Properties of a quasi-one-dimensional system below the superconducting transition temperature

Yu. A. Firsov and G. Yu. Yashin

A. F. Ioffe Physico-Technical Institute, Academy of Sciences of the USSR
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We study the superconducting properties of a quasi-one-dimensional model which is a system of periodically spaced parallel metal filaments whose superconducting phases are linked directly through non-stationary Josephson junctions. We assume that the filaments are not too thin: $\xi \gg d \gg a$, where d is the filament diameter, a the lattice constant of the metal, and ξ the coherence length of the bulk metal. We study the behavior of the correlation functions, elucidate some features of ODLRO and discuss the specific features of a phase transition in such a quasi-one-dimensional system. We study the Meissner effect in such a system and show that it cannot be described by a trivial generalization of the Meissner equations to the anisotropic case. The dependence of the critical temperature T_k on the transverse resonance integral $J_\perp$ (which characterizes the tunneling rate of the electrons from filament to filament) which was found earlier by Larkin and Efetov when T_k is approached from above (i.e., from the region where the phases in the different filaments are uncorrelated, and the phase system behaves as a one-dimensional one) is in the present paper cofirmed by approaching T_k from below (i.e., from the region where the system is essentially quasi-one-dimensional).

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1. INTRODUCTION

At present different quasi-one-dimensional substances such as the TCNQ salts,[1] compounds with a mixed valence, such as KCP,[2] quasi-one-dimensional magnetics,[3] compounds of the A-15 type,[4] synthetic quasi-one-dimensional objects—secondary crystals and so on[5] are studied intensively experimentally. It is convenient to characterize the degree of anisotropy of the physical properties of such substances by the magnitude of the small dimensionless parameter

$$\gamma=\frac{J_\perp}{J_\parallel}\approx\left(\frac{a_\parallel}{a_\perp}\right)^2\frac{m_\parallel}{m_\perp},$$

where $J_\perp$ and $J_\parallel$ are transverse and longitudinal resonance integrals which characterize the rate of tunneling of electrons at right angles to and along the filaments, and $m_\parallel$, $m_\perp$ and $a_\parallel$, $a_\perp$ are, respectively, the longitudinal and transverse effective masses and lattice constants.

The usual theory of strongly anisotropic substances is based upon the same hypotheses as the theory of standard semiconductors: the self-consistent field method, the Bloch theorem, the description in terms of single-particle excitations, neglecting collective effects, and the assumption that fluctuations play a minor role at all temperatures except a narrow region near the phase transition temperature.

From the above-mentioned experiments the conclusion follows that for sufficiently small γ these approximations are too rough. Attempts to "approach from the other side" and to construct a theory of strongly anisotropic substances, completely ignoring linking between the filaments ($\gamma=0$) turned out to be untenable. Firstly, this is a consequence of the fact that many approximation methods which yield good results in the three-dimensional case lead to incorrect results in the one-dimensional case. Secondly, even when the one-dimensional model can be solved exactly it has not always a meaning to compare the results obtained with experimental results for actual quasi-one-dimensional objects for which γ, although small, is not exactly equal to zero. Taking only one potential link (the Coulomb interaction between electrons in different filaments) does not get rid of the most specific features of one-dimensional solutions: although the branch of the collective excitation is separated by a gap, the gap is anisotropic and vanishes in the direction at right angles to the filaments,[6] and the single-particle excitations are therefore, as before, absent (the single-particle Green function has no poles), the momentum distribution function of the particles does not have the Fermi step-function form,[7] and the admittances have the same frequency and temperature features as in the one-dimensional case, i.e., the temperature for the transition to any new state T_k is equal to zero.

Taking a weak ($\gamma \ll 1$) kinetic coupling into account (i.e., taking into account the possibility of tunneling or hopping of electrons from filament to filament) radically changes these results. The phase transition temperature becomes finite, albeit small for small γ. The dependence of T_k on $J_\perp$ is different for different models,[8–10] but in agreement with Landau's theorem about the impossibility of phase transitions, if the interaction is not a long-range one, $T_k\to 0$ as $J_\perp\to 0$. We note that all these results are obtained using some kind of self-consistency with respect to the transverse coupling; the degree of accuracy of this method has not been established.

When $J_\perp\neq 0$ ($\gamma\neq 0$) there occur single-particle excitations in the spectrum of the quasi-one-dimensional system,[11] but for small γ the fraction of individual degrees of freedom will be very small as compared with the collective ones. We have chosen a model described in[8] because in it the single-particle branches of the

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spectrum are practically not excited for $T < T_k$ (since for $2J_\perp \ll kT_c^0$ they are separated by a gap $\Delta \sim kT_c^0 \gg kT_k$) and it is sufficient to consider only the collective degrees of freedom. So far for this model only an expression for T_k had been obtained, but we study the collective spectrum, we find the correlation functions, we investigate the ODLRO features and the Meissner effect, and we make some observations about the nature of the phase transition.

2. THE MODEL

Efetov and Larkin[8] considered in 1974 the following model of a quasi-one-dimensional system: metallic filaments of diameter d are placed in a dielectric matrix. Under conditions where n ($n \gg 1$) atoms are fitted into the transverse cross section of each filament, there are n subzones in each of them and an individual filament is stable against Peierls splitting.[8,12] The period of the two-dimensional square lattice, formed by the filaments in the cross section perpendicular to them, is $a_\perp$. It has been shown[8] that when $(T_c^0 - T)/T_c^0 \gg n^{-2/3}$ one can write the partition function of such a system in the form

$$Z=\int D\varphi \exp[-F(\varphi)], \tag{2.1}$$

where

$$F(\varphi)=\int dz\int dz'\int_0^\beta d\tau \sum_{i,j}\mathscr{K}_{ij}(z-z')\dot\varphi_i(z,\tau)\dot\varphi_j(z,\tau)+\int dz\int_0^\beta d\tau\left\{\frac{1}{8}\right. \cdots$$

$$\left.\times\frac{N_s\hbar^2}{m}\sum_i\left(\frac{\partial\varphi_i}{\partial z}\right)^2+\sum_{i,g}W(g)\left[1-\cos[\varphi_i(z,\tau)-\varphi_{i+g}(z,\tau)]\right]\right\}. \tag{2.2}$$

Here φ is the phase of the order parameter, z is a coordinate reckoned along the filament, i, j, g characterize the number of the filament, $\beta = 1/kT$, $\dot\varphi$ is the derivative with respect to the imaginary time τ, N_s is the linear density of superconducting electrons, near T_c^0

$$N_s(T)\approx N_s(0)\left(\frac{T_c^0-T}{T_c^0}\right)^{1/2},$$

T_c^0 is the formally evaluated (e.g., by the mean field method) transition temperature for a single filament, neglecting fluctuations and dimensional effects. The remaining notation has the same meaning as in[8]. Moreover, it is convenient for us to introduce a quantity $E_\perp$:

$$E_\perp N_s(T)=W(g)\approx J_\perp^2/\xi_\parallel kT_c^0.$$

The heuristic considerations which lead to (2.2) are rather obvious: if one assumes that each filament is described by the usual Ginzburg–Landau (G-L) functional and if there is an electrostatic interaction between the filament, the strength of which is characterized by the quantity $\mathscr{K}(z-z')$, the dynamics of the system are described by the Lagrangian

$$\mathscr{L}[\psi(t)]=\int dz\int dz'\sum_{i,j}\frac{\hbar^2}{8}\mathscr{K}_{ij}(z-z')\dot\varphi_i(z)\dot\varphi_j(z')$$

$$+\int\frac{dz}{\xi_0}\sum_i\left\{a\frac{T_c^0-T}{T_c^0}|\psi_i|^2+b|\psi_i|^4+\frac{\hbar^2}{2m}\left|\frac{\partial\psi_i}{\partial z}\right|^2\right.$$

$$\left.+\sum_g W(g)\left[1-\cos[\varphi_i(z)-\varphi_{i+g}(z)]\right]\right\}. \tag{2.3}$$

In contrast to (2.2) $\dot\varphi$ is here the derivative with respect to the normal time. The quantity $\mathscr{K}_{ij}(z-z')$ is the microscopic analog of the specific electrostatic induction coefficient $C_{ij}(z-z')$. In[8] it was assumed that if $q_\perp \sim 1/a_\perp$, $q_z \ll 1/a_\perp$, one can put[1)]

$$\mathscr{K}_q\approx\text{const}=K\approx\frac{1}{\pi}\frac{n}{\hbar v_0}\left[1+\frac{e^2}{\hbar v_0}n\ln\left(\frac{a_\perp}{d}\right)\right]^{-1}. \tag{2.4}$$

Using standard methods we find that the partition function of the system described by the Lagrangian (2.3) has the form

$$Z=\int D\psi\exp\left\{-\int_0^\beta d\tau\,\mathscr{L}[\psi(\tau)]\right\}.$$

If we now assume that the dependence of the modulus of ψ is a steep one and that we can integrate over $D|\psi|$ by Laplace's method, we are led to Eq. (2.1).

3. CHOICE OF THE ZEROTH APPROXIMATION

The results for the correlation functions were obtained in[8], neglecting the term

$$W[1-\cos(\varphi_i-\varphi_{i+g})],$$

which may be valid only above T_k. It is physically clear that the coupling between filaments must lead to some breakdown of the one-dimensional specific features of the problem (see the Introduction), but the problem of what this breakdown is or how strongly it affects the physical properties of the system has practically not been studied up to the present. Earlier studies[9,10] of this problem neglect the possibility that electrons may tunnel from filament to filament. We show below that the interaction between phase fluctuations at different filaments due to this tunneling radically affects the physical properties of the system and imposes on it a behavior which can be called with complete justification quasi-one-dimensional. The basic idea of the following calculations is to take into account this new form of the coupling which arises below the temperature

$$T_k\sim T_c^0\left(\frac{J_\perp}{kT_c^0}\right)^{1/(1-\alpha/2)}=J_\perp\left(\frac{J_\perp}{kT_c^0}\right)^{\alpha/(2-\alpha)}$$

of the superconducting phase transition. (We bear in mind that $\alpha \ll 1$.)

It is clear that one can perform calculations with the functional (2.2) only in the framework of perturbation theory, i.e., through splitting F into F_0 and F_1 ($F = F_0 + F_1$) and the subsequent expansion of all calculated quantities in a power series in F_1. One must then aim at having taken into account the coupling between the filaments already to some extent in F_0. This can be achieved by expanding $\cos(\varphi_i - \varphi_{i+g})$ in a power series of its argument and retaining in F_0 the term $(\varphi_i - \varphi_{i+g})^2$, putting all other terms in F_1. However, a consistent consideration of the operator structure of $\cos(\varphi_i - \varphi_{i+g})$ forces us to bring it first into normal form and only afterwards separate from it terms quadratic in the creation and annihilation operators. We explain this in some detail.

In the Hamiltonian formalism (2.1) takes the form

$$Z=\mathrm{Sp}\{\exp(-\beta\mathscr{H})\},$$

where $\mathscr{H}$ is the Hamiltonian in the second quantization representation which is obtained from the Lagrangian (2.3) when $|\psi|=\text{const}$:

$$\mathscr{H}=\sum_i\int dz\left[\frac{\delta L}{\delta\dot\varphi_i(z)}\dot\varphi_i(z)-L\right].$$

Here L is the density of $\mathscr{L}(\varphi)$,

$$\mathscr{L}(\varphi)=\int dz\sum_i L.$$

Introducing a characteristic longitudinal length via the formula $\hbar^2 N_s/4m\xi_\parallel=kT_c^0$ ($\xi_\parallel$ differs by a factor from the usual GL coherence length, $\xi_\parallel=1.4\ \xi_{GL}$) and using the fact that $\mathscr{K}_q\approx K$, we have:

$$\mathscr{H}=\sum_j\int dz\left\{\frac{2\pi_j^2}{K\xi_\parallel}+\xi_\parallel N_s\left[\frac{\hbar^2}{8m\xi_\parallel^2}\left(\frac{\partial\varphi_j}{\partial z}\right)^2+E_\perp\sum_g(1-\cos(\varphi_j-\varphi_{j+g}))\right]\right\}, \tag{3.1}$$

where $\pi_j=\hbar^{-1}\delta L/\delta\dot\varphi_j$. The difference in the coefficients of $(\partial\varphi/\partial z)^2$ in (3.1) and (2.3) is explained by the fact that, in correspondence with[13], $N_s=4|\psi|^2$. In this normalization m and e are the mass and charge of a single electron, and z in (3.1) is already dimensionless.

Quantization is accomplished through the usual rules: we take φ_q and π_q to be operators such that $[\pi_q\varphi_q]=i$ and we write them in the form

$$\pi_q=i\left(\frac{\omega_q}{2\lambda_q}\right)^{1/2}[b_{-q}-b_q^+],\quad \varphi_q=\left(\frac{\lambda_q}{2\omega_q}\right)^{1/2}[b_{-q}+b_q^+]. \tag{3.2}$$

Substituting (3.2) into (3.1) we see that when we perform the reduction to normal form there appear terms quadratic in the operators b_q and b_q^+ in the expansion of $\cos(\varphi_i-\varphi_{i+g})$ not only from $(\varphi_i-\varphi_{i+g})^2$ but also from any power of $(\varphi_i-\varphi_{i+g})$. In order to split off such terms we reduce $\cos(\varphi_i-\varphi_{i+g})$ to its normal form, using the Feynman identity

$$\exp\{\hat A+\hat B\}=\exp\{\hat A\}\exp\{\hat B\}\exp\{{}^1/_2[\hat B\hat A]\}.$$

As a result we get

$$\cos(\varphi_i-\varphi_{i+g})=\frac{1}{2}\left\{\exp\left[\frac{i}{\sqrt N}\sum_q[A_q(i,g)b_q^++A_q^*(i,g)b_q]\right]+\exp\left[-\frac{i}{\sqrt N}\sum_q[A_q(i,g)b_q^++A_q^*(i,g)b_q]\right]\right\}$$

$$=\frac{1}{2}\exp[-S_0(g)]\left\{\exp\left[\frac{i}{\sqrt N}\sum_q A_q(i,g)b_q^+\right]\exp\left[\frac{i}{\sqrt N}\sum_q A_q^*(i,g)b_q\right]+\exp\left[-\frac{i}{\sqrt N}\sum_q A_q(i,g)b_q^+\right]\exp\left[-\frac{i}{\sqrt N}\sum_q A_q^*(i,g)b_q\right]\right\}.$$

Here $N=N_\perp L\xi_\parallel$ ($N_\perp$ is the number of filaments),

$$A_q(i,g)=\exp(iq_\perp i_\perp+iq_z z)[1-\exp(iq_\perp g)]\left(\frac{\lambda_q}{2\omega_q}\right)^{1/2},$$

$$S_0(g)=\frac{1}{N}\sum_q\frac{\lambda_q}{2\omega_q}(1-\cos qg).$$

From this it is clear that in the term in the expansion of $\cos(\varphi_i-\varphi_{i+g})$ which is quadratic in b_q and b_q^+ the important factor $\exp(-S_0)$ has been split off. Choosing ω_q and λ_q such as to diagonalize the quadratic Hamiltonian which one obtains when expanding up to second-order terms in the operators, we get

$$H_0=\sum_q\hbar\omega_q[b_q^+b_q+{}^1/_2], \tag{3.3}$$

$$\omega_q=\bar\omega\Omega_q,\quad \Omega_q^2=[q_z^2+2\delta^2(2-\cos q_x-\cos q_y)];$$
$$\delta^2=\delta_{cl}^2\exp\{-S_0(g)\}, \tag{3.4}$$

$$\delta_{cl}^2=E_\perp\Big/\frac{\hbar^2}{8m\xi_\parallel^2}\approx J_\perp^2\Big/\frac{\hbar^2N_s}{8m\xi_\parallel}kT_c^0\approx2\left(\frac{J_\perp}{kT_c^0}\right)^2,$$
$$\bar\omega=\left(\frac{N_s}{m\xi_\parallel K}\right)^{1/2},\quad \lambda_q=\frac{4}{K\hbar\xi_\parallel}. \tag{3.5}$$

It is clear that the quantity $\exp(-S_0)$ enters into δ and appreciably changes the original "classical" anisotropy of the problem: if we linearize the classical Hamiltonian (3.1), we see that the sound velocity along and at right angles to the filaments differs by a factor δ_{cl}, while in the quantum problem it is decreased by the phase fluctuations and equals $\delta<\delta_{cl}$.

We introduce the important dimensionless parameter of the problem α as follows:

$$\alpha=\frac{2}{\pi}\left(K\frac{\hbar^2N_s}{m}\right)^{-1/2}=\frac{2}{\pi}\frac{1}{K\xi_\parallel\hbar\bar\omega}=\frac{\hbar\bar\omega}{2\pi kT_c^0}. \tag{3.6}$$

In what follows the quantities α, δ, δ_{cl}, $kT_c^0=\hbar^2N_s/4m\xi_\parallel$ are the basic parameters of the problem; all quantities of interest to us can be expressed in terms of them, e.g.:

$$\varphi_j(z)=\frac{1}{\sqrt N}\sum_q\left(\frac{\pi\alpha}{\Omega_q}\right)^{1/2}[b_q^++b_{-q}]\exp(iq_\perp j+iq_z z). \tag{3.7}$$

Equation (3.4) is the equation for δ. This quantity enters in the definition of $S_0(g)$ through $\lambda_q/2\omega_q=\pi\alpha/\Omega_q$:

$$S_0(g)=\frac{1}{N}\sum_q\frac{\pi\alpha}{\Omega_q}[1-\cos qg]\to\frac{\pi\alpha}{(2\pi)^3}\int_{-1}^{1}dq_z\int_{-\pi}^{\pi}dq_x\int_{-\pi}^{\pi}dq_y[1-\cos qg]\times[q_z^2+2\delta^2(2-\cos q_x-\cos q_y)]^{-1/2}. \tag{3.8}$$

(Here and henceforth q_z is measured in units $\xi_\parallel^{-1}$, $q_\perp$ in $a_\perp^{-1}$; with respect to q_z we introduce a cut-off at $\xi_\parallel^{-1}$ as the GL equation is inapplicable at smaller length scales.) We shall show below that $\delta(0)=\delta_{cl}^{1/(1-\alpha/2)}$. Using (2.4) and (3.6) we can see that α is small when the number of zones n is large; using an order of magnitude estimate for δ_{cl} we see that

$$\delta(0)\approx\left(\frac{J_\perp}{kT_c^0}\right)^{1/(1-\alpha/2)}.$$

for small $J_\perp$ and large n, α and δ are thus small parameters.

We can somewhat improve the similarity of H_0 and $\mathscr{H}$ by choosing H_0 in a self-consistent way (see[21]). This leads to a replacement of the quantity $\delta=\delta(0)$ by $\delta(T)$ in all Eqs. (3.3) to (3.5), (3.7), and (3.8):

$$\delta^2(T)=\delta_{cl}^2\exp\{-S_T(g)\}, \tag{3.9}$$

and one must introduce in $\mathscr{H}$ the cancelling term

$$\sum_{lg}\int dz[\exp(-S_T)-1]W(g),$$

while the quantity $S_T(g)$ will have the form

$$S_T(g)=\frac{\pi\alpha}{N}\sum_{q}\frac{1-\cos qg}{\Omega_q(T)}\operatorname{cth}\left(\frac{\hbar\omega_q\beta}{2}\right).$$

We shall use in what follows a Hamiltonian which has the form (3.3) where the quantity $\delta(T)$ must be obtained from Eq. (3.9).

4. TRANSITION TEMPERATURE

We estimate the magnitude of $S_T(g)$ in order to solve Eq. (3.9). We replace, as in (3.8) the sum over q by an integral. In that case

$$S_T(g)=\frac{\pi\alpha}{(2\pi)^3}\int_{-l}^{l}dq_z\int_{-\pi}^{\pi}dq_x\int_{-\pi}^{\pi}dq_y\frac{1-\cos qg}{\Omega_q}\operatorname{cth}\left(\frac{\hbar\omega_q\beta}{2}\right)$$

$$=\frac{\pi\alpha}{(2\pi)^3}\int_{-1/\delta}^{1/\delta}dq_z\int_{-\pi}^{\pi}dq_x\int_{-\pi}^{\pi}dq_y\frac{1-\cos qg}{[q_z^2+2(2-\cos q_x-\cos q_y)]^{1/2}}$$
$$\times\operatorname{cth}\left\{\frac{\hbar\bar{\omega}\delta\beta}{2}[q_z^2+2(2-\cos q_x-\cos q_y)]^{1/2}\right\}. \quad (4.1)$$

We introduce the temperature $T_c=\delta(0)T_c^0$, where $\delta(0)$ is $\delta(T)$ for $T=0$ and takes the form (3.4). Furthermore, when evaluating the correlators we shall see that T_c (or $\pi\alpha T_c$) is the characteristic temperature scale of the problem. We note that the quantity $2^{1/(1-\alpha/2)}T_c$ coincides with the transition temperature evaluated in[8] taking quantum fluctuations into account.

The integral (4.1) can be estimated for different temperature regions, as a result of which we get (here $q_0=T/\pi\alpha T_c$)

$$S_T(g)\approx\alpha q_0^4A\left[\frac{\delta(0)}{\delta(T)}\right]^2+\alpha\ln\frac{1}{\delta}+\alpha B\quad\left(T<\pi\alpha T_c\frac{\delta(T)}{\delta(0)}\right),$$
$$S_T(g)\approx\alpha\ln\frac{\pi\alpha T_c^0}{T}+\frac{T}{4T_c}\frac{\delta(0)}{\delta(T)}\mathscr{J}\quad\left(\pi\alpha T_c^0>T>\pi\alpha T_c\frac{\delta(T)}{\delta(0)}\right).$$

Here A is a number; it turned out to be equal to $(144\pi)^{-1}$, i.e., this temperature correction is small; $B\sim 1$;

$$\mathscr{J}=\frac{2\sqrt{2}}{\pi^2}\int_0^{\pi}dq_1\int_0^{\pi}dq_2\frac{1-\cos q_1}{(2-\cos q_1-\cos q_2)^{1/2}}$$
$$=4\sqrt{\frac{2}{\pi}}\int_0^{\infty}dt\,e^{-2t^2}I_0(t^2)[I_0(t^2)-I_1(t^2)]\approx 1.$$

Using Eq. (3.4) the result mentioned in the preceding section follows from these formulae: $\delta(0)\approx\delta_{cl}^{1/(1-\alpha/2)}$. If $T\neq 0$ it is more convenient to solve Eq. (3.9) not for $\delta(T)$ but for the quantity $\rho(T)=\delta^2(T)/\delta^2(0)$. For $\pi\alpha T_c^0>T>\pi\alpha T_c\,\delta(T)/\gamma(0)$ this equation has the form

$$\rho(T)=\left[\frac{\delta(T)}{\delta(0)}\right]^2=\frac{\exp\{-S_T(g)\}}{\exp\{-S_0(g)\}}=\left(\frac{TM}{\pi\alpha T_c}\right)^{\alpha}\exp\left[-\frac{T}{4T_c}\mathscr{J}\rho^{-1/2}(T)\right]. \quad (4.2)$$

A similar equation appears in the plane-rotator model,[14] where it serves for the definition of the transition temperature. The fact is that the equation

$$y=\exp\{-x/y\}$$

has a non-zero solution only when $x<e^{-1}$; when $x=e^{-1}$ the solution $y(x)$ vanishes jumpwise; the magnitude of the jump is e^{-1}. Solving (4.2) we have for the critical temperature T_k:

$$T_k=T_c\left(\frac{8}{\mathscr{J}e}\right)^{1/(1-\alpha/2)}\left(\frac{M}{\pi\alpha}\right)^{\alpha/(2-\alpha)},$$
$$\Delta\rho=\left(\frac{8}{\mathscr{J}}\frac{M}{\pi\alpha}\right)^{\alpha/(1-\alpha/2)}\left(\frac{1}{e}\right)^{2/(1-\alpha/2)}. \quad (4.3)$$

Here M is a number of the order of unity, $\Delta\rho$ the discontinuity in $\rho(T)$ at the point T_k. The expression for T_k is in the case of small α parametrically the same as the results of[8]. The question of the numerical agreement of the values of T_k must be studied carefully, since a large difference of the factors would point to not too successful a choice of one of the approximations. Although the transition in our calculations looks like a first-order transition, it is more likely that this is a consequence of the approximations made when constructing the model; it may also turn out that taking the fluctuations in the order parameter density into account is important for evaluating $\rho(T)$ near T_k. The parametric agreement of the results for T_k and the presence of exact (without any numerical differences) limiting transitions as $\delta_{cl}\to 0$ in the one-dimensional results of[8], where the transition is explicitly sought as a second-order transition, convinces us that the transition in the model is more likely of second order and that no jump $\Delta\rho$ (even a very small one!) should appear in more correct calculations.

5. NATURE OF THE SPECTRUM AND FEATURES OF ODLRO IN THE MODEL STUDIED

If we examine the Hamiltonian (3.1) it becomes clear that states with the lowest energy are those states with a constant phase from filament to filament and along a filament. The classical equation of motion which would stem from (3.1) if the rules of Hamiltonian mechanics are used in the case of a single filament has been well studied: it is a sine-Gordon type of equation (see, e.g.,[15]). It describes, in particular, wave phenomena in a Josephson contact.[16] In contrast to the case studied here, the spectrum of its small oscillations starts from a limiting frequency ω_0. The appearance of an infinite number of filaments "lowers" the frequency ω_0 to zero. The spectrum of the small oscillation type excitations of the classical Hamiltonian (3.1) are phase oscillations propagating in the system of filaments. The sound velocity is anisotropic: $v_\perp/v_\parallel=\delta_{cl}$; if one extends the analogy with sound oscillations one can say that the "rigidity" of the system in the transverse direction is less than that in the lognitudinal direction by a factor δ_{cl}^2. Taking quantum effects into account (the Hamiltonian (3.3)) leads to the fact that the zero-point phase oscillations increase this anisotropy in the velocity and rigidity; now

$$v_\perp/v_\parallel=\delta(T)<\delta_{cl}.$$

The fact that $\delta(T)$ vanishes in the point T_k means that in the chosen approximation the system of filaments decays into independent filaments in the sense that the various correlation functions above T_k have purely one-dimensional properties. At the point T_k there is a transition from a state with strongly correlated order parameters (in terms of the GL equation) in different filaments and along a single filament to a state where there is no ordering. We shall show below that the quantity $\delta(T)$ is directly connected with a very general characteristic of superconductivity—the existence of ODLRO and that the vanishing of $\delta(T)$ is connected with its vanishing; this can serve as a justification (but not a proof) of the hypothesis about the purely one-dimensional nature of the correlation above T_k.[8]

The ODLRO concept was introduced in 1962 by Yang.[17] According to a theorem proved in[17] the existence of a non-vanishing limit

$$\langle\psi^+(\mathbf{x}_2)\psi^+(\mathbf{x}_1)\psi(\mathbf{x}_1')\psi(\mathbf{x}_2')\rangle\to A(T)\neq0 \tag{5.1}$$

as $\mathbf{x}_1=\mathbf{x}_2$, $\mathbf{x}_1'=\mathbf{x}_2'$, $|\mathbf{x}_1-\mathbf{x}_1'|\to\infty$ leads in a system of charged fermions to the appearance of superconductivity and the quantization of magnetic flux. The phase transition in a system with ODLRO (e.g., in the BCS model) occurs in such a way that the quantity A(T) smoothly turns to zero in the transition point.[18] In the language of the GL theory condition (5.1) can be written in the form

$$\langle\psi(\mathbf{x}_2)\psi^*(\mathbf{x}_1)\rangle\to A(T)\neq0 \quad \text{as} \quad |\mathbf{x}_1-\mathbf{x}_2|\to\infty. \tag{5.2}$$

In purely one- and two-dimensional systems (5.1) and (5.2) are not satisfied (see[19–21]). In those papers it is shown that the breaking of ODLRO occurs basically due to phase fluctuations. In our model where $|\psi|$ is fixed conditions (5.1) and (5.2) look like

$$|\psi|^2F(\mathbf{j}_\perp,z)=|\psi|^2\langle\exp i[\varphi(\mathbf{j}_\perp,z)-\varphi(0,0)]\rangle\to A(T) \tag{5.3}$$

where $\mathbf{j}_\perp^2+z^2\to\infty$. One can easily calculate such a correlator if we use H_0 to average (see[21]):

$$\langle\exp i[\varphi(\mathbf{j}_\perp,z,\tau)-\varphi(0,0,0)]\rangle=\exp\Big\{-\frac{\pi\alpha}{(2\pi)^3}\int_{-1}^{1}dq_z\int_{-\pi}^{\pi}dq_y\int_{-\pi}^{\pi}dq_x$$
$$\times\frac{1-\cos\mathbf{q}_\perp\mathbf{j}_\perp\cos q_zz}{\Omega_\mathbf{q}}\frac{\mathrm{ch}[\hbar\omega_\mathbf{q}(\beta/2-\tau)]}{\mathrm{sh}(\hbar\omega_\mathbf{q}\beta/2)}\Big\}=\exp\Big\{-\frac{T}{T_c^0}\sum_{n=-\infty}^{\infty}\frac{1}{(2\pi)^3}\int_{-1}^{1}dq_z$$
$$\times\int_{-\pi}^{\pi}dq_x\int_{-\pi}^{\pi}dq_y\frac{1-\cos\mathbf{q}_\perp\mathbf{j}_\perp\cos q_zz\cos2\pi nkT\tau}{(T/\alpha T_c^0)^2n^2+\Omega_\mathbf{q}^2}\Big\}. \tag{5.4}$$

The last form of the correlator (with $\sum_n$) is given to facilitate a comparison with Eq. (40) of[8]: it is clear that the results of[8] are obtained from (5.4) as $\delta=0$.

The correlator (5.3) which interests us is obtained from (5.4) for $\tau=0$; we can evaluate it is the appropriate temperature ranges. When $T<\pi\alpha T_c\rho^{1/2}(T)$ we have

$$F(\mathbf{j}_\perp,0,0)=\begin{cases}\delta^\alpha\exp\{\alpha A\} & (|\mathbf{j}_\perp|<\pi),\\ \delta^\alpha\exp\left\{\dfrac{\alpha}{2\pi\mathbf{j}_\perp^2}+\alpha B\right\} & \left(\dfrac{\pi\alpha T_c}{T}\rho^{1/2}(T)>|\mathbf{j}_\perp|>\pi\right),\\ \delta^\alpha\exp\left\{\dfrac{T}{2T_c}\rho^{-1/2}(T)\dfrac{1}{|\mathbf{j}_\perp|}\right\} & \left(|\mathbf{j}_\perp|>\dfrac{\pi\alpha T_c}{T}\rho^{1/2}(T)\right),\end{cases} \tag{5.5}$$

$$F(0,z,0)=\begin{cases}\exp\left\{-\dfrac{\alpha z^2}{4}\right\} & (z<1),\\ z^{-\alpha}\exp\{-\alpha C\} & \left(\dfrac{1}{\delta(T)}>z>1\right),\\ \delta^\alpha\exp\left\{\alpha\dfrac{\sin z}{z}-\alpha D\right\} & \left(\dfrac{\pi\alpha T_c^0}{T}>z>\dfrac{1}{\delta(T)}\right),\\ \delta^\alpha\exp\left\{\dfrac{2}{\pi}\dfrac{T}{T_c}\rho^{-1/2}(T)\dfrac{1}{z\delta(T)}+\dfrac{\alpha\sin z}{z}\right\} & \left(z>\dfrac{\pi\alpha T_c^0}{T}\right).\end{cases} \tag{5.6}$$

In the case $\pi\alpha T_c^0\gg T\gg\pi\alpha T_c\rho^{1/2}(T)$ we have

$$F(0,z,0)=\begin{cases}\exp\{-\alpha z^2/4\} & (z<1),\\ z^{-\alpha}\exp\{-\alpha E\} & (\pi\alpha T_c^0/T>z>1),\\ \left(\dfrac{T}{\pi\alpha T_c^0}\right)^\alpha\exp\left\{-\dfrac{z}{r_c}\right\};\ r_c=\dfrac{2T_c^0}{T} & \left(\dfrac{1}{\delta(T)}>z>\dfrac{2T_c^0}{T}\right)\\ \left(\dfrac{T}{\pi\alpha T_c^0}\right)^\alpha\exp\left\{-\dfrac{T}{2T_c}\rho^{-1/2}(T)F+\alpha\dfrac{\sin z}{z}+\right.\\ \left.+\dfrac{2}{\pi}\dfrac{T}{T_c}\rho^{-1/2}(T)\dfrac{1}{z\delta(T)}\right\} & \left(z>\dfrac{1}{\delta(T)}\right).\end{cases} \tag{5.7}$$

Here A to F are numbers of order unity. In the range $\pi\alpha T_c^0/T<z<2T_c^0/T$ and when $\pi\alpha T_c^0\gg T\gg\pi\alpha T_c\rho^{1/2}(T)$ the function $F(0,z,0)$ changes little and is approximately equal to $(T/\pi\alpha T_c^0)^\alpha$.

If we attempt to evaluate the temperature corrections to Eqs. (5.5) and (5.6) we can see that the temperature will in all calculations occur in a combination of the form $(T/\pi\alpha T_c)\rho^{-1/2}(T)$, e.g., when

$$T<\pi\alpha T_c\rho^{1/2}(T)$$

the corrections will be of the form

$$F_T(\mathbf{j}_\perp,z,0)=F(\mathbf{j}_\perp,z,0)\exp\left\{-\frac{\alpha}{2\pi}A\left(\frac{T}{\pi\alpha T_c}\right)^2\rho^{-1}(T)\right\}.$$

Analyzing the behavior of the longitudinal and transverse correlators in the case $T<\pi\alpha T_c\rho^{1/2}(T)$ we see that when the arguments tend to infinity they tend to one and the same quantity δ^α, but in different ways: the transverse correlator reaches its asymptotic value practically at once at neighboring filaments while the longitudinal correlator decreases like a power law up to values $z\approx\delta^{-1}(T)$ (in dimensional units up to $\xi_\parallel\delta^{-1}(T)$) and only afterwards the nature of the decrease changes to $\exp(1/z)$. Even though longitudinal distances less than $\xi_\parallel$ are not described by our model, the fact that the decreases of $F(0,\mathbf{j}_\perp,0)$ and of $F(0,0,z)$ for small arguments have a different character reflects the anisotropy of the problem: since $a_\perp/\xi_\parallel$ can be relatively small, the dimensions of the region of strong correlations (i.e., the region where the correlator is appreciably larger than its limiting value) in the z-direction and in the x,y-directions can be different. The correlators (5.5) to (5.7) possess the property (5.2), i.e., there is ODLRO in the system. It is clear that the vanishing of the parameter $A(T)$ (see (5.1)) and of $\delta(T)$ are connected facts which we mentioned at the beginning of this section. One sees easily the limiting transition to a purely one-dimensional case, which was analyzed in[8], in the longitudinal correlator when $\pi\alpha T_c^0\gg T\gg\pi\alpha T_c\rho^{1/2}(T)$: when $\delta=0$ the region of the exponential decrease in

(5.7) stretches to infinity; when $\delta=0$ the plateau which the correlators reach in all temperature ranges also becomes zero. When $\delta=0$ (5.6) also has the form $z^{-\alpha}$, correct for the purely one-dimensional case, in the case $T=0$ (as in the case $\delta=0$, $T<\pi\alpha T_c$ is equivalent to $T=0$).

6. EXISTENCE OF THE MEISSNER EFFECT AND ANISOTROPY OF THE PENETRATION DEPTH

In the GL scheme the existence of the Meissner effect follows from the linear connection between the current **j** and the vector potential **A**:

$$\mathbf{j}(\mathbf{x})=\frac{e\hbar}{im}(\psi_0^*\nabla\psi_0-\psi_0\nabla\psi_0^*)-\frac{4e^2}{mc}|\psi_0|^2A,$$

where ψ_0 is the solution of the GL equations. When fluctuations are taken into account this connection takes the form

$$j_\alpha(\mathbf{x})=-\frac{e^2n_s}{mc}\left\{A_\alpha(\mathbf{x})-\frac{\hbar^2n_s}{4mkT}\int d^3\mathbf{x}'\left\langle\frac{\partial\varphi(\mathbf{x})}{\partial x_\alpha}\frac{\partial\varphi(\mathbf{x}')}{\partial x_\beta'}\right\rangle A_\beta(\mathbf{x}')\right\}. \tag{6.1}$$

The convolution

$$\left\langle\frac{\partial\varphi(\mathbf{x})}{\partial x_\alpha}\frac{\partial\varphi(\mathbf{x}')}{\partial x_\beta'}\right\rangle$$

is calculated, neglecting the $|\psi|^4$ term in the GL functional. In the Fourier representation (6.1) is clearly gauge invariant:

$$j_\alpha(\mathbf{q})=-\frac{c}{4\pi\lambda_L^2}\left[\delta_{\alpha\beta}-\frac{q_\alpha q_\beta}{\mathbf{q}^2}\right]A_\beta(\mathbf{q}), \tag{6.2}$$

where $\lambda_L=(mc^2/4\pi n_s e^2)^{1/2}$ is the longitudinal penetration depth. The generalization of (6.1) and (6.2) to the anisotropic case looks as follows:

$$\frac{j_\alpha(\mathbf{x})}{c}=-\frac{e^2n_s}{m_\alpha c^2}\left\{A_\alpha(\mathbf{x})-\frac{\hbar^2n_s}{4kT}\sum_\beta\frac{1}{m_\beta}\int\mathscr{K}_{\alpha\beta}(\mathbf{x},\mathbf{x}')A_\beta(\mathbf{x}')d^3\mathbf{x}'\right\}, \tag{6.3}$$

where

$$\mathscr{K}_{\alpha\beta}(\mathbf{x},\mathbf{x}')=\left\langle\frac{\partial\varphi(\mathbf{x})}{\partial x_\alpha}\frac{\partial\varphi(\mathbf{x}')}{\partial x_\beta'}\right\rangle, \tag{6.4}$$

$$\frac{j_\alpha(\mathbf{q})}{c}=-\frac{e^2n_s}{m_\alpha c^2}\sum_\beta\left(\delta_{\alpha\beta}-\frac{q_\alpha q_\beta}{m_\beta\sum_\gamma q_\gamma^2m_\gamma^{-1}}\right)A_\beta(\mathbf{q})$$

$$=-\frac{1}{4\pi\lambda_{L\alpha}^2}\sum_\beta\left(\delta_{\alpha\beta}-\frac{q_\alpha q_\beta}{m_\beta\sum_\gamma q_\gamma^2m_\gamma^{-1}}\right)A_\beta(\mathbf{q}). \tag{6.5}$$

Here m_α is the effective mass for motion along the α-axis, $\lambda_{L\alpha}=(m_\alpha c^2/4\pi e^2n_s)^{1/2}$; clearly $\lambda_{L\alpha}/\lambda_{L\beta}=(m_\alpha/m_\beta)^{1/2}$.

The change from (6.1) to (6.2) and from (6.3) to (6.4) and (6.5) can be accomplished by changing to Fourier transforms in x and noting that

$$\langle\varphi_{\mathbf{q}}\varphi_{-\mathbf{q}}\rangle=\frac{1}{2}\frac{kT}{\mathbf{q}^2}\frac{8m}{n_s\hbar^2},$$

$$\langle\varphi_{\mathbf{q}}\varphi_{-\mathbf{q}}\rangle=\frac{1}{2}\frac{kT}{\sum_\alpha q_\alpha^2m_\alpha^{-1}}\frac{8}{n_s\hbar^2}$$

respectively, in the isotropic and anisotropic cases. All averages are performed using the usual GL functional in which one takes into account only terms which are quadratic in the deviations from the equilibrium value ψ_0. In order to obtain the magnitude of the penetration depth in the case studied we must fact the analog of Eqs. (6.3) to (6.5). We use the find that the phase occurring in (2.2) is a gauge invariant phase[16] and we separate explicitly from it the vector potential, using the standard substitution:

$$\frac{\partial\varphi}{\partial z}\to\frac{\partial\varphi}{\partial z}-\frac{2e}{\hbar c}A_z,$$

$$\varphi_j-\varphi_{j+g}\to\varphi_j-\varphi_{j+g}-\frac{2e}{\hbar c}\int_{j+g}^{j}\mathbf{A}(\mathbf{x})d\mathbf{x},$$

the last integral we replace simply by $\mathbf{g}\cdot\mathbf{A}(\mathbf{j})$. The partition function is now an explicit function of **A**. For the current we have according to[22]

$$\frac{\mathbf{j}(\mathbf{x})}{c}=-kT\frac{\delta}{\delta\mathbf{A}}\ln Z(\mathbf{A}). \tag{6.6}$$

We note that the transverse current then has the purely Josephson form, as expected. Evaluating (6.6) in the approximation which is linear in **A**, we get for the current an expression such as (6.3) with $n_s=N_s/a_\perp^2$ (the linear electron density changed to a volume density) and with an effective transverse mass $m_\perp=4\hbar^2/E_\perp a_\perp^2$; $m_\parallel=m$, the function $\mathscr{K}_{\alpha\beta}$ then has a more complicated form than (6.4): besides correlators of the form (6.4) one finds, e.g.,

$$\mathscr{K}_{xy}=\langle\sin[\varphi_j-\varphi_{j+g_x}]\sin[\varphi_n-\varphi_{n+g_y}]\rangle$$

and so on. All averages are defined as

$$\langle\hat{A}\rangle=\mathrm{Sp}\{e^{-\beta\mathscr{H}}\hat{A}\},$$

where $\mathscr{H}$ is the second-quantization Hamiltonian. In the present paper all these correlators are evaluated in the zeroth approximation in $H_{\rm int}$. One can find details of analogous calculations, e.g., in[21,23]. For instance, we have for the quantity $\tilde{\mathscr{K}}$

$$\tilde{\mathscr{K}}=\langle\sin\lambda_1[\varphi_{j_1}(z_1\tau_1)-\varphi_{j_1+g_1}(z_1\tau_1)]\sin\lambda_2[\varphi_{j_2}(z_2\tau_2)-\varphi_{j_2+g_2}(z_2\tau_2)]\rangle$$

$$=\exp\{-\lambda_1^2S_T(\mathbf{g}_1)-\lambda_2^2S_T(\mathbf{g}_2)\}\cdot$$

$$\times\mathrm{sh}\left\{\lambda_1\lambda_2\frac{\pi\alpha}{(2\pi)^3}\int_{-1}^{1}dq_z\int_{-\pi}^{\pi}dq_x\int_{-\pi}^{\pi}dq_y\,a_{12}(\mathbf{q})\frac{\Phi_{\mathbf{q}}(\tau_1-\tau_2)}{\Omega_{\mathbf{q}}}\right\}, \tag{6.7}$$

where

$$\Phi_q(\tau_1-\tau_2)=\frac{\mathrm{ch}\{\hbar\bar{\omega}\Omega_{\mathbf{q}}[\beta/2-(\tau_1-\tau_2)]\}}{\mathrm{sh}[\hbar\bar{\omega}\Omega_{\mathbf{q}}\beta/2]},$$

$$a_{12}(\mathbf{q})=\cos q_z(z_1-z_2)[\cos\mathbf{q}(\mathbf{j}_1-\mathbf{j}_2)+\cos\mathbf{q}(\mathbf{j}_1+\mathbf{g}_1-\mathbf{j}_2-\mathbf{g}_2)$$
$$-\cos\mathbf{q}(\mathbf{j}_1+\mathbf{g}_1-\mathbf{j}_2)-\cos\mathbf{q}(\mathbf{j}_1-\mathbf{j}_2-\mathbf{g}_2)].$$

From this correlator we can obtain all $\mathscr{K}_{\alpha\beta}$ by differentiation with respect to the appropriate parameters and arguments.

At large distances the argument of the sinh function in (6.7) decreases fast and we can thus obtain the Fourier transform for small q_z and $\mathbf{q}_\perp$ by expanding the sinh up to the first term. Taking the Fourier transform of (6.3) with respect to x, y, z we see that

$$\frac{j_\alpha(q)}{c}=-\frac{1}{4\pi\lambda_{L\alpha}^2}\sum_\beta\left\{\delta_{\alpha\beta}-\frac{q_\alpha q_\beta}{\bar{m}_\beta\sum_\gamma q_\gamma^2\bar{m}_\gamma^{-1}}\right\}A_\beta, \tag{6.8}$$

where $\bar{m}_z=m_\parallel=m$, $\bar{m}_x=\bar{m}_y=\bar{m}_\perp=\langle\cos(\varphi_n-\varphi_{n+g})\rangle^{-1}m_\perp$.

In our approximation $m_\perp$ is renormalized due to phase fluctuations:

$$\bar{m}_\perp = e^{S_T(g)} m_\perp = e^{S_T(g)} (E_\perp a_\perp^2/4\hbar^2)^{-1}.$$

The quantity $\bar{m}_\perp^{-1}$ vanishes at the phase transition point which corresponds to the vanishing of rigidity in the transverse direction. The expressions for the $\lambda_{L\alpha}$ have the form

$$\lambda_{Lz}^2 = \lambda_\parallel^2 = \frac{mc^2}{4\pi n_s e^2}, \qquad \lambda_{Lx}^2 = \lambda_{Ly}^2 = \lambda_\perp^2 = \frac{\bar{m}_\perp c^2}{4\pi n_s e^2}.$$

The anisotropy of the penetration depth in our case is

$$\eta^2(T) = \left(\frac{\lambda_\parallel}{\lambda_\perp}\right)^2 = \frac{m_\parallel}{m_\perp} = \frac{E_\perp a_\perp^2}{4\hbar^2} e^{-S_0(g)} \rho(T) m_\parallel \approx \left(\frac{a_\perp J_\perp}{a_\parallel \varepsilon_F}\right)^2 \delta_{c1}^{\alpha/(1-\alpha/2)} \rho(T). \tag{6.9}$$

We see that $\lambda_\perp^{-1}$ and $\lambda_\parallel^{-1}$ vanish at different points: $\lambda_\perp^{-1}$ at T_k, and $\lambda_\parallel^{-1}$ at T_c^0. This difference is purely formal in character: looking at Eq. (6.8) we see that when $\bar{m}_\perp$ becomes infinite the expression within braces vanishes for any **q** so that the Meissner effect disappears simultaneously in both directions as $T \to T_k$.

7. DISCUSSION OF THE RESULTS

All calculations of this paper are, in fact, based upon the Hamiltonian (3.1). The reliability of the approximations used in its derivation has so far not been thoroughly elucidated (see the earlier footnote). However, (3.1) contains very important features of a quasi-one-dimensional system with a complex order parameter so that we can forget its origin (thereby disregarding the problem of the degree of adequacy of its starting model) and simply consider (3.1) as a convenient model Hamiltonian which is amenable to a correct discussion provided $(T_k - T)/T_k$ is not too small.

Larkin and Efetov[8] studied only the case $T > T_k$ when the phases on different filaments are no longer coherent. However, this turned out to be sufficient to find an expression for T_k taking into account the destructive role of the phase fluctuations near T_k. In our paper we succeeded to construct a rather complete description of the model for $T < T_k$. As far as we know this was the first time this was done for a quasi-one-dimensional model with a complex order parameter.

1. We studied the spectrum of low-lying collective excitations—the "sonic" phase oscillations. The transverse rigidity in such a system tends to zero as $T \to T_k$. It is, strictly speaking, not known how this happens—discontinuously or smoothly. In the framework of the variant of the self-consistent field method used in the paper we found that the jump $\Delta\rho$ was equal to e^{-1}, i.e., small (this corresponds to a first-order phase transition which is close to a second-order one) but it is fully possible that a more exact calculation gives a smooth vanishing of $\rho(T)$ as $T \to T_k$ (second-order phase transition).

2. First of all we found the actual form of the correlation functions in quasi-one-dimensional superconductors for $T < T_k$ and studied the law according to which they reach saturation. This limiting value corresponds to the establishment of isotropic ODLRO. It shows a maximum permissible degree of phase coherence in such a system and must be determined, taking quantum phase fluctuations into account. The correlation lengths (over which ODLRO is established) along and at right angles to the filaments in such a system differ by many orders:

$$\frac{l_\perp}{l_\parallel} = \frac{a_\perp}{\xi_\parallel/\delta} = \frac{a_\perp \delta(T)}{0.25\hbar v_{cp}/kT_c^0} \approx \frac{a_\perp J_\perp}{a_\parallel J_\parallel} \rho^{1/2}(T) \left(\frac{J_\perp}{kT_c^0}\right)^{\alpha/(1-\alpha/2)} \ll 1.$$

An interesting conclusion is that as $T \to T_k$ both the long-range and the short-range order vanish, i.e., as $T \to T_k$

$$\langle\cos[\varphi_0 - \varphi_1]\rangle \to 0, \qquad \langle\cos[\varphi_0 - \varphi_{l_\perp}]\rangle \xrightarrow[|l_\perp| \to \infty]{} 0.$$

It is more likely that this conclusion is not merely a consequence of the inaccuracy of the self-consistent field method, but corresponds to the physics of the breaking of phase coherence in such a model (see below).

3. We studied the Meissner effect in a weak magnetic field. It vanishes not at $T = T_c^0$, where there must occur a smeared-out peak in the heat capacity,[9] but at $T = T_k$ where the coherence of the phases in different filaments vanishes. (It is possible that there appears a second peak corresponding to the fact that ODLRO is broken at $T = T_k$.) The Meissner effect is strongly anisotropic in such a system:

$$\left(\frac{\lambda_\perp}{\lambda_\parallel}\right)^2 = \eta^{-2}(T) = \frac{m_\perp}{m_\parallel} \exp\{S_T(g)\} \approx \left(\frac{a_\parallel J_\parallel}{a_\perp J_\perp}\right)^2 \exp\{S_T(g)\}.$$

It is clear that the penetration depth of the magnetic field into a sample made from such a quasi-one-dimensional substance will depend strongly on the angle between the magnetic field H and the direction of the filaments. Let, for example, the surface of the sample be ground parallel to the direction of the filaments and the magnetic field be directed such that the magnetic field lines are parallel to the surface of the sample (they may then be at an angle to the filaments). If H is at right angles to the filaments (but parallel to the surface of the sample) the penetration depth is $\lambda_\parallel = (4\pi n_s e^2/mc^2)^{-1/2}$ and has the normal value of 10^{-5} to 10^{-6} cm. This is connected with the fact that the superconducting currents screening this field must flow along the filaments, i.e., the conditions are hardly different from those for a bulk sample. If, however, H is parallel to the filaments, the screening current must flow from filament to filament, i.e., the conditions widely differ from those in the case of a bulk sample, and the penetration depth is

$$\lambda_\perp = \lambda_\parallel \eta^{-1} \gg \lambda_\parallel.$$

If the coefficient η were equal to $m_\parallel/m_\perp$, we could, notwithstanding the fact that the masses $m_\parallel$ and $m_\perp$ have different physical origins ($m_\perp$ is caused by the Josephson nature of the coupling between the phases in neighboring filaments) and that the expressions for them are different:

$$m_\parallel^{-1} = \frac{J_\parallel a_\parallel^2}{\hbar^2}, \qquad m_\perp^{-1} \approx \frac{J_\perp}{\hbar^2} a_\perp^2 \frac{J_\perp}{J_\parallel},$$

talk about a certain analogy with the normal anisotropic Meissner effect. The additional factor $\exp(-S_T)$ in $\eta(T)$ (see (6.69)) is non-trivial; it reflects the degree of phase coherence in different filaments and vanishes at $T = T_k \ll T_c^0$. Generally speaking, the paramagnetic term in $\mathbf{j}$ is determined by the contribution to the correlators $\mathscr{K}_{\alpha\beta}$ at long distances (the singularity of the form

$$q_\alpha q_\beta \left(\sum_\gamma q_\gamma^2 \bar{m}_\gamma^{-1}\right)^{-1}$$

in (6.8) is connected with just this point) and $\mathbf{j}_{\rm para}$ might thus, in contrast to $\mathbf{j}_{\rm dia}$, be proportional to the factor $\exp\{-S_T(\infty)\}$ which describes ODLRO in the system. However, it follows from the form of Eqs. (6.3), the condition of gauge invariance, and the current conservation law div $\mathbf{j} = 0$, i.e., $\mathbf{q}\cdot\mathbf{j} = 0$, that these factors must be the same in $\mathbf{j}_{\rm para}$ and $\mathbf{j}_{\rm dia}$ and must simultaneously vanish as $T \to T_k$.

4. All results described above were obtained for an infinite sample; for the solution of the problem of the penetration depth in an actual sample of finite dimensions the boundary conditions are very important.

5. All results of the present paper were obtained, taking into account dynamic fluctuation effects (quantal phase fluctuations), the intensity of which is characterized by the magnitude of the dimensionless parameter α (see[8]), e.g.,

$$T_k^{\rm cl} = A T_c^0 \delta_{\rm cl} = \sqrt{2AJ_\perp/k}, \quad T_k^{\rm cl}/T_k^{\rm cl} = (J_\perp/kT_c^0)^{\alpha/(2-\alpha)}, \qquad (7.1)$$

$$\lambda_\perp^{\rm qu}/\lambda_\perp^{\rm qu} = \exp\{-1/2S_T(g)\} \approx (J_\perp/kT_c^0)^{\alpha/(2-\alpha)}\rho^{-1/2}(T).$$

Here $A \sim 1$; $A = 2$ according to Ref. 8, and $A \approx 8/e$ according to the estimates (4.3) of the present paper (carried out in the spirit of Ref. 14). We achieve a correct description of the quantum corrections by directly including in (2.2) contributions of the form $\mathscr{K}_{ij}\dot{\varphi}_i\dot{\varphi}_j$ and by correctly splitting off the terms which are quadratic in $b_\mathbf{q}$ and $b_\mathbf{q}^+$ (see Sec. 3). We remember, however, that spatial and temporal dispersion effects in $\mathscr{K}_\mathbf{q}$ have not been completely taken into account so that the results obtained refer rather to the model Hamiltonian (3.1) than to the original physical system (see earlier footnote).

We were able to obtain all results given here thanks to a felicitous choice of the zeroth approximation in which the effect of the fluctuations on the form of the spectrum was already taken into account (the transverse rigidity and, hence, also the velocity of the collective oscillations tend to zero as $T \to T_k$).

APPENDIX

We can exactly split off from the Hamiltonian (3.1) the diagonal part[21]:

$$\mathscr{H} = \mathscr{H}_d + \mathscr{H}_{nd}$$

$$\mathscr{H}_d = \sum_q \frac{\hbar\bar{\omega}}{2}\left[\Omega_\mathbf{q} + \frac{q_z^2}{\Omega_\mathbf{q}}\right]\left(b_\mathbf{q}^+ b_\mathbf{q} + \frac{1}{2}\right) - \frac{\hbar\bar{\omega}}{2}\delta_{\rm cl}^2 \frac{1}{2\pi\alpha} \times \sum_\mathbf{g} N \exp\left\{-\frac{2\pi\alpha}{N}\sum_\mathbf{q}\frac{1-\cos\mathbf{qg}}{\Omega_\mathbf{q}}\left(\hat{N}_\mathbf{q} + \frac{1}{2}\right)\right\},$$

$$\mathscr{H}_{nd} = -\frac{\hbar\bar{\omega}}{4\pi\alpha}\delta_{\rm cl}^2\left\{\int dz \sum_{i,\mathbf{g}}\left[\prod_\mathbf{q}\exp\left(i[A_\mathbf{q} b_\mathbf{q}^+ + A_\mathbf{q}^* b_\mathbf{q}]\frac{1}{\sqrt{N}}\right) + \prod_\mathbf{q}\exp\left(-i[A_\mathbf{q} b_\mathbf{q}^+ + A_\mathbf{q}^* b_\mathbf{q}]\frac{1}{\sqrt{N}}\right)\right] - N\sum_\mathbf{g}\exp\left\{-\frac{2\pi\alpha}{N}\sum_\mathbf{q}\frac{1-\cos\mathbf{qg}}{\Omega_\mathbf{q}}\left(\hat{N}_\mathbf{q} + \frac{1}{2}\right)\right\}\right\}$$
$$-\frac{\hbar\bar{\omega}}{2}\delta_{\rm кл}^2\sum_\mathbf{g}\exp\{-S_T(\mathbf{g})\}\sum_\mathbf{q}\frac{1-\cos\mathbf{qg}}{\Omega_\mathbf{q}}(b_\mathbf{q}^+ b_{-\mathbf{q}}^+ + b_\mathbf{q} b_{-\mathbf{q}}). \qquad (A.1)$$

The approximation given by us (see section 3) corresponds to neglecting $\mathscr{H}_{nd}$ and expanding the exponent in (A.1); see[21] for a more detailed derivation of $\mathscr{H}_d$ and $\mathscr{H}_{nd}$ and also for a discussion of the approximation.

[1)]The approximation $\mathscr{K}_\mathbf{q}$ constant, which was made in [8] enables us to describe the electrostatic induction between filaments, including Coulomb screening, rather correctly. However, the quantity $C_{ij}(z)$ decreases weakly with increasing z even when we take into account the screened interaction between electrons at other filaments, and this may lead to singularities (most likely logarithmic) in $\mathscr{K}_\mathbf{q}$ for small $q_\perp$, q_z. Moreover, it is not excluded that taking the Coulomb effects into account may lead to more radical consequences. We recall that the plasma spectrum of such a quasi-one-dimensional model in the normal state has an anisotropic gap which vanishes in the perpendicular direction. [6] This means that in a quasi-one-dimensional superconductor (in the phase system) the collective branches of the spectrum may intersect. In principle this may lead to the formation of mixed modes which are neglected in the model considered. If $\mathscr{K}_\mathbf{q} \neq$ const (but finite as $|\mathbf{q}| \to 0$) the form of Eqs. (5.5), (5.6) is qualitatively conserved. For the existence of ODLRO it is sufficient that $\mathscr{K}_\mathbf{q} < q^{-a}$ as $q \to 0$, where a is an arbitrary number, less than unity.

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Energy spectrum of acceptors in germanium and its response to a magnetic field

E. M. Gershenzon, G. N. Gol'tsman, and M. L. Kagane

V. I. Lenin State Pedagogical Institute
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We investigated the spectrum of the submillimeter photoconductivity of p-Ge at helium temperatures and the effects of a magnetic field up to 40 kOe on the spectrum. A large number of lines of transitions between the excited states of the acceptors was observed, some of the lines were identified, and the energies of a number of spectral levels B, Al, Ga, In, and Tl in Ge were identified. The results are compared with calculations and with experimental data obtained from the spectra of the photoexcitation of the ground state of the impurities. Using one transition as an example, we discuss the splitting of the excited states of acceptors in the magnetic field and under uniaxial compression.

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INTRODUCTION

Besides the study of the energy spectrum of donors in semiconductors,[1] great interest attaches to an investigation of shallow acceptors. This, however, is a more difficult task both theoretically and experimentally. The calculation of the acceptor spectrum in Ge by the effective-mass method[2–5] and the study of the effect of a magnetic field and of uniaxial deformation of the sample on this spectrum within the framework of perturbation theory[6–12] is quite complicated. It has been carried out for a limited number of states and yields less reliable results than in the case of donors (e.g.,[13–15]). A group-theoretical analysis of the influence exerted on the spectrum by a magnetic field[16] and by uniaxial deformation (F)[17] yields new information and a number of exact results, but is not sufficient.

The spectrum of acceptors of group III in Ge was experimentally investigated with long-wave infrared grating spectrometers with registration of the absorption of the radiation,[12,18–22] including studies in the presence of a field H and a deformation F and photoconductivity,[23,24] and also with Fourier-transformation spectrometers with registration of the photoconductivity.[25,26] Transitions from the ground state were investigated, and some of them, to the nearest excited state, could be identified with those calculated. The resolution and sensitivity of the spectrometers used in[12,20–22] were adequate for a detailed investigation of the Zeeman and piezosplitting of only an insignificant number of the spectral lines and under sufficiently strong perturbations. At the same time, the theory is applicable most fully only in the region of small perturbations.

The purpose of the present work was to study the energy spectrum of shallow acceptors in Ge, the effect exerted on this spectrum by a magnetic field, and in a number of cases the effect of uniaxial compression, by using a sensitive high-resolution submillimeter spectrometer based on backward-wave tubes (BWT). We first investigated several series of transitions between excited states of acceptors, but unfortunately, the short-wave limit of the employed spectrometer ($\lambda \approx 250$ μm) did not make it possible to study the spectrum of the transitions from the ground state. In view of the limitations of the theory the most detailed investigations of the Zeeman effect and of the effect of uniaxial compression were restricted to line splitting under small perturbations. The measurements were performed by determining the photoconductivity due to photothermal ionization[27,28] of the excited states of the impurity.

EXPERIMENTAL CONDITIONS AND PROCEDURE

The photoconductivity spectra of the acceptors in Ge were measured mainly in the same way as those of the donors.[1] However, the more complicated character of the acceptor center, and the fact that it has been less thoroughly studied, led to a number of modifications of the measurement procedure. The following factors become significant: the determination of the spectrum of the excited states at $H=0$ and its response to uniaxial compression, the study of the Zeeman effect following compression of the sample, the determination of the relative intensities of the Zeeman and piezoelectric components when various polarizations of the radiation are used, and the measurement of the anisotropy of the Zeeman effect.

