[17]C. J. Lumdsen and L. E. H. Trainor, Can. J. Phys. **54**, 2340 (1976).
[18]H. W. Huang, Phys. Rev. A **8**, 2553 (1973); B **11**, 1144 (1975)
[19]I. Prigogine, Nonequilibrium Statistical Mechanics, Wiley, New York, 1963 (Russ. Transl., Mir, M., 1964).
[20]M. J. R. Hoch, S. P. McAlister, and M. I. Gordon, J. Phys. C **8**, 53 (1975).
[21]E. Litov and E. A. Uehling, Phys. Rev. B **1**, 3713 (1970).
[22]Z. Czapla and L. Sobszyk, Acta Phys. Pol. A **51**, 189 (1977).
[23]L. P. Pereverzeva, Yu. M. Poplavko, I. S. Rez, and L. I. Kuznetsova, Kristallografiya **21**, 981 (1976) [Sov. Phys. Crystallogr. **21**, 562 (1976)].

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Deformation of electron-hole drops in a magnetic field

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The stationary shape of an electron-hole drops (EHD) in a constant magnetic field is calculated in the hydrodynamic approximation with allowance for the recombination magnetization and the surface tension.

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It was previously shown[1] that in electron-hole drops (EHD) application of an external magnetic field should produce circular currents that lead to recombination magnetization of the drops. This phenomenon was used in a number of papers[2–4] to explain the magnetostriction of the EHD. It was assumed in these papers that the drops are ellipsoids of revolution. In the present paper the drop shape is determined by solving a differential equation obtained from the EHD equilibrium conditions.

Consider an EHD in a magnetic field under conditions of stationary photoexcitation. The carrier recombination in the drop leads to the appearance of electron and hole fluxes from the surface to the interior of the EHD. We assume that the carrier densities n vary little inside the drops. In this case the carrier fluxes are given by

$$\operatorname{div}\mathbf{v}=-1/\tau. \qquad (1)$$

Here $\mathbf{v}$ is the velocity of the carrier flux and τ is the EHD lifetime. The solution of (1) in a cylindrical coordinate frame ($\rho, \varphi, z \parallel \mathbf{H}$) is

$$v_\rho=-\frac{B\rho}{\tau},\quad v_z=-\frac{(1-2B)z}{\tau},\quad v_\varphi=0, \qquad (2)$$

where B is a constant that specifies the distribution of the fluxes in the drop.

The distribution of the pressure p in the EHD is described by the equation[5]

$$\operatorname{grad} p=[\mathbf{j}\times\mathbf{H}]/c. \qquad (3)$$

Here $j=\sigma\mathbf{v}\times\mathbf{H}/c$ is the current density, σ is the conductivity, and c is the speed of light. From (3), with account taken of (2), we obtain the following expression for the pressure:

$$p=p(0)+\frac{4\alpha\varepsilon}{a}u^2,\quad u=\frac{\rho}{a}, \qquad (4)$$

where $p(0)$ is the pressure on the z axis, $\varepsilon=a^3\sigma BH^2/8\alpha c^2\tau$ is a dimensionless parameter, α is the surface-tension coefficient, and a is the value of ρ on the EHD equator.

At equilibrium, the pressure p at each point of the EHD drop should equal $-2\alpha k$ (k is the average curvature of the surface). For the curve $z=af(u)$, rotation around the z-axis of which describes the surface of the drop, this condition can be written in the form

$$f''(1+f'^2)^{-3/2}+f'u^{-1}(1+f'^2)^{-1/2}+ap/\alpha=0. \qquad (5)$$

The solution of (5) with allowance for the boundary conditions $f'(0)=0$ on the poles of the EHD and $f(1)=0$ and $f'(1)=-\infty$ on the equator, can be represented in the form

$$f(u)=\int_1^u\frac{K\,dt}{(1-K^2)^{1/2}},\qquad K(t)=-\frac{a}{\alpha t}\int_0^t p(u)u\,du. \qquad (6)$$

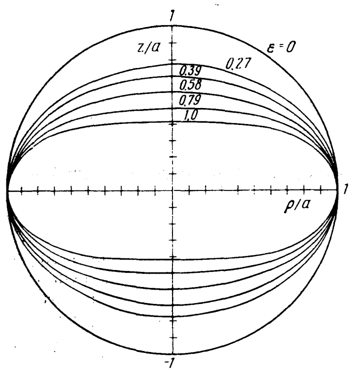

FIG. 1. Shape of the intersection of the EHD with the plane passing through the z axis, as calculated from formulas (6) for different values of ε.

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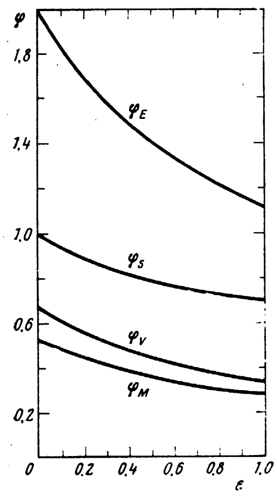

FIG. 2. Plots of the functions φ_M, φ_S, φ_V and φ_E against ε.

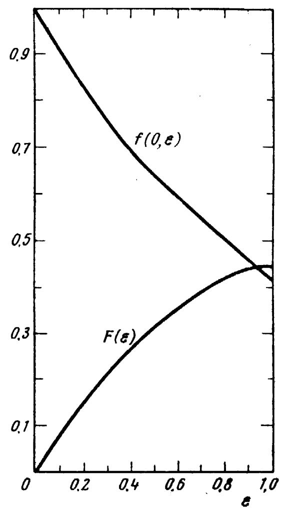

FIG. 3. Plots of the functions F and $f(0, \varepsilon)$ against ε.

It follows from (4) and (6) that

$$K(t) = -[\varepsilon(t^3-t)+t],$$
$$p(0) = \frac{2\alpha}{a}(1-\varepsilon).$$

We consider the case when the pressure p in the EHD is positive, i.e., $0 \leq \varepsilon \leq 1$, and the problem has a stationary solution. Figure 1 shows the shapes of the curve $f(u)$ at different values of the parameter ε. It is seen from the figure that the EHD becomes flatter with increasing ε.

The magnetic moment M, the surface area S, the volume V, and the sum E of the surface and magnetic energies of the drop can be calculated from the formulas

$$M = \frac{4\pi a^2 \varepsilon \alpha}{H}\varphi_M(\varepsilon), \quad \varphi_M = -\int_0^1 \frac{Kt^2\,dt}{(1-K^2)^{1/2}},$$
$$S = 4\pi a^2 \varphi_S(\varepsilon), \quad \varphi_S = \int_0^1 \frac{t\,dt}{(1-K^2)^{1/2}}, \tag{7}$$
$$V = 2\pi a^3 \varphi_V(\varepsilon), \quad \varphi_V = -\int_0^1 \frac{Kt^2\,dt}{(1-K^2)^{1/2}},$$
$$E = 2\pi a^2 \alpha(2\varphi_S - \varepsilon\varphi_M) = 2\pi a^2 \alpha \varphi_E.$$

The functions φ_M, φ_S, φ_V and φ_E are shown in Fig. 2.

Under stationary conditions, at each point on the EHD surface the electron and hole flux should be continuous, i.e., the exciton flux density Φ on the EHD surface should equal the density of the recombination flux from the surface:

$$\Phi = \frac{na}{\tau}[-BKu + (1-2B)f(1-K^2)^{1/2}]. \tag{8}$$

We assume for simplicity that Φ is constant on the EHD surface. Then for

$$B = f(0, \varepsilon)/[1+2f(0, \varepsilon)] \tag{9}$$

the condition (8) is satisfied exactly on the poles and on the equator, and within not more than 2% over the rest of the EHD surface.

We introduce the notation

$$F(\varepsilon) = \frac{\varepsilon[1+2f(0,\varepsilon)]}{f(0,\varepsilon)}\left(\frac{\varphi_V}{\varphi_S}\right)^3, \quad \gamma = \frac{\Phi^3\tau^2\sigma H^2}{n^3\alpha c^2}. \tag{10}$$

We then obtain from (7) and (9) the following final relations:

$$F(\varepsilon) = \gamma, \tag{11}$$
$$a = \frac{2\Phi\tau}{n}\frac{\varphi_S}{\varphi_V}. \tag{12}$$

The functions $F(\varepsilon)$ and $f(0, \varepsilon)$ are shown in Fig. 3. By calculating the value of γ at the specified values of Φ and H we can determine from the condition (11) and from Fig. 3 the value of ε and next the quantity $f(0, \varepsilon)$ that characterizes the deformation of the EHD.

By way of example, we consider the case when the exciton density is constant when the magnetic field is varied. Then[6]

$$\Phi = R_0 n/3\tau$$

(R_0 is the radius of the EHD at $H=0$). We determine the conditions under which an EHD compression corresponding to $\varepsilon = 1$ is obtained (see Fig. 1). For $\varepsilon = 1$ we obtain from (7), (11), and (12) and from Figs. 2 and 3 the value $a \approx 1.42 R_0$ and next

$$S \approx 1.39 S_0, \quad V \approx 1.39 V_0, \quad H = \left[\frac{27\alpha c^2 \tau F(1)}{R_0^3 \sigma}\right]^{1/2}$$

(S_0 and V_0 are the values of S and V at $H=0$). In germanium at 2 °K[6,3] we have $R_0 = 5\times10^{-4}$ cm, $\tau = 4\times10^{-5}$ sec, $n = 2\times10^{17}$ cm^{-3} and $\sigma = 2\times10^{16}$. Substituting these values in the expression for the magnetic field we obtain $H \approx 6$ kG.

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[1]A. S. Kaminskiĭ and Ya. E. Pokrovskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. 21, 431 (1975) [JETP Lett. 21, 197 (1975)].

[2] J. P. Wolfe, J. E. Furneaux, Markiewiez. Proc. Thirteenth Intern. Conf. on Physics of Semiconductors, Rome, 1976, p. 954.
[3] A. S. Kaminskiĭ, Ya. E. Pokrovskiĭ, and A. E. Zhidkov, Zh. Eksp. Teor. Fiz. 72, 1960 (1977) [Sov. Phys. JETP 45, 1030 (1977)].
[4] V. L. Kononenko and V. N. Murzin, Pis'ma Zh. Eksp. Teor. Fiz. 24, 590 (1976) [JETP Lett. 24, 548 (1976)].
[5] H. Alfvén, Cosmical Electrodynamics, Clarendon Press, Oxford, 1950.
[6] Ya. E. Pokrovsky, Phys. Status Solidi A 11, 385 (1972).

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Nonlinear Cyclotron absorption of a hole doppleron in cadmium

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We investigated experimentally the nonlinear behavior of the impedance of a cadmium plate in the region of existence of the hole doppleron. It is shown theoretically that this phenomenon can be attributed to nonlinear cyclotron absorption of the wave in the metal. A theory of nonlinear cyclotron absorption of a hole doppleron in cadmium is constructed. The nonlinearity is due to the influence of the wave magnetic field H that alters the trajectories of the resonant electrons responsible for the cyclotron asorption. The Lorentz force connected with the field H modulates the particle velocity along the magnetic field at a characteristic frequency ω_0 proportional to the square root of the wave amplitude. The modulation of the longitudinal particle velocity leads to violation of the condition of their resonant interaction with the wave, as a result of which the absorption coefficient decreases. The nonlinearity is significant when the frequency ω_0 is large compared with the electron-collision frequency. A decrease of the cyclotron absorption changes radically the picture of the surface-impedance oscillations of the plate in the magnetic field. We studied in the experiment the influence of the temperature, of the angle of inclination of the magnetic field, and of the frequency on the nonlinear-effect threshold field that separates the regions of linear and nonlinear behavior of the sample impedance. The measurement results are in qualitative agreement with the conclusions of the theory.

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1. INTRODUCTION

We have previously reported[1] observation of nonlinear effects in the propagation of a large-amplitude hole doppleron in cadmium. A change in the picture of the oscillations of the surface resistance R and of its derivative with respect to the static magnetic field was observed with increasing amplitude of the exciting field. The waves propagated in the plate along a magnetic field parallel to the [0001] hexagonal axis. No nonlinear effects were observed when an electron doppleron propagated in the cadmium. It is known that a hole doppleron exists in cadmium near the boundary of the doppler-shifted cyclotron resonance (DSCR) of the holes of the monster, and is subject to cyclotron damping by the electrons of the lens.[2] There is no cyclotron damping in the region of the existence of an electron doppleron due to the DSCR of electrons with maximum displacement. Since the nonlinear growth of the oscillation amplitudes was observed only for the propagation of a hole doppleron, it is natural to assume that this nonlinear effect is connected with a decrease of collisionless cyclotron absorption. We construct in this paper a theory of the nonlinear effect and present results of an experimental study of the peculiarities of its manifestation at various temperatures and various angles between the magnetic field and the [0001] axis. Good agreement was observed between the deductions of the theory and the experimental data.

We examine first the physical picture of nonlinear cyclotron absorption. In the case when the wave propagates along a constant magnetic field $\mathbf{H}_0$, the cyclotron absorption is due to resonant interaction with those electrons which revolve on the Larmor orbit together with the electric field of the waves. For these electrons, the angle between the velocity $\mathbf{v}_\perp$ in the plane of revolution and the electric field, which also lies in this field, remains constant and the particle effectively draws (or gives up) energy from (to) the wave. Since the electrons and the hole-doppleron electric field revolve in opposite directions, the resonant electrons are those that overtake the wave and travel along $\mathbf{k} \parallel \mathbf{H}_0 \parallel z$ with a velocity

$$v_z = (\omega_c + \omega)/k. \quad (1)$$

Here $\mathbf{k}$ and ω are the wave vector and frequency of the doppleron, and ω_c is the cyclotron frequency of the resonant electrons. In plasma physics it is customary to call the energy absorption by particles that satisfy condition (1) "anomalous cyclotron absorption." From the equations of motion of a particle in a constant field $\mathbf{H}_0$ it is

