Two plasmon decay and generation of the $(3/2)\omega_0$ harmonic

V. P. Silin and A. N. Starodub

P. N. Lebedev Physical Institute, USSR Academy of Sciences
(Submitted March 2, 1977)
Zh. Eksp. Teor. Fiz. **73**, 884–890 (September 1977)

The dependence of the threshold value of the field intensity on the wave vector of growing plasma perturbations is studied. The spatial region of development of absolute parametric instability is determined. It is shown that the size of the region is of the order of the size of the plasma corona produced of a laser-irradiated solid target. It is predicted that the generation intensity of the $(3/2)\omega_0$ harmonic should increase sharply and have a threshold. The increase is due to the pump field attaining such high values that longwave plasmons, capable of producing the harmonic as a result of resonance merging with the pump wave, are formed in the decay.

PACS numbers: 52.35.Mw, 52.35.Py, 52.50.Jm

Studies of the generation of the harmonic $(3/2)\omega_0$ when a plasma is exposed to laser radiation with frequency ω_0 are being carried out intensively at the present time (see, for example, Refs. 1–4). Studies of this sort have great value for the understanding of phenomena which take place in the plasma corona of a target irradiated by a laser. Generation of the harmonic $(3/2)\omega_0$ is connected with the process of parametric instability which arises in the plasma region with a density close to one-fourth critical.

It is assumed at the present time that an absolute instability is developed in the vicinity of such a region in a spatially inhomogeneous plasma,[5-7] due to the decay of the pump wave into two plasmons. In the present communication we evaluate, on the basis of analysis of the boundary of such an absolute instability, the region of its spatial localization. The size of this region is essential for understanding the possibility of absorption of the pump wave by the plasma as a consequence of two-plasmon decay, since the corresponding optical thickness τ of the plasma increases with increase in the size of the region of localization of the instability. On the other hand, analysis of the instability boundary allows us to deduce the possibility of a sudden increase (having a threshold) in the generation of the harmonic $(3/2)\omega_0$ at fluxes exceeding a certain value, which is generally greater than the threshold of parametric instability.

1. The boundary of the instability of an inhomogeneous plasma relative to decay of the pump wave into two plasmons is, according to our previous work,[5] determined by the following equation:

$$\frac{v_E}{c}=\frac{8}{3^{1/2}}\frac{\tilde{\gamma}(k_y)}{\omega_0}+\frac{\pi(2n+1)}{2\cdot3^{1/2}\varphi(s)\,|k_y|L_N}. \qquad (1)$$

Here

$$\tilde{\gamma}(k_y)={}^1/_2\nu_{ei}+{}^1/_{16}\left(\frac{\pi}{8}\right)^{1/2}\omega_0(|k_y|r_D)^{-3}\exp[-{}^1/_4(k_yr_D)^{-2}]$$

is the damping decrement of the plasmon with wave number k_y, ν_{ei} is the frequency of the electron-ion collisions, v_E is the velocity of oscillations of the electron in the electric field of the pump wave, c is the velocity of light, and L_N is the characteristic size of the plasma inhomogeneity in the vicinity of the point corresponding to one-fourth the critical density. The function $\varphi(s)$ is expressed in terms of the complete elliptic integral of the first $[K(s)$ and second $[E(s)]$ kind:

$$\varphi(s)=K(s)+s^{-1}[K(s)-E(s)],\qquad s=\frac{v_Ec^{-1}-8\tilde{\gamma}/3^{1/2}\omega_0}{v_Ec^{-1}+8\tilde{\gamma}/3^{1/2}\omega_0}.$$

Finally, n is an integer equal to the number of plasmon waves that fit inside the region of localization of the plasmon.

It follows from Eq. (1) that the effect of the inhomogeneity of the density on the excitation of plasmons with wave numbers $k_y>k_i$ [where $k_i=(2n+1)(4L_N\nu_{ei})^{-1}\omega_0$] is insignificant. For these values of k_y the instability boundary is described by the formula of the theory of a homogeneous plasma (see Ref. 8)

$$\frac{v_E}{c}=\frac{4}{3^{1/2}}\frac{\nu_{ei}}{\omega_0}+\frac{1}{2}\left(\frac{\pi}{24}\right)^{1/2}\times(|k_y|r_D)^{-3}\exp\left[-\frac{1}{4}(k_yr_D)^{-2}\right]. \qquad (2)$$

In Fig. 1 (case a) this dependence is shown as curve I. With decrease in the quantity k_y the effect of the plasma inhomogeneity increases and at $k_y<k_i$ it becomes decisive. For such plasmons, the last term in Eq. (1) is the principal one. At $k_i\gg k_y$, this leads to the following value of the instability boundary.

$$\frac{v_E}{c}=\frac{(2n+1)\pi}{2\cdot3^{1/2}|k_y|L_N}\times\ln^{-1}\left[\frac{\pi\omega_0(2n+1)}{\nu_{ei}|k_y|L_N}\right]. \qquad (3)$$

Curve II corresponds to the dependence described by Eq. (3).

The dependence of the instability boundary on the quantity k_y, which is a solution of Eq. (1), is shown in the drawing as curve III and represents the result of the addition of the asymptotically continued curves I and II. It must be noted that such a dependence of the instability boundary on the quantity k_y takes place at

 0038-5646/77/4603-0465$02.40

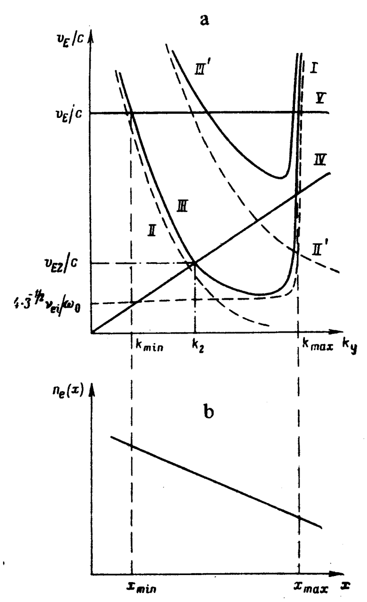

FIG. 1. Instability threshold of the plasma (a). The region of localization of the instability (b). The relatively small change in the amplitude of the pump wave inside the region of localization of the instability is neglected.

$$k_1 < k_{st} \sim r_D^{-1} \ln^{-1/2}(\omega_0/\nu_{ei}),$$

i.e., at

$$L_N > l \ln^{1/2}(\omega_0/\nu_{ei}),$$

where $l = r_D(\omega_0/2\nu_{ei})$ is the free path length of the electrons. In such a case, we shall speak of a weakly inhomogeneous plasma.

In the opposite case of a strongly inhomogeneous plasma, when

$$L_N < l \ln^{1/2}(\omega_0/\nu_{ei}),$$

we have $k_l \sim k_{st}$. Therefore the first term in formula (1) in such a case is not very important. This means, in particular, that the instability boundary at all $k_y < k_{st}$ will be determined by Eq. (3), which, for the case considered, corresponds to curve II′. For $k_y > k_{st}$, the instability boundary is described as before by the portion of the curve I corresponding to Cherenkov interaction of the electrons with the plasmons. As a result, for the instability boundary at

$$L_N < l \ln^{1/2}(\omega_0/\nu_{ei})$$

we have curve III′.

The instability threshold at a given value $k_y = k_m$ corresponds to a minimum in the boundary curve. In a weakly inhomogeneous plasma, this threshold corresponds to the result of the theory of a homogeneous plasma[8]

$$v_{E,\,\mathrm{thr}}\, c^{-1} = 4\nu_{ei}(3^{1/2}\omega_0)^{-1}. \tag{4}$$

Here

$$k_m = (2r_D)^{-1} \ln^{-1/2}[2(2\pi)^{1/2} L_N/(2n+1) r_D].$$

We note that in a weakly inhomogeneous plasma, the instability boundary is close to the threshold value (4) over a wide range of wave numbers k_y. The quantity k_m is close to the right boundary of this region. The left boundary $k^{(1)}$ can be determined by requiring that the boundary value, for example, should not exceed the threshold (4) by more than a factor of two. Then

$$k^{(1)} = L_N^{-1}\omega_0\nu_{ei}^{-1}[2\pi(2n+1)\varphi^{-1}(1/3)].$$

Since $L_N \gg l$ in the case (4), $k_m \gg k^{(1)}$, which corresponds to a wide region in which the boundary value is close to (4).

In the case of a strongly inhomogeneous plasma, the instability threshold significantly exceeds the value (4) and is equal to[5]

$$\frac{v_{E,\,\mathrm{thr}}}{c} = \frac{\pi(2n+1)}{\sqrt{3}} \frac{r_D}{L_N} \ln^{1/2}\left[\frac{4}{\sqrt{2\pi}} \frac{L_N}{r_D(2n+1)}\right] \ln^{-1} 8. \tag{5}$$

Here

$$k_m = (2r_D)^{-1} \ln^{-1/2}[4L_N/(2\pi)^{1/2}(2n+1) r_D].$$

The line IV in the drawing corresponds to the result of Ref. 7, in which the effects of thermal motion of the particle are considered, which are important for a collisionless plasma, and according to which the absolute instability arises at

$$v_E c^{-1} \geqslant 3 v_{Te} c^{-1} |k_y| r_D,$$

where v_{Te} is the thermal velocity of the electrons. Comparing this boundary value with the values (4) and (5) for the instability threshold, we can conclude that curve III (or III′) will not cross the line IV if

$$\frac{\nu_{ei}}{\omega_0} > \frac{3\cdot 3^{1/2}}{8} \frac{v_{Te}}{c} \ln^{-1/2}\left[\left(\frac{2}{\pi}\right)^{1/2} \frac{L_N}{(2n+1) r_D}\right] \tag{6}$$

in the case (4) and if

$$(2n+1)\frac{\lambda_0}{L_N} > 3\cdot 3^{1/2} \ln(8) \ln^{-1}\left[\frac{4}{(2n+1)(2\pi)^{1/2}} \frac{L_N}{r_D}\right] \tag{7}$$

(λ_0 is the pump wavelength) in case (5). It follows from the inequality (6) that the curve II will not intersect the curve I in a weakly inhomogeneous plasma in the case in which the plasma is sufficiently cold, and the ion charge Z of the plasma is large. Under conditions of a strongly inhomogeneous plasma, as follows from the inequality (7), intersections will be lacking in the excitation of plasmons only with a sufficiently large number n. For example, at $L_N = 100\ \lambda_0$, $\lambda_0 = 1.06\ \mu$ and plasma electron temperature 1 keV, the inequality (7) is satisfied at $n \geqslant 110$. However, as the density profile becomes steeper, the inequality (7) can exist for smaller values of n. Thus, at $L_N = 10\ \lambda_0$, this inequality is already satisfied at $n \geqslant 12$.

If conditions exist for the plasma that are the opposite of the conditions (6) and (7), then intersection of the line IV with the curve III will take place, In this case, the point of intersection with coordinates (cf. Ref. 7)

$$\frac{v_{E2}}{c}=\frac{3^{1/2}\pi}{2^{1/2}}(2n+1)^{1/2}\frac{r_D}{(\lambda_0 L_N)^{1/2}}\ln^{-1/2}\left[\frac{\pi(2n+1)^{1/2}l}{(\lambda_0 L_N)^{1/2}}\right],$$

$$k_y=k_2=\frac{1}{3^{1/4}}\frac{(2n+1)^{1/2}}{2^{1/2}}\left(\frac{\lambda_0}{L_N}\right)^{1/4}r_D^{-1}\ln^{-1/2}\left[\frac{\pi(2n+1)^{1/2}l}{(\lambda_0 L_N)^{1/2}}\right] \quad (8)$$

determines the instability threshold and the threshold value of the wave number. Since, by definition, $V_{E2}c^{-1} > 4\nu_{ei}(3^{1/2}\omega_0)^{-1}$, the threshold (8) is realized at $l > (\lambda_0 L_N)^{1/2}$.

The curves shown in the drawing allow us to answer the important question of the spatial localization of the two-plasmon parametric instability. Thus, at sufficient excess over threshold, curve III′ or the curve composed of portions of curves III and IV allows us to find the maximum and minimum values of the wave vector of the plasmons whose generation is allowed at a given field of the pump wave (in the drawing, this value of the field is shown by the horizontal line V). Actually, according to this Eq. (1), we obtain the result that at a given value of v_e excitation of the plasmons takes place, with wave vectors k_y such that $k_{\max}>k_y>k_{\min}$, where

$$k_{max}=\frac{1}{2r_D}\ln^{-1/2}\left[\left(\frac{2\pi}{3}\right)^{1/2}\frac{c}{v_E}\right]$$

$$k_{min}=\frac{1}{L_N}\frac{\pi(2n+1)}{2\cdot 3^{1/2}}\frac{c}{v_E}\ln^{-1}\left[3^{1/2}\frac{v_E}{c}\frac{\omega_0}{\nu_{ei}}\right]. \quad (9)$$

According to the previous work,[9] a plasmon with given wave vector is localized in the region $\Delta_1 x = (3^{1/2}/2)v_E c^{-1}L_N$. However, the region of localization of the plasmon is not identical, generally speaking, with the region of localization of the instability. The reason is that the position of the region $\Delta_1 x$ on the density profile depends on the value of k_y. This dependence leads to its own definition of the width of the region of localization of the instability, which is equal to

$$\Delta_2 x=6(k_{max}+k_{min})\delta k r_D^2 L_N,$$

where $\delta k = k_{\max} - k_{\min}$ is the spread of wave numbers of the plasmons excited at a given value of v_E. The region $\Delta_2 x$ is illustrated in the drawing (case b). As a result we have for the region ΔX of instability localization the interpolation formula $\Delta X = \Delta_1 x + \Delta_2 x$. In the immediate vicinity of threshold, when $k_{\max} \approx k_{\min}$, the second term is small, i.e., $\Delta X \approx \Delta_1 x$. However, the situation changes qualitatively with increase in the quantity v_e. Since $k_{\max} > k_{\min}$ at $v_E \gg v_{e\,\text{thr}}$ it follows that

$$\Delta_2 x \sim {}^3/_2 L_N \ln^{-1}[(2\pi)^{1/2}c/3v_E].$$

Comparing the values of $\Delta_1 x$ and $\Delta_2 x$, we can conclude that since $cv_E^{-1} > 1$ the size of the region of instability localization at $v_E > v_{e\,\text{thr}}$ greatly exceeds the size of the region of localization of the individual plasmons. It follows from the expression for $\Delta_2 x$ that at $v_E > v_{E\,\text{thr}}$ the size of the region of localization of the instability is comparable with the plasma inhomogeneity dimension L_N. Thus, at an Nd-laser radiation flux $\sim 10^{14}$ W/cm^2 we have $\Delta_2 x L_N^{-1} = 30\%$.[1)]

We note that the increase in the region of localization of the instability in excitation of plasma waves with different wave number is a general phenomenon. In Ref. 10, it was studied for the case of parametric decay of the pump wave into a Langmuir wave and an ion-sound wave. Comparing the value of $\Delta_2 x$ with the value of Δx from Ref. 19, we can conclude that in the case

$$L_N > \frac{c}{\omega_0}\left(\frac{\gamma_s}{\omega_s}\frac{\omega_0}{\nu_{ei}}\right)^{1/2}\frac{v_{Te}}{v_E}\ln^{-1}\left(\frac{c}{v_E}\right)\ln^{-1/2}\left(\frac{\omega_0}{\nu_{ei}}\right)$$

(here γ_s and ω_s are the decrement and frequency of the ion-sound wave) the region of localization of the two-plasmon decay exceeds the region of localization of the parametric decay.

2. We shall concern ourselves briefly with analysis of the decrement of two-plasmon decay, the equation for which is obtained from (1) by the substitution $\tilde{\gamma} \to \tilde{\gamma} + \gamma$, i.e.,

$$\gamma=-\tilde{\gamma}-\pi(2n+1)\omega_0[8|k_y|L_N\varphi(s)]^{-1}+\frac{3^{1/2}}{8}v_E c^{-1}\omega_0.$$

In the case of a weakly inhomogeneous plasma, it then follows that

$$\gamma=\frac{3^{1/2}\omega_0(v_E-v_{E,b})}{8c}, \quad (10)$$

where $v_{E,b}$ is determined by the formula (2). According to (10), at $v_E \gg v_{E,b}$ the increment takes on the value

$$\gamma=\frac{3^{1/2}v_E\omega_0}{8c}. \quad (11)$$

If the plasma is strongly inhomogeneous, then the increment is determed by Eq. (10 at $|v_E - v_{E,b}| < \tilde{\gamma}\omega_0^{-1} v_E$, where the boundary value $v_{E,b}$ is determined by Eq. (3). With increase in v_E, when already $|v_E - v_{E,b}| < \tilde{\gamma}\omega_0^{-1} v_{E,b}$, the increment of the instability is determined from the equation

$$\gamma=\frac{3^{1/2}\omega_0 v_E}{8c}-\frac{\pi(2n+1)}{8|k_y|L_N}\omega_0\ln^{-1}\left[\frac{4}{3^{1/2}}\frac{c}{v_E}\frac{\omega_0}{\gamma+\tilde{\gamma}}\right]-\tilde{\gamma}. \quad (12)$$

It then follows, in particular, that the increment again takes the value (11) in the case of increase of v_E,

In such an analysis of the increment we have assumed the value k_y to be fixed and the amplitude of the pump wave to vary. The obtained formulas allow us to understand the behavior of the increment for different values of k_y, excited at a fixed value of v_E. For this, we return to the drawing and consider v_E to be given and equal to a quantity shown by the line V. Then the increment of growth at $k_y \approx k_{\min}$ will be determined by the expression (10) with $v_{E,b}$ given by Eq. (3). With increase in k_y, the growth increment of the plasmons increases. At

$$|k_y-k_{min}|>\nu_{ei}\omega_0^{-1}k_{min}$$

the increment is described by Eq. (12), and at k_y significantly different from $k_{\min}$ (but still $k_y < k_{\max}$) we obtain the formula (11) for the increment. Upon further increase in the value of k_y, Cherenkov interaction of plasmons with electrons of the plasma becomes significant. Therefore, the contribution of $\tilde{\gamma}$ will increase with increasing k_y and the increment will decrease. As a consequence, near $k_y \approx k_{max}$ the increment will again be described by the formula (10), where $v_{E,b}$ is determined by the formula (2).

3. The above discussion allows us now to make clear one important dependence of the intensity of radiation of the harmonic $(3/2)\omega_0$ on the energy flux density of the light incident on the plasma. For this we use the formula for the flux density $q_{3/2}$ of the harmonic $(3/2)\omega_0$, obtained under the assumption that it arises as a result of nonresonant action of the pump wave with the plasmon (see Ref. 11):

$$q_{3/2} \approx q v_E^2 (k_y \lambda_0 c)^{-2} \tau_T, \qquad (13)$$

where q is the flux of laser radiation with wavelength λ_0, $\tau_T = \Delta X \nu_{turb}/c$ is the turbulence optical thickness (ν_{turb} is the turbulence frequency of the absorption). Substituting the value k_{min} in formula (13), we find the upper bound of the transformation coefficient $K_{3/2} = q_{3/2} q^{-1}$ of the radiation of the Nd laser in the harmonic $(3/2)\omega_0$:

$$K_{3/2} \leqslant [2 \cdot 10^{-18} q]^2 \tau_T.$$

It then follows that even for such a small transformation coefficient as found in Ref. 3 it is necessary to assume that τ_T is not very small in comparison with unity. But even under this assumption, the results of the experiments of Refs. 1, 2, and 4 cannot be explained by means of the formula (13), which describes coalescence with a plasmon whose the wavelength is much smaller than the pump wavelength.

In this connection, for an understanding of the results of the experiments of Refs. 1, 2, and 4, we can assume that under the conditons of these researches, the energy flux density reaches such a value for which the minimum value of the wave vector of the plasmon becomes comparable with $2\pi/\lambda_0$. In particular, in accord with the expression (9), which for such longwave plasmons can be used only as an estimate, this is possible at

$$\frac{v_{E,3}}{c} \sim \frac{(2n+1)}{6} \frac{\lambda_0}{L_N} \ln^{-1}\left[\frac{(2n+1)\lambda_0}{L_N} \frac{\omega_0}{\nu_{ii}}\right]$$

which corresponds to a laser-radiation flux

$$q_3 \approx 5 \cdot 10^{16} L_N^{-2} \lambda_0^2,$$

where the size of the inhomogeneity L_N and the laser wavelength λ_0 are measured in microns and the flux q_3 in W/cm^2. For the longwave plasmon waves excited in such fluxes, resonance of the plasmons with the pump wave turns out to be possible. As a result, the flux $q_{3/2}$ increases by several orders of magnitude in comparison with the right side of Eq. (13), thank to the appearance of a factor $\sim \omega_0/\nu_{ei}$.

It follows with certainty from our consideration that the region of localization of the longwave plasmons can be much smaller than the region of two-plasmon parametric instability. This assertion can help us to understand the possibility of use of the harmonic $(3/2)\omega_0$ for the study of local properties of plasma corona.

[1)]According to the results of Ref. 4, we can assume that two-plasmon decay takes place in a region with size $\sim 100\ \mu$. Evidently, the characteristic size of the inhomogeneity of the plasma density under experimental conditions[4] amounts to hundreds of microns. We can therefore discern an agreement between the measured value of the region of generation and the value Δx calculated by us.

[1]P. Lee, D. V. Giovanielli, R. P. Godwin and G. H. McCall, Appl. Phys. Lett. P4, 406 (1974).

[2]H. C. Pant, E. Eidmann, P. Sacksenmaier and R. Sigel, Optic Comm. 16, 396 (1976).

[3]A. I. Avrov, V. Yu. Bychenkov, O. N. Krokhin, V. V. Pustovalov, A. A. Rupasov, V. P. Silin, G. V. Sklizkov, V. T. Tikhonchuk and A. S. Shikanov, Pis'ma Zh. Eksp. Teor. Fiz. 24, 293 (1976) [JETP Lett. 24, 262 (1976)].

[4]V. V. Aleksandrov, V. D. Vikharev, V. P. Zotov, N. G. Kovalskiĭ and M. I. Pergament, Pis'ma Zh. Eksp. Teor. Fiz. 24, 551 (1976) [JETP Lett. 24, 508 (1976)].

[5]V. P. Silin and A. N. Starodub, Zh. Eksp. Teor. Fiz. 66, 176 (1974) [Sov. Phys. JETP 39, 82 (1974)].

[6]Y. C. Lee and P. Kaw, Phys. Rev. Lett. 32, 135 (1974).

[7]C. S. Liu and M. N. Rosenbluth, Phys. Fluids 19, 967 (1976).

[8]E. A. Jackson, Phys. Rev. 153, 235 (1966).

[9]V. P. Silin, A. N. Starodub and M. V. Filippov, Preprint. Phys. Inst. Acad. Sci. USSR No. 158, 1976; Zh. Eksp. Teor. Fiz. 73, 188 (1977) [Sov. Phys. JETP 46, 97 (1977)].

[10]V. P. Silin and A. N. Starodub, Fiz. Plasmy 3, 280 (1977) [Sov. Phys. Plasma Phys. 3, 156 (1977)].

[11]A. I. Avrov, V. Yu. Bychenkov, O. N. Krokhin, V. V. Pustavalov, A. A. Rupasov, V. P. Silin, G. V. Sklizkov, V. T. Tikhonchuk and A. S. Shikanov, Zh. Eksp. Teor. Fiz. 72, 970 (1977) [Sov. Phys. JETP 45, 507 (1977)].

Translated by R. T. Beyer