employed by Pfennig.[12]

[2)]The more rigorous criterion is $\Delta\omega \gg \gamma$, where γ is the collisional linewidth. For electron broadening, this region is commonly referred to as the single-electron region.[1,2]

[1]I. I. Sobel'man, Vvedenie v teoriyu atomnykh spektrov (Introduction to the Theory of Atomic Spectra), Fizmatgiz, M., 1963 (English transl. publ. by Pergamon Press, Oxford, 1972).

[2]R. G. J. Breene, The Shift and Shape of Spectral Lines, Pergamon Press, Oxford, 1961.

[3]A. Jablonski, Phys. Rev. **68**, 78 (1945).

[4]I. I. Sobel'man, Opt. Spektrosk. **1**, 617 (1956).

[5]M. Baranger, Phys. Rev. **111**, 481 (1958); **112**, 855 (1958).

[6]J. Szudy, Acta Phys. Pol. A **40**, 361 (1971).

[7]N. Tran Minh and H. Van Regemorter, J. Phys. B **5**, 903 (1972).

[8]N. Tran Minh, N. Feautrier, and H. Van Regemorter, J. Phys. B **8**, 1810 (1975).

[9]J. Szudy and W. E. Baylis, J. Quant. Spectrosc. Radiat. Transfer **15**, 641 (1974).

[10]V. S. Lisitsa and G. V. Sholin, Zh. Eksp. Teor. Fiz. **61**, 912 (1971) [Sov. Phys. JETP **34**, 484 (1972)].

[11]L. Spitzer, Phys. Rev. **58**, 348 (1940).

[12]H. Pfennig, J. Quant. Spectrosc. Radiat. Transfer **12**, 821 (1972).

[13]M. J. Seaton, Proc. Phys. Soc. London **77**, 174 (1961).

[14]Yu. N. Demkov, V. N. Ostrovskiĭ, and E. A. Solov'ev, Zh. Eksp. Teor. Fiz. **66**, 125 (1974) [Sov. Phys. JETP **39**, 57 (1974)].

[15]V. N. Ostrovskiĭ and E. A. Solov'ev, Zh. Eksp. Teor. Fiz. **66**, 1590 (1974) [Sov. Phys. JETP **39**, 779 (1974).

[16]A. Hautot, J. Math. Phys. (N.Y.) **14**, 1320 (1973).

[17]L. D. Landau and E. M. Lifshitz, Kvantovaya mekhanika (Quantum Mechanics), Nauka, M., 1974 (English transl. publ. by Pergamon Press, Oxford, 1975).

[18]G. N. Watson, Theory of Bessel Functions, Cambridge University Press, 1944 (Russ. Transl., IIL, 1949).

[19]I. S. Gradshteĭn and I. M. Ryzhik (Gradshteyn and Ryshik), Tablitsy integralov, summ, ryadov, i proizvedeniĭ (Table of Integrals, Series, and Products) Nauka, M., 1971 (English trans. of an earlier ed. publ. by Academic Press, New York, 1966).

[20]H. Bateman and A. Erdelyi, Higher Transcendental Functions, McGraw-Hill, New York, 1953 (Russ. Transl., Nauka, 1966, Vol. 1).

[21]N. Feautrier, N. Tran Minh, and H. Van Regemorter, J. Phys. B **9**, 1871 (1976).

[22]N. Tran Minh, N. Feautrier, and H. Van Regemorter, J. Quant. Spectrosc. Radiat. Transfer **16**, 849 (1976).

Translated by S. Chomet

Finite amplitude Langmuir oscillations in the plasma resonance region

S. V. Bulanov, L. M. Kovrizhnykh, and A. S. Sakharov

P. N. Lebedev Physical Institute, USSR Academy of Sciences

(Submitted September 20, 1976)

Zh. Eksp. Teor. Fiz. **72**, 1809–1823 (May 1977)

We consider, in the multi-fluid hydrodynamical approximation, induced Langmuir oscillations of an inhomogeneous plane plasma layer in an external harmonic hf field. We study the limitation of the electric field amplitude near the plasma resonance point as the result of a linear transformation into plasma waves, the self-intersection of electron trajectories, the anharmonicity of the Langmuir oscillations, and the nonstationarity of the plasma for various density profiles. We study numerically the generation of non-linear plasma waves and the acceleration of particles when resonance break down takes place which leads to an effective dissipation of the energy of the oscillations in the resonance region. We analyze qualitatively the role of the ion motion which is the result of striction forces and we indicate conditions under which modulational, parametric, and other instabilities which are connected with ion motion turn out to be unimportant.

PACS numbers: 52.35.Mw

1. The amplification of an hf field near the plasma resonance point is of great interest in connection with the problem of the anomalous absorption of electromagnetic waves in a non-uniform plasma and the generation of accelerated particles.[1,2] The magnitude of the field at the resonance is determined by collisions, the linear transformation into plasma waves, and in strong fields by the electron non-linearity and the change in the plasma density under the influence of striction forces.[3–11]

In the present paper we consider the establishment of the field in the plasma resonance region under conditions where the modulational, parametric, and other instabilities, connected with ion motion in the self-consistent field, turn out to be unimportant (see inequalities (68), (70)).

We consider a plane layer of plasma with ion density n_i which is non-uniform in x. The plasma is in a uniform external electric field which depends on the time as $E = E_0 \sin\omega t$ and is parallel to the inhomogeneity gradient. We choose the origin $x = 0$ at the plasma resonance point, i.e., $\omega = \omega_p(x = 0)$. We denote by s the ratio of the amplitude of the acting field to the maximum

0038-5646/77/4505-0949$02.40

© 1978 American Institute of Physics

value of the field in the resonance, i.e., s is equal to the reciprocal of the magnitude of the amplification:

$$E_m=E_0/s. \tag{1}$$

We shall describe the phenomena at plasma resonance in the hydrodynamic approximation, neglecting collisions:

$$\frac{\partial n_\alpha}{\partial t}+\frac{\partial n_\alpha v_\alpha}{\partial x}=0, \tag{2}$$

$$\frac{\partial v_\alpha}{\partial t}+v_\alpha\frac{\partial v_\alpha}{\partial x}=\frac{e_\alpha}{m_\alpha}(E+E_0\sin\omega t)$$
$$-\frac{1}{n_\alpha}\frac{\partial}{\partial x}v_{T\alpha}^2 n_\alpha(x,0)\,(n_\alpha/n_\alpha(x,0))^\gamma, \tag{3}$$

$$\frac{\partial E}{\partial x}=4\pi\sum_\alpha e_\alpha n_\alpha. \tag{4}$$

Here $v_{T\alpha}^2=T\alpha/m_\alpha$; $n_\alpha(x,0)$ and $T\alpha$ are the initial density and temperature of particles of kind α ($\alpha=i, e, \ldots$). The rest of the notation is the normally used one.

When considering electron non-linearities in a cold plasma it is convenient to change in Eqs. (2) to (4) from the Euler variables x, t to the Lagrangian coordinates x_α, t. The density of particles of kind α is equal to

$$n_\alpha(x_\alpha,t)=n_{\alpha 0}(x_\alpha)\,|dx_\alpha/dx|. \tag{5}$$

The function $n_{\alpha 0}(x_\alpha)$ determines the connection between the Lagrangian and Euler variables. If we take $n_{\alpha 0}(x_\alpha)=n_\alpha(x_\alpha,0)$ we can determine the connection between x and x_α as follows:

$$x=x_\alpha+\sigma_\alpha(x_\alpha,t),\quad v_\alpha=\dot\sigma_\alpha(x_\alpha,t), \tag{6}$$

where x_α is the initial position and $\sigma_\alpha(x_\alpha,t)$ the displacement of an element of fluid of kind α; $n_\alpha(x_\alpha,t)=n_{\alpha 0}(x_\alpha)/|1+\sigma'_\alpha(x_\alpha,t)|$. We shall denote time derivatives by a dot and coordinate derivatives by a prime.

It follows from Eqs. (2), (4) that

$$(\dot E)_{x_\alpha}=4\pi\sum_{\beta\neq\alpha}e_\beta n_\beta(x_\beta,t)\,[(\dot\sigma_\alpha)_{x_\alpha}-(\dot\sigma_\beta)_{x_\beta}]. \tag{7}$$

The summation is over kinds of particles. The subscript x_α (or x_β) means that the derivative is taken for constant values of x_α (or x_β). The values of the functions in (7) are evaluated for the same Euler coordinate, determined by Eq. (6), which we rewrite in the form of an equation for x_β:

$$x_\beta=x_\alpha+\sigma_\alpha(x_\alpha,t)-\sigma_\beta(x_\beta,t). \tag{8}$$

We differentiate it with respect to time for constant x_α. We find that

$$(\dot\sigma_\alpha)_{x_\alpha}-(\dot\sigma_\beta)_{x_\beta}=(1+\sigma_\beta')(\dot x_\beta)_{x_\alpha}. \tag{9}$$

Substituting this expression into (7) and using Eq. (5) we find the electric field:

$$E(x_\alpha,t)=4\pi\sum_{\beta\neq\alpha}e_\beta\int_{x_\beta(0)}^{x_\beta(t)}n_{\beta 0}(y)\,dy. \tag{10}$$

It follows from (6) and (8) that $x_\beta(0)=x_\alpha$. The equations of motion become

$$\ddot\sigma_\alpha(x_\alpha,t)=\frac{4\pi e_\alpha}{m}\sum_{\beta\neq\alpha}e_\beta\int_{x_\alpha}^{x_\beta(t)}n_{\beta 0}(y)\,dy$$
$$-\frac{1}{n_{\alpha 0}(x_\alpha)}\left(\frac{v_{T\alpha}^2 n_{\alpha 0}(x_\alpha)}{(1+\sigma_\alpha'(x_\alpha,t))^\gamma}\right)'+\frac{e_\alpha E_0}{m_\alpha}\sin\omega t. \tag{11}$$

$x_\beta(t)$ in Eqs. (10), (11) is the solution of Eq. (8) for given x_α.

It is convenient when expanding (11) in powers of the non-linearity to introduce a function $\xi_{\alpha\beta}$ which is equal to the difference of the Euler coordinates of elements of fluid of kinds α and β which have the same Lagrangian coordinates $x_\alpha=x_\beta=x_0$:

$$\xi_{\alpha\beta}(x_0,t)=\sigma_\alpha(x_0,t)-\sigma_\beta(x_0,t). \tag{12}$$

One can write the electric field as a power series in $\xi_{\alpha\beta}$:

$$E(x_\alpha,t)=4\pi\sum_{\beta\neq\alpha}e_\beta\sum_{n=1}^{\infty}\frac{\xi_{\alpha\beta}^n}{n!}\left[\frac{\partial^{n-1}n_\beta(x_\beta,t)}{\partial x^{n-1}}\right]\bigg|_{x_\beta=x_\alpha}$$
$$=4\pi\sum_{\beta\neq\alpha}e_\beta\sum_{n=1}^{\infty}\frac{\xi_{\alpha\beta}^n}{n!}\left[\left(N_\beta\frac{\partial}{\partial x_\beta}\right)^{n-1}n_{\beta 0}N_\beta\right]\bigg|_{x_\beta=x_\alpha}, \tag{13}$$

where

$$N_\beta=1/(1+\sigma_\beta'). \tag{14}$$

For an electron-ion plasma the electron oscillations are in zeroth approximation considered on the background of a given ion motion (in particular, of fixed ions). As the expression for the electric field acting on the electrons does not contain the derivatives ξ'_{ei}, the electron equations of motion will in a cold plasma when $\xi/L\ll 1$ be weakly non-linear up to the moment when the electron trajectories intersect themselves, i.e., when the Jacobian $|dx/dx_e|=|1+\sigma_e'|$ vanishes, after which these equations are inapplicable.

2. We consider the plasma resonance on the background of fixed ions in the linear approximation. We approximate the ion density distribution by the function

$$n_i(x)=n_i(0)\,[1-(-x/L)^k], \tag{15}$$

where

$$L=\left(\frac{\partial^k n_i(0)}{\partial x^k}\bigg/ n_i(0)\,k!\right)^{-1/k},\quad k=1,2,3,\ldots. \tag{16}$$

When $k>1$ the resonance lies close to the maximum or the point of inflection of $n_i(x)$. We study in most detail the case $k=1$. The limitation on the field is caused in the linear approximation by the absorption due to collisions with a frequency $\nu=\max\{\nu_{ei},\nu_{en}\}$ or the transformation into plasma waves.[3,4] The field at the resonance is determined by Eq. (1) where the parameter s equals

$$s=\max\{s_\nu=\nu/\omega,\ s_T=(r_{De}/L)^{2/3}\},\qquad (17)$$

where $r_{De}=(3T_e/m_e\omega^2)^{1/2}$. The size of the region where the field is localized is of the order $\Delta x=sL$, the time for the establishing of the field at the resonance $\tau\approx\pi/\omega s$. It is shown in Ref. 12 that when $k>1$ taking dissipation due to collisions into account leads to the width of the resonance turning out to be of the order of

$$\Delta x=s^{1/k}L.\qquad (18)$$

We study the limitations on the field at the resonance due to the linear transformation of the oscillations into plasma waves. For the stationary case we write the field in the form

$$E(x,\ t)={}^1/_2(iE(x)e^{-i\omega t}+\text{c.c.}).\qquad (19)$$

Linearizing the set (2) to (4) we get

$$r_{De}{}^2d^2E/dx^2+(-x/L)^kE=E_0.\qquad (20)$$

We neglected here the term $r_{De}^2n'E'/n$ in comparison with the other terms which can be done near the resonance.

We formulate the boundary conditions for (20). The field does not penetrate the region where the density is larger than the critical one and in the region with a smaller density the field has the form of an outgoing wave.

We introduce the notation:

$$y=-x/\Delta x=-(x/L)(L/r_{De})^{2/(k+2)},\qquad (21)$$

$$u(y)=y^{1/2}J_{1/(k+2)}\left(\frac{2}{k+2}y^{(k+2)/2}\right),\qquad (22)$$

$$v(y)=y^{1/2}J_{-1/(k+2)}\left(\frac{2}{k+2}y^{(k+2)/2}\right).\qquad (23)$$

Here $J_\nu(x)$ is a ν-th order Bessel function. We write the solution of Eq. (20) in the form

$$E(y)=E_0\left(\frac{r_{De}}{L}\right)^{-2k/(k+2)}\left\{\frac{\pi}{(k+2)\sin[\pi/(k+2)]}\right.$$
$$\left.\times\left[u(y)\int_0^y v(y')dy'-v(y)\int_0^y u(y')dy'\right]-A\Phi(y)\right\},\qquad (24)$$

where A depends on k:

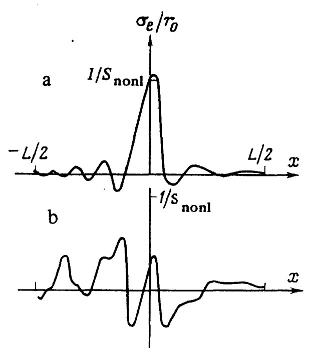

FIG. 1. Curves illustrating the possibility of the carrying away of the energy of the oscillations from the plasma resonance region by non-linear Langmuir waves: a) $\omega t=12\pi$, b) $\omega t=20\pi$; $s_{\rm nonl}=0.05$, $s_T=0.04$.

$$A=\Gamma\left(\frac{1}{k+2}\right)\Gamma\left(\frac{2}{k+2}\right)\Big/(k+2)^{(2k-1)/(2k+2)}.\qquad (25)$$

The function $\Phi(y)$ equals

$$\Phi(y)=\begin{cases}e^{2i\pi/(k+2)}v(y),\ k\ \text{even}\\ e^{i\pi/(k+2)}v(y)-(1-e^{i\pi/(k+2)})u(y),\ k\ \text{odd}\end{cases}\qquad (26)$$

For symmetric plasma density profiles (even values of k) $E(y)$ is an even function of y. Its asymptotic behavior as $y\to\pm\infty$ is:

$$E(y)\approx E_0\left(\frac{r_{De}}{L}\right)^{-2k/(k+2)}\left\{|y|^{-k}\right.$$
$$\left.-iA\sin\left(\frac{2\pi}{k+2}\right)e^{i\pi/(k+2)}|y|^{1/2}H^{(1)}_{1/(k+2)}\left(\frac{2}{k+2}|y|^{(k+2)/2}\right)\right\}.\qquad (27)$$

For odd values of k we have as $y\to+\infty$

$$E(y)\approx E_0\left(\frac{r_{De}}{L}\right)^{-2k/(k+2)}\left\{\frac{1}{y^k}-iA\sin\left(\frac{\pi}{k+2}\right)\right.$$
$$\left.\times(1+e^{i\pi/(k+2)})y^{1/2}H^{(1)}_{1/(k+2)}\left(\frac{2}{k+2}y^{(k+2)/2}\right)\right\}.\qquad (28)$$

As $y\to-\infty$ the field decreases as a power:

$$E(y)\approx-E_0(r_{De}/L)^{-2k/(k+2)}|y|^{-k}.\qquad (29)$$

$H_\nu^{(1)}(x)$ in Eqs. (27), (28) is a Hankel function of the first kind.

The amplification of the field at resonance can be expressed in terms of the quantity s which in the case considered is equal to

$$s_T=E_0/E_m=(r_{De}/L)^{2k/(k+2)}.\qquad (30)$$

The characteristic size of the field localization near resonance is equal to $\Delta x=s_T^{1/k}L$, i.e., has the same form as Eq. (18).

3. At sufficiently low temperatures and rare collisions (or relatively large external fields) it is necessary to take into account electron non-linearities.[5-7] As before we assume the ion motion to be given.

We consider to begin with the cold plasma approximation ($v_{Te}=0$) when we get from (11) an ordinary differential equation for $\sigma_e(x_e,t)$. Let the ions be fixed, so that $\sigma_e=\xi_{ei}$ and

$$\ddot\sigma_e+\frac{4\pi e^2}{m}\int_0^{\sigma_e}n_i(x_e+\sigma)d\sigma=-\frac{eE_0}{m_e}\sin\omega t.\qquad (31)$$

The coordinate x_e occurs in Eq. (31) solely as a parameter.

We discuss first of all the self-intersection of the electron trajectories. For this it is sufficient to restrict ourselves to the approximation linear in σ_e/L. The solution of Eq. (31) for zero initial conditions ($\sigma_e(t=0)=0$, $\dot\sigma_e(t=0)=0$) is elementary (see also Ref. 6):

$$\sigma_e(x_e,t)=\left(r_0\Big/\left(1-\frac{\omega_p{}^2(x_e)}{\omega^2}\right)\right)\left(\sin\omega t-\frac{\omega}{\omega_p(x_e)}\sin\omega_p(x_e)t\right).\qquad (32)$$

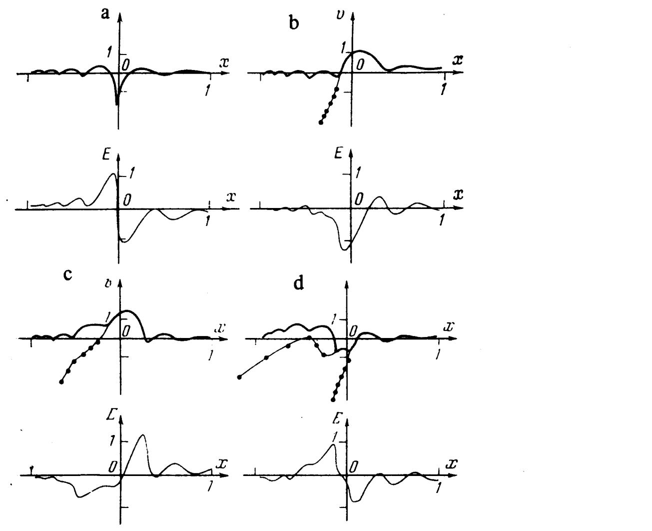

FIG. 2. Break down of the resonance in a cold plasma: a) $\omega t = 10.5\pi$; b) $\omega t = 11\pi$; c) $\omega t = 11.5\pi$; d) $\omega t = 12.5\pi$; $s_{\rm nonl} = 0.1$.

Here

$$r_0 = eE_0/m_e\omega^2, \quad \omega_p^2(x_e) = 4\pi e^2 n_i(x_e)/m_e.$$

In the resonance point, where $\omega = \omega_p(x_e)$, the amplitude of the oscillations increases linearly with time: $\sigma_e \approx \frac{1}{2} r_0 \omega t \cos\omega t$, and the resonance width decreases as $1/t$. The moment of self-intersection when the oscillations have an essentially non-linear character can be found from the condition that the Jacobian vanishes, $|dx/dx_e| = 0$, i.e., $\sigma_e' = -1$. The ratio of the acting field to the field at resonance is then of the order of

$$s_{\rm nonl} = (r_0/L)^{k/(k+1)}. \tag{33}$$

The width of the resonance is comparable with the amplitude of the oscillations: $\Delta x \approx \sigma_{em} \approx s_{\rm nonl}^{1/k} L$. For a linear plasma profile ($k=1$) $s = (r_0/2L)^{1/2} = s_{\rm nonl}/2^{1/2}$.[5-7]

As $\sigma_e' \to -1$, the electron density (5) becomes infinite and the neglect of the pressure in (31) may become unfounded or, what amounts to the same, the neglect of the plasma wave dispersion. If $s_{\rm nonl} \gg s_T$, the pressure affects the non-linear stage and the limitation on the field is connected with the carrying away of energy by the non-linear plasma waves. The amplification of the hf field is then determined by the quantity $s_{\rm nonl}$ of (33). To illustrate this statement we give the results of a numerical calculation on a computer of the hydrodynamical Eqs. (2) to (4).

We considered the fixed ion approximation, a linear density profile ($k=1$), the isothermal case ($\gamma = 1$), $s_{\rm nonl} = 0.05$ and $s_T = 0.04$. The initial values of σ_e and $\dot\sigma_e$ we defined from the solution (32) for the cold plasma at time $t = 12\pi/\omega$ when the field is already comparable to the maximum $\sim E_0/s_{\rm nonl}$ (Fig. 1a); the x-dependence of σ_e has a strikingly expressed resonance character. We give in Fig. 1b the solution for $t = 20\pi/\omega$ and it is clear that the amplitude of the oscillations at the resonance is somewhat diminished and in the region $x<0$ there appeared non-linear plasma waves. We note that in a cold plasma resonance would break down at $t = 18.5\pi/\omega$.

One can estimate the characteristic size of the non-uniformity of the electric field corresponding to the "width of the front" of the non-linear wave $l = E_m/(\partial E/\partial x)_m$. For the isothermal case $\gamma = 1$ and a linear density profile $l \sim r_{De}^2/r_0$. If $l < r_{De}$, i.e., $r_0 > r_{De}$ the hydrodynamic description is inapplicable and the pressure cannot prevent the destruction of the resonance, i.e., after a time $\tau \approx \pi/\omega\sigma_{\rm nonl}$ the resonance breaks down.

In Fig. 2 we give the results of a computer simulation using an electrostatic code[13] of the break-down of the resonance in a cold plasma on a fixed ion background.[1)] The upper curves depict the position of elements of the electron plasma in the x, v phase plane (x in units L, v in units $v_{E_0}/s_{\rm nonl}$, the heavy line connects the points which in the given scale blend together). The lower curves depict the x-dependence of $s_{\rm nonl} E/E_0$. It is clear that the break-down of the resonance is expressed in the peaking of the line in the phase plane near $x=0$ (Fig. 2a) after which the line is deflected in the direction of $x<0$ (Fig. 2b) and, finally a group of electrons detaches itself from the bulk and flies to the boundary of the plasma (Fig. 2c). The energy of the particles is of order $\sim eE_0L$, and the number of particles accelerated in a single burst of order $n_0 s_{\rm nonl}^{3/2} L$.

Similar particle bursts from the resonance region occur also in subsequent periods of the oscillations (Fig. 2d). The field at the resonance then ceases to increase. One may expect that regular electron bursts (through the period) will continue until there is an appreciable energy loss due to the departure of accelerated particles and the distortion of the plasma motion near the resonance by the reverse flow (after a time $\sim s_{\rm nonl}^{-1/2}\omega^{-1}$). The total energy of the particles accelerated up to that time is comparable to the energy contained in the resonance ($\sim s_{\rm nonl} eE_0 L^2 n_0$), and the total number of accelerated particles is of order $n_0 s_{\rm nonl} L$. If these particles leave the plasma, the field amplitude at the resonance reaches after a time $\sim \pi/\omega s_{\rm nonl}$ again values for which the self-intersection of trajectories starts and there appear a new series of bursts of accelerated particles of length $\sim 1/\omega s_{\rm nonl}^{1/2}$.

We note that the self-intersection of electron trajectories discussed here in the plasma resonance region in final reckoning occurs due to the non-uniformity of the plasma. Firstly, there is a separate region (near $x=0$) where the amplitude of the oscillations increases until it becomes comparable with the dimensions of the localization of the hf field. Secondly, the coordinate dependence of the frequency $\omega_p(x_e)$ of the eigenoscillations leads to an increase in the phase difference of the oscillations in different points and as a consequence to a decrease in the dimensions of the field non-uniformity:

$$\partial q/\partial t = -\partial\omega_p/\partial x_e;$$

here q is the local value of the wavenumber. Self-intersection of electron trajectories occurs when q^{-1} is comparable to the amplitude of the oscillations.

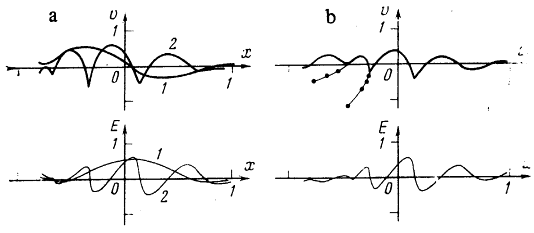

FIG. 3. Interaction of an hf field pulse with a non-uniform cold plasma: a) curves 1—$\omega t = 4\pi$, curves 2—$\omega t = 10\pi$; b) $\omega t = 12\pi$; $s_{\rm nonl} = 0.1$.

We consider now the problem of the effect of an hf field pulse of length t_0 on a non-uniform plasma with a linear density profile ($k=1$). Let $\omega^{-1} < t_0 < \tau$; τ is the time after which the resonance would break-down in the case of a continuous action of the hf field. After the field is switched on the absolute magnitude of the derivative $\sigma_e'(x_e)$ increases proportional to the time (although the amplitude remains constant), and after a time

$$t = \frac{2}{\omega r_0 t_0 \omega_p'(x_e)} + \frac{t_0}{2} \quad (34)$$

self-intersection of electron trajectories begins.

We simulated the behavior of the plasma after the trajectories intersect with themselves on a computer using the code, employed above. We depict in Fig. 3 the x-dependence of the electric field E and the phase diagram for the problem of the interaction of an hf field pulse with a non-uniform plasma for $s_{\rm nonl} = (r_0/L)^{1/2} = 0.1$, $t_0 = 4\pi/\omega$. It is clear that the electric field which is weakly non-uniform at time $t = t_0$ (Fig. 3a, curve 1) without changing its amplitude becomes appreciably more inhomogeneous (Fig. 3a, curve 2) and after that there appear regions of many-current flow (Fig. 3b). Bursts of accelerated particles appear at several places practically simultaneously.

4. We consider now the effect of anharmonicity on the phenomena at plasma resonance. The anharmonicity of the oscillations leads to the amplitude of the forced oscillations (and the field E) to turn out to be finite at the resonance point ($\omega_p = \omega$; see Fig. 4). In the region $\omega_p(x_e) > \omega_p(x_e^*)$ there are three solutions for the stationary amplitude. The solution of the initial problem is unique. For zero initial conditions $\sigma_e(t=0) = 0$, $\dot\sigma_e(t=0) = 0$ the solution shown by a heavy line in Fig. 4 is established.

We expand in Eq. (31) the expression for the electric field in powers of σ_e (see (13); we remember that $\xi_{ei} = \sigma_e$ in the case considered). Anharmonicity may play an important role if it becomes necessary to take it into account for amplitudes of the oscillations less than the width of the resonance region as for amplitudes comparable to the dimensions of the resonance the limitation of the field occurs due to the self-intersection of the trajectories. We restrict ourselves therefore in the expansion (13) merely to terms up to third order in σ_e. We get

$$\ddot\sigma_e + \omega_p^2(x_e)\sigma_e = -\alpha\sigma_e^2 - \beta\sigma_e^3 - \frac{eE_0}{m_e}\sin\omega t, \quad (35)$$

where

$$\alpha = \frac{2\pi e^2}{m_e}\frac{\partial n_i(x_e)}{\partial x_e} \qquad \beta = \frac{2\pi e^2}{3m_e}\frac{\partial^2 n_i(x_e)}{\partial x_e^2}.$$

The method for solving such equations is well known.[15] We find the magnitude of the amplitude of the oscillations at resonance σ_m from the equation

$$\sigma_e(\varepsilon - \varkappa\sigma_e^2) = eE_0/2m_e\omega_p(x_e),$$
$$\varkappa = (3\beta/8\omega_p - 5\alpha^2/12\omega_p^2), \qquad \varepsilon = \omega - \omega_p. \quad (36)$$

When $k=1$ the maximum value of the field E_m at the resonance as a result of the limitation due to anharmonic effects should be determined in the expression (15) for $n_i(x_e)$ by the ratio $E_0/(r_0/L)^{2/3}$, but this is preceded by self-intersection of the trajectories (for $E_m = E_0(r_0/L)^{1/2}$). This is connected with the vanishing of the cubic non-linearity for $k=1$ while the quadratic non-linearity does in first order in σ_e/L not contribute to the frequency shift (see also Ref. 5). If $k>1$ anharmonic effects lead to a limitation of the field for values determined by the parameter $s_A = (r_0/L)^{k/(k+1)}$ and the width of the resonance is of order $s^{1/k}L$, i.e., effects of anharmonicity and of the self-intersection of trajectories turn for $k>1$ to be of the same order of magnitude.

It is well known (see, e.g., Ref. 15) that non-linearity of oscillations not only changes resonance effects near $\omega_p = \omega$, but also leads to the appearance of new resonances. We restrict our study to resonances at frequencies $\omega_p(0) = \frac{1}{2}\omega$ and $\omega_p(0) = 2\omega$.

Let the plasma profile be given by Eq. (15). We consider first the multiple resonance $\omega_p(0) = 2\omega$. We find the function $\sigma_e(x_0)$ from the equation

$$\sigma_e(2\varepsilon - \varkappa\sigma_e^2) = 4\alpha r_0/9\omega^5. \quad (37)$$

Here $\varepsilon = \omega - \omega_p(x_e)/2$, and the other notation is as before. It is necessary to add to Eq. (37) the condition $|\sigma_e'| < 1$ which takes into account the effect of the self-intersection of the electron trajectories. Estimates for the maximum field amplitude and for the width of the resonance at the double frequency for $k>1$ are as follows:

$$E_m = E_0(r_0/L)^{(1-k)/(1+k)}, \qquad \Delta x = L(r_0/L)^{2/(k+2)}. \quad (38)$$

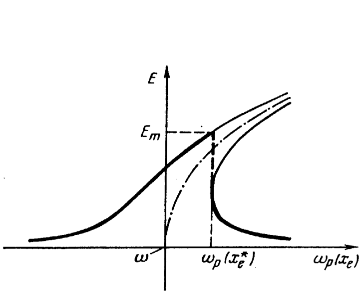

FIG. 4. Limitation of the field at resonance due to the anharmonicity of the oscillations.

If $k=1$ there is no field amplification. In that case there is self-intersection of electron trajectories when $E\sim E_0$ in the region of width r_0 near the point $\omega=\frac{1}{2}\omega_p$.

For resonance at half the frequency $\omega_p(0)=\frac{1}{2}\omega$ the possible values of the field amplitude and the resonance width can be found from the equation

$$\sigma_e^2(\varepsilon/2-\varkappa\sigma_e^2)=\alpha^2\sigma_0^2r_0^2/36\omega_p^2, \quad (39)$$

where $\varepsilon=\omega-2\omega_p(x_e)$. We get for the resonance width

$$\Delta x=kr_0/6. \quad (40)$$

The condition $|\sigma_e'|<1$ leads to the fact that for not too large k $E_m\sim E_0$, i.e., there is no field amplification and, hence, no resonance $\omega_p(0)=\frac{1}{2}\omega$.

5. Under actual experimental conditions the ion density is as a rule a function both of the spatial and of the time coordinate. Recently papers have appeared devoted to experimental studies and numerical simulation of plasma resonance under non-stationary conditions.[16,17,22] In this connection it is of interest to obtain analytical expressions and characteristic parameters for the field at resonance in a deformed plasma.

We consider firstly the cold plasma approximation ($T_e=T_i=0$). If we neglect, as before, the effect of the self-consistent electric field on the ion motion (zeroth approximation in m_e/m_i) we get for the ion and electron displacements in Lagrangian variables

$$\ddot{\sigma}_i(x_0,t)=0,\quad \ddot{\sigma}_e(x_0,t)=-\frac{e}{m}(E(x_0,t)+E_0\sin\omega t). \quad (41)$$

We give the initial conditions in the form

$$E(x_0,0)=0,\quad \sigma_e(x_0,0)=\sigma_i(x_0,0)=0,$$
$$\dot{\sigma}_e(x_0,0)=\dot{\sigma}_i(x_0,0)=V_0(x_0), \quad (42)$$

i.e., there are no currents and there is no charge separation at $t=0$.

We subtract the first of Eqs. (41) from the second and use the expansion (13) of the electric field E in powers of $\xi=\xi_{ei}$, assuming that $\xi\ll L$. In the approximation which is linear in ξ we retain, however, the electron non-linearities of the self-intersection of electron trajectories type, and we have

$$\ddot{\xi}+\omega_p^2(x_0,t)\xi=-\frac{eE_0}{m_e}\sin\omega t, \quad (43)$$

where

$$\omega_p^2(x_0,t)=\frac{4\pi e^2n_i(x_0,t)}{m_e}=\omega_p^2(x_0)N_i(x_0,t), \quad (44)$$

$$N_i(x_0,t)=1/(1+\sigma_i')=1\bigg/\left(1+\frac{\partial V_0}{\partial x_0}t\right). \quad (45)$$

We fix the value x_0. Let at some time $t_1(x_0)$ the plasma frequency $\omega_p(x_0,t)$ pass through the value which is equal to the frequency of the external field. In that volume element oscillations are then excited. If $\omega_p(x_0,t)$ is a slowly varying function of the time i.e., $2\pi\dot{\omega}_p/\omega_p^2\ll1$, we get by writing the time dependence of ω_p near t_1 as

$$\omega_p(x_0,t)=\omega+\dot{\omega}_p(x_0,t_1)(t-t_1), \quad (46)$$

near the resonance region ($|\omega_p(x_0,t)-\omega|\ll\omega$)

$$\xi(x_0,t)=\frac{r_0\omega}{2}\int_{-t_1}^{t-t_1}\cos\left(\omega t_1+\frac{\dot{\omega}_p}{2}[(t-t_1)^2-\tau^2]\right)d\tau. \quad (47)$$

The excitation of oscillations near $t=t_1$ occurs during a time $\Delta t\approx\dot{\omega}_p^{-1/2}$, and if $t_1\gg\Delta t$ the amplitude of the oscillations after the excitation is equal to

$$\xi_0=r_0\pi^{1/2}/s_V, \quad (48)$$

where

$$s_V=(2|\dot{\omega}_p|/\omega^2)^{1/2}|_{t=t_1}. \quad (49)$$

The further evolution of the oscillations is described by the solution of the homogeneous equation

$$\xi(x_0,t)=\frac{r_0\pi^{1/2}}{s_V(x_0)}\left(\frac{\omega}{\omega_p(x_0,t)}\right)^{1/2}\cos\left(\omega t_1(x_0)+\int_{t_1(x_0)}^{t}\omega_p(x_0,t')dt'\right), \quad (50)$$

i.e., the amplitude ξ_0 of the oscillations is proportional to $\omega_p^{-1/2}(x_0,t)$, and the electric field $E\propto\omega_p^{3/2}(x_0,t)$. The moment when self-intersection of electronic trajectories arises can be found from the condition[2)]

$$1+\partial\sigma_i/\partial x_0+\partial\xi/\partial x_0=0. \quad (51)$$

One verifies easily that the breakdown of the oscillations for $s_V\gg s_{\rm nonl}$ occurs for times which are larger than $1/s_V\omega$, i.e., outside the region of excitation.

For a non-stationary plasma with a non-vanishing temperature T_e we restrict ourselves to the linear approximation in the electric field amplitude. The ion density n_i will be assumed to be a given function of x and t. We denote the velocity with which the resonance point shifts relative to the ions by v_R:

$$v_R=(n_iv_i'/n_i')|_{\omega=\omega_p}. \quad (52)$$

We discuss the case when the resonance region does not lie close to the extrema of n_i and v_i, i.e., at the resonance $n_i'/n_i\sim1/L$, $v_i'\sim1/T$, where L and T are characteristic length and time scales for the change in the ion density which are much larger than the corresponding scales for the field E. To fix the ideas we put $v_{Te}^2\gg v_R^2$ and $n_i'>0$. The latter corresponds to an expansion of the plasma in the resonance region when $v_R>0$ and to a compression when $v_R<0$.

We change to a system of coordinates moving with the resonance point. Close to the resonance region we can write (for $\gamma=3$)

$$3v_{Te}^2E''+2v_R\dot{E}'-\ddot{E}-\omega^2(1+x/L)=\omega^2E_0\sin\omega t. \quad (53)$$

As v_R and L are slowly changing quantities we can as-

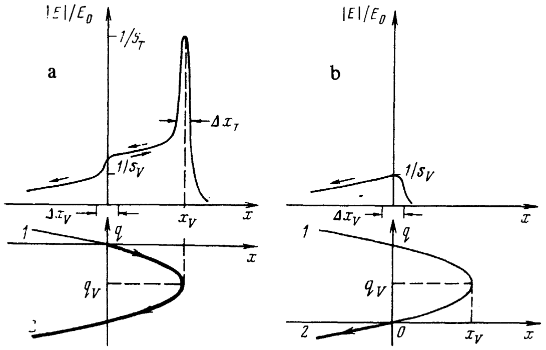

FIG. 5. x-dependence of the amplitude and of the local value of the wavenumber in the plasma resonance region: a) in contracting plasma; b) in an expanding plasma.

sume them to be constant when solving Eq. (53) (and depending on the time merely as a parameter). Writing the field in the form (19) we then get for the stationary complex amplitude

$$3v_{Te}^2E''-2i\omega v_RE'-(\omega_p^2-\omega^2)E=E_0\omega^2. \tag{54}$$

Here $\omega_p^2=\omega^2(1+x/L)$.

If the quantity

$$s_V=\left|\frac{v_R}{\omega L}\right|^{1/2}=\left|\frac{v_i'}{\omega}\right|^{1/2}=\left|\frac{2\dot\omega_F}{\omega^2}\right|^{1/2} \tag{55}$$

is much smaller than $s_T=(r_{De}/L)^{2/3}$ we can neglect the velocity v_R and we get the solution of the linear theory with fixed ions.[3] We consider the opposite case $s_V > s_T$.[3)]

In the region $|x| \ll Lv_R^2/v_{Te}^2$ the solution has the form

$$E=\begin{cases}(E_0/s_V)f(x/s_VL), & v_R<0\\ -(E_0/s_V)f^*(-x/s_VL). & v_R>0\end{cases}, \tag{56}$$

where

$$f(z)=ie^{iz^2}\int_{-\infty}^{z/2}e^{-iy^2}\,dy. \tag{57}$$

The width of the region of excitation is of order s_VL and the field amplitude is of order E_0/s_V. When $v_R<0$ to the right of the excitation region (with $v_R>0$ to the left) the field amplitude is equal to

$$E_m=\pi^{1/2}E_0/s_V. \tag{58}$$

Outside the excitation region the propagation of the oscillations is described by the solution of the homogeneous equation. We write the complex amplitude of the field in the form

$$E=u\exp\left(i\int^x q\,dx\right), \tag{59}$$

where $u(x)$ is a real amplitude and $q(x)$ the local value of the wavenumber. Putting $u'' \ll q^2u$, we get

$$q_{1,2}=q_V(1\mp(1-x/x_V)^{1/2}), \tag{60}$$

where

$$q_V=v_R\omega/3v_{Te}^2,\quad x_V=Lv_R^2/3v_{Te}^2. \tag{61}$$

The group velocity of the propagation of the oscillations is equal to[4)]

$$v_{gr}=\partial\omega/\partial q=-v_R+3qv_{Te}^2/\omega=3(q-q_V)v_{Te}^2/\omega. \tag{62}$$

The x-dependence of $|E|$ and q is shown in Fig. 5. The heavy line shows the propagation of the oscillations after they are excited near $x=0$, and the arrows indicate the direction of the group velocity.

We have everywhere, except in the vicinity of the points $x=0$, $x=x_V$, for the amplitude $u(x)$

$$u(x)=\frac{\pi^{1/2}E_0}{s_V}\left|\frac{q_V}{q-q_V}\right|^{1/2}. \tag{63}$$

When $v_R<0$ this expression refers to any of the two branches of oscillations (see (60) and Fig. 5a).

If $v_R<0$ the wave undergoes reflection near x_V (Fig. 5). In that region the solution of the homogeneous equation can, apart from a phase constant, be expressed in terms of the Airy function Ai(z):

$$E=\frac{2\pi E_0}{s_T}\mathrm{Ai}\left(\frac{x-x_V}{s_TL}\right)e^{iq_Vx}. \tag{64}$$

We have thus for $v_R<0$ in the region of the reflection point x_V a resonance amplification of the field (see Fig. 5a):

$$E_m\sim E_0/s_T, \tag{65}$$

as for $v_R=0$ and with the same resonance width $\Delta x \sim s_TL$. The velocity v_R leads in this case only to the fact that the excitation points and the points of the resonance amplification turn out to be spatially separated.

If $v_R>0$, the wave propagates immediately after its excitation (Fig. 5b) into the region of lower density and the group velocity increases. Near the excitation point the amplitude is then a maximum and determined by Eq. (58). The width of the region in which the amplitude is comparable to the maximum is of the order x_V, i.e., much larger than the width of the excitation region.

Above we considered potential oscillations near the plasma resonance in the zeroth approximation in m_e/m_i, i.e., we neglected the effect of the self-consistent field on the ion motion. However, it is well known that under certain conditions the effect of striction forces may have to be taken into account.[8–10]

We write down the equation for the electrons and ions in the Lagrangian form, restricting ourselves to the quadratic terms in the expansion (13) of the field E in powers of ξ and assume that

$$(\xi+r_{De})(\xi'/\xi+\sigma_i'/\sigma_i)\ll 1,\quad (m_e/m_i)^{1/2}\xi\ll r_{De}.$$

Separating the fast-oscillating part (with frequency $\sim \omega_p$) of the electron displacement $\tilde{\xi}$ from the slowly varying part, we get for $\tilde{\xi}$ and σ_i (for $\gamma = 1$)

$$\frac{\partial^2 \tilde{\xi}}{\partial t^2} + \omega_p^2 \tilde{\xi} = v_{Te}^2 \frac{\partial}{\partial \rho}\left(n_i^2 \frac{\partial}{\partial \rho}\tilde{\xi}\right) - \frac{eE_0}{m_e}\sin\omega t, \qquad (66)$$

$$\frac{\partial^2 \sigma_i}{\partial t^2} = -\frac{m_e}{m_i}\frac{\partial}{\partial \rho}\left(n_i\left(v_{Te}^2 + \omega_p^2 \bar{\tilde{\xi}}^2/2\right)\right), \qquad (67)$$

where $\partial/\partial\rho = n_{i0}^{-1}\partial/\partial x_0$, the horizontal bar indicates time averaging, and $n_i = n_{i0}/(1+\sigma_i')$.

If we assume that $\sigma_i' = (n_{i0} - n_i)/n_i \ll 1$ and $\omega_p \tilde{\xi} \ll v_{Te}$, these equations go over into the well known equations for the high-frequency potential and the amplitude variation in the ion density.[18]

We consider now qualitatively the consequences of taking into account the displacement of the ions under the action of striction forces over the time of the establishment of the resonance $\tau \sim \pi/\omega s$. We discuss two characteristic cases: the subsonic (stationary) regime when we can neglect in Eq. (67) the time derivative, and the supersonic regime in which case the term with the thermal pressure is small in (67). The relative change in the ion density in the resonance region turns out to be unimportant if it is less than the characteristic parameter $s = E_0/E_m$ which determines the structure of the field in the resonance. In the quasi-stationary regime ($(m_e/m_i)^{1/2} v_{Te} \gg s^2 \omega L$) we can neglect the change in the plasma density under the action of striction forces, provided (see Refs. 7, 8)

$$s > s_i = (r_0/r_{De})^{1/2}. \qquad (68)$$

One sees easily that in the case when the field at the resonance is limited by the linear transformation into Langmuir waves ($s = (r_{De}/L)^{2/3}$) inequality (68) corresponds to the condition under which there is no modulational instability: $q^{-1} \sim \Delta x \sim sL$.

In the supersonic regime ($(m_e/m_i)^{1/2} v_{Te} < s^2 \omega L$) the displacement of the ions due to the striction forces does not appear during the time when the field increases at the resonance, provided

$$m_e/m_i < 4s^7/s_{\rm nonl}^4,\quad s_{\rm nonl} = (r_0/L)^{1/4}. \qquad (69)$$

For a cold plasma when $s = s_{\rm nonl}$ one can write this condition as

$$s_{\rm nonl} > (m_e/4m_i)^{1/3}. \qquad (70)$$

Satisfying inequality (70) guarantees the relative smallness of the growth rate of the parametric instability $\gamma \sim \omega_p (m_e/m_i)^{1/3}$,[20] i.e., during the characteristic time for establishing the resonance the parametric instability does not succeed in developing.

Inequalities (68), (70) indicate the limits of the applicability of the results obtained above. In other words, these results are valid for a plasma resonance either in weak external fields or if the pumping field is sufficiently large. However, in the latter case, only during a limited period $t < \gamma^{-1}$. Nonetheless during that period such an interesting effect as a burst of accelerated particles with energies of the order of eE_0L can develop in the plasma resonance region.

6. The results obtained in this paper give a picture of the role of various effects at plasma resonance: linear transformation for various plasma density profiles, self-intersection of electron trajectories, anharmonicity of the non-linear oscillations, and non-stationarity of the ion density. These effects do not, of course, exhaust all possibilities for limiting the hf field in the plasma resonance region. For instance, in Ref. 7 we considered the influence of relativistic effects and in Ref. 21 the dissipation of the oscillations due to the development of the modulational instability was considered. We note that taking the effect of the modulational instability into account as well as other non-linear effects connected with the ion motion on the phenomena in the plasma resonance region is a complicated problem which is very far from having been completed. Numerical experiments[22] show that the break down of the plasma density profile occurring due to strictional forces may lead to the occurrence of modulational and ion-acoustic decay instabilities which indicates the necessity for observing well-known caution in transferring results obtained for a uniform plasma to the plasma resonance region in a non-uniform plasma.

We give some numerical estimates. Let $\omega = 10^{10}$ s^{-1}, $E_0 = 1$ kV/cm, $L = 2$ cm, $T_e = 1$ eV. For those parameters $s_{\rm nonl} = 0.1$; $s_T = 3\times10^{-2}$, $(m_e/4m_i)^{1/3} \approx 0.05$ (hydrogen plasma); $v_{Te}(m_e/m_i)^{1/2} < s^2\omega L$ (supersonic regime). The maximum field amplitude at resonance is determined by the self-intersection of the electron trajectories.

If $\omega = 2\times10^{10}$ s^{-1}, $E_0 = 10$ V/cm, $L = 5$ cm, $T_e = 1$ eV, we have $s_{\rm nonl} \approx 3\times10^{-3}$, $s_T = 10^{-2}$, $s_i = 5\times10^{-2}$ and the action of the striction forces is the determining factor.

For the given values of the plasma parameters s_ν (i.e., the effect of collisions) is negligibly small.

1) A similar study of the break down of the waves and of the acceleration of particles has been given earlier in Refs. 14, 23. To the same extent in which the statement of the problem was the same as that given here the results turn out to be identical.

2) As $\partial\xi/\partial x_0$ is a fast changing function of the time, it follows from condition (51) that the break down of the electron oscillations precedes the occurrence of the self-intersection of ion trajectories ($1+\sigma_i'=0$).

3) Together with the condition $v_{Te}^2 > v_R^2$ this gives $s_V^3 > s_T^3 > s_V^4$.

4) As the propagation of the oscillations from the point $x = 0$ along the region $x/L \ll 1$ with the group velocity (62) occurs during a time much smaller than T, the stationary state of the field will, in the region considered, be established much faster than the gradients of n_i and v_i will be able to change appreciably, i.e., one can indeed assume that in Eq. (53) v_R and L depend on the time as on a parameter.

1V. I. Barinov, I. R. Gekker, K. F. Sergeĭchev, O. V. Sizukhin, and V. S. Trofimov, Proc. Lebedev Inst. 73, 37 (1973) [Translation published by Consultants Bureau].

2G. M. Batanov, K. A. Sarksian, and V. A. Silin, 9th Internat. Conf. on Phenomena in Ionized Gases, 1969, Bucharest, Contributed Papers, p. 541 (1969).

[3]V. L. Ginzburg, Rasprostranenie elektromagnitnykh voln v plazme (Propagation of electromagnetic waves in a plasma) Nauka, 1967 [translation published by Pergamon Press].
[4]N. G. Denisov, Zh. Eksp. Teor. Fiz. **31**, 609 (1956) [Sov. Phys. JETP **4**, 544 (1957)].
[5]N. S. Erokhin, V. E. Zakharov, and S. S. Moiseev, Zh. Eksp. Teor. Fiz. 56, 179 (1969) [Sov. Phys. JETP **29**, 101 (1969)].
[6]P. Koch and J. Albriton, Phys. Rev. Lett. **32**, 1420 (1974).
[7]S. V. Bulanov and L. M. Kovrizhnykh, Fiz. Plazmy 2, 105 (1976) [Sov. J. Plasma Phys. 2, 58 (1976)].
[8]V. B. Gil'denburg, Zh. Eksp. Teor. Fiz. **46**, 2156 (1964) [Sov. Phys. JETP 19, 1456 (1964)].
[9]G. J. Morales and J. C. Lee, Phys. Rev. Lett. **33**, 1016 (1974).
[10]V. B. Gil'denburg and G. M. Fraĭman, Zh. Eksp. Teor. Fiz. **69**, 1601 (1975) [Sov. Phys. JETP 42, 816 (1976)].
[11]R. L. Stenzel, A. Y. Wong. and U. C. Kim, Phys. Rev. Lett. 32, 654 (1974).
[12]I. G. Kondrat'ev and M. A. Miller, Izv. Vyssh. Uchebn. Zaved. Radiofiz. **6**, 885 (1968).
[13]J. Dawson, in Vychislitel'nye metody v fizike plazmy (Computational methods in plasma physics) Mir, 1974, p. 11.
[14]J. Albriton and P. Koch, Phys. Fluids **18**, 1136 (1975).
[15]L. D. Landau and E. M. Lifshitz, Mekhanika (Mechanics) Nauka, 1965 [translation published by Pergamon Press].
[16]N. Ikezi, K. Mima, K. Nishikawa, and M. Inutake, Phys. Rev. Lett. 36, 794 (1976).
[17]D. W. Forslund, J. M, Kindel, K. Lee, and E. L. Lindeman, Phys. Rev. Lett. **36**, 35 (1976).
[18]V. E. Zakharov, Zh. Eksp. Teor. Fiz. 62, 1745 (1972) [Sov. Phys. JETP 35, 908 (1972)].
[19]A. A. Vedenov and L. I. Rudakov, Dokl. Akad. Nauk SSSR 159, 767 (1964) [Sov. Phys. Dokl. **9**, 1073 (1965)]
[20]V. P. Silin, Parametricheskoe vozdeĭstvie izlucheniya bol'shoĭ moshchnocti na plazmy (Parametric action of intense radiation on a plasma) Nauka, 1973.
[21]A. A. Galeev, R. Z. Sagdeev, V. D. Shapiro, and V. I. Shevchenko, Pis'ma Sh. Eksp. Teor. Fiz. 21, 539 (1975) [JETP Lett. 21, 251 (1975)].
[22]K. G. Estabrook, E. J. Valeo, and W. L. Kruer, Phys. Fluids, **18**, 1151 (1975).
[23]J. P. Freidberg, R. W. Mitchell, R. L. Morse, and L. I. Rudsinski, Phys. Rev. Lett. **28**, 795 (1972).

Translated by D. ter Haar

Bremsstrahlung of relativistic electrons in a plasma in a strong magnetic field

A. V. Akopyan and V. N. Tsytovich

Institute of Radiophysics and Electronics, Armenian Academy of Sciences
(Submitted October 19, 1976)
Zh. Eksp. Teor. Fiz. **72**, 1824–1832 (May 1977)

The bremsstrahlung of relativistic electrons in a plasma in a strong magnetic field is considered, with effects of dynamic screening and of a new mechanism of transition bremsstrahlung [A. V. Akopyan and V. N. Tsytovich, Zh. Eksp. Teor. Fiz. **71**, 166 (1976) [Sov. Phys. JETP **44**, 87 (1977)] taken into account. It is shown that in a strong magnetic field the effects of dynamic screening strongly diminish the intensity of ordinary bremsstrahlung at high frequencies $\omega < \omega_{max}$; here $\omega_{max} = 2\delta^2\omega^*$, where $\omega^* = \omega_{pe}c/v_{Te}$, ω_{pe} is the electronic plasma frequency, $\delta/ = \epsilon/m_e c2$, ϵ is the energy of the relativistic electron, and v_{Te} is the mean thermal velocity of the plasma electrons ($v_{Te} \ll c$). The angular distribution of the bremsstrahlung spectral intensity and the total intensity are studied in detail, both for the usual bremsstrahlung mechanism and for transition bremsstrahlung.

PACS numbers: 52.25.Ps

1. INTRODUCTION: GENERAL STATEMENT OF THE PROBLEM

Bremsstrahlung in a plasma which is in a very strong magnetic field has recently been intensively studied in connection with the problem of interpreting the radiation of pulsars.[1–6] Actually, as will be seen from what follows, bremsstrahlung is altered by a magnetic field even at relatively small field strengths (but for sufficiently small frequencies). Therefore bremsstrahlung in a magnetic field is also of immediate interest for laboratory experiments on the magnetic containment of a plasma.

In the papers already referred to,[1–6] and also in[7,8], the influence of a magnetic field on bremsstrahlung was not analyzed completely, attention being given mainly to the case of nonrelativistic particles in a quantizing magnetic field. At the same time, as we shall show, the effect of a plasma on bremsstrahlung is more important in a strong magnetic field than with no field. The results we shall give properly apply not only to plasmas, but to other media as well, namely in all cases in which the plasma approximation can be used for the dielectric constant.

In constructing a more or less complete theory of bremsstrahlung in a nonequilibrium magnetoactive plasma it is necessary to take into account at least four effects: 1) screening by the plasma of the fields of colliding particles, 2) effects of the plasma on the propa-

 0038-5646/77/4505-0957$02.40 © 1978 American Institute of Physics