On the change in the behavior of formfactors of compound systems as high transferred momenta are approached asymptotically

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A study is made of the behavior of the elastic formfactor $F_N(q)$ and of the matrix element for virtual decay $G_N(q)$ of a compound system of N strongly interacting particles. On the example of a one-dimensional model with a δ-like attractive potential it is shown that for an arbitrary number of components N at high transferred momenta F_N and G_N fall off as a power of the transferred momentum, with $F_N \sim G_N \sim q^{-2(N-1)}$. Such a power-type fall-off corresponds completely to the quantum asymptotic behavior of the N-body problem. The exact expression for $F_N(q)$ contains N-1 pole singularities. The nature of these poles is discussed. In the domain of low transferred momenta F_N and G_N exhibit exponential fall-off with the slope of the exponent being determined by the radius of the soliton solution of the classical nonlinear equation corresponding to the quantum equation. The quasiclassical correction to the soliton formfactor is obtained. A change in the behavior of the functions F_N and G_N is observed in the region $q_{cr} = q_0 N$, where q_0 is the characteristic hadron momentum determined by the dynamics of the system and independent of the number of components. Thus, for large N the approach of the form factors to the power-law asymptotic behavior is delayed.

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§1. INTRODUCTION

Processes with high transferred momenta, and in particular the asymptotic behavior of the elastic formfactor of a hadron system, are determined by that region of configuration space where all the components are close to one another, i.e., knowledge of the hadron wave function with large relative momenta between components is required. If we are given the number of components N of the hadron wave function and the form of the interaction between the components $v(q)$, then the asymptotic behavior of the formfactor is determined by the simple formula

$$F_N(q) \sim [v(q)/q^2]^{N-1}. \quad (1)$$

This formula was obtained in Ref. 1 on the basis of dimensional considerations for the case $v(q) = \text{const}$. In the nonrelativistic case this formula was proved for an arbitrary N and for functions $v(q)$ which fall off according to a power law in the domain of high q.[2] In the relativistic case it was obtained in the ladder approximation for a system of interacting scalar particles with $N = 2, 3$ and with the interaction $v(q)$ approaching a constant for high q,[3] and also for an interaction of the type $\lambda\varphi^3$ ($v(q) \sim 1/q^2$).

The validity of the asymptotic behavior (1) is apparently confirmed by the experimental data for a nucleon ($N = 3$)[4] and for a π meson ($N = 2$).[5] It is generally assumed that a necessary condition for the approach to the asymptotic regime is that the following inequality for the transferred momentum q should hold[6]:

$$q^2 \gg q_0^2, \quad (2)$$

where q_0 is the average momentum of the component in the hadron. From experimental data for the nucleon it follows that $q_0 \sim 2.5$ GeV/c.

The number of components N does not appear at all in the estimate (2), and yet from physical considerations it seems natural that the asymptotic regime (1) should arise only when the momentum q transferred to one component of the hadron is redistributed between the other components in equal amounts, and for this $N-1$ interactions are needed. The final fraction of the momentum transferred to each of the components is q/N. Therefore condition (2) for the approach to the asymptotic regime is replaced by

$$q^2 \gg N^2 q_0^2, \quad (3)$$

where q_0 has the same meaning as before.

We note that condition (3) is qualitatively already confirmed by data on the nucleon and the π meson: the approach to the asymptotic regime for the π-meson formfactor occurs earlier than for the nucleon, in the region $q \sim 1.5$ GeV/c. If one regards the deuteron at small distances as a system of six quarks then the boundary for the asymptotic behavior must be at a distance which is double that for the nucleon, i.e., at $q \sim 5$ GeV/c. Therefore the available experimental data on the electromagnetic formfactor for the deuteron[7] ($q^2 \leqslant 6$ GeV/c^2) do not refer to the asymptotic domain and do not serve as an experimental confirmation of the six-quark model of the deuteron.

The problem arises of the behavior of the formfactor in the region lying below the asymptotic region and of the change in the regime at $q \sim Nq_0$. The following picture appears to be entirely natural: for $q \ll Nq_0$ the effective number of the components of the system is infinite and the quasiclassical field description of the hadron is valid. In this case the hadron can be described by the soliton solution of the classical field equation for its components. Recently active attempts were made to utilize solitons for describing the properties of hadrons.[8] A study of the formfactors of classical solitons shows that with an increase in the transferred momentum they fall off exponentially.[9] At the same time the known formfactors of hadron systems and also the exclusive and inclusive hadron cross sections in the domain of high transferred momenta fall off according to a power law. Remaining within the framework of the soliton assumptions regarding the properties of hadron formfactors the power-type functions can be obtained if we take into account the inapplicability of the classical description to the domain of small distances and make an exact calculation of the quantum asymptotic behavior of the solution. The transition from the exponential soliton regime to the power law quantum regime must occur specifically at $q \sim q_0 N$.

Below on the example of a model that can be solved exactly both in the quantum and the classical case we consider the properties of the elastic formfactor of a compound system. In §2 we study the formfactor of the quantum problem. In §3 we obtain the relation of the quantum solution to the formfactor of the classical soliton. In §4 we consider the possibilities of generalizing our model solution to more realistic cases.

A part of the material presented here has been published previously.[10]

§2. THE FORMFACTOR OF A QUANTUM COMPOUND SYSTEM

We consider the one-dimensional nonrelativistic quantum theory of a complex field $\hat{\Psi}(x, t)$, defined by the Hamiltonian[1)]

$$H=\frac{1}{2m}\int \hat{\Psi}_x^+\hat{\Psi}_x dx-\frac{g}{2}\int \hat{\Psi}^+(x)\hat{\Psi}^+(y)\delta(x-y)\hat{\Psi}(x)\hat{\Psi}(y)dx\,dy. \quad (4)$$

In such an exactly soluble model[11] in each N-particle sector there exists only a single bound state of all the N particles with the wave function

$$\Psi_N(x_1, x_2, \ldots, x_N)=C_N\exp\left\{-i\frac{p}{N}(x_1+\ldots+x_N)-\frac{g}{4}\sum_{i>j}^{N}|x_i-x_j|\right\}. \quad (5)$$

The formfactor is defined by the single-particle density $\rho_N(x)$ in the usual manner:

$$F_N(q)=\int_{-\infty}^{\infty}\rho_N(x)e^{iqx}dx. \quad (6)$$

The function $\rho_N(x)$ for the wave function (5) was obtained in Ref. 12:

$$\rho_N(x)=\frac{g}{2N}\sum_{n=1}^{N-1}(-1)^{n+1}\frac{n(N!)^2\exp(-gnN|x|/2)}{(N-1-n)!(N-1+n)!}. \quad (7)$$

From this we immediately obtain

$$F_N(q)=\frac{g}{2}\sum_{n=1}^{N-1}(-1)^n\frac{n^2(N!)^2}{(q^2+{}^1/_4 g^2N^2n^2)(N-1-n)!(N-1+n)!}. \quad (8)$$

For an explicit calculation of $F_N(q)$ it is convenient to extend the summation to the values $-(N-1)\leq n<1$ and to write the sum so obtained in the form of an integral over the contour C shown in Fig. 1:

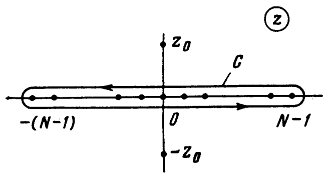

FIG. 1. The contour of integration C for the expression (9). The dots indicate the poles of the integrand.

$$F_N(q)=[(N-1)!]^2 i\oint_C\frac{z^2dz}{[\alpha^2+z^2]\Gamma(N-z)\Gamma(N+z)\sin\pi z}. \quad (9)$$

Taking the residues outside the contour C we obtain in accordance with the Cauchy theory

$$F_N(q)=\prod_{n=1}^{N-1}\frac{[(N-1)!]^2}{(n^2+4q^2/g^2N^2)} \quad (10)$$

$$=\frac{2\pi q[(N-1)!]^2}{gN\,\mathrm{sh}(2\pi q/gN)\Gamma(N+2iq/gN)\Gamma(N-2iq/gN)}. \quad (11)$$

We examine the behavior of $F_N(q)$ defined by (10) and (11) under the condition that the quantity $q_0=gN/2$ is fixed. For

$$q\ll q_{cr}=Nq_0 \quad (12)$$

we obtain

$$F_N(q)\cong\frac{\pi q}{q_0\,\mathrm{sh}(\pi q/q_0)}. \quad (13)$$

In the opposite limiting case $q\gg q_{cr}$ we have

$$F_N(q)\cong\frac{[(N-1)!]^2}{(q^2/q_0^2)^{N-1}}, \quad (14)$$

and this agrees with the asymptotic behavior of (1) since in our case $v(q)=\text{const}$. The nature of the transition from one asymptotic behavior to the other is shown in Fig. 2 for $N=3$, 6, 10, 20. On the same diagram we also show the form of the function $\ln F(q)$ (13). As will be seen from §3 in the domain of low q the functions $\ln F_N(q)$ differ from the function $\ln F(q)$ which falls off linearly by terms $\sim q^2/N$.

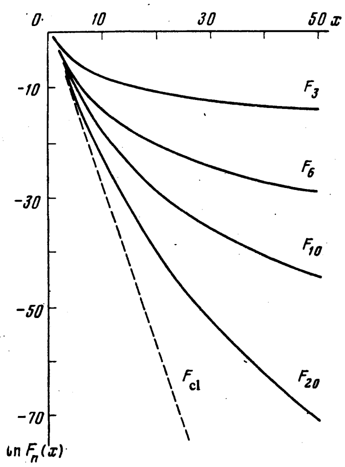

FIG. 2. The dependence of the formfactors of the compound system (the number of quantum components is $N=3$, 6, 10, 20) and of the soliton (dotted line) on the transferred momentum (in dimensionless units, $x=q/q_0$).

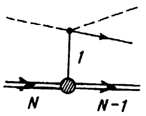

FIG. 3.

It must be noted that the existence of two different regimes for the behavior of the formfactor (13) and (14) for this model was recently noted independently of our work by Amado and Woloshyn.[13]

An analogous discussion can be applied to the matrix elements $\langle \Psi_N | \Psi_{N-1} \Psi_1 \rangle \equiv G_n$ which enters, for example, into the expression for the matrix element of the process shown in Fig. 3 where G_N corresponds to the lower part of the diagram.

Calculations similar to those given above (cf., the exact derivation in the Appendix), lead to the following expression:

$$G_N(q)=\frac{\delta(P_N-P_1-P_{N-1})}{(2\pi)^{1/2}}\left(\frac{2}{g}\right)^{1/2}\times\left\{\prod_{n=1}^{N-1}\left[1+\frac{q^2}{q_0^2(2n-1)^2}\right]\right\}^{-1}. \tag{15}$$

Just as before in the region (12) we have an asymptotic behavior of the form

$$G_N(q)\sim \mathrm{ch}^{-1}(\pi q/q_0), \tag{16}$$

while for $q \gg Nq_0$ we obtain a power law falling-off analogous to (14).

We now discuss the meaning of the poles of the formfactor $F_N(q)$ in the expression (10). Our assertion consists of the fact that to each n-th pole of $F_N(q)$ there corresponds an anomalous threshold of the triangle diagram of Fig. 4 with a prescribed choice of the numbers N_1 and N_2. The fact that an anomalous threshold leads to a pole and not to a cut is associated with the one-dimensional nature of the problem.

We introduce the dimensionless angles θ and χ:

$$\cos\theta=\frac{M_1^2+M_2^2-M_N^2}{2M_1M_2},\quad \cos\chi=\frac{2M_1^2-t}{2M_1^2}. \tag{17}$$

The condition for the anomalous threshold $2\theta+\chi=2\pi$ leads us to the following value for the position of the singularity with respect to $t=q^2$:

$$t=[(M_1+M_2)^2-M_N^2][M_N^2-(M_1-M_2)^2]/M_2^2. \tag{18}$$

Denoting the masses of the particles by $M_{N_i}=N_i m-\varepsilon_{N_i}$, going over in (18) to the nonrelativistic limit and taking into account the fact that $\varepsilon_N=g^2(N^3-N)/48m$, we obtain

$$q_{N_i}^2=t_{N_i}={}^1/_4 g^2N^2N_i^2, \tag{19}$$

and this agrees exactly with the position of the poles in (10).

§3. THE FORMFACTOR OF THE CLASSICAL SOLITON

We now show that the exponential regime of the formfactor for low q corresponds to the behavior of the formfactor of a classical soliton. Proceeding in analogy with the work of Ishikawa[9] we introduce into the Lagrangian the interaction of our field Ψ with an external scalar field $A(x)$:

$$L_{int}=\int\Psi^*(x)A(x)\Psi(x)\,dx=\int j(x)A(x)\,dx, \tag{20}$$

and from this we obtain the expression for the formfactor

$$F(q)=\int dx\,e^{iqx}j(x)=\int e^{iqx}|\Psi(x,0)|^2dx. \tag{21}$$

The classical equation for the function $\Psi(x,t)$, determined by the Hamiltonian H (4) has the form

$$i\frac{\partial\Psi}{\partial t}+\frac{\partial^2\Psi}{\partial x^2}+\varkappa\Psi|\Psi|^2=0 \tag{22}$$

and is completely integrable.[14] The soliton of Eq. (22) has the following form:

$$\Psi(x,t)=\frac{\sqrt{\varkappa n}}{2\sqrt{2}}\exp\left\{i\frac{p}{n}(x-vt-x_0)+i\mu t+i\varphi\right\}\mathrm{ch}^{-1}\left[\frac{\varkappa n}{4}(x-vt-x_0)\right], \tag{23}$$

where x_0, φ are the parameters of the soliton, $n=\int\Psi^*\Psi dx$ is an integral of the motion of Eq. (22). The relation of the N-particle wave function (5) to the soliton (23) was investigated in Ref. 15. In particular it was shown that the properties of the quantum system (5) go over into the properties of solitons as $N\to\infty$ and under the condition of weak coupling $Ng=n\varkappa=$ const.

First of all we note that in this case the slope of the exponent in expression (13) is determined by the quantity $q_0=gN/2=\varkappa n/2$, i.e., it is related to the radius of the classical soliton (13). Further substituting the soliton solution into the formfactor (21) we obtain exactly the expression (13). Thus, the form of the quantum formfactor with a large number of components in the domain of low transferred momenta agrees with the formfactor of the classical soliton in the limit $N\to\infty$ and our intuitive considerations in §1 are completely confirmed by the model investigation.

We now obtain the quasiclassical correction to the classical formfactor $F(q)$ (13). This can be done most conveniently by expanding the Γ functions in formula (11) into a Taylor series in the neighborhood of the point N (taking into account the fact that $N\gg q/q_0$). We obtain in the first nonvanishing approximation

$$F_N(q)=\frac{\pi q}{q_0\,\mathrm{sh}(\pi q/q_0)}\left\{1+\frac{q^2}{q_0^2}\sum_{m=0}^{\infty}\frac{1}{(m+N)^2}\right\}. \tag{24}$$

Taking into account the fact that for large N the sum $\sum^\infty(m+N)^2\approx 1/N$ we obtain

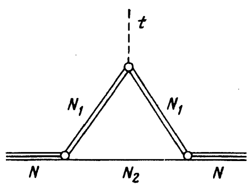

FIG. 4.

$$\ln F_N(q)=\ln F(q)+\frac{q^2}{q_0^2}\frac{1}{N}, \qquad (25)$$

i.e., the quantum corrections to the soliton solution have the form of an expansion in powers of $1/N$.

Formula (24) shows that it is not possible to reconstruct the exact quantum solution (10) for the formfactor of an N-particle system on the basis of quantizing small oscillations in the neighborhood of the classical solution. For the transition from an exponential falloff into one described by a power law it is necessary to take into account all the terms in the series expansion in terms of q^2/q_0^2, and not of a finite number of them as in formula (24).

§4. CONCLUSION

We discuss the possibility of generalizing our model problem to more realistic cases. Here the following questions arise: a) what will occur in the transition to the three-dimensional case, b) what will be introduced by a transition to the relativistic case, and c) what will be altered when the finite range of interaction is taken into account?

In answering the first question we note first of all that the asymptotic behavior of the formfactor $F_N(q) \sim q^{-2(N-1)}$ certainly does not depend on the dimensionality of the problem, since the answer is related to the enumeration of the number of propagators carrying the momentum q transferred to the system. But the form of the propagator $G\sim q^{-2}$ in the momentum representation does not depend on the dimensionality of the space. The behavior of $F_N(q)$ in the domain lying below the asymptotic domain in accordance with Ref. 16 is also determined by the properties of the propagators, but taking into account the mass terms and therefore also does not depend on the dimensionality of the problem. As regards the domain of low transferrred momenta, the problem here is related to the existence of three-dimensional soliton solutions which were recently found in numerical experiments.[17]

Taking into account the relativistic nature of the motion leads first of all to a change in the kinematics. Thus, for example, in the work of Ishikawa[9] an expression was obtained for the formfactor of the classical soliton using the model of a scalar field of the Ginzburg-Landau-Higgs type which differs from expression (13) merely by the replacement $q^2 \to [t(t-4)M^2]^{1/2}$, and is given by:

$$F(t)=\frac{\pi[t(t-4M^2)]^{1/2}}{2\sqrt{2}}\,\mathrm{sh}^{-1}\left(\frac{\pi}{2\sqrt{2}}\left[\frac{t(t-4M^2)}{M^2\mu^2}\right]^{1/2}\right), \qquad (26)$$

where M is the soliton mass. From (26) it can also be seen that the root singularity of $F(t)$ at $t=4M^2$ is illusory, since it appears both in the numerator and the denominator of formula (26) and cancels out as the point $4M^2$ is approached. On the other hand, the threshold root singularity must enter the correct relativistic expression for the formfactor. Therefore, although the nonrelativistic limit $t \ll 4M^2$ of expression (26) agrees with our expression (13), nevertheless formula (26) may turn out to incorrect in the region of large t.

We investigate to what can a finite range of interaction r_0 lead. In the case of $q_{cr}r_0 \ll 1$ all our formulas apparently remain valid right up to $q\sim r_0^{-1}$ and the asymptotic behavior of the solution in the region $q \gtrsim r_0^{-1}$ will be determined by formula (1). If $q_{cr}r_0\sim 1$, then the behavior of the formfactor in the domain of low q will be determined by the soliton solution of the classical equations with a point interaction, and the approach to the asymptoric behavior (1) will have a more complicated form.

We consider what kind of qualitative predictions can be obtained for the change in the nature of the asymptotic behavior of the formfactor for nuclei. Associating q_0 in formula (13) with the nuclear radius by $q_0R=\pi$, we obtain

$$q_{cr}=A^{1/3}\pi\cdot 0.8\ \mathrm{F}^{-1}, \qquad (27)$$

which yields 5.9 GeV/c for Ca^{40} and 2.9 GeV/c for the C^{12} nucleus, i.e., the values of q_{cr} lie in a domain accessible to experiment. The values of $F_N(q)$ calculated according to (14) turn out to be of the order of 10^{-9}. In the case of estimating the real formfactor for the nucleus it is necessary to take into account the Pauli principle for nucleons,[18] and this greatly reduces the asymptotic value of the formfactor (14).

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APPENDIX

We calculate the matrix element corresponding to the virtual decay of the bound state of N particles into a bound state of $N-1$ particles and one free particle. Such a matrix element

$$G_N=\langle\Psi_N|\Psi_{N-1}\Psi_1\rangle \qquad (A.1)$$

$$=\int_{-\infty}^{\infty} dx_1\,dx_2\ldots dx_N\Psi_N(x_1,\ldots,x_N)\Psi_{N-1}(x_1,\ldots,x_{N-1})\Psi_1(\bar{x}_N) \qquad (A.2)$$

enters into the amplitude of a reaction of the type of a quasielastic knock-out. The Feynman diagram corresponding to such a reaction is shown in Fig. 3.

For the calculation of G_N it is convenient first of all to consider the contribution to the integral in (A.2) from the domain of the configuration space $\{x_N\}$, determined by the condition $x_1<x_2<\ldots<x_{N-1}<x$ (the sector a_1). In this domain the integral of (A.2) has the form

$$I_N^{a_1}=\int\left(\prod_1^N dx_i\right)\exp\left\{-\frac{ip}{N}(x_1+\ldots+x_N)+\frac{g}{4}\sum_{k=1}^{N}(N+1-2k)x_k\right.$$

$$\left.+\frac{ip_{N-1}}{N-1}(x_1+\ldots+x_{N-1})+\frac{g}{4}\sum_{m=1}^{N-1}(N-2m)x_m+ip_1\bar{x}_N\right\} \qquad (A.3)$$

and can be easily calculated by consecutive integration over all the x_i. As a result we obtain

$$I_N^{a_1}=2\pi\delta(p_N-p_{N-1}-p_1)\Big/\prod_{k=1}^{N-1}\Big[\frac{g}{4}k(N-k)+\frac{g}{4}k(N-1-k)-\frac{ip_Nk}{N}+\frac{ip_{N-1}k}{N-1}\Big]. \quad (A.4)$$

The interchange of any two particles other than $\bar{x}_N$ does not alter the result of the integration. As a result the contribution of all the regions of $\{x_N\}$, in which $\bar{x}_N$ occurs to the right of all the other particles is equal to $I_N^a=(N-1)!\,I_N^{a_1}$.

We now consider the region $x_1<x_2<\ldots<x_{N-2}<\bar{x}_N<x_{N-1}$ (the sector b_1). A consecutive integration leads to the result

$$I_N^{b_1}=2\pi\delta(p_N-p_{N-1}-p_1)\Big\{\prod_{k=1}^{N-2}\Big[\frac{g}{4}k(N-k)+\frac{g}{4}k(N-k-1)-\frac{ip_Nk}{N}+\frac{ip_{N-1}k}{N-1}\Big]\Big\}^{-1}\Big[\frac{g}{4}(N-1)1+\frac{g}{4}(N-2)1-\frac{ip_N(N-1)}{N}+\frac{ip_{N-1}(N-2)}{N-1}+ip_1\Big]^{-1}. \quad (A.5)$$

Just as earlier for the sectors a, $I_N^b=(N-1)!\,I_N^{b_1}$. Subsequently the evaluation of the integrals for other sectors of the configuration space $\{x_N\}$ is carried out in an analogous manner and the complete integral I_N can be represented in the form of the sum

$$I_N=\sum_{M=0}^{N-1}\frac{(N-1)!\,2\pi\delta(p_N-p_{N-1}-p_1)}{(N-1-M)!\,M!\prod_{l=1}^{M}\prod_{k=1}^{N-1-M}[(2N-2k-1)-i\alpha][(2N-2l-1)+i\alpha]} \quad (A.6)$$

where we have used the notation $4q/gN=\alpha$ and $q=p_1-p_{N-1}/N-1)$ is the transferred momentum. The expression (A.6) is real due to the symmetry with respect to the indices k and l and can be easily brought to the form

$$I_N=\frac{(N-1)!\,2^{N-1}\cdot 2\pi\delta(p_N-p_{N-1}-p_1)}{\prod_{k=1}^{N-1}[(2N-2k-1)^2+\alpha^2]}. \quad (A.7)$$

After taking into account the normalization constants of the wave functions Ψ_N, Ψ_{N-1}, and Ψ_1 it is easy to make the transition from (A.7) to (15) given in the main text.

[1)]Both here and in the rest of this paper we use the system of units in which $\hbar=c=1$.

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