Properties of Josephson thin film variable-thickness microbridges

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The properties of submicron variable-thickness Josephson junctions are investigated experimentally in a direct current and in a UHF field. In a certain temperature range, the volt-ampere characteristics of the junctions possess at the beginning of the resistive region a characteristic section with a small differential resistance, which increases with decrease in temperature. The shape and evolution of the volt-ampere characteristics are in accord with the theoretical predictions of Aslamazov and Larkin [Sov. Phys. JETP **43**, 698 (1976)]. The theory takes into account the presence of nonequilibrium electrons in the weak coupling region and also the stimulation of superconductivity by a direct current, which is a consequence of this presence. Some features of gap-like volt-ampere characteristics are investigated.

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It is well known that the most widely used theoretical model for the description of the properties of weakly coupled superconductors is the resistive model, proposed by Aslamasov and Larkin[1] for junctions of small dimensions, when their length and transverse dimensions are considerably smaller than the coherence length ξ. This model generally gives a satisfactory description of the properties of real junctions, in particular, the peculiarities of their behavior at ultra high frequencies (UHF) (the Josephson radiation,[2] the anomalous behavior of the UHF impedance,[3] the excitation of "non-Josephson" oscillations when connected in a resonant system,[4] and so on), although a more detailed comparison reveals certain qualitative and quantitative divergences in the predictions of the theory[1] and experimental results.

In the theoretical work of Likharev and Yakobson,[5] an attempt was made, using an approach similar to that developed in Ref. 1, to take into account the relaxation processes of the order parameter in the weak coupling region by means of a simple generalization of the nonlinear Ginzburg–Landau equations for the nonstationary case. And although the phenomenological theory of Refs. 5 and 6 is rigorously valid only for gapless superconductors, its conclusions explain certain peculiarities of behavior in a direct current and in the UHF field of the Josephson junctions and in the presence of an energy gap, for example, the effect of an excess current.

In a recent work, Aslamazov and Larkin,[7] working within the framework of the microscopic theory, analyzed the effect of nonequilibrium electrons in the weak coupling region on the properties of the junction. It turned out that, as a result of this effect, the shape of the volt-ampere characteristic (VAC) can differ significantly from that predicted earlier.[1]

In this connection, it is of interest to analyze the experimental features of the superconducting weak bonds and to compare the results with theory, the more so that departures are rather frequently observed from the resistive model,[1] similar to those discussed in Refs. 5–7, for example, discontinuities in the voltage on the VAC in plane bridges,[8] characteristic singularities of VAC of bridges of variable thickness in the region of small voltages,[9] and so on. However, there is meaning in the correct comparison with theory, in all probability, only for those junctions whose electrical and geometric parameters can be determined sufficiently accurately, for example, for superconducting thin film bridges of variable thickness.

In our previous work,[9] we investigated the conditions of transition from ordered motion of quanta of the magnetic flux, using bridges of variable thickness, to the case in which the width W and the length L of the bridge become smaller than or of the order of ξ and the formation of a vortical structure in the weak coupling region is impossible. It has been found that the description of coherent phenomena in bridges with the help of the vortical model is clearly inapplicable, if $W, L < (2-3)\xi$. However, even for bridges of smaller dimensions, at a temperature close to the critical T_c (i.e., when the quantity ξ is sufficiently large), a significant difference is noted in the properties from those predicted by the theory of Ref. 1, although detailed descriptions and more discussions of these differences did not appear in [9]. Therefore, the aim of our work has been the further detailed investigation of the properties of bridges of variable thickness of submicron dimensions at direct current and in a UHF field, and a comparison of the results with models which take into account the processes of the relaxation of the order parameter and the effect of nonequilibrium electrons in the weak-coupling region.

EXPERIMENTAL RESULTS AND THEIR DISCUSSION

1. We investigated tin bridges of variable thickness with dimensions $L \sim 0.5\ \mu$, $W \sim 1\ \mu$, thickness of the film bridge $d = (300-1000)$ Å, thickness of the edges[1)] $d_e = 5 \times 10^3$ Å and resistance in the normal state $R_N = 0.05-0.3$ ohm. For better matching of the UHF impedance of the bridge to the external electrodynamic system, the bridge was connected in the central lead of a coaxial resonator of the three-centimeter band.[10] Measurements of the VAC were carried out by the usual four-point scheme in

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TABLE I. Bridge parameters.*

Sample	L, μm	W, μm	R_N, ohms	$\xi(0)$, Å	$\delta_\perp \times \Delta T$ μm K	T_c, K
CS-52	0.5	1.2	0.14	820	0.31	3.77
CS-53	0.5	1.1	0.4	900	0.8	3.77
CS-55	0.5	0.7	0.055	1100	0.08	3.79
CS-58	0.5	1.2	0.26	780	0.62	3.77

*$\delta_\perp$ is the penetration depth of a weak magnetic field perpendicular to the plane of the film, ξ (0) is the coherence length at $T=0$. The critical temperature of the bridge T_c was determined by linear extrapolation of the temperature dependence of the critical current of the bridge $I_c(T)$ in the region of high temperatures. This temperature corresponded to the value T_c determined from the temperature transition in terms of the resistance and, as a rule, was smaller by several hundredths of a degree than the critical temperature of the film edges T_c^e.

the given-current regime. To decrease the effect of external static, we used filters on all the wiring of the circuit and careful screening of the experimental apparatus.

The measurements were carried out in the temperature range 3–4 K; stabilization of the temperature was maintained by a disk monostat with accuracy to within 3×10^{-4} K. All the bridges were prepared by the method of Ref. 10 and possessed similar properties. In the present paper we discuss typical experimental results obtained for bridges whose parameters were calculated by the method described in Ref. 9 and are given in Table I.

We note that the Abrikosov vortices were not obtained in the film bridge in the temperature range $\Delta T = T_c - T \lesssim 1$ K.[9] This allows us to make a correct comparison of our results with theory[1,5-7] at least near T_c.

2. Figure 1 shows the temperature change of the VAC of the bridge CS-52 in the temperature range $\Delta T = 0$–0.33 K. It is seen that as ΔT increases, the shape of the VAC changes materially: whereas the shape of the VAC at $T\approx T_c$ and not too high voltages is close to hyperbolic (Fig. 1b), in accord with the theory,[1] at T somewhat lower than T_c a linear portion appears at the beginning of the resistive region with differential resistance $R_d < R_N$ (Fig. 1a). Such a shape and the evolution of the VAC upon decrease in the temperature agree qualitatively with the result obtained in Ref. 7 for a three-dimensional structure of the bridge.

As is well known, in the presence of transport current, the Δ gap in the film bridge becomes smaller than the gap Δ_e in the film edges, which governs the localization of the incipient electric field at $I > I_c$ in the weak coupling region. However, as is shown in Ref. 7, under certain conditions when direct current flows the distribution function of the normal electrons, shifts strongly toward higher energies, as a result of which an effective "increase" occurs in the gap, and, consequently there is an increase in the superconducting component of the current through the bridge (i.e., a stimulation of superconductivity). This should lead to the appearance

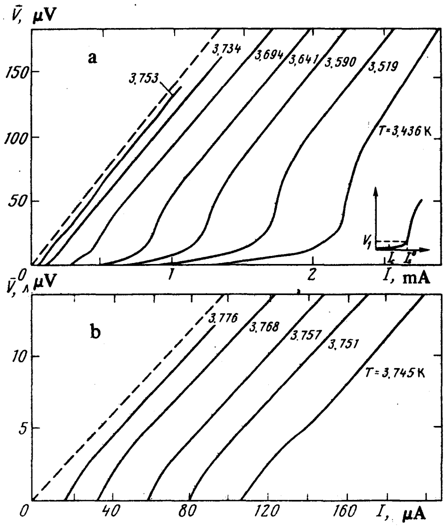

FIG. 1. Family of VAC of the sample CS-52 with changing temperature: a—in large scale (characteristic voltages $\bar{V} \sim 50\,\mu$V), in the insert is shown the VAC obtained in the work of Aslamazov and Larkin[7]; b—in small scale (characteristic voltages $\bar{V}\sim 5\,\mu$V). The dashed curves are the VAC of the bridge in the normal state.

of a linear portion of the VAC with $R_d < R_N$ at the beginning of the resistive region if the period of the Josephson oscillations is less than the energy relaxation time τ_e of the electrons[2)] due to interaction with the phonons. When the characteristic value of the current I_s^0 is reached (see the caption on Fig. 1a), a sharp increase occurs in the time-averaged voltage, due to the growth in the normal component of the current. The stimulation effect occurs if the dimensions of the junction are larger than the characteristic length $\eta = \xi(T)(\Delta T/T_c)^{1/4}$, at which electrons with energies less than Δ_e can diffuse from the bridge. Since the quantity η depends on the temperature, the inequality $L > \eta$ begins to be satisfied at some departure downward from T_c, i.e., the stimulation of the superconductivity in direct current in bridges has a temperature threshold, which is also observed experimentally. Actually, as is seen from Fig. 1, the portion of the VAC with $R_d < R_N$ becomes significant at $\Delta T\approx 0.04$ K, when $\eta \simeq 0.3\ \mu$, i.e., less than L. Upon further decrease in the temperature, the quantity R_d of the linear portion decreases and the portion itself becomes broader, as follows from Ref. 7. Moreover, for some samples, the temperature dependence of the ratio $(I_s^0/I_c)^4$ is linear in correspondence with Ref. 7, for example, in the range $\Delta T = 0.08$–0.2 K for the bridge CS-52 (Fig. 2). The de-

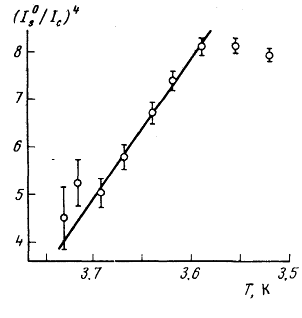

FIG. 2. Dependence of the characteristic current I_s^0, normalized to the critical current I_c, on the temperature T for the sample CS-52.

parture of $(I_s^0/I_c)^4$ from a linear dependence at small ΔT is probably associated with the increase in error in the determination of I_s^0, and at large ΔT—with the appearance of voltage jumps on the VAC.[3)]

Subsequent, more detailed investigations of the functional dependence of the VAC, undertaken with the help of measurements of $d\overline{V}/dI=f(I)$, have confirmed the presence of a linear resistive portion in the region $I\sim I_c$ and have shown that as I approaches I_s^0, the region of the VAC is described by a law that is close to hyperbolic. The values of R_d of the linear portion of the VAC at $I\sim I_c$ change in proportion to $(\Delta T)^{-1}$, as follows from the expression (25) of Ref. 7 if we use in it the experimental values of I_c, I_s^0 and τ_s.

The time τ_ε for electrons whose energy is less than Δ_e can be estimated from the experimental data. Using the expressions (13) and (21) of Ref. 7 for bridge currents higher and lower than the characteristic current I_s^0, we get

$$\tau_\varepsilon=\frac{\pi}{4}\frac{\hbar}{kTe^3}\frac{\Delta^3}{V_1^3}, \tag{1}$$

where V_1 is the characteristic value of the bias voltage, corresponding to $I=T_s^0$. The value of τ_ε of the bridge CS-52, estimated from (1), was $\approx 10^{-9}$ sec in the temperature range $0.08<\Delta T<0.2$ K, where the model of Ref. 7 can be assumed to be applicable for qualitative estimates, and agrees in order of magnitude with the value of the scattering of the electrons τ_0 from phonons in the normal metal at $T=T_c$ (see, for example, Ref. 12).

It is interesting to note that we have observed the well known[13] effect of the increase in the critical current under the action of UHF radiation with frequency $f=\omega/2\pi\approx 10$ GHz, also with a temperature threshold. For example, in the bridge CS-53, this effect appears and increases as the temperature decreases only at $\Delta T>0.02$ K. With increase in the dimensions of the bridge, there was a tendency toward decrease in the threshold value of ΔT both in the case of an autonomous bridge and upon stimulation of the superconductivity by UHF radiation.

Figure 3 shows typical temperature dependences near T_c of the critical current I_c,[4)] of the characteristic current I_s^0, and of the maximum critical current $I_c^{\max}$ following the action of the UHF radiation with power P, for the bridge CS-53. It is seen (see also Fig. 1) that the characteristic change in the VAC of an autonomous bridge and the significant increase in the critical current in an UHF field appear at some distance below T_c. It can be shown that the stimulation of superconductivity by UHF radiation in the investigated bridges has causes similar to those considered in Ref. 7, the more so that the condition $\omega\tau_\varepsilon\gg 1$ was used in the region $T\approx T_c$.

The stimulation of superconductivity by the UHF field in plane bridges has been observed by several groups of investigators, and some of them have explained this phenomenon by a model developed by Éliashberg and coworkers.[14] As is well known, the basic idea of Éliashberg is that the presence of the UHF field leads to a dis-

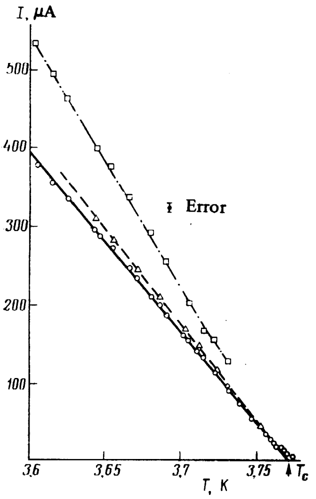

FIG. 3. Temperature dependence of the critical current I_c(○), of the characteristic current I_s^0(□) (see Fig. 1a), and of the maximum critical current $I_c^{\max}$(Δ) under the action of the UHF power for the sample CS-53.

placement of the distribution function of the excited states in homogeneous films towards the high energy region and, by the same token, to an effective increase of the gap. According to the calculation of Ref. 14, the action of the UHF field can lead to a significant increase in the critical temperature of the superconductor. However, in contrast to Ref. 14 and a number of experimental researches, in which the increase in I_c in the UHF field was studied in long homogeneous films,[15,16] the stimulation of the superconductivity considered by Aslamazov and Larkin appears in the VAC in the absence of an external UHF signal and upon achievement of some threshold temperature somewhat below T_c, and only in the presence of weak coupling of the superconductors.

3. Upon further increase in the current in the region $I\gg I_c$ the VAC of the bridges over the entire studied temperature range have the form of straight lines shifted relative to the VAC in the normal state by an amount $\Delta V(T)$ (Fig. 1). The explanation of this effect of "excess" current in the bridges of variable thickness was given by Likharev and Yakobson.[5] The explanation is as follows: in currents significantly larger than I_c, because of the high frequency of the Josephson oscillations, the order parameter does not relax to its equilibrium state and as a consequence a destruction of the superconductivity takes place in the central part of the bridge. However, near the edges, because of the closeness effect, the superconductivity in the film bridge is preserved at the characteristic distance

$$x_0=\left[\frac{\tau_\Delta}{\tau_j}\frac{\xi}{L}\frac{\partial\varphi}{\partial t}\right]^{-1/2}\xi\ll\xi$$

from the bridge-edge interface (here τ_Δ is the relaxation time of the order parameter, φ is the phase difference of the wave functions of the superconducting edges, τ_j is the relaxation time of the current). The change of the VAC into a straight line parallel to IR_N takes place when x_0 becomes less than $L/2$ and the temperature dependence $\Delta V(T)$ is linear.

The theory of Ref. 5 correctly describes in general the effect of "excess current," observed in bridges of variable thickness of submicron dimensions[5)]: the dependence $\Delta V(T)$ for most samples is actually linear ($\Delta V \sim \Delta T$), and, in correspondence with the conclusions of theory, ΔV tends to zero upon approach of the temperature to the critical temperature of the film edges ($T_c^e > T_c$). However, it is not quite correct to make a quantitative comparison of the results of experiment with the theory of Ref. 5 over the entire temperature range, since first, the equations used in this theory are rigorously justified only for gapless superconductors and, second, in the experiment, some effect on the quantity ΔV can be exerted by the geometric shape of the bridge-edge boundary, which generally differs somewhat in each individual sample.

It should be noted that the nonlinear phenomena considered by Aslamazov and Larkin[7] can lead to a similar effect of "excess current." Nevertheless, in the immediate vicinity of T_c (in the region $\Delta T \leqslant 0.04$ K for the sample CR-52), where according to experimental results (Figs. 1 and 2) and in correspondence with theoretical estimates[7] the nonequilibrium electrons do not significantly affect the shape of the VAC, in all probability, the use of the equations of Ref. 5 for approximate estimates is possible. For short bridges, under the condition

$$\frac{\tau_\Delta}{\tau_j}\left(\frac{L}{\xi}A\right)^2\frac{I}{I_c}\ll 1,\quad A^2=\frac{A_0^2}{\Delta^2}=\frac{(T_c^e-T)T_c^e}{|T_c-T|T_c},\tag{2}$$

the equation for φ has the following form:

$$\frac{d\varphi}{dt}\left[\frac{1}{A^2}+\frac{\tau_\Delta}{15\tau_j}\left(\frac{L}{\xi}\right)^2(1-\cos\varphi)\right]+\sin\varphi=\frac{I}{I_c}.\tag{3}$$

The appearance of the second term on the left side of Eq. (3) is due to allowance for processes of relaxation of the order parameter in the film bridge. Its effect reduces to a decrease in the effective normal resistance R_N to the value R:

$$\frac{1}{R}=\frac{1}{R_N}\left[\frac{1}{A^2}+\frac{\tau_\Delta}{15\tau_j}\left(\frac{L}{\xi}\right)^2\right].\tag{4}$$

The values of R were determined by analogy with Ref. 6 from the experimentally measured VAC with the help of the relation $\overline{V}=R(I^2-I_c^2)^{1/2}$ for $\overline{V}=1$ μV. An estimate of τ_Δ from the expression (4) for all the investigated samples gave

$$\tau_\Delta[\text{sec}]\approx(2\pm1)\cdot 10^{-10}\frac{1[\text{K}\cdot\text{sec}]}{\Delta T[\text{K}]},\tag{5}$$

which is in excellent agreement with the results of Ref. 6.

We note that the condition (2) of applicability of Eqs. (3) is not strictly satisfied even in the immediate vicinity of T_c, since at $T\approx T_c$ we have $A^2\propto\xi^2\propto(\Delta T)^{-1}$; therefore, the estimate of the quantity τ_Δ (5) is approximate.

The effect of a finite value of the quantity τ_Δ can be seen in the UHF properties of the bridges. It is known that the simplest method of study of coherent effects at UHF is to investigate the behavior of the amplitude of the critical current T_c and the Josephson steps of the current $I_{1,2,\ldots}$ on the UHF power P. Calculation of $I_1(\sqrt{P})$ on the basis of an equation of type (3) by numerical methods[18] has shown that there are differences between these dependences and those calculated earlier in the framework of the ordinary resistive model (see for example, Ref. 19), in particular, at $\Omega=hf/2eI_cR_N\gtrsim 1$, the value of I_1 is much smaller than follows from the theory of Ref. 1. The results of the calculation were confirmed experimentally in the study of the behavior of plane bridges of indium in a UHF field.[18]

We have investigated the dependences $I_{c,1,2,\ldots}(\sqrt{P})$ for bridges of variable thickness irradiated by a signal in the three-centimeter band. Figure 4 shows the dependences on Ω of the quantity $I_1^{\max}$ and of the parameter $\varkappa_0=(a_2-a_1)/a_1$, where $a_i=\sqrt{P_i}$ is the amplitude of the UHF current at which $I_c(\sqrt{P})$ has its ith zero. It is seen that in the immediate neighborhood of T_c, when the condition $\Omega\gtrsim 1$ is satisfied, these dependences differ from those calculated from the resistive model; at the same time, they are in qualitative agreement with calculation.[18] On going to the region $\Omega<1$, the differences decrease, although there is a significant divergence of the shape of the VAC of the autonomous bridge from that predicted in Ref. 2. The latter circumstance indicates that, in all probability, the singularities in the behavior of the autonomous bridges, due to nonequilibrium electrons in the weak coupling region, cease to appear in a strong UHF field.

4. The VAC given in Fig. 1 were typical for bridges with a residual resistance of the square film bridge $R_\square \lesssim 0.1$ ohm. For bridges with smaller values of $R_\square$, on the portions of the VAC from I_s^0 to the region of the excess current, singularities appear in the form of sharp increases in R_d (Fig. 5). Upon a decrease in T they become more pronounced, gradually undergoing a transition to hysteresis-like voltage jumps. The temperature dependences of the voltages of the upper and lower edges of the more strongly pronounced singularities, shown in Fig. 5 by the arrows, are shown in the insert. It is seen that the upper edge agrees very well

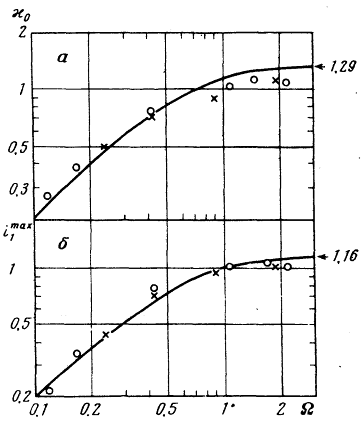

FIG. 4. Dependence of the characteristic quantity $\varkappa_0$ and the maximal amplitude of the first Josephson current step $I_1^{\max} = I_1^{\max}/T_c$ on the quantity $\Omega=hf/2eI_cR_N$ (f is the frequency of the UHF effect); o—for the sample CS-53, ×—for the sample CS-58. The experimental error did not exceed the dimensions of the circles and crosses. The continuous curves show the corresponding theoretical dependences.

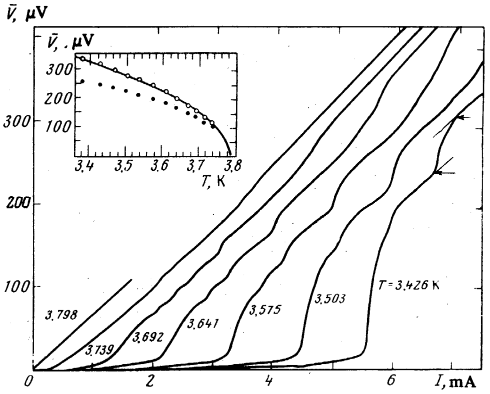

FIG. 5. Family of VAC of the sample CS-55 with changing temperature. The arrows on the curve at the temperature $T = 3.426$ K show the characteristic points, the values of $\bar{V}$ at which are plotted in the insert as a function of the temperature. The continuous curves in the insert show the theoretical dependence of the second harmonic of the gap $\frac{1}{2}(2\Delta(T)/e)$.

with the theoretical dependence $\bar{V}=\frac{1}{2}(2\Delta(T)/e)$ ($\Delta(T) = 3.06k(T_c\Delta T)^{1/2}$, $T_c = 3.79$ K). It can be assumed that these singularities are connected with the gap in the excitation spectrum of the electrons. This conclusion is confirmed by the presence of singularities of a similar character at voltages corresponding to the fundamental and other subharmonics $V_n = 2\Delta/en(n = 1, 3, 4, 5, \ldots)$ of the gap. Strictly speaking, there is no explanation yet of the effect of the gap on the VAC of bridges and point contacts, although the gap singularities have been observed frequently on their VAC (see, for example, Refs. 20 and 21), and in the point contacts they become more pronounced as a rule with increasing R_N, and are connected with the presence of an oxide layer and an increase in the quasiparticle current at $\bar{V} = 2\Delta/en$. We note that singularities similar to those shown in Fig. 5 for the fundamental and odd harmonics of the gap were obtained recently within the framework of the microscopic theory for tunnel junctions of small dimensions.[22,23] Therefore, we can assume that we can use the results of the calculations given in Refs. 22 and 23 for the description of several properties of bridges with long free path length ($\gtrsim L$) i.e., smaller values of $R_\square$.

CONCLUSION

1. The temperature changes in the shape of the VAC, in the region of small voltages, of bridges of variable thickness of submicron dimensions can be attributed to the appearance of stimulation of superconductivity by a constant current, due to the presence of nonequilibrium electrons in the weak-coupling region. The experimental value of the temperature threshold of the appearance of characteristic changes in the VAC is in excellent agreement with the theoretical.

2. Comparison of the singularities in the behavior of the investigated bridge in the absence and presence of UHF radiation allows us to assume that some of them have similar causes.

3. The behavior of the bridges under the action of a strong UHF signal and under the condition $\Omega \lesssim 1$ is well described by the resistive model,[1] which indicates the smallness of the effect of the nonequilibrium electrons on the properties of the bridges in strong UHF fields. The departure of the UHF properties of the bridges from those predicted in Ref. 1 for $\Omega > 1$ can be explained by taking into account the finite value of the relaxation time of the order parameter of the superconducting condensate.

4. It is found that the gap singularities in the VAC, present in the form of jumps in the voltage, become more clearly evident upon decrease in the resistance because of the increase in the free path length and preservation of the smallness of the transverse dimensions of the bridges.

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[1)] Such a ratio of the film thicknesses ($d_e \gg d$) and also the results of previous experiments[9,10] allow us to regard the investigated bridges as three-dimensional weakly coupled structures.

[2)] The latter condition was used in obtaining the basic equations of Ref. 7.

[3)] As is shown in the work of Tinkham *et al.*,[11] in similar bridges at $\Delta T \lesssim 0.2$ K, the thermal effects do not influence the shape of the VAC, at least up to voltages of $\bar{V} \sim 100$ μ.

[4)] At $T < T_c$, the critical current dependes linearly on ΔT in correspondence with the theory of Ref. 1. At $T_c < T < T_c^e$ the critical current, decreasing to zero upon increase in the temperature to T_c^e, is due to the fact that in this range of temperatures, the bridge is itself an SNS structure.

[5)] Previously,[17] we discussed this effect in bridges of variable thickness with dimensions W and L much greater than ξ.

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Functional for the hydrodynamic action and the Bose spectrum of superfluid Fermi systems of the He^3 type

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A hydrodynamic-action functional is constructed by a continual-integration method for Fermi systems with pairing in the p state. A simplified model is considered in which it is possible to introduce local fields that describe tensor Bose condensates. It is shown that the most stable is the B phase, which undergoes a second-order phase transition into a planar $2D$ phase in a sufficiently strong magnetic field. The A phase is metastable in the considered model and is destroyed by an arbitrarily weak magnetic field. The Bose spectrum of the system is investigated. In the model in question it contains four phonon branches in the B phase, six in the $2D$ phase, and nine in the A phase. In the more general case of a type-He^3 Fermi system there are four branches each in the B and $2D$ phases and five in the A phase. Qualitative conclusions are deduced for real superfluid He^3. In particular, arguments are advanced favoring a second-order phase transition from the B phase to a planar $2D$ phase in a sufficiently strong magnetic field.

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1. INTRODUCTION

At temperatures on the order of 10^{-3} K, several superfluid phases can exist in He^3 and go over into one another when the external parameters of the system are changed.[1–5] The difficulty of constructing a complete microscopic theory of He^3 makes it expedient to study simplified models similar to the Bose-gas model for He^4.

We investigate here a simplified He^3 model using a continual-integral formalism that is convenient, in particular, for the description of collective excitations. We change for this purpose from an integral over Fermi fields to an integral over an auxiliary Bose field that in fact corresponds to collective excitations. This device was used in[6,7] for a model of a superfluid Fermi gas with pairing in the s state. In this approach, the transition of a Fermi system into the superfluid state constitutes Bose condensation of the model Bose system.

We have considered a technically more complicated case, that of pairing in the p state. In this case we deal of necessity not with a scalar wave function, but with a tensor one that describes the superfluid state, i.e., 9 complex or 18 real independent functions (18 degrees of freedom). This makes possible the coexistence of several superfluid phases (including the phases A and B typical of He^3) and to a rich spectrum of collective excitations. In the simplified model we can get along with a local Bose field that describes the collective excitations. In the general case, however, a bilocal formalism is needed (see, e.g.,[8]).

In Sec. 2 we describe a simplified model that admits of a transition from an integral over Fermi fields to an integral over an auxiliary local Bose field, and construct a "hydrodynamic action" functional S_h that describes the collective excitations.

In Sec. 3 the functional S_h is investigated in the Ginzburg–Landau region $|\Delta T| \ll T_c$. We obtain the Bose-condensation point and the density of the Bose condensate for various superfluid phases. The investigation shows that in the absence of a magnetic field the energywise most favored and stable (with respect to small perturbations) is the B phase. The A phase is energywise less profitable, but is also stable to small perturbations. However, application of an arbitrarily weak magnetic field destroys the A phase. With increasing magnetic field, the B phase becomes deformed and goes over con-

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