen (and muonium) in strongly doped semiconductors. Expressions have been obtained for the precession of muionium in a transverse field and it has been shown that the anisotropic characteristics of the hyperfine interaction can be obtained from the variation of the temperature dependence of the amplitudes and the frequencies of muon precession.

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An experimental study of the anisotropy of the hyperfine interaction of impurity hydrogen and semiconductors has not been possible until now. In the present paper a theory has been developed of the muon method of investigating such anisotropy in those cases when it must be most strongly pronounced.

In order to have a shift in the muon precession frequency, it is necessary to have a sufficiently high density of conduction electrons.[1] In principle, this is possible both in metals and in strongly doped semiconductors. However in metals one should expect either that the muonium is ionized, or that its dimensions increase greatly and thereby weaken the hyperfine interaction constant. As will be seen from the following, this makes it practically impossible to observe effects associated with anisotropy. Therefore everywhere in the following we shall speak only of strongly doped semiconductors, having in mind at the same time that formally the theory is also applicable to metals.

We consider a strongly doped semiconductor at a low temperature and a situation in which the muonium either has not formed a diamagnetic compound at all or has entered into a chemical reaction only partially (for example during slowing down).

Strong doping gives rise to a relatively high density of conduction electrons. This leads to two consequences. Firstly, the conduction electrons will be scattered by the electrons in the muonium atoms and therefore the situation arises of "rapid electron spin exchange" in the muonium atom. Secondly, the considerable density of electrons of the medium leads to an increase in the dielectric constant which in a semiconductor, even without this occurring, differs from unity even at distances of the order of the Bohr radius. This leads to a swelling of muonium, whose dimensions become comparable with the characteristic dimensions of an atomic cell. At the same time the spherical symmetry of the hyperfine interaction of muonium and the electron disappears. We first consider the case when the hyperfine interaction preserves the symmetry of an ellipsoid of rotation but, as will be seen later, our results will be valid also for the general case.

In the case of rapid electron spin exchanges the muon precession occurs as if the muon were free. However

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