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Phase diagram of a ferromagnetic plate in an external magnetic field

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The phase diagram, in coordinates H and l, is investigated for a ferromagnetic plate located in an external magnetic field parallel to the surface of the plate. It is shown that in the high-anisotropy case, $\beta > 4\pi$, the phase transition from the uniform state to a domain state on change of the plate thickness occurs as a second-order phase transition when $H > (\beta - 4\pi)M_0$, and as a first-order transition when $H < (\beta - 4\pi)M_0$. The phase diagram of a plate with large anisotropy, $\beta > 4\pi$, is substantially more complicated than in the low-anisotropy case investigated earlier, and it has a number of peculiarities. In particular, an equilibrium domain structure exists for an arbitrarily small plate thickness.

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INTRODUCTION

A quantitative theory of the domain structure of ferromagnets was first developed by Landau and Lifshitz[1] (see also[2]). The domain structure of a ferromagnetic plate was investigated in the case in which the thickness of the plate greatly exceeds the thickness of the domain boundary. A criterion was also formulated for the single-domain state of a ferromagnet: a specimen of ferromagnetic material should be in the uniform state if $L \ll \alpha^{1/2}$, where L is a characteristic dimension of the specimen and α is an exchange constant. Later, domain structures of ferromagnets were extensively studied experimentally; this included intensive study of the properties of thin ferromagnetic films (those with parameters close to the single-domain criterion). For films with the axis of easiest magnetization perpendicular to the surface, the so-called stripe domain structure was discovered experimentally (see, for example, [3]), and also a transition from the uniform state of a film to a state with a stripe domain structure upon change of the plate thickness l (or of the external magnetic field H applied in the plane of the film). The experimental discovery of the transition from the uniform state of a film to a domain state led to a number of theoretical studies.[4,5] It was shown[4,5] that in ferromagnetic films with small anisotropy ($\beta < 4\pi$, where β is an anisotropy constant), the state with a stripe domain structure occurs in consequence of an instability of the uniform state of the film, in which the magnetization lies in the plane of the film, with respect to small inhomogenous perturbations. A phase diagram, in coordinates H and l, was given for a ferromagnetic film with small anisotropy, $\beta < 4\pi$.[5] The stability regions of the uniform phase $\Phi_{\parallel}$ and the nonuniform (domain) phase Φ_D were separated by a line of second-order phase transitions $H = H_c(l)$.

The form of the domain structure in the vicinity of the curve $H_c(l)$ has been investigated.[6] The variation of the parameters of the magnetization distribution (amplitude, period, etc.) was found, and also an analytical expression for $H_c(l)$ was obtained for large plate thicknesses:

$$\frac{H_c(l)}{M_0} = \beta - \frac{4\pi^{3/2}}{l}\left(\frac{\alpha\beta}{\beta+4\pi}\right)^{1/2}. \tag{1}$$

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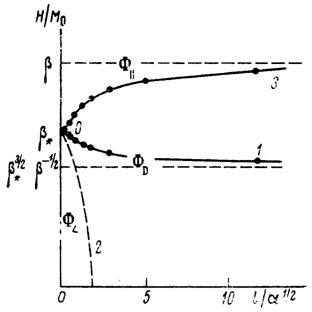

FIG. 1. Phase diagram of a ferromagnetic plate with large anisotropy ($\beta > 4\pi$) (schematic). 1, instability line of the phase $\Phi_<$, $H_1(l)$. 2, line of first-order phase transitions $\Phi_< \rightleftarrows \Phi_D$, $H_{tr}(l)$. 3, line of second-order transitions $\Phi_\parallel \rightleftarrows \Phi_D$, $H_c(l)$. The points show values of $H_1(l)$ obtained by numerical methods when $l \sim \alpha^{1/2}$.

It was shown that for large plate thicknesses, the transition $\Phi_\parallel \rightleftarrows \Phi_D$ occurs for the case of large anisotropy, $\beta > 4\pi$, as for the case $\beta < 4\pi$, as a second-order transition. It can be shown, however, that when $H = 0$ the uniform state of a film with $\beta > 4\pi$ is stable with respect to small perturbations for arbitrary plate thicknesses (see[7], § 11.5). From this fact it is clear that when ever $H = 0$, the transition from the uniform state to a domain state cannot be a second-order phase transition.

We have investigated the states of a ferromagnetic plate with large anisotropy, in an external magnetic field parallel to the surface of the plate. We have constructed, in the (H, l) plane, a phase diagram that differs significantly from the phase diagram for a plate with small anisotropy (see Fig. 1). We have shown that when $H < (\beta - 4\pi)M_0$, the transition from the uniform state of the plate to a state with a domain structure occurs by a first-order phase transition. It turned out that when $\beta \gg 4\pi$, a thermodynamic-equilibrium domain structure exists at arbitrarily small plate thickness. We have shown that for an infinite plane-parallel plate of thickness l, the width of the interval of magnetic-field values within which there is an equilibrium domain structure approaches zero as $l \to 0$.

§1. UNIFORM PHASES OF A FERROMAGNETIC PLATE, AND THEIR STABILITY

We write the simplest expression for the thermodynamic potential of a uniaxial plane-parallel ferromagnetic plate in an external magnetic field:

$$W = \int dV \left\{ \frac{1}{2} \alpha \left(\frac{\partial M_i}{\partial x_k} \right)^2 - \frac{1}{2} \beta (\mathbf{M}\mathbf{n})^2 - \mathbf{H}\mathbf{M} - \frac{1}{2} \mathbf{H}_m \mathbf{M} \right\}, \qquad (2)$$

where α is the exchange constant, β is the anisotropy constant, $\mathbf{n}$ is a unit vector along the axis of anisotropy (the z axis), $\mathbf{H}$ is the external magnetic field, and $\mathbf{H}_m$ is the demagnetizing field. We shall suppose that the anisotropy axis is perpendicular to the surface of the plate and that the external magnetic field is applied in the plane of the plate, in the direction of the y axis.

The demagnetizing field $\mathbf{H}_m$ is determined by the equations of magnetostatics

$$\operatorname{rot} \mathbf{H}_m = 0, \quad \operatorname{div} \mathbf{H}_m = -4\pi \operatorname{div} \mathbf{M} \qquad (3)$$

with the usual boundary conditions at the plate boundary

$$\mathbf{H}^i_{m,t} = \mathbf{H}^e_{m,t}, \quad H^e_{m,n} = (H_m{}^i + 4\pi M)_n, \qquad (4)$$

where the indices i and e denote the fields inside and outside the plate, and the indices t and n denote the tangential and normal components of the vectors.

In order to investigate the phase diagram of the plate, it is necessary to find and to investigate with respect to stability the static solution of the Landau-Lifshitz equation

$$[\mathbf{M} \times \mathbf{H}_{\text{eff}}] = 0, \quad \mathbf{H}_{\text{eff}} = -\frac{\delta W}{\delta \mathbf{M}}, \qquad (5)$$

taking into account the usual conditions on the plate boundary ($z = \pm l/2$)

$$\frac{\partial \mathbf{M}}{\partial x_n} = 0 \qquad (6)$$

together with the magnetostatic equations (3) and (4).

Equations (3)–(6) have both uniform static solutions and also nonuniform static solutions, corresponding to a domain structure. In order to construct the phase diagram, we shall begin with investigation of the uniform states of the plate. Here $\mathbf{H}_m = -4\pi \mathbf{n}(\mathbf{n}M)$, and it is easily shown that, depending on the value of the external field, two uniform phases of the plate are possible for $\beta > 4\pi$: besides the phase $\Phi_\parallel$, in which $\mathbf{M} \parallel \mathbf{H}$ and which exists also for $\beta < 4\pi$, there exists in the case of large anisotropy ($\beta > 4\pi$) a phase $\Phi_<$ determined by the following relations:

$$\mathbf{M} = M_0(\mathbf{e}_y \cos\theta_0 + \mathbf{n} \sin\theta_0), \quad \cos\theta_0 = H/\beta_* M_0, \quad \beta_* = \beta - 4\pi. \qquad (7)$$

The phase $\Phi_<$ is stable when $H < \beta_* M_0$. When $H = \beta_* M_0$, there should occur a phase transition of the second kind $\Phi_\parallel \rightleftarrows \Phi_<$.

It is clear that with increase of the plate thickness, a nonuniform (domain) phase Φ_D can become energetically advantageous. In order to construct the phase diagram with allowance for nonuniform states, it is necessary to investigate the stability of the phases $\Phi_\parallel$ and $\Phi_<$ with respect to nonuniform perturbations. The stability of the phase $\Phi_\parallel$ has been investigated in a number of papers[4–6] (in particular, in[4,5] a value of the instability field of the phase $\Phi_\parallel$ was found by use of numerical methods). It was shown[6] that the transition from the uniform phase to the domain phase when $H > \beta_* M_0$, i.e. the transition $\Phi_\parallel \rightleftarrows \Phi_D$, occurs at $H = H_c(l)$ as a second-order phase transition, and the analytical expression (1) was obtained for $H_c(l)$ when $l \gg \alpha^{1/2}$. As we shall show below,

$$H_c(l) \to \beta_* M_0 + \pi^2 l^2 M_0/\alpha \quad \text{as} \quad l \to 0,$$

that is, when $H \to \beta_* M_0$ the value of the plate thickness at which the transition $\Phi_\parallel \rightleftarrows \Phi_D$ occurs approaches zero (see Fig. 1).

We shall investigate the stability of the uniform phase of the plate when $H < \beta_* M_0$, i.e. the stability of the phase $\Phi_<$; for this purpose, we shall find the spectrum of small oscillations of the magnetization in the plate by use of the dynamic Landau-Lifshitz equation

$$\frac{\partial \mathbf{M}}{\partial t} = g[\mathbf{M} \times \mathbf{H}_{\text{eff}}],$$

taking into account formulas (3)–(6) (g is the gyromagnetic ratio).

Taking into account that the magnetization of unit volume is conserved, we write

$$M_x = M_0 \sin\varphi, \quad M_y = M_0 \cos\theta \cos\varphi, \quad M_z = M_0 \sin\theta \cos\varphi,$$

where $\theta = \theta_0 + \vartheta$, θ_0 is the equilibrium angle between the magnetization and the external field in the phase $\Phi_<$ $\times(\cos\theta_0 = H/\beta\ M_0)$, and ϑ and $\phi \ll 1$. We represent the demagnetizing field $\mathbf{H}_m$ in the form

$$\mathbf{H}_m = \mathbf{H}_m{}^0 + M_0 \mathbf{h}_m,$$

where $\mathbf{H}_m^0$ is the value of the demagnetizing field in equilibrium,

$$\mathbf{H}_m{}^0 = -4\pi M_0 \sin\theta_0 \mathbf{n}.$$

It can be shown that instability of phase $\Phi_<$ is produced by oscillations of the magnetization propagated along the x axis

$$(\vartheta = \vartheta(x, z, t), \quad \varphi = \varphi(x, z, t), \quad \mathbf{h}_m = \mathbf{h}_m(x, z, t)).$$

The nonvanishing components of the magnetic field $\mathbf{h}_m$ are then those along the x and z axes. By use of the Landau-Lifshitz equations, is it easy to obtain the relations between the oscillations of the magnetization and of the magnetic field,

$$h_{mz} = -\left[\frac{\alpha\Delta}{\cos\theta_0} + \xi\right]\vartheta + \frac{1}{gM_0\cos\theta_0}\frac{\partial\varphi}{\partial t},$$
$$h_{mx} = \left[\frac{h}{\cos\theta_0} - \alpha\Delta\right]\varphi - \frac{1}{gM_0}\frac{\partial\vartheta}{\partial t}, \tag{8}$$

where $h = H/M_0$, Δ is the Laplacian operator, and $\xi = \beta\cos\theta_0 - h/\cos^2\theta_0$.

In order to find the spectrum of small oscillations of the magnetization, it is necessary to solve (3) and (8) inside the plate and the magnetostatic equations (3) outside the plate, and to take into account the conditions (4) on the boundary of the plate ($z = \pm l/2$). This is a very unwieldy problem, and the resulting dispersion equation for the oscillation frequencies can be investigated only numerically. But this problem simplifies significantly for the case of large plate thickness, and when $l \gg \alpha^{1/2}$ the investigation of the stability of the phase $\Phi_<$ can be carried out analytically. We shall suppose that $l \gg \alpha^{1/2}$ (we shall return in the last subsection of this section to the important case of small thicknesses, $l \lesssim \alpha^{1/2}$).

1. *Stability of* $\Phi_<$. *Large thicknesses*. As we shall show below, when $l \gg \alpha^{1/2}$, the field $H_1(l)$ at which the phase $\Phi_<$ becomes unstable is close to $M_0\beta_*^{3/2}\beta^{-1/2}$; that is, $\xi \ll 1$. It is evident from equations (8) that $\alpha\Delta\vartheta \sim \xi\vartheta$. Furthermore, as we shall show below during analysis of the boundary conditions (4), the instability of the phase phase $\Phi_<$ is determined by oscillations of the magnetization for which

$$\alpha\frac{\partial^2\vartheta}{\partial z^2} \sim \alpha\xi\frac{\partial^2\vartheta}{\partial x^2} \sim \xi^2\vartheta. \tag{9}$$

By substituting (8) in the magnetostatic equations (3) and taking account of (9), one can easily show that the magnetization oscillations satisfy the following equation:

$$\left[-(gM_0)^{-2}\frac{\partial^2}{\partial t^2} + \xi\beta\cos\theta_0 + \alpha\beta\frac{\partial^2}{\partial x^2}\right]\frac{\partial^2\vartheta}{\partial x^2} = 4\pi h\cos\theta_0\frac{\partial^2\vartheta}{\partial z^2}. \tag{10}$$

Similar equations are obtained also for the components of the magnetic field $\mathbf{h}_m$ inside the plate.

We write the solutions of (3) and (10) in the form

$$\vartheta(x, z, t) = (a\cos qz + b\sin qz)e^{i(kx - \Omega t)},$$
$$\mathbf{h}_m{}^i = -\nabla\Psi^i. \tag{11}$$

By using (3), we find

$$\Psi^i = -\left[\frac{\omega^2}{h} + \xi - \frac{\alpha k^2}{\cos\theta_0}\right]\left[\left(bq + \frac{\omega k}{h}a\right)\cos qz + \left(aq - \frac{\omega k}{h}b\right)\sin qz\right]\left[q^2 + \left(\frac{\omega k}{h}\right)^2\right]^{-1} e^{i(kx - \Omega t)}.$$

The relation between the constants a and b is determined by the boundary conditions (4). In formulas (11), k is the wave vector of the oscillation, $\Omega = \Omega(k)$ is the frequency of the magnetization oscillations,

$$\omega^2 = \frac{\Omega^2}{(gM_0)^2} = 4\pi h\cos\theta_0\frac{q^2}{k^2} + \beta(\alpha k^2 - \xi\cos\theta_0), \tag{12}$$

the value of the parameter q for a solution with a prescribed wave vector k and frequency Ω is determined by the relation (12).

Outside the plate, the potential Ψ^e of the magnetic field satisfies the homogeneous Laplace equation:

$$\Psi^e = e^{i(kx - \Omega t)}\begin{cases}\Psi^+ e^{-kz}, & z \geqslant l/2,\\ \Psi^- e^{kz}, & z \leqslant -l/2.\end{cases} \tag{13}$$

By use of the boundary conditions (4), it is easy to obtain the equation that determines the dependence of the frequency of the magnetization oscillations on the wave vector k,

$$\left[1 + \left(\frac{\omega k}{qh}\right)^2\right]\operatorname{tg} ql = \frac{k}{q}\frac{\alpha k^2 - \xi\cos\theta_0 - \omega^2/\beta}{2\pi\cos^2\theta_0}. \tag{14}$$

In writing the dispersion equation (14), we have used the inequality $\xi \ll 1$ and the conditions (9).

By use of (9) it is easily seen that $\operatorname{tg} ql \sim \xi^{1/2} \ll 1$; that is,

$$q = (\pi n/l)(1 - 2\beta_*/\beta kl), \tag{15}$$

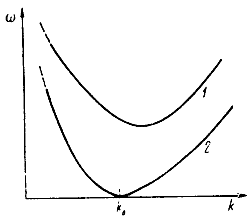

FIG. 2. Dependence of the frequency of small magnetization oscillations on their wave vector k, for different plate thicknesses: 1) $l<l_1(H)$; 2) $l=l_1(H)$.

and the expression for the frequency of the magnetization oscillations takes the form

$$\omega_n^2=\frac{4\pi^{3/2}h\cos\theta_0}{(kl)^2}n^2+\beta(\alpha k^2-\xi\cos\theta_0). \tag{16}$$

Formula (16) is correct only when $kl\gg n$, since in its derivation we have used the inequality $k\gg q$ (see (9)).

It is evident that the value of $\omega_n^2(k)$ as a function of the wave vector k reaches a minimum at $k=k_0$:

$$k_0=\left(\frac{4\pi^3h\cos\theta_0}{\alpha\beta l^2}\right)^{1/4}n^{1/2}, \tag{17}$$

and the minimum value of the square of the frequency is determined by the expression

$$\omega_{n0}^2=\omega_n^2(k_0)=2nl^{-1}(4\pi^3h\alpha\beta\cos\theta_0)^{1/2}-\xi\beta\cos\theta_0. \tag{18}$$

From formula (18) it is easily seen that at a definite value of the plate thickness, ω_0^2 may change sign (Fig. 2); that is, the uniform state $\Phi_<$ of the plate becomes unstable with respect to inhomogeneous perturbations with wave vector $k=k_0$; this must be interpreted as instability with respect to formation of a domain structure. The smallest critical thickness l_1 of the plate corresponds to $n=1$.

Thus the uniform state becomes unstable at $l>l_1(h)$,

$$l_1(h)=\frac{4\pi^{3/2}}{\xi}\left(\frac{\alpha h}{\beta\cos\theta_0}\right)^{1/2}, \tag{19}$$

or at

$$h>h_1(l)=\frac{\beta_*^{3/2}}{\beta^{1/2}}+\frac{2\alpha^{1/2}}{l}\left(\frac{\pi\beta_*}{\beta}\right)^{3/2}.$$

It must be mentioned that the value of the critical thickness $l_1(h)$ can also be found by analysis of the static solutions of the Landau-Lifshitz equations, i.e., with $\Omega=0$. Then $l_1(h)$ is defined as that value of the plate thickness for which the system (14) and (12) can first be solved when $\Omega=0$. It is easy to show that these equations have a solution when $l\geq l_1(h)$, where $l_1(h)$ is defined by formula (19).

2. *Stability of* $\Phi_<$. *Arbitrary thicknesses.* Despite the fact that investigation of the instability curve of the phase $\Phi_<$ at an arbitrary value of the plate thickness l requires use of numerical methods (an analytical expression for $H_1(l)$ can be obtained when $\beta\gg 4\pi$ and $l\to 0$), it is possible to obtain exact relations between the values of the instability fields $H_c(l)$ and $H_1(l)$ of the phases $\Phi_\parallel$ and $\Phi_<$.

As has already been mentioned, investigation of the stability of a phase can be carried out on the basis of the static Landau-Lifshitz equations (3) and (5). We shall seek a solution of (3) and (5) in the form

$$\mathbf{m}=\mathbf{m}(z)e^{ikx},\quad \mathbf{h}_m=\mathbf{h}_m(z)e^{ikx}.$$

The functions $\mathbf{m}(z)$ and $\mathbf{h}_m(z)$ can be expressed in the form of a superposition of Fourier harmonics of the form

$$\mathbf{m}_qe^{iqz},\quad \mathbf{h}_{mq}e^{iqz}. \tag{20}$$

The relation of the parameters q to the wave vector k and to the parameters of the problem is determined, as in the preceding subsection by Eqs. (3) and (5) with use of the boundary conditions (4) and (6).

By use of (3) and (5) it is easy to show that the values of the parameter q corresponding to the static case ($\Omega=0$) satisfy the equation

$$\left(4\pi+\alpha k^2+\alpha q^2+\frac{h}{\cos\theta_0}\right)(\alpha q^2+\alpha k^2-\xi\cos\theta_0)$$
$$+\frac{4\pi q^2}{q^2+k^2}\left[\beta\cos^2\theta_0+h\cos\theta_0-\frac{h}{\cos\theta_0}+\alpha\sin^2\theta_0(k^2+q^2)\right]=0. \tag{21}$$

It is evident that when $l\gg\alpha^{1/2}$ and $q^2\ll k^2\sim\xi/\alpha\ll 1$ (see (9)), the relation (21) becomes (12) with $\Omega=0$. But in contrast to (12), equation (21) is cubic in q^2; and the expansion (20) must contain a superposition of Fourier harmonics with all q that satisfy Eq. (21).

The solutions of (3) and (5) have the form

$$\vartheta(z)=\sum_{i=1}^{3}\vartheta(q_i)e^{iq_iz}+\vartheta(-q_i)e^{-iq_iz},$$
$$\varphi(q)=-\frac{k^2+q^2}{4\pi q^2}\left(\alpha q^2+\alpha k^2-\xi\cos\theta_0+\frac{4\pi q^2}{k^2+q^2}\cos\theta_0\right)\vartheta(q),$$
$$h_z(q)=(\alpha q^2+\alpha k^2-\xi\cos\theta_0)\vartheta(q),\quad h_x(q)=(k/q)h_z(q), \tag{22}$$

where q_i^2 are the roots of (21), $i=1, 2, 3$. The values of $\vartheta(q_i)$ are determined by the boundary conditions (4) and (6).

On substituting (22) in (4) and (6), we obtain for the values of $\vartheta(q_i)$ a system of homogeneous linear equations in which the coefficients depend on the plate thickness l:

$$\sum_{i=1}^{3}q_i\sin\frac{1}{2}q_il\vartheta^+(q_i)=0, \tag{23}$$

$$\sum_{i=1}^{3}\left[(4\pi\cos\theta_0+\alpha k^2+\alpha q_i^2-\xi\cos\theta_0)\cos\frac{1}{2}q_il\right.$$
$$\left.+\frac{k}{q}\left(\alpha k^2+\alpha q^2-\xi\cos\theta_0+\frac{4\pi q_i^2}{k^2+q_i^2}\right)\sin\frac{1}{2}q_il\right]\vartheta^+(q_i)=0, \tag{24}$$

$$\sum_{i=1}^{3}(k^2+q_i^2)\left(\alpha k^2+\alpha q_i^2-\xi\cos\theta_0+\frac{4\pi q_i^2}{k^2+q_i^2}\right)\vartheta^+(q_i)\cos\frac{1}{2}q_il=0. \tag{25}$$

where $\vartheta^+(q_i)=\vartheta(q_i)+\vartheta(-q_i)$. Equation (23) is a consequence of the condition $(\partial m_z/\partial z)_{\pm l/2}=0$; Eq. (25) is a consequence of the condition $(\partial m_x/\partial z_{\pm l/2}=0$ (see (6)); Eq. (24) follows from the boundary condition (4). For the values of $\vartheta(q_i)-\vartheta(-q_i)$ equations analogous to (23)–(25) can be obtained, but we shall not write them, since it can be shown that the stability of the uniform state $\Phi_<$ breaks down with respect to nonuniform oscillations that satisfy the condition

$$\vartheta(q_i)=\vartheta(-q_i),$$

that is, the condition $m_z(z)=m_z(-z)$.

The condition for appearance of a nonuniform static solution of the equation for the magnetization is the compatibility condition of the system (23)–(25), i.e., the vanishing of its determinant. Investigation of the system (23)–(25) is significantly simplified at large anisotropy, and we shall hereafter suppose that $\beta\gg 4\pi$. Then the roots of the equation (21) satisfy the inequality

$$q_1^2\ll|q_3|^2, \quad |q_2|^2\ll|q_3|^2,$$

and it is easily shown that in equation (25) the coefficients of $\vartheta^+(q_1)$, and $\vartheta^+(q_2)$ are much smaller than the coefficient of $\vartheta^+(q_3)$; therefore

$$\vartheta^+(q_3)\ll\vartheta^+(q_2), \quad \vartheta^+(q_1).$$

Then the magnetization distribution is determined solely by the Fourier harmonics with $q=q_1$ and $q=q_2$ (see (22)).

To within errors of the order of the small parameter $4\pi/\beta$, the compatibility condition of the system (23)–(25) takes the form

$$-2(4\pi-\delta-\alpha k^2)^{1/2}\left[\left(\frac{\delta}{2}\right)^2+4\pi\alpha k^2\right]\operatorname{th}\frac{\varkappa l}{2}\operatorname{tg}\frac{ql}{2}+\frac{\delta\alpha^{1/2}}{2}\left(q\operatorname{tg}\frac{ql}{2}+\varkappa\operatorname{th}\frac{\varkappa l}{2}\right)+\alpha^{1/2}\left[\left(\frac{\delta}{2}\right)^2+4\pi\alpha k^2\right]^{1/2}\left(\varkappa\operatorname{th}\frac{\varkappa l}{2}-q\operatorname{tg}\frac{ql}{2}\right)=0, \tag{26}$$

where

$$q\equiv q_1=\alpha^{-1/2}[((\delta/2)^2+4\pi\alpha k^2)^{1/2}-\delta/2-\alpha k^2]^{1/2},$$
$$\varkappa=|q_2|=\alpha^{-1/2}[((\delta/2)^2+4\pi\alpha k^2)^{1/2}+\delta/2+\alpha k^2]^{1/2},$$
$$\delta=4\pi-\xi\cos\theta_0.$$

In writing Eq. (26), we have used the fact that when $\beta\gg 4\pi$, the breakdown of stability of the phase $\Phi_<$ occurs within the narrow field interval $(\beta_*^{3/2}\beta_*^{-1/2}<h<\beta)$, so that $1-\cos\theta_0\sim 4\pi/\beta\ll 1$.

The stability of the phase $\Phi_<$ with respect to nonuniform perturbations breaks down for that value of the plate thickness at which there first appears a real root $k^2=k^2(l)$ of equation (26). We consider the case of extremely small plate thicknesses ($l\ll\alpha^{1/2}$). It can be shown that when $\delta\to 0$ $(h\to\beta_*)$, the root $k^2(l)\to 0$; that is, the inequalities

$$ql\ll 1, \quad \varkappa l\ll 1.$$

are satisfied.

Equation (26) then simplifies significantly:

$$\alpha k^2-2\pi kl+\delta=0.$$

Hence it follows that Eq. (26) has a real root $k^2(l)$ when $l>l_1(h)=(\delta\alpha)^{1/2}\pi$.

Thus we have shown that when $l\ll\alpha^{1/2}$, the expression for the instability field of the phase $\Phi_<$ has the form

$$H_1(l)=\beta_*M_0-\pi^2l^2M_0/2\alpha. \tag{27}$$

If $\delta\to 0$, it can be shown that $\vartheta^+(q_1)\to\vartheta^+(q_2)$, and the magnetization distribution (22) has the form

$$\vartheta(x,z)=\vartheta_0e^{ikx}(\cos qz+\operatorname{ch}\varkappa z),$$
$$\alpha k^2=\delta, \quad \alpha q^2=\alpha\varkappa^2=(4\pi\delta)^{1/2}. \tag{28}$$

An investigation of the conditions for solvability of Eq. (24) for the case $l\sim\alpha^{1/2}$ was carried out numerically; the results of the investigation are shown in Fig. 1.

We note that if we set $\cos\theta_0=1$ and $h>\beta_*$ in (26), then this equation describes the stability of the uniform phase $\Phi_\parallel$. In this case it is easy to deduce that $\delta=h-\beta_*$. In the case $h<\beta_*$ (phase $\Phi_<$), $\cos\theta_0=h/\beta_*$, and

$$\delta=\beta(1-h^2/\beta_*^2)\approx 2(\beta_*-h).$$

Since the magnetic field enters Eq. (26) only through the value of δ, it is evident that there is a relation between the values of the instability fields $H_c(l)$ and $H_1(l)$:

$$H_c(l)-\beta_*M_0=-2[H_1(l)-\beta_*M_0]. \tag{29}$$

To within a quantity of order $4\pi/\beta$, this relation is satisfied over the whole field range within which loss of stability can occur for the uniform phases $\Phi_\parallel$ and $\Phi_<$ $(\beta_*^{3/2}\beta^{-1/2}<h<\beta)$.

§2. DOMAIN STRUCTURE OF A FERROMAGNETIC PLATE NEAR THE PHASE-TRANSITION LINE $\Phi_\parallel\rightleftarrows\Phi_D$. SMALL THICKNESSES

In the preceding section, we investigated the stability of the uniform phases $\Phi_\parallel$ and $\Phi_<$ of a ferromagnetic plate on the basis of the linearized Landau-Lifshitz equations (3) and (5). In order to investigate the properties of the phase Φ_D, it is necessary to find nonlinear solutions of the static Landau-Lifshitz equations, in particular to find the dependence of the amplitude of the magnetization variation on the plate thickness and the magnetic field when $H<H_c(l)$. The magnetization distribution in the domain phase when $l\gg\alpha^{1/2}$ was obtained in[6].

As we shall show in the following section, knowledge of the variation of the amplitude of the domain structure and of its energy when $H<H_c(l)$ and $l\lesssim\alpha^{1/2}$ enables one to investigate the stability of the phase Φ_D on the basis of Landau's theory of phase transitions.[8] We consider a nonuniform magnetization distribution when $H<H_c(l)$. Equations (5) can be written in the form

$$\alpha\Delta m_x+h_{mx}=\frac{m_x}{m_y}(h+\alpha\Delta m_y),$$
$$\alpha\Delta m_z+h_{mz}+\beta m_z=\frac{m_z}{m_y}(h+\alpha\Delta m_y), \tag{30}$$
$$\mathbf{m}=\frac{\mathbf{M}}{M_0}.$$

Since the amplitude of the domain structure approaches zero when $H\to H_c(l)$, near the transition field $H_c(l)$ we may suppose that $m_z(x, z)$ and $m_x(x, z) \ll 1$ and may set

$$m_y = 1 - {}^1/_2 (m_x^2 + m_z^2). \quad (31)$$

We shall be interested in the properties of the domain structure when $l \ll \alpha^{1/2}$ (then $h_c(l) - \beta_* \ll 4\pi$). Since we are supposing that $H_c(l) - H \ll H_c(l)$, the amplitude of the domain structure is small. (We recall that the phase transition $\Phi_\parallel \rightleftarrows \Phi_D$ is a transition of second order.) Therefore the magnetization distribution is close to the distribution described by the linear equations considered in the preceding section. In particular, we may assert that (see (29))

$$\frac{\partial^2 m_i}{\partial x^2} \sim \delta m_i, \quad \frac{\partial^2 m_i}{\partial z^2} \sim (\overline{\sqrt{4\pi\delta}})^{1/2} m_i, \quad \delta = h - \beta_* \ll 1. \quad (32)$$

By use of (3) and (30) and of the relations (32), it is easy to show that

$$m_x \approx \delta^{1/4} (4\pi/\beta) m_z \ll m_z,$$

and to put (3) and (30) into the form

$$(h-\beta_*) \frac{\partial^2 m_z}{\partial z^2} + (h-\beta) \frac{\partial^2 m_z}{\partial x^2} - \alpha \Delta^2 m_z + \frac{h}{2} \Delta m_z^3 = 0,$$
$$h_{m_z} = (h-\beta) m_z - \alpha \Delta m_z + {}^1/_2 h m_z^3. \quad (33)$$

We seek a solution of (33) in the form

$$m_z(x, z) = A_1(z) \cos kx + A_3(z) \cos 3kx + \ldots \quad (34)$$

As can be shown[6], $A_3(z) \sim (H_c(l) - H) A_1(z)$, and in order to find the energy of the domain phase near the phase transition we need to keep only the first term of the expansion (34). The quantity $A_1(z)$ satisfies the equation

$$\alpha \left(\frac{d^2}{dz^2} - k^2 \right)^2 A_1 - (h-\beta_*) \frac{d^2 A_1}{dz^2} - (\beta - h) k^2 A_1 + \frac{3h}{8} \frac{d^2}{dz^2} A_1^3 = 0. \quad (35)$$

We shall seek a solution of Eq. (35), when $H \to H_c$, in the form

$$A_1(z) = \eta (a_1 \cos \tilde{q}_1 z + a_2 \cos \tilde{q}_2 z)$$
$$+ \eta b_1 \cos 3\tilde{q}_1 (z) + \eta b_2 \cos (2\tilde{q}_1 + \tilde{q}_2) z + \ldots, \quad (36)$$

the quantity η has the meaning of amplitude of the domain structure. It turns out that $b_i \sim a_i (H - H_c) \ll a_i$, and only the first two terms are needed in (36). As in the linear case considered in the preceding section, the relation between the values of a_1 and of a_2 is determined by the boundary conditions. The values of $\tilde{q}_1$ and $\tilde{q}_2$ are determined by Eqs. (35):

$$\alpha (\tilde{q}_1^2 + k^2)^2 + \tilde{q}_1^2 (h - \beta_*) - (\beta - h) k^2 + {}^9/_{32} h (\tilde{q}_1^2 + k^2) \eta^2 (a_1^2 + 2a_1 a_2) = 0,$$
$$\alpha (\tilde{q}_2^2 + k^2)^2 + \tilde{q}_2^2 (h - \beta_*) - (\beta - h) k^2 + {}^9/_{32} h (\tilde{q}_2^2 + k^2)$$
$$\times \eta^2 (a_2^2 + 2a_1 a_2) = 0. \quad (37)$$

As we can show by analysis of the boundary conditions, when $l \ll \alpha^{1/2}$, $a_1 = a_2$, and we may take $a_1 = a_2 = 1$.

For the values of $\tilde{q}_1$ and $\tilde{q}_2$ we get

$$\tilde{q}_1 \equiv \tilde{q} = \alpha^{-1/2} [((\tilde{\delta}/2)^2 + 4\pi\alpha k^2)^{1/2} - \tilde{\delta}/2 - \alpha k^2]^{1/2},$$
$$\tilde{q}_2 = i\tilde{\varkappa} = i\alpha^{-1/2} [((\tilde{\delta}/2)^2 + 4\pi\alpha k^2)^{1/2} + \tilde{\delta}/2 + \alpha k^2]^{1/2}, \quad (38)$$

where

$$\tilde{\delta} = h - \beta_* + {}^{27}/_{32} h \eta^2.$$

It is easily seen that the expression (38) for $\tilde{q}_1$ or $\tilde{q}_2$ differs formally from the expression (26) for q_1 or q_2 only by replacement of δ by $\tilde{\delta}$. It can be shown that analysis of the boundary conditions in the nonlinear case also leads to a relation that differs from (26) only by replacement of δ by $\tilde{\delta}$. From this fact it is clear (see (38)) that if we have found the instability curve $H_c(l)$ of the phase $\Phi_\parallel$, then when $H < H_c(l)$ the amplitude η of the domain structure is determined by the relation

$$\eta^2 = \frac{32}{27} \frac{H_c(l) - H}{H_c(l)}. \quad (39)$$

By substituting the solution (34), with use of (35), (36), and (39), in the expression (2) for the thermodynamic potential of the plate, we obtain an expression that determines the energy of the domain phase of the ferromagnetic plate when $l \ll \alpha^{1/2}$ and $H - H_c(l) \ll H_c(l)$:

$$W = \frac{V}{3\beta} \left[\frac{8}{9} (H_c(l) - H) \right]^2, \quad (40)$$

where V is the volume of the plate.

§3. STABILITY OF THE DOMAIN PHASE. PHASE DIAGRAM OF A PLATE NEAR THE TRIPLE POINT

Instability of the phase $\Phi_<$ when $H > H_1(l)$ may be indicative either of a second-order phase transition at $H = H_1(l)$ or of a first-order phase transition at some field value $H = H_{tr}(l)$, where $H_{tr}(l) < H_1(l)$. It is clear that in order to resolve the question of the order of the transition $\Phi_< \rightleftarrows \Phi_D$, it is necessary to investigate the stability of the domain phase Φ_D.

For simplicity and clarity, we shall not investigate with respect to stability the nonuniform nonlinear solutions of (3) and (5), corresponding to a domain structure, but shall use symmetry considerations in the spirit of Landau's theory of phase transitions.[8] By virtue of the relations (27) and (29), the regions of existence of three phases ($\Phi_\parallel$, $\Phi_<$, and Φ_D) come together at the point $H = \beta_* M_0$, $l = 0$ (the point O in Fig. 1).

As was mentioned in the preceding section, the transitions $\Phi_\parallel \rightleftarrows \Phi_D$ and $\Phi_\parallel \rightleftarrows \Phi_<$ occur as second-order phase transitions. The order parameter for the transition $\Phi_\parallel \rightleftarrows \Phi_<$ is the angle θ that the magnetization vector makes with the external field **H** (see (7)). As order parameter in the transition $\Phi_\parallel \rightleftarrows \Phi_D$ we may take the amplitude of the domain structure in the phase Φ_D, which vanishes when $H = H_c(l)$.[6] The most symmetric of the three phases is the phase $\Phi_\parallel$, since a transition to phase $\Phi_<$ or Φ_D entails loss of various symmetry elements characteristic of phase $\Phi_\parallel$ (reflection in the plane of the plate for the transition $\Phi_\parallel \to \Phi_<$, and symmetry with respect to translation by an arbitrary vector along the x axis for the transition $\Phi_\parallel \to \Phi_D$).

Thus the phase $\Phi_\parallel$, depending on the plate thickness, changes on change of the magnetic field to different less symmetric phases $\Phi_<$ and Φ_D. As was shown by Landau[8] from symmetry considerations, in such a situation two types of phase diagrams may occur (see Fig. 2 and 3 of[8]). There can be a phase diagram in which the transition $\Phi_< \rightleftarrows \Phi_D$ occurs as a first-order phase transition; in this case only the three phases indicated above ($\Phi_\parallel$, $\Phi_<$, and Φ_D) exist (see[8], Fig. 2). Or else there can be a phase diagram in which the transition $\Phi_< \rightleftarrows \Phi_D$ occurs by two second-order transitions; $\Phi_< \rightleftarrows \Phi_4$, $\Phi_4 \rightleftarrows \Phi_D$ (see[8], Fig. 3). Here Φ_4 is a certain least symmetric state of the ferromagnetic plate, whose symmetry group is a subgroup simultaneously of the symmetry groups of phases $\Phi_<$ and Φ_D; that is, a certain nonuniform state of the ferromagnetic plate, unsymmetric with respect to reflection in the plane of the plate.

Hereafter we shall show that for the case $\beta \gg 4\pi$, the phase Φ_4 cannot occur, and the phase transition $\Phi_< \rightleftarrows \Phi_D$ occurs as a first-order phase transition. We shall investigate the state of the plate near the point O; for this purpose, we shall write the simplest expression for the thermodynamic potential Φ of the plate that describes the existence near the point O of three phases of the plate, in the form of an expansion in powers of the order parameters θ and η:

$$\Phi=\Phi_0+{}^4/_9(h-h_c(l))\eta^2+{}^1/_2(h-\beta_*)\theta^2 + {}^1/_2\beta\gamma\theta^2\eta^2+{}^3/_{16}h\eta^4+{}^1/_8\beta\theta^4. \tag{41}$$

The coefficients of θ^2 and θ^4 in the expansion (41) can be easily obtained. To determine the coefficients of η^2 and η^4, we used the expressions obtained in the preceding section for the equilibrium values of the order parameter η^2 in the phase Φ_D, (39), and for the equilibrium energy of the domain phase, (40).

The coefficient γ determines the form of the phase diagram of the plate near the point O, in particular the form of the instability curves $H_1(l)$ and $H_2(l)$ of the phases $\Phi_<$ and Φ_D, and consequently the nature of the phase transition $\Phi_< \rightleftarrows \Phi_D$.

By minimizing (41), we can express the values of the instability fields of the phases $\Phi_<$ and Φ_D in terms of the instability field $H_c(l)$ of the phase $\Phi_\parallel$. For the field $H_1(l)$ we get

$$\Delta H_1(l)\left(1-\frac{9}{4}\gamma\right)=\Delta H_c(l), \tag{42}$$

where $\Delta H = H - \beta_* M_0$. By using the relation $\Delta H_c/\Delta H_1 = -2$ (see (29)), we find the value of the coefficient γ:

$$\gamma={}^4/_3.$$

Thus all the coefficients in the expansion (41) are known, and we can obtain the conditions for stability of the domain phase Φ_D without direct investigation of the stability of the nonlinear solutions of the Landau-Lifshitz equation, of the type (34).

By substituting the value of γ that we have found in (41), we can show that the domain phase is stable when the condition

$$H<H_c(l),$$

is satisfied; that is, the stability of the phase Φ_D is destroyed only on the second-order phase-transition line $\Phi_\parallel \rightleftarrows \Phi_D$. The stability regions of phases $\Phi_<$ and Φ_D overlap (see Fig. 1), and consequently the transition $\Phi_< \rightleftarrows \Phi_D$ occurs as a first-order phase transition of the first kind.

To find the first-order phase-transition line, we equate the energies of phases Φ_D, (40), and $\Phi_<$:

$$3\left[\frac{8}{27}(H_c-H)\right]^2=\frac{1}{2}(H-\beta_*M_0)^2. \tag{43}$$

On solving (43), we get the following expression for $H_{tr}(l)$:

$$H_{tr}(l)=\beta_*M_0-[H_c(l)-\beta_*M_0]\left(1-\frac{2^{7/2}}{3^{5/2}}\right)^{-1}\approx\beta_*M_0-(\pi^2l^2\alpha^{-1/2})\left(1-\frac{2^{7/2}}{3^{5/2}}\right)^{-1}. \tag{44}$$

It is evident that

$$H_{tr}(l)<H_1(l).$$

CONCLUSION

The phase diagram of a ferromagnetic plate with $\beta \gg 4\pi$ is shown schematically in Fig. 1. On this phase diagram there is a triple point, at which the existence regions of three phases make contact: the uniform phases $\Phi_\parallel$ and $\Phi_<$ and the domain phase. The transition $\Phi_< \rightleftarrows \Phi_D$ occurs as a first-order phase transition. It was found that for $\beta \gg 4\pi$ an equilibrium domain phase can exist at an arbitrarily small plate thickness l (see Fig. 1). It is found, however, that as $l \to 0$, the amplitude of the domain structure diminishes to zero, as does also the width of the interval of magnetic-field values within which a domain structure exists.

It must be mentioned that when $\beta \gtrsim 4\pi$, it is not possible to give a complete analysis of the phase diagram without resort to numerical methods. In particular, the possibility is not excluded that within a restricted region of the diagram the phase Φ_4 occurs (see § 3), and the transition $\Phi_< \rightleftarrows \Phi_D$ occurs by two second-order phase transitions. It is also not excluded that for $\beta \gtrsim 4\pi$, domain structure is absent when $l \ll \alpha^{1/2}$; that is, it exists only when $l \geq l_c$ ($l_c \to 0$ when $4\pi/\beta \to 0$). But the relation (29) between the instability fields of phases $\Phi_\parallel$ and $\Phi_<$ remains correct as $H \to (\beta - 4\pi)M_0$ regardless of the value of β.

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Investigation of ferromagnetic resonance in $Y_3Fe_5O_{12}$ by the method of optical spectroscopy

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Results are presented of an investigation of inelastic scattering of linearly polarized monochromatic light ($\lambda = 6328$ Å) by homogeneous-precession magnons excited by FMR in a thin $Y_3Fe_5O_{12}$ plate. By separating the contributions of the magneto-optical effects to the intensities of the spectral lines with combination frequencies, the amplitudes and polarizations of the precessions are reconstructed as functions of the microwave magnetic field. It is shown that when the threshold for the second-order parametric process is exceeded, besides saturation of the amplitude of the homogeneous mode, a substantial change takes place also in its polarization.

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High-frequency modulation of light in ferromagnetic resonance (FMR) in magnetically ordered crystals has been the subject of relatively few studies. Mention should be made of a paper by Hanlon and Dillon,[1] where direct spectroscopic proof was obtained that microwave satellites appear when monochromatic linearly polarized light passes through a $CrBr_3$ single crystal. High-frequency modulation of light in antiferromagnetic resonance in a $CoCO_3$ crystal was observed by Borovid-Romanov *et al.*[2]

At the same time, the study of magneto-optical phenomena due to FMR is of considerable interest, especially in crystals having comparable values of the Faraday rotation (FE) and the linear magnetic birefringence—the Cotton-Mouton effect (CME). It will be shown below that in this case optical measurements yield sufficiently complete information on the most important parameters of the homogeneous magnetization oscillations in FMR.

We consider the interaction of linearly polarized monochromatic light with a ferromagnetic crystal, in which homogeneous precession of magnetization is excited. We assume for simplicity that the crystal has cubic symmetry (for example, $Y_3Fe_5O_{12}$) and is magnetized along the [001] axis.

We choose the coordinate system such that its axes coincide with the fourfold axes of the crystal, and the constant magnetic field H_0 is directed, as usual, along the Z axis. In this case the crystal is optically uniaxial with the optical axis directed along the magnetization.[3] Inasmuch in FMR the deviation of the magnetic moment from its equilibrium position is small, it is accurate enough to state that the optical axis of the crystal precesses. Assume that the light propagates along the X axis. The vector $\mathbf{E}$ of the incident light can then be represented in the form

$$\mathrm{E}_0 = \{E_{0z}, E_{0y}\} = \{\cos\psi, \sin\psi\} e^{i\omega_0 t}, \quad (1)$$

where ψ is the angle between the vector $\mathbf{E}$ and the Z axis; ω_0 is the frequency; the intensity is set equal to unity. The change of the polarization of the light passing through the crystal can be described by the corresponding Jones matrix $\hat{Q}$, the elements of which have been calculated with allowance for the principle of superposition of the FE and the CME. By a procedure similar to that used by Tron'ko[4] we obtain expressions for the matrix element that describe the light emerging from the sample

$$Q_{zz} = Q_{yy}^* = \cos\Delta l + i\delta\cos 2\theta_y \sin\Delta l/2\Delta, \quad (2)$$

$$Q_{zy} = -Q_{yz}^* = \Delta^{-1}(\tfrac{1}{2} i\delta \sin 2\theta_y - \rho_x)\sin\Delta l, \quad (3)$$

$$\Delta = (\tfrac{1}{4}\delta^2 + \rho_x^2)^{1/2}, \quad (4)$$

where ρ_x is the Faraday rotation per unit thickness of the crystal and is due to the component m_x of the magnetization vector (in the absence of precession we have $\rho_x \equiv 0$, for in this case the light propagates perpendicular to $\mathbf{M}_0$); δ is the specific phase shift of the CME ($\delta \sim |M|^2$); θ_y is the angle between the projection of the

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