Nonleptonic decays of K mesons and hyperons

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Nonleptonic decays of ordinary hadrons are considered within the framework of the renormalizable vector schemes for weak interactions. An asymptotically free theory is put forward for strong interactions. All the independent operators in the effective Hamiltonian are enumerated, and expansion coefficients which take into account strong interactions at short distances are determined. The valence quark model is used to estimate the matrix elements of the operators. Transitions with $\Delta T = 3/2$ are evaluated in terms of a single parameter. The large magnitude of the amplitudes for $\Delta T = 1/2$ transitions is explained in terms of an operator producing the product of two scalars (or pseudoscalars), which is due to the presence of strong interaction in any model. This explanation rests on the assumption of small bare quark masses, which characterize violations of chiral symmetry. A number of relationships, which agree with experiment, are obtained. Estimates show that the introduction of right-handed currents is not essential for nonleptonic decays.

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1. INTRODUCTION

Substantial progress has recently been achieved in the theory of nonleptonic weak interactions. Thus, it has been shown that strong-interaction effects at short distances can be calculated within the framework of asymptotically free theories of the quark-gluon interaction. In particular, it turns out that the strong interaction enhances the $\Delta T = 1/2$ part of the Hamiltonian and reduces the $\Delta T = 3/2$ part.[1] Secondly, convincing evidence has been obtained for the existence of new charmed particles, and there have been various speculations about the possible role of these new particles and (or) new currents in the explanation of the $\Delta T = 1/2$ rule.[2–4]

Roughly speaking, analysis of nonleptonic weak decays is carried out in three stages. It starts with some some definite assumptions about the structure of the bare Hamiltonian. The next step is to modify this Hamiltonian to take into account strong-interaction effects at short distances. This results in the so-called effective weak interaction Hamiltonian. Finally, in the third stage, some scheme has to be used to evaluate the matrix elements of the effective Hamiltonian in order to obtain information about the relative contributions of the different operators to physical amplitudes. Unfortunately, there has been very little progress in recent years in the techniques available for the evaluation of matrix elements and, therefore, the $\Delta T = 1/2$ problem is currently tackled either by using very approximate estimates of the matrix elements or the problem is not considered at all.

Our aim in this paper is to obtain the most general form of the effective Hamiltonian for nonleptonic decays of ordinary hadrons, and to estimate the corresponding matrix elements within the framework of the simple quark model. In our analysis of the effects occurring at short distances, we use the standard scheme in which the strong interaction is connected with the exchange of an octet of massless gluons interacting with the color degrees of freedom of the quarks (this is the so-called quantum chromodynamics).

It is important to emphasize that the entire variety of models involving new quarks and new currents can be reduced to a limited number of operators in the effective Hamiltonian for weak decays of ordinary hadrons, i.e., hadrons made up of u, d, and s quarks. The point is that we are not considering operators of the form $(\bar{c}c)(\bar{s}d)$, where c is a charmed quark. In fact, since charmed quarks have a large mass, they can appear in ordinary hadrons only for a short time and at short distances. Interactions at short distances, on the other hand, can be completely taken into account in asymptotically free theories because the quark-gluon coupling constant is small for large virtual momenta.[5] An indirect experimental confirmation of the latter proposition is the small width of the decays of ψ particles (provisionally identified as bound states of heavy quarks[6]) into ordinary hadrons.

Thus, within the framework of the approach which we have adopted, heavy quarks can be seen in the decay of ordinary hadrons only in closed loops, for example, as indicated in Figs. 1 and 2, and the contribution of these loops is taken into account in the coefficients of the expansion of the effective Hamiltonian in terms of operators constructed from light quark and gluon fields. In other words, we are assuming zero admixture of heavy quarks to the wave function for ordinary hadrons because, by definition, the wave function describes the components with relatively small virtuality, $p \lesssim m$, where m is a characteristic hadronic mass.

It will be shown below that the total number of operators in the effective Hamiltonian with $\Delta S = 1$ is seven when certain definite requirements are imposed on the commutator of the Hamiltonian and the axial charge. These requirements have been verified for nonleptonic decays of K mesons.

We shall calculate the strong-interaction effects at short distances in all cases. These calculations depend on the isolation of the contribution of the two regions $m^{-1} > x > m_c^{-1}$ and $m_c^{-1} > x > m_W^{-1}$, where m_W and m_c are the masses of the W boson and the c quark, and m is the

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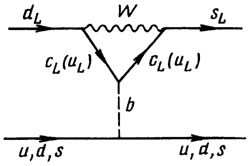

FIG. 1.

characteristic hadronic mass. We shall suppose, for simplicity, that the masses of all the new quarks are of the order of m_c since, otherwise, a larger number of regions will have to be isolated.

For $m^{-1} > x > m_c^{-1}$, the calculation reduces to the determination of the matrix of anomalous dimensionalities of the above operators. For $m_c^{-1} > x > m_W^{-1}$, one must include operators containing the heavy-quark fields since they mix with operators that do not contain such fields. It is precisely in this way that the existence of new quarks can be seen in the magnitude of the coefficients in the effective Hamiltonian. It is important to note that an increase in the number of quarks has little effect on the coefficients.

To evaluate the matrix elements of the operators in the effective Hamiltonian, we shall use what might be called the valence quark approximation. We shall suppose that the dominant contribution to the meson wave function is that due to the valence quark-antiquark pair, whereas the dominant contribution to the baryon wave function is that due to three valence quarks. However, we shall take into account (phenomenologically) all those gluon exchanges that are connected with the retention of the quarks.

Our aim is to combine the evaluation of the strong-interaction effects at short distances with the quark model for the description of large distances. This procedure cannot be exact mainly because, to estimate the coefficients, we extrapolate expressions valid for small coupling constants right up to values $g^2/4\pi = 1$. This is why the true values of the coefficients in the effective Hamiltonian may differ from the calculated values. However, the prior onset of scaling in electroproduction, and of the Zweig rule for φ and ψ decays, suggest to us that this discrepancy is not very large and cannot substantially exceed, say, 50%. In fact, if the necessary correction were to be found to be large, this would be evidence for the failure of our model.

We shall show that, in the approximation which we have adopted, the matrix elements of certain operators in the effective Hamiltonian for nonleptonic interactions reduce to the product of known amplitudes for leptonic decays and, in this sense, can be evaluated. In particular, all the $\Delta T = 3/2$ transitions in the decays of K mesons and hyperons reduce to the product of matrix elements of currents and can be expressed in terms of one parameter. The resulting predictions are in agreement with the sign of the experimental data on $K \to 2\pi$ decays and s-wave amplitudes for hyperon decays. More detailed verification will require better experimental data.

The operator producing $\Delta T = 3/2$ transitions contains only the left-handed components of the fermion field $\frac{1}{2}(1+\gamma_5)\psi$. Operators with $\Delta T = 1/2$ contain both left- and right-handed components of the Fermi fields. In the valence quark approximation, the matrix elements of these operators can be expressed in terms of the matrix elements of the densities $\bar{q}q$, $\bar{q}\gamma_5 q$. Using the equations of motion, these densities can be expressed in terms of the divergences of currents and the bare quark masses. We shall use the masses proposed in[7,8], namely, $m_u = m_d = 5.4$ MeV and $m_s = 150$ MeV.

We emphasize that the parameters m_u, m_d, and m_s cannot be identified with the physical masses of the quarks. They appear in the evaluation of the divergences of currents and thus characterize the violation of chiral symmetry. The relationships which we shall use are also valid for interacting fields and, in particular, the interaction may result in the spontaneous violation of symmetry and the appearance of a meson of small mass, whereas the quark becomes relatively heavy even in the limit as $m_q \to 0$.

When the quark masses are, in fact, small, the $\Delta T = 1/2$ rule is due to the contribution of operators containing both left-handed and right-handed components of the Fermi fields. The possibility of this explanation was discussed in our previous paper.[9] Here, we report more detailed calculations of the amplitudes for nonleptonic decays of K mesons and hyperons. In particular, a new relationship will be established between the observed amplitudes for hyperon decays. It relates the s and p waves and is in good agreement with experimental data. By normalizing the corresponding coefficient in the effective Hamiltonian to the absolute value of, say, the s wave, we can determine, using certain additional assumptions, the probability of $K \to 2\pi$ decay. The result of this is in agreement with the experimental value.

We shall also consider some alternative explanations of the $\Delta T = 1/2$ rule, put forward in[2-4] and assuming the existence of right-handed currents connecting light and heavy quarks. Within the framework of our approach, the corresponding contribution to the amplitudes turns out to be small.

Our plan is as follows. In the second section, we list all the operators which can appear in the effective Hamiltonian. The expansion coefficients which take into account strong interactions at short distances are found in Sec. 3. Section 4 discusses the $\Delta T = 3/2$ transitions in the decay of K mesons and hyperons. We then consider the $\Delta T = 1/2$ amplitudes in the decays of hyperons (Sec. 5) and K mesons (Sec. 6). Finally, in Sec. 7 we estimate the matrix elements of the operator representing the introduction of right-handed currents into the theory.

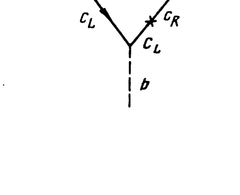

FIG. 2.

2. GENERAL STRUCTURE OF EFFECTIVE HAMILTONIAN

In this section, we enumerate the operators that can appear in the effective Hamiltonian for nonleptonic decays. We begin by considering the conditions which the Hamiltonian must satisfy. We shall suppose that it commutes with the $V^i - A^i$ generators of the group $SU(2)_L \times SU(2)_R$:

$$[V^i - A^i, H^w] = 0 \quad (i = 1, 2, 3). \tag{1}$$

This relation, taken together with the hypothesis of partial conservation of axial current (PCAC), leads to well known relationships for $K \to 2\pi$, 3π decays, which are known to be confirmed experimentally both for $\Delta T = 1/2$ and $\Delta T = 3/2$ transitions. Condition (1) means that the right-handed components of u and d quarks $[q_{L,R} = \frac{1}{2}(1 \pm \gamma_5)_q]$ can appear in the Hamiltonian only in a combination with zero isotopic spin. Insofar as the strange quark is concerned, the Hamiltonian can contain both its left-handed and right-handed components. In addition to the quark fields, there may also be the gluon field b^a ($a = 1, \ldots, 8$), and the Hamiltonian should be invariant under gauge transformations of gluon fields.

We note that, when the Hamiltonian is constructed, we may omit operators appearing in the form of total derivatives because such operators do not contribute to the matrix elements of real physical processes.

Following these preliminary remarks, let us now examine systematically the possible operators which we shall characterize by the number of quark fields which they include. It is readily seen that the dimensionality d of bilinear operators is greater than four. In fact, the only operators of dimensionality 3 and 4 are the following:

$$\bar{s}_R d_L, \quad i\bar{s}_R \gamma_\mu D_\mu d_L, \tag{2}$$

where D_μ is the covariant derivative given by $D_\mu = \partial_\mu - \frac{1}{2} i g t^a b^a_\mu$, and t^a are the Gell-Mann matrices acting in color space and normalized by $\mathrm{Sp}(t^a t^b) = 2\delta^{ab}$. It is show shown in[10] that such operators are removed by redefining the fields d and s and by renormalizing the parameters in the Lagrangian. They need not, therefore, be considered here.

The only operator of dimensionality 5 has the form

$$i b_{\mu\nu}{}^a \bar{s}_R \sigma_{\mu\nu} t^a d_L, \tag{3}$$

where $b^a_{\mu\nu}$ is the gluon field tensor

$$b_{\mu\nu}{}^a = \partial_\mu b_\nu{}^a - \partial_\nu b_\nu{}^a + g C^{abc} b_\mu{}^b b_\nu{}^c \tag{4}$$

and C^{abc} are the structure constants of the group $SU(3)'$, $[t^a, t^b] = 2iC^{abc}t^c$. We note that the operator given by (3) satisfies the $\Delta T = 1/2$ rule and is a component of the $SU(3)$ octet. The analogous operator made up of the fields S_L and d_R need not be included because it does not satisfy (1).

This exhausts the list of operators that are bilinear in the quark fields. For example, the operator $\bar{s}_R D_\mu D_\mu d_L$, which has the dimensionality of 5, contains two derivatives and can be reduced to $m_d^2 \bar{s}_R d_L$ with the aid of the equations of motion. An analogous procedure can be applied to the more complicated operators of dimensionality 6, which take the form

$$\bar{s} D_\mu D_\nu D_\gamma \Gamma_{\mu\nu\gamma} d, \quad \bar{s} D_\mu b_{\nu\gamma}{}^a t^a \Gamma_{\mu\nu\gamma}' d,$$

where $\Gamma_{\mu\nu\gamma}$ is made up of $g_{\mu\nu}$, $\varepsilon_{\mu\nu\gamma\delta}$, and γ matrices. In fact, using the equations of motion and the relation

$$(D_\mu D_\nu - D_\nu D_\mu)\psi = -{}^1/_2 i g b_{\mu\nu}{}^a t^a \psi \tag{5}$$

we can reduce these operators either to one of those considered above or to four-fermion operators which we shall now consider.

Let us first take the four-fermion operators constructed exclusively from left-handed fields:

$$\begin{aligned}
O_1 &= \bar{s}_L\gamma_\mu d_L \bar{u}_L \gamma_\mu u_L - \bar{s}_L \gamma_\mu u_L \bar{u}_L \gamma_\mu d_L \quad (\{8_f\}, \Delta T = {}^1/_2),\\
O_2 &= \bar{s}_L\gamma_\mu d_L \bar{u}_L \gamma_\mu u_L + \bar{s}_L \gamma_\mu u_L \bar{u}_L \gamma_\mu d_L + 2\bar{s}_L\gamma_\mu d_L \bar{d}_L \gamma_\mu d_L\\
&\quad + 2\bar{s}_L\gamma_\mu d_L \bar{s}_L \gamma_\mu s_L \quad (\{8_d\}, \Delta T = {}^1/_2),\\
O_3 &= \bar{s}_L\gamma_\mu d_L \bar{u}_L \gamma_\mu u_L + \bar{s}_L \gamma_\mu u_L \bar{u}_L \gamma_\mu d_L + 2\bar{s}_L\gamma_\mu d_L \bar{d}_L \gamma_\mu d_L\\
&\quad - 3\bar{s}_L\gamma_\mu d_L \bar{s}_L \gamma_\mu s_L \quad (\{27\}, \Delta T = {}^1/_2),\\
O_4 &= \bar{s}_L\gamma_\mu d_L \bar{u}_L \gamma_\mu u_L + \bar{s}_L \gamma_\mu u_L \bar{u}_L \gamma_\mu d_L - \bar{s}_L\gamma_\mu d_L \bar{d}_L \gamma_\mu d_L, \quad (\{27\}, \Delta T = {}^3/_2).
\end{aligned} \tag{6}$$

The properties of the operators in relation to the unitary and isotopic spin are indicated in parentheses. Color indices are omitted throughout, for example, the product $\bar{s}_L \gamma_\mu d_L \bar{u}_L \gamma_\mu u_L$ represents, in fact, $\bar{s}_{Li} \gamma_\mu d^i_L \bar{u}_{Lj} \gamma_\mu u^j_L$, where i, j run through the values 1, 2, 3.

Operators containing right-handed fields and satisfying condition (1) have the form

$$\begin{aligned}
O_5 &= \bar{s}_L \gamma_\mu t^a d_L (\bar{u}_R \gamma_\mu t^a u_R + \bar{d}_R \gamma_\mu t^a d_R + \bar{s}_R \gamma_\mu t^a s_R) \quad (\{8\}, \Delta T = {}^1/_2),\\
O_6 &= \bar{s}_L \gamma_\mu d_L (\bar{u}_R \gamma_\mu u_R + \bar{d}_R \gamma_\mu d_R + \bar{s}_R \gamma_\mu s_R) \quad (\{8\}, \Delta T = {}^1/_2),\\
O_7 &= \bar{s}_L \gamma_\mu d_L \bar{s}_R \gamma_\mu s_R, \qquad O_8 = \bar{s}_L \gamma_\mu t^a d_L \bar{s}_R \gamma_\mu t^a s_R,\\
O_9 &= \bar{s}_R d_L \bar{s}_R s_L, \qquad O_{10} = \bar{s}_R t^a d_L \bar{s}_R t^a s_L.
\end{aligned} \tag{7}$$

Any other four-fermion operator satisfying condition (1) can be reduced to a linear combination of O_1–O_{10} with the aid of the field transformations.

It is important to emphasize that the operators O_7–O_{10} cannot appear in a realistic weak-interaction model. In fact, on the one hand, these operators consist of fields with the same charge and, on the other, the operators O_7–O_{10} cannot appear in a bare Hamiltonian which is a product of currents because there are not neutral currents with a change of strangeness. They could appear only when gluon exchanges of the form shown in Fig. 1 were to be taken into account. However, since gluons are singlets with respect to the unitary group, all operators connected with such annihilation graphs must be $SU(3)$ octets. On the other hand, the operators O_7–O_{10} are not of this kind, and an octet component cannot be taken out of them without violating (1). This is why we shall not consider the operators O_7–O_{10}.

We have enumerated all operators of dimensionality $d = 6$. Operators of higher dimensionality enter the effective Hamiltonian with coefficients proportional to m^2/m_W^2 or m^2/m_c^2 and their contribution is, therefore, negligible.

Thus, the matrix elements for the decay of strange particles are, in general, determined by seven operators, namely: six four-fermion operators O_1–O_6 and the "magnetic moment" type operator introduced in (3). Since the last operator is multiplied by m_c, it is convenient to introduce the mass of the charmed quark directly into the definition:

$$T = i m_c \bar{s}_R \sigma_{\mu\nu} t^a d_L b_{\mu\nu}^a. \tag{8}$$

The effective Hamiltonian then takes the form

$$H^{eff}(\Delta S=1) = \sqrt{2} G \sin\theta_c \left\{ \sin\varphi\, c_T T + \cos\theta_c \sum_{i=1}^{6} c_i O_i \right\}, \tag{9}$$

where G is the Fermi constant, c_T, c_i ($i = 1, \ldots, 6$) are coefficients, and θ_c and φ are the mixing angles defining the form of the left- and right-handed currents:

$$j_\mu^L = \bar{u}_L \gamma_\mu (\cos\theta_c d_L + \sin\theta_c s_L) + \bar{c}_L \gamma_\mu (-\sin\theta_c d_L + \cos\theta_c s_L) + \ldots.$$
$$j_\mu^R = \bar{c}_R \gamma_\mu s_R \sin\varphi + \ldots \tag{10}$$

The dots in (10) represent other possible terms with heavy quarks which, in general, either do not contain the fields d, s or contain them in the form, say $\bar{\tilde{c}} d$, where $\tilde{c}$ is the combination orthogonal to the field c. [1)]

In conclusion, we note that the operator T will appear in models with only left-handed currents but it will then have a small coefficient (see the next section). This contribution is not taken into account in (9).

3. STRONG INTERACTION AT SHORT DISTANCES AND COEFFICIENTS OF THE OPERATOR EXPANSION

In the last section, we enumerated all the possible operators that predominate in the Wilson expansion and form the effective Hamiltonian. Here, we shall consider the influence of the strong interaction at short distances on the coefficients c_{1-6}, c_T that define the relative contribution of the various operators. The procedure for evaluating the coefficients is as follows. The first step is to determine them for the bare Hamiltonian. The gluon corrections are then introduced in the lowest order in the quark-gluon constant g. Summation of the leading logarithmic terms in all orders of perturbation theory is carried out with the aid of the renormalization group, as applied to the coefficients in the Wilson expansion. [11,12] Let us consider some of the relationships. The equations of the renormalization group for a column c of coefficients c_i are

$$\left[\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g} - \gamma(g)\right] c\left(g, \frac{m_W}{\mu}\right) = 0, \tag{11}$$

where μ is the normalizing parameter, $\beta(g)$ is the Gell-Mann–Low function, and $\gamma(g)$ is the matrix of anomalous dimensionalities. The solution of (11) can be written in the form

$$c\left(g, \frac{m_W}{\mu}\right) = G \exp\left[\int_{\bar{g}(m_W)}^{\bar{g}(\mu)} dg \frac{\gamma(g)}{\beta(g)}\right] c(\bar{g}(\mu), 1), \tag{12}$$

where $\bar{g}$ is the effective charge and we have introduced the matrix ordering operation

$$G\{\gamma(g_1)\gamma(g_2)\} = \begin{cases} \gamma(g_1)\gamma(g_2) & g_1 > g_2 \\ \gamma(g_2)\gamma(g_1) & g_2 > g_1 \end{cases}.$$

Equation (12) is valid only for $\mu > m_c$, for which we can neglect all masses and find the coefficients c_i for $\mu \sim m_c$ with logarithmic precision. We note that operators containing heavy-quark fields are important for $\mu > m_c$.

Our problem is to determine the coefficients of the operators containing only the light-quark fields at the point $\mu = m \ll m_c$. These quantities satisfy (11) (subject to the substitution $m_W \to m_c$) but, when the functions $\beta(g)$ and $\gamma(g)$ are evaluated for $\mu < m_c$, we need not include diagrams containing the heavy quark lines. The solution is then given by (12) with $m_W \to m_c$, and the initial data for $\mu = m_c$ are found for $\mu > m_c$, as explained above.

The evaluation of the coefficients c_{1-6} in the Weinberg–Salam model with u, d, s, and c quarks was considered in our previous paper. [9] Here, we shall generalize these results and show that they apply to a broad class of models. The inclusion of new heavy quarks requires only minor modifications.

When the strong interaction is turned off, the Hamiltonian for weak nonleptonic processes in the Weinberg–Salam model is given in the local limit by

$$H^{(0)}(\Delta S=1) = \sqrt{2} G \sin\theta_c \cos\theta_c \sum_{i=1}^{4} c_i^{(0)} (O_i - O_i(u\to c)), \tag{13}$$

where

$$c_1^{(0)} = -1, \quad c_2^{(0)} = {}^1/_5, \quad c_3^{(0)} = {}^2/_{15}, \quad c_4^{(0)} = {}^2/_3 \tag{14}$$

and $O_i(u \to c)$ are operators obtained from O_i [see (6)] by introducing the field replacement $u \to c$.

The results obtained in[1] are valid for the evaluation of the strong interaction corrections for the virtual momentum region $p > m_c$. The important point is that the annihilation graphs of the form shown in Fig. 1 are suppressed in these regions because of the reduction in the contributions due to the u and c quarks in the intermediate state. The matrix $\gamma(g)$ for the coefficients of $O_i - O_i(u \to c)$ is determined by the graphs of Fig. 3 and turns out to be diagonal:

$$\gamma = -\frac{2g^2}{16\pi^2}\begin{pmatrix} 4 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix}. \tag{15}$$

Since $\beta(g) = -bg^3/16\pi^2 + \cdots$, we can use (12) to determine the coefficients c_i $(\mu = m_c) = \tilde{c}_i$:

$$\tilde{c}_1 = \varkappa_1^{4/b} c_1^{(0)}, \quad \tilde{c}_{2,3,4} = \varkappa_1^{-2/b} c_{2,3,4}^{(0)}, \tag{16}$$

where $c_i^{(0)}$ are given by (14) and

$$\varkappa_1 = \frac{\bar{g}^2(m_c)}{\bar{g}^2(m_W)} = 1 + b\frac{\bar{g}^2(m_c)}{16\pi^2}\ln\frac{m_W^2}{m_c^2}, \tag{17}$$

$$b = 11 - {}^2/_3 N, \tag{18}$$

in which N is the number of types of quark (aromas) which is equal to 4 in the Weinberg–Salam model.

The result given by (16) is valid for the contribution of left-handed currents and for models with a large number of quarks. The introduction of additional quarks reduces to the replace of the field c by a linear combi-

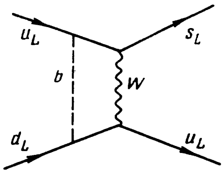

FIG. 3.

nation of quark fields so that there is a change only in the quantity b in the expression for the effective charge.[2)]

For momenta $p<m_c$, on the other hand, which we shall now consider, the introduction of new quarks will not, in general, affect the functions $\beta(g)$ and $\gamma(g)$ because the logarithmic contribution is determined only by the three light quarks and $b=9$.

The particular feature of this region is that the annihilation graph of Fig. 1 does not now give rise to a power-type reduction. The result for this graph for external momenta $p\sim\mu\ll m_c$ has the form

$$M_1=\frac{\sqrt{2}}{3}G\sin\theta_c\cos\theta_c\frac{g^2}{16\pi^2}\ln\frac{m_c^2}{\mu^2}\bar{s}_L\gamma_\mu t^a d_L[\bar{u}\gamma_\mu t^a u+\bar{d}\gamma_\mu t^a d+\bar{s}\gamma_\mu t^a s], \quad (19)$$

i.e., we have the O_5 operator containing the right-handed components of quark fields. The local form of this is connected with the fact that the effective vertex $d\to sb$ in the graph in Fig. 1 is proportional to q^2 (where q is the 4 momentum of the gluon).

If we evaluate the graphs in Figs. 1 and 3 with different four-fermion vertices, we can obtain the matrix $\gamma(g)$ for momenta $m<p<m_c$. It is diagonal for O_3, O_4 which can easily be seen from their unitary and isotopic classification and, therefore, the final answer for the coefficients c_3 and c_4 is quite simple:

$$\begin{pmatrix} c_3 \\ c_4 \end{pmatrix}=\varkappa_2^{-2/9}\begin{pmatrix} \tilde{c}_3 \\ \tilde{c}_4 \end{pmatrix}=\varkappa_2^{-2/9}\varkappa_1^{-2/b}\begin{pmatrix} 2/15 \\ 2/3 \end{pmatrix}, \quad (20)$$

where

$$=\frac{\bar{g}^2(m)}{\bar{g}^2(m_c)}=1+9\frac{\bar{g}^2(m)}{16\pi^2}\ln\frac{m_c^2}{m^2}. \quad (21)$$

For the column (c_1, c_2, c_5, c_6), the matrix $\gamma(g)$ is given by

$$\gamma(g)=-\frac{2g^2}{16\pi^2}\rho, \quad \rho=\begin{pmatrix} 34/9 & 10/9 & 4/3 & 0 \\ 1/9 & -23/9 & -2/3 & 0 \\ 1/6 & -5/6 & 6 & 3/2 \\ 0 & 0 & 16/3 & 0 \end{pmatrix} \quad (22)$$

and the coefficients (c_1, c_2, c_5, c_6) for $\mu=m\ll m_c$ are given by

$$\begin{pmatrix} c_1 \\ c_2 \\ c_5 \\ c_6 \end{pmatrix}=\exp\left(\frac{\rho}{9}\ln\varkappa_2\right)\begin{pmatrix} -\varkappa_1^{4/b} \\ 1/5\,\varkappa_1^{-2/b} \\ 0 \\ 0 \end{pmatrix}. \quad (23)$$

Before we can use the matrix exponent, we must find the eigenvalues ρ_i and eigenvectors of the matrix ρ. These vectors correspond to linear combinations of operators for which the effect reduces to multiplication by $\varkappa_2^{\rho_i/9}$.

The final expression for the coefficients c_1, c_2, c_5, c_6 is somewhat cumbersome:

$$\begin{aligned}
c_1&=-\varkappa_1^{4/b}(0{,}98\varkappa_2^{0{,}42}+0{,}01\varkappa_2^{0{,}80})+0{,}04\varkappa_1^{-2/b}(\varkappa_2^{0{,}42}-\varkappa_2^{-0{,}30}),\\
c_2&=0{,}20\varkappa_1^{-2/b}(0{,}96\varkappa_2^{-0{,}30}+0{,}03\varkappa_2^{-0{,}12})-0{,}02\varkappa_1^{4/b}(\varkappa_2^{0{,}42}-\varkappa_2^{-0{,}30}), \quad (24)\\
c_5&=10^{-2}\varkappa_1^{4/b}(3{,}3\varkappa_2^{0{,}42}+0{,}3\varkappa_2^{-0{,}30}-3{,}9\varkappa_2^{0{,}80}+0{,}3\varkappa_2^{-0{,}12})\\
&+10^{-2}\varkappa_1^{-2/b}(-0{,}1\varkappa_2^{0{,}42}+2{,}9\varkappa_2^{-0{,}30}-1{,}4\varkappa_2^{0{,}80}-1{,}4\varkappa_2^{-0{,}12}),\\
c_6&=10^{-2}\varkappa_1^{4/b}(4{,}8\varkappa_2^{0{,}42}-0{,}6\varkappa_2^{-0{,}30}-2{,}9\varkappa_2^{0{,}80}-1{,}3\varkappa_2^{-0{,}12})\\
&+10^{-2}\varkappa_1^{-2/b}(-0{,}2\varkappa_2^{0{,}42}-5{,}8\varkappa_2^{-0{,}30}-1{,}0\varkappa_2^{0{,}80}+7{,}0\varkappa_2^{-0{,}12}),
\end{aligned}$$

where $\varkappa_1$ and $\varkappa_2$ are defined by (17) and (21).

The numerical values of the coefficients c_{1-6} are listed in Table I, from which it is clear that c_{1-4} are stable against a variation in parameters, while c_5, c_6 change by a factor of roughly two. The reason for this is that the logarithmic contribution to c_5, c_6 originates only in the region of virtual momenta $m<p<m_c$, so that the answer depends very strongly on the choice of the normalization point m. On the other hand, the logarithmic contribution to c_1-c_4 is provided by the more extended region of virtual momenta $m<p<m_W$ so that a change in the parameter m within the range $m_\pi-m_\rho$ has very little effect.

It is important to note that the coefficients c_5 and c_6 are numerically quite small. This is a reflection of the small value of the numerical coefficient in the vacuum polarization operator ($\sim 1/3\pi$), to which the diagram given in Fig. 1 reduces in the local limit. The small numerical values of the coefficients ensure that the contribution of O_5 and O_6 is negligible provided only the matrix elements of O_5 and O_6 are not enhanced in comparison with O_1. As shown in[9], there is reason to expect that the enhancement does, in fact, occur in K-meson decays so that the contribution of O_5 and O_6 may predominate.

The formulas given by (24) reflect the dynamic enhancement by the strong interaction of the operator O_1, and the dynamic suppression of the operators O_2-O_4. The phenomenon was first discovered in[1]. The numerical values of the corresponding factors are 2.5 and 1/1.5 for c_1 and c_4, respectively.

Let us now consider the coefficient c_T. The appearance of the operator T due to the combined action of left- and right-handed currents is described in the lowest order in g by the diagram shown in Fig. 2 (the cross corresponds to expansion over the mass). The corre-

TABLE I. Coefficients in the operator expansion.

N	c_1	c_2	c_3	c_4	c_5	c_6	η_T
$m=0.7$ GeV, $m_W=70$ GeV, $m_c=2$ GeV, $\bar{g}^2(m)/4\pi=1$							
4	−2.41	0.089	0.085	0.423	−0.063	−0.014	0.764
6	−2.49	0.086	0.083	0.415	−0.064	−0.014	0.757
8	−2.59	0.082	0.081	0.407	−0.066	−0.015	0.764
$m=0.14$ GeV, $m_W=100$ GeV, $m_c=2$ GeV, $\bar{g}^2(m)/4\pi=1$							
4	−2.70	0.062	0.079	0.396	−0.135	−0.048	0.756
6	−2.75	0.060	0.078	0.392	−0.137	−0.048	0.742
8	−2.80	0.059	0.078	0.389	−0.138	−0.049	0.727

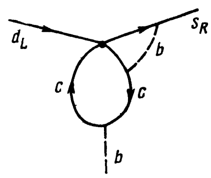

FIG. 4.

sponding expression for c_T, which is defined by (8) and (9), is[13, 14]

$$c_T = g/16\pi^2.$$

The order of magnitude of c_T can be estimated from this result. The summation of corrections due to the higher orders is carried out in[13] and involves, in particular, the evaluation of two-loop diagrams of the form shown in Fig. 4 (the point denotes the weak vertex). The final expression is

$$c_T = \bar{g}(m)\,\eta_T/16\pi^2,$$
$$\eta_T = \varkappa_2^{-2/27}\,\varkappa_1^{-14/3b}\left[1 + {}^1/_{19}\left(\varkappa_1^{38/3b} - 1\right) + {}^5/_{11}\left(\varkappa_1^{11/3b} - 1\right)\right]. \qquad (26)$$

The numerical values of η_T are listed in Table I.

If the model does not include right-handed currents, the operator T, containing the field s_R, may, nevertheless, appear because of the mass term of the s quark field. Thus, a result proportional to m_s is given by Fig. 5 which involves a u quark in the intermediate state. Although this contribution is completely cancelled out by the c quark contribution, this compensation does not hold in the subsequent orders (diagrams of the form shown in Fig. 4). Summation of all orders leads to the following expression for the contribution of left-handed currents to the coefficient of the operator T:

$$H^{(L)} = \sqrt{2}\,G \sin\theta_C \cos\theta_C \frac{\bar{g}(m)}{16\pi^2}\, i m_s \bar{s}_R \sigma_{\mu\nu} t^a d_L b_{\mu\nu}^a$$
$$\times\left[-\frac{1}{104}\varkappa_1^{4/b}\left(\varkappa_2^{4/9} - \varkappa_2^{-14/27}\right) + {}^{17}/_{32}\varkappa_1^{-2/b}\left(\varkappa_2^{-2/9} - \varkappa_2^{-14/27}\right)\right]. \qquad (27)$$

The derivation of this expression is analogous to the case of right-handed currents, considered in[13].

Apart from the small values connected with the mass of the strange quark (in the case of right-handed currents, m_s is replaced by m_c), the expression given by (27) contains a further small quantity because the quantity in the brackets is roughly equal to 0.07. This mean means that, in models incorporating only left-handed currents, the operator T can be neglected.

In conclusion, we reproduce the approximate expression for the effective Hamiltonian:

$$H^{eff}(\Delta S=1) \approx \sqrt{2}G \sin\theta_C \cos\theta_C[-2.5O_1 + 0.08O_2 + 0.08O_3$$
$$+0.4O_4 - (0.06-0.14)O_5 - (0.01-0.05)O_6] + \sqrt{2}G \sin\theta_C \sin\varphi \cdot 0.06T. \qquad (28)$$

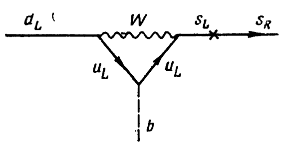

FIG. 5.

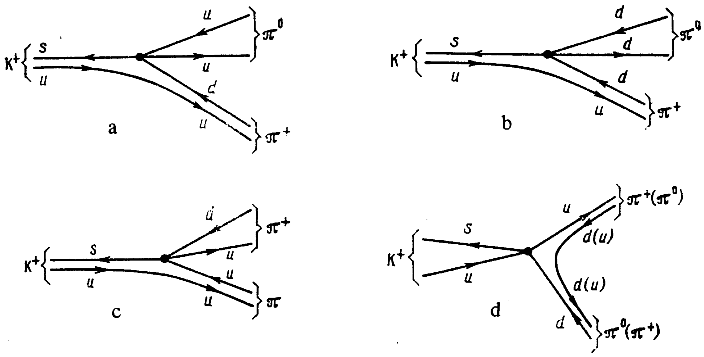

FIG. 6.

4. TRANSITIONS WITH $\Delta T = 3/2$

Processes in which the isotopic spin changes by 3/2 are determined by the matrix element of the operator O_4 [see (6)]. An example of this is the $K^+ \to \pi^+\pi^0$ decay. Its amplitude can be written in the form

$$M(K^+ \to \pi^+\pi^0) = \langle\pi^+\pi^0|H^{eff}(\Delta S=1)|K^+\rangle$$
$$= \sqrt{2}G \sin\theta_C \cos\theta_C c_4 \langle\pi^+\pi^0|O_4|K^+\rangle. \qquad (29)$$

We shall use the valence quark approximation to evaluate the matrix element. The mesons π^+, π^0, K^+ have the following quark structure:

$$\pi^+ \sim u\bar{d}, \quad \pi^0 \sim (u\bar{u} + d\bar{d})/\sqrt{2}, \quad K^+ \sim u\bar{s},$$

and we replace the meson states in (29) by the corresponding quark operators. Applying Wick's rule to the T product of the operator O_4 and the quark fields corresponding to the external lines, we obtain the diagrams shown in Fig. 6.

Let us consider in greater detail the contribution of diagram a in Fig. 6 as an example. The corresponding matrix element of O_4 can be written in the form

$$M_4^a = {}^4/_3\langle\pi^0|\bar{u}_L\gamma_\mu u_L|0\rangle\langle\pi^+|\bar{s}_L\gamma_\mu d_L|K^+\rangle, \qquad (30)$$

where the factor 4/3 appears because of the inclusion of color indices. In fact, the operator O_4 contains the two terms $\bar{s}d\bar{u}u$ and $\bar{s}u\bar{u}d$ [see (6)]. The first is characterized by a color singlet in $\bar{u}u$, $\bar{s}d$ and, therefore, enters (30) with a weight of 1. To find the contribution of the second term, we must use the Fierz transformation

$$\bar{s}_{Li}\gamma_\mu u_L^i \bar{u}_{Lj}\gamma_\mu d_L^j = \bar{s}_{Li}\gamma_\mu d_L^j \bar{u}_{Lj}\gamma_\mu u_L^i \qquad (31)$$

and take into account the fact that, for example,

$$\langle\pi^+|\bar{s}_{Li}\gamma_\mu d_L^j|K^+\rangle = {}^1/_3\delta_i^j\langle\pi^+|\bar{s}_L\gamma_\mu d_L|K^+\rangle.$$

The matrix elements which appear on the right-hand side of (30) can be expressed in terms of the constant f_π of the $\pi \to \mu\nu$ decay and the formfactors $f_\pm$ of the $K^+ \to \pi^0 e\nu$ decay:

$$\langle\pi^0|\bar{u}_L\gamma_\mu u_L|0\rangle = -\frac{if_\pi}{2\sqrt{2}}q_\mu, \qquad f_\pi = 0{,}95m_\pi, \qquad (32)$$

where q_μ is the pion momentum and

$$\langle\pi^+|\bar{s}_L\gamma_\mu d_L|K^+\rangle = -{}^1/_2[(p+q_+)_\mu f_+ + (p-q_+)_\mu f_-], \qquad (33)$$

where p and q_+ are the K and π^+ meson momenta.

The final result is

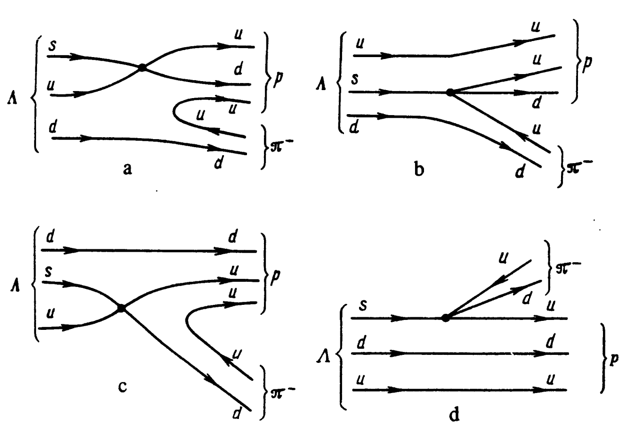

FIG. 7.

$$M_1^a=\frac{if_\pi}{3\sqrt{2}}[(m_K^2-m_\pi^2)f_+ + m_\pi^2 f_-]\approx\frac{if_\pi}{3\sqrt{2}}m_K^2, \tag{34}$$

where we have neglected m_π^2/m_K^2 and have assumed that $f_+(m_\pi^2)=1$.

The evaluation of the contribution of graphs b and c of Fig. 6 is quite similar, and the contribution of diagram d is zero. The point is that this contribution is proportional to $p_\mu\langle\pi^+\pi^0|\bar{u}\gamma_\mu d|0\rangle$, where p is the 4-momentum of the K meson and is zero because of the conservation of vector current without change in strangeness. The final result is

$$M(K^+\to\pi^+\pi^0)=ic_4 G m_K^2 f_\pi \sin\theta_C\cos\theta_C. \tag{35}$$

We note that the matrix element in (35) is zero in the limit when $m_K^2=m_\pi^2=0$. This reflects the well-known fact that $\Delta T=3/2$ transitions are forbidden in K-meson decays in the limit of zero pion momentum. In order not to violate the conservation law, we must also put $m_k^2=0$.

The matrix element $\langle\pi^+\pi^0|O_4|K^+\rangle$ vanishes even for nonzero but equal masses of π and K mesons, i.e., in the $SU(3)$ limit [see (34) before m_π^2 is neglected]. This is a direct consequence of the valence quark approximation.

Comparison with the experimental value

$$|M(K^+\to\pi^+\pi^0)|_{\text{exper}}=0.05 G m_K^2 m_\pi \tag{36}$$

shows that ($\sin\theta_C=0.22$)

$$c_4\approx 0.25, \tag{37}$$

which is smaller by a factor of 1.6 than the theoretical estimate [see (28) and Table I].

The agreement with the sign of the theoretical estimate is confirmed as follows. The expressions for the $\Delta T=1/2$ amplitude (see Sec. 6) and the formula given by (35) yield a positive number for the ratio $M(K^+\to\pi^+\pi^0)/M(K_s\to\pi^+\pi^-)$. In the experiment, the sign is found from the deviation of the ratio of the $K_s\to\pi^0\pi^0$ and $K_s\to\pi^+\pi^-$ decay probabilities from the value predicted by the $\Delta T=1/2$ rule. It is readily verified that the theoretical and experimental signs are the same.

The discrepancy by the factor of 1.6 in the magnitude of c_4 seems to us to be connected with the inaccuracy in the highest logarithmic approximation for the sum of the perturbation theory series. This hypothesis can be verified by considering deviations from the $\Delta T=1/2$ rule in hyperon decays which we shall now consider.

Quark diagrams describing, for example, the $\Lambda\to p\pi^-$ decay are shown in Fig. 7. The symmetry in color space of the various operators in the effective Hamiltonian, and of the quark wave functions of the baryons, is important for the evaluation of the diagrams.

In particular, the operators $O_{2,3,4}$ are symmetric under the interchange of the color indices of the quarks in the quark-quark scattering channel, and the operator O_1 is antisymmetric under the same interchange.[3)] The operators $O_{5,6}$, on the other hand, have no definite symmetry. However, the nucleon and hyperon wave functions are antisymmetric in color space and, therefore, in diagrams a–c of Fig. 7, the only nonzero contribution is that due to the antisymmetric operators. Thus, for the symmetric operator O_4 describing the $\Delta T=3/2$ transitions, all that remains is diagram d in Fig. 7, in which the pion is emitted from the weak vertex. The corresponding matrix element reduces to the multiplication of currents:

$$\langle\pi^- p|O_4^+|\Lambda\rangle=\frac{1}{\sqrt{2}}\langle n\pi^0|O_4^+|\Lambda\rangle=\frac{4}{3}\langle\pi^-|\bar{d}_L\gamma_\mu u_L|0\rangle\langle p|\bar{u}_L\gamma_\mu s_L|\Lambda\rangle$$
$$\approx-\frac{i}{\sqrt{6}}f_\pi q_\mu\bar{u}_f\left(\gamma_\mu+\frac{5}{9}g_A\gamma_\mu\gamma_5\right)u_i. \tag{38}$$

We have used the $SU(3)$ symmetry to describe the matrix element $\langle p|\bar{u}_L\gamma_\mu s_L|\Lambda\rangle$. This description is known to be in agreement with experiment.

The amplitude for $\Delta T=3/2$ in other hyperon decays can be found in the same way. The predictions for deviations from the $\Delta T=1/2$ rule are listed in Table II. We have used the value (37) of c_4, determined from the $K^+\to\pi^+\pi^0$ decay, and the amplitudes were written in the form[15]

$$M=-iGm_\pi^2\bar{u}_f(A+B\gamma_5)u_i. \tag{39}$$

We emphasize that we are predicting not only the absolute values of the amplitudes but also their signs.[4)]

Comparison with experimental data,[15] also listed in Table II, will show that the theoretical predictions for the s waves are in agreement with the experimental values. In the case of p waves, the situation is less clear because of experimental uncertainties.

It is also important to remember the lack of precision in the calculations, which is connected with electromagnetic corrections. These corrections are at a maximum

TABLE II. Deviations from the $\Delta T=1/2$ rule in hyperson decays.

Decay	Amplitude A		Amplitude B	
	Theory	Experiment	Theory	Experiment
$\Lambda_-^0+\sqrt{2}\Lambda_0^0$	0.12	0.09±0.03	−0.95	0.66±0.81
$\sqrt{2}\Sigma_0^+-\Sigma_+^++\Sigma_-^-$	0.14	0.22±0.09	0.49	2.7±1.1
$\Xi_-^-+\sqrt{2}\Xi_0^0$	−0.14	−0.15±0.07	0.23	1.7±2.2

in the case of Λ decays because of the Coulomb interaction in the final state[16] in the $\Lambda \to p\pi^-$ decay and $\Sigma^0\Lambda$ mixing.[17] Effects associated with radiative corrections in Σ-hyperon decays are much smaller. We emphasize that it will be important to improve the experimental situation in the case of the $\Delta T = 3/2$ transitions in hyperon decays because the corresponding amplitudes can be calculated unambiguously.

We have found that, in the approximation of valence quarks, the amplitudes for the $\Delta T = 3/2$ transitions reduce to the product of the matrix elements of leptonic decays. Such estimates were first carried out by Schwinger[18] for K decays and by Kobzarev and Okun'[19] for hyperon decays. The difference between these estimates is that we have taken into account the vacuum state in the channel $\langle \pi^0 | j_\mu | 0 \rangle \langle \pi^+ | j'_\mu | K^+ \rangle$ although there is, of course, no corresponding leptonic decay of π^0 mesons. Moreover, we have taken into account the color indices and dynamic effects associated with strong interactions at short distances (through the coefficient c_4). It is also important to note that, in the model which we have adopted, only transitions with $\Delta T = 3/2$ reduces to the multiplication of currents. Amplitudes with $\Delta T = 1/2$ gave a more complicated structure and do not reduce to the simple multiplication of current matrix elements.

We note that Pati and Woo[20] were the first to discuss the color properties of the operator O_4 in connection with the $\Delta T = 1/2$ rule (see also[21]). They noted that the matrix elements $\langle B_f | H^{eff} | B_i \rangle$, which are connected with the s-wave amplitudes for hyperon decays in the limit of zero pion 4-momentum, vanished for the operator O_4^+ in the valence quark approximation.

In contrast to[20], we have calculated the amplitudes for the $\Delta T = 3/2$ transitions and have considered both s and p waves. The corresponding amplitudes turned out to be small. We shall show in the next section that $\Delta T = 1/2$ transitions are much greater and are comparable with experimental values.

5. TRANSITIONS WITH $\Delta T = 1/2$ IN HYPERON DECAYS

In this section, we shall consider amplitudes with $\Delta T = 1/2$ in hyperson decays (the corresponding analysis for K decays is given in the next section). We shall obtain a relationship between observed amplitudes that do not contain the coefficients c_i. These relationships agree with experimental data and, in fact, comparison with experiment will be used to determine the coefficients $c_{5,6}$. The agreement with theoretical estimates is to within a factor of about 2.

In the case of $\Delta T = 1/2$ transitions, there are six operators, namely, T^+, O_1^+, O_2^+, O_3^+, O_5^+, O_6^+. Let us consider, to begin with, which graphs are important for O_{1-3}, $O_{5,6}$ (the operator T is discussed in Section 7).

The operator O_1 is antisymmetric under the interchange of color indices in the quark-quark scattering channel. For this reason, graphs of the kind shown in Figs. 7a–7c do not vanish for the operator O_1 in contrast to the case of symmetric operators (see last section). The contribution of such graphs depends on the wave function of the baryon and does not reduce to the multiplication of the matrix elements for leptonic decays. We shall not use any assumptions on the explicit form of the wave functions. However, the valence quark approximation and the $SU(3)$ symmetry enable us to find the relationship between the contribution of O_1^+ to different decays. More precisely, it will be shown below how the sign of these contributions can be established. Insofar as the absolute magnitudes are concerned, we shall restrict our attention here to very rough estimates. These estimates result in values that are comparable with the observed amplitudes.

The operator O_1^+ is also found to contribute to graphs such as graph d in Fig. 7, where the pion leaves a weak vertex. These graphs are readily evaluated exactly in the same way as they were evaluated in the last section for the operator O_4^+. The corresponding contribution vanishes in the $SU(3)$ limit and is numerically small. The contribution of the symmetric operators O_2^+ and O_3^+ is completely determined by graphs such as graph d, and is also numerically small.

On the other hand, in the case of O_5^+, graphs such as graph d in Fig. 7 do not reduce to zero in the $SU(3)$ limit and provide the large factor $m_\pi^2/m_u m_s$ which compensates the small value of the coefficient c_5. However, a small value remains in the case of graphs a–c. We shall therefore neglect such graphs for O_5^+. Inclusion of O_6^+ ensures that all the amplitudes contain the combination $c_5 + (3/16)c_6$ (the factor 3/16 is connected with the color structure).

We must now summarize. We expect that the dominant contribution to transitions with $\Delta T = 1/2$ in hyperon decays is provided by the operators O_1^+ and O_5^+. The contribution of O_1^+ is largely determined by graphs such as a, b, and c in Fig. 7 and the contribution of O_5^+ by graphs such as d.

To derive the necessary relationships, it is important to remember that there is a combination of amplitudes for hyperon decays to which the operator O_1^+ does not contribute. This is seen most simply by considering the amplitude for the transition $\Xi^- \to \Sigma^-\pi^0$, which, with the aid of the $SU(3)$ symmetry, can be expressed in terms of the amplitudes for the observed decays as follows:

$$M(\Xi^- \to \Sigma^-\pi^0) = \sqrt{3}\Lambda_-^0 - \Sigma_0^+ + \sqrt{3}\Xi_-^-. \tag{40}$$

The upper index in this expression refers to the charge of the decaying baryon and the lower to the charge of the resulting pion.

Since Ξ^- and Σ^- hyperons do not contain u quarks, the amplitude (40) does not include a contribution due to O_1, which represents the $su \to du$ interaction, and this amplitude is determined exclusively by the operators $O_{5,6}$. The corresponding contribution is described by graphs such as d in Fig. 7 and can be written in the form

$$\langle \Sigma^-\pi^0 | O_5^+ | \Xi^- \rangle = -{}^{32}/{}_9 \langle \pi^0 | \bar{d}_L d_R | 0 \rangle \langle \Sigma^- | \bar{d}_R s_L | \Xi^- \rangle. \tag{41}$$

where we have used for O_5^+ the Fierz transformation

$$\bar{\psi}_{1L}\gamma_\mu\psi_{2L}\bar{\psi}_{3R}\gamma_\mu\psi_{4R} = -2\psi_{1L}\psi_{4R}\bar{\psi}_{3R}\psi_{2L}. \tag{42}$$

The densities in (41) can be expressed in terms of the divergences of currents and the bar quark masses:

$$\bar{d}\gamma_5 d = -\frac{i}{2m_d}\partial_\mu(\bar{d}\gamma_\mu\gamma_5 d), \quad \bar{u}\gamma_5 s = -\frac{i}{m_u+m_s}\partial_\mu(\bar{u}\gamma_\mu\gamma_5 s),$$
$$\bar{u}s = \frac{i}{m_s-m_u}\partial_\mu(\bar{u}\gamma_\mu s). \tag{43}$$

These results are valid even in the presence of strong interactions. Substituting (43) and (41), we obtain

$$\langle\Sigma^-\pi^0|O_5{}^+|\Xi^-\rangle = -i\frac{4}{9\sqrt{2}}\frac{f_\pi m_\pi^2}{m_u}\bar{u}_f\left(\frac{m_\Xi-m_\Sigma}{m_s-m_u}+g_A\gamma_5\frac{m_\Xi+m_\Sigma}{m_s+m_u}\right)u_i. \tag{44}$$

For the ratio of the s and p waves in this amplitude, we find [the parametrization of the amplitude is given by (39)]

$$\frac{B(\Xi^-\to\Sigma^-\pi^0)}{A(\Xi^-\to\Sigma^-\pi^0)} = g_A\frac{m_\Xi+m_\Sigma}{m_\Xi-m_\Sigma}\approx 25. \tag{45}$$

On the other hand, using the expression given by (40) together with the experimental data, we obtain[5]

$$\frac{\sqrt{3}B(\Lambda_-{}^0)-B(\Sigma_0{}^+)+\sqrt{3}B(\Xi_-{}^-)}{\sqrt{3}A(\Lambda_-{}^0)-A(\Sigma_0{}^+)+\sqrt{3}A(\Xi_-{}^-)} = 33\pm 10, \tag{46}$$

which is in agreement with the theoretical prediction given by (45).

Comparison of the theoretical prediction given by (44) for the s wave with the experimental value (see Footnote 5)

$$\sqrt{3}A(\Lambda_-{}^0)-A(\Sigma_0{}^+)+\sqrt{3}A(\Xi_-{}^-) = -0.51\pm 0.10 \tag{47}$$

yields

$$c_5+{}^3/_{16}c_6\approx -0.25. \tag{48}$$

To obtain these results, we have used the following quark masses[7,8]:

$$m_u=m_d=5.4\,\text{MeV}, \quad m_s=150\,\text{MeV}. \tag{49}$$

The value given by (48) has the same sign as the theoretical prediction and differs from the second variant in Table I by a factor of only 1.7. The agreement seems quite satisfactory because the magnitude of $c_{5,6}$ is sensitive to the low-energy contribution and to quark mass values.

Using (48), we can calculate the contribution of $O^+_{5,6}$ to all observed decays. The results are listed in Table III. It is clear that this contribution is very substantial. We note that, for p waves, the above contribution corresponds to the K meson and not to baryon pole graphs. The difference between the experimental values and the contribution of $O^+_{5,6}$ is determined by the matrix elements of O^+_1. Theoretically, we find only the signs of the matrix elements of O^+_1 in the case of s waves. For all decays, these signs agree with experiment.

The sign of the matrix elements of O^+_1 is established as follows. Consider the s-wave amplitude for the decay $\Sigma^+\to p\pi^0$ in the limit of zero pion 4-momentum. At this point,

$$\langle p\pi^0|O_1{}^+|\Sigma^+\rangle_{q_{\pi^0}=0} = \frac{i}{\sqrt{2}f_\pi}\langle p|O_1{}^+|\Sigma^+\rangle. \tag{50}$$

The operator O^+_1 can be written in the following form with the aid of the Fierz transformations:

$$O_1{}^+ = 2[j^k(us)]^+[j^k(ud)], \tag{51}$$

where

$$j^k(ud) = \varepsilon^{kji}\bar{u}_j\frac{1-\gamma_5}{2}d_i{}^c, \quad d^c=\gamma_2\gamma_0\bar{d}^T. \tag{52}$$

Using the U-spin reduction operator U^-, we can express the state $|\Sigma^+\rangle$ in terms of $|p\rangle$ as follows:

$$|\Sigma^+\rangle = U^-|p\rangle. \tag{53}$$

After substituting (53), we find that the matrix element $\langle p|O^+_1|\Sigma^+\rangle$ reduces to

$$\langle p|O_1{}^+|\Sigma^+\rangle = \langle p|[O_1{}^+, U^-]|p\rangle$$
$$= 2\langle p|[j^k(ud)]^+j^k(ud)-[j^k(us)]^+j^k(us)|p\rangle, \tag{54}$$

where we have used the fact that the s and d quarks form a doublet in the group of U spin. Since the s-quark impurity in the proton is small, the sign of the right-hand part of (54) is determined by the term $[j^k)ud)]^+j^k(ud)$, which is positive definite. The matrix element (54) and, correspondingly, the contribution of O^+_1 to $A(\Sigma^+_0)$ is, therefore, positive and this is in agreement with experiment. The contribution of O^+_1 to s waves for other decays is connected with the $SU(3)$ symmetry relations established above:

$$\Lambda_-{}^0(O_1) = -\frac{1}{\sqrt{3}}\Sigma_0{}^+(O_1), \quad \Xi_-{}^-(O_1) = \frac{2}{\sqrt{3}}\Sigma_0{}^+(O_1), \tag{55}$$

where we have allowed the matrix element $\langle\Sigma^-\pi^0|O^+_1|\Xi^-\rangle$ to vanish [see the discussion after (40)].

The expression given by (54) is convenient for estimating the matrix element of O^+_1. This formula relates O^+_1 to the diquark density $\epsilon^{jki}u_k d_i$ inside the proton. Assuming that the ratio of diquark and quark densities is equal to the reciprocal of the proton volume, we obtain

$$\langle p|O_1{}^+|\Sigma^+\rangle \sim m_\pi{}^3\bar{u}_f u_i, \tag{56}$$

where we have assumed that the proton volume is m_π^{-3}. The estimate given by (56) yields a contribution of 0.6 for O^+_1 in $A(\Sigma^+_0)$, which is to be compared with the value of 0.4 (see Table III). A similar result is obtained when, in the product of currents in O^+_1, we restrict our attention to single-nucleon intermediate states.

In conclusion of this section, we note one further possible verification of the scheme, namely, $\Omega^-\to\Xi\pi$ decay. As in the case of the $\Xi^-\to\Sigma^-\pi^0$ transition, the operator O^+_1 does not provide a contribution, and the amplitude is determined by diagrams such as d in Fig. 7. The matrix element $\langle\Xi^0|\bar{u}\gamma_5 s|\Omega^-\rangle$, which appears when the contribution of $O^+_{5,6}$ is evaluated, can be expressed in terms of the quark mass and the strong constant for the $\Omega^- \to \Xi^0 K^-$ transition. This constant can be found with the aid of $SU(3)$ from the width for the $\Xi^*(1530)\to\Xi\pi$ decay, and when this is done we obtain

TABLE III. Contribution of the operators $O_{5,6}$ to the amplitudes for hyperon decays.

Decay	Amplitude A		Amplitude B	
	Contribution of $O_{5,6}$	Experiment	Contribution of $O_{5,6}$	Experiment
$\Lambda_-{}^0$	−1.33	−1.48±0.01	−10.69	−10.17±0.24
$\Sigma_0{}^+$	1.10	1.48±0.05	−3.34	−12.04±0.59
$\Xi_-{}^-$	1.52	2.04±0.02	2.53	−6.73±0.41

$$\Gamma(\Omega^-\to\Xi^0\pi^-)=2\Gamma(\Omega^-\to\Xi^-\pi^0)=\Gamma(\Xi^*\to\Xi\pi)\left(\frac{p_\Omega}{p_{\Xi^*}}\right)^3$$
$$\times\left[\frac{8}{9}Gf_Kf_\pi\sin\theta_c\cos\theta_c\frac{m_\pi^2}{m_u m_s}\left(c_5+\frac{3}{16}c_6\right)\right]^2\approx 0.45\cdot 10^{10}\ \text{sec}^{-1};\qquad(57)$$

where p_Ω and $p_{\Xi*}$ are the momenta of the pions in the corresponding decays, and for $(c_5+(3/16)c_6)/m_u m_s$ we have used the value that follows from hyperon decay [see (44), (47)–(49)]. The experimental fraction of decays to $\Xi\pi$ is not well known (it is roughly equal to one-half), and the total width of the Ω^-, according to recent data,[22] is $(1.3\pm 0.2)\times 10^{10}$ sec^{-1} [it was previously assumed[15] that it was $(0.77\pm 0.15)\times 10^{10}$ sec^{-1}].

6. THE $K_S^0\to\pi^+\pi^-$ DECAY

Let us estimate the matrix element for the $K_S^0\to\pi^+\pi^-$ decay. It turns out that the usual operators, i.e., those present in the bare Hamiltonian, do not explain the experimental data despite the enhancement of the $\Delta T=1/2$ term in the Hamiltonian by the strong interaction.[1] Moreover, the operators $O_{5,6}$, which contain both left-handed and right-handed components of the fermion fields and are present in $H^{eff}(\Delta S=1)$ with small coefficients, are found to predominate. We shall estimate their contribution to the $K_S^0\to\pi^+\pi^-$ decay by using the results obtained from the analysis of hyperon decays. The theoretical prediction is in good agreement with the experimental value. However, when the estimates were carried out, it was found necessary to introduce an additional assumption about the so-called σ term in $\pi\pi$ scattering.

Thus, let us consider the $K_S^0\to\pi^+\pi^-$ decay by examining, to begin with, the matrix element of the operator O_1. This calculation can be carried out in exactly the same way as in the case of the $K^+\to\pi^+\pi^0$ decay, and the answer can be expressed in terms of the product of the matrix elements of leptonic decays:

$$\langle\pi^+\pi^-|O_1|K_s^0\rangle=-\frac{if_\pi\sqrt{2}}{6}(m_K^2-m_\pi^2)\left(f_++f_-\frac{m_\pi^2}{m_K^2-m_\pi^2}\right).\qquad(58)$$

The matrix elements of O_2, O_3, and O_4 can be found similarly. To determine the coefficients c_1–c_3, we used the theoretical estimates given by (28). The contribution of O_{1-4} is then given by

$$\sqrt{2}G\sin\theta_c\cos\theta_c\left\langle\pi^+\pi^-\left|\sum_{i=1}^{4}c_iO_i\right|K_s^0\right\rangle\approx iGm_K^2m_\pi\cdot 0.21,\qquad(59)$$

which is much less than the experimental value expressed in the same units, i.e., 1.02. Such a large discrepancy cannot, of course, be connected with uncertainties in the coefficients c_1–c_3, and we arrive at the conclusion that some other operators must be included. Direct estimates of the contribution of $O_{5,6}$, given below, confirm this conclusion.

Before we proceed to this estimate, let us emphasize that the enhancement of O_1 due to the strong interaction at short distances does not improve the comparison between theory and experiment to any great extent. In fact, for the bare Hamiltonian, it can be shown that

$$\langle\pi^+\pi^-|H^0|K_s^0\rangle\approx iGm_K^2m_\pi\cdot 0.20,\qquad(60)$$

which is practically the same as (59). The reason for this is twofold. Firstly, terms with $\Delta T=1/2$ and $\Delta T=3/2$ provide comparable contributions in the calculation with the bare Hamiltonian. The first is enhanced by the strong interaction and the second is suppressed by it. Secondly, in view of the antisymmetry in the color indices of the operator O_1, the matrix elements of this operator contain the additional factor of one-half which practically eliminates the entire enhancement.

Let us now consider the operators $O_{5,6}$.

By standard methods, we can show that

$$\langle\pi^+\pi^-|O_5|K_S^0\rangle={}^{16}/_3\langle\pi^+\pi^-|O_6|K_S^0\rangle=-\frac{32\sqrt{2}}{9}\{\langle\pi^+|\bar u_Rd_L|0\rangle\langle\pi^-|\bar s_Lu_R|K^0\rangle$$
$$+\langle\pi^+\pi^-|\bar d_Rd_L|0\rangle\langle 0|\bar s_Ld_R|K^0\rangle\},\qquad(61)$$

where, in the derivation of this expression, we have used the Fierz transformation (42).

Three of the four matrix elements on the right-hand side of (61) can be expressed with the aid of (43) in terms of the current matrix elements and the quark masses

$$\langle\pi^+|\bar u_Rd_L|0\rangle=-\frac{if_\pi m_\pi^2}{2(m_u+m_d)},\quad\langle 0|\bar s_Ld_R|K^0\rangle=i\frac{f_Km_K^2}{2(m_s+m_u)},$$
$$\langle\pi^-|\bar s_Lu_R|K^0\rangle=[f_+(m_K^2-m_\pi^2)+f_-m_\pi^2]/2(m_s-m_u),\qquad(62)$$

where, for numerical estimates, we use the quark masses given by (49).

The estimated matrix element $\langle\pi^+\pi^-|\bar d_R\bar d_L|0\rangle$ is subject to a greater degree of ambiguity. In the limit of zero pion momenta, this matrix element is expressed in terms of m_π^2, as follows:

$$\langle\pi^+\pi^-|\bar d_Rd_L|0\rangle|_{q_\pi=0}=\frac{1}{4}\langle\pi^+\pi^-|\bar dd+\bar uu|0\rangle=\frac{m_\pi^2}{2(m_u+m_d)}.\qquad(63)$$

In actual fact, m_π^2 is equal to the average of the total strong-interaction Hamiltonian over the π-mesonic state. Since all the terms in the bare Hamiltonian other than the mass term conserve $SU(2)_L\times SU(2)_R$ symmetry, we arrive at (63).

However, in $K_S^0\to\pi^+\pi^-$ decay, the pion momenta are quite large and it is, therefore, desirable to take into account the associated departures from (63). The dependence on the momenta can be estimated as follows. If the operator for the π^+-meson field is proportional to $\bar u\gamma_5 d$, the isoscalar σ-field operator in the limit of $SU(2)_L\times SU(2)_R$ symmetry is

$$\sigma\propto[A^-,\pi^+]\propto\bar uu+\bar dd,\qquad(64)$$

where A^- is the axial charge.

It seems, therefore, natural to associate the dependence on the pion momenta with the finite mass of the σ meson. In the approximation that is linear in $(q_++q_-)^2$, where $q_\pm$ are the 4-momenta of the $\pi^\pm$ mesons, we have

$$\langle\pi^+\pi^-|\bar d_Rd_L|0\rangle=\frac{m_\pi^2}{2(m_u+m_d)}\left[1+\frac{(q_++q_-)^2}{m_\sigma^2}\right].\qquad(65)$$

Substituting (62) and (65) in (61), we have

$$\langle\pi^+\pi^-|O_5|K_S^0\rangle = {}^{16}/_3\langle\pi^+\pi^-|O_6|K_S^0\rangle$$

$$= i\frac{4\sqrt{2}}{9}\frac{f_\pi m_K^2 m_\pi^2}{m_u m_s}\left[\frac{f_K}{f_\pi} - 1 + \frac{f_K}{f_\pi}\frac{m_K^2}{m_\sigma^2}\right], \qquad (66)$$

where we have neglected m_π^2 in comparison with m_k^2, and m_u in comparison with m_s.

We note that the matrix elements of $O_{5,6}$ vanish in the limit of exact $SU(3)$ symmetry, which is a consequence of the well-known selection rule for the Gell-Mann $\tilde{G}$ parity.[23] Since we have neglected m_π^2, the limit of $SU(3)$ symmetry in (66) corresponds to $m_k^2 = 0$. The suppression of the corresponding matrix element reduces to its proportionality to either $(f_k/f_\pi - 1)$ or m_k^2.

We assume that $m_\sigma = 700$ MeV in numerical estimates. This value of the mass will, in practice, characterize not the mass of the physical σ meson but the energies at which the $\pi\pi$ interaction in the s wave becomes strong although the approximation which we have used may turn out to be too rough.

As a result, the contribution of $O_{5,6}$ to the amplitude for the $K_s^0 \to \pi^+\pi^-$ decay is given by

$$\sqrt{2}G\sin\theta_C\cos\theta_C(c_5 + {}^3/_{16}c_6)\langle\pi^+\pi^-|O_5|K_S^0\rangle = iGm_K^2 m_\pi \cdot 0.85, \qquad (67)$$

where we have used (48) for the coefficient $c_5 + (3/16)c_6$.

If we add the contribution of O_{1-4} to (67), we obtain

$$\langle\pi^+\pi^-|H^{eff}(\Delta S=1)|K_S^0\rangle \approx iGm_K^2 m_\pi \cdot 1.05, \qquad (68)$$

which is practically the same as the corresponding numerical value.

Even if we allow for some ambiguity in these estimates, it can be shown that the theoretical and experimental values for the amplitudes with $\Delta T = 1/2$ in K-meson decays are comparable. Since, for the amplitudes with $\Delta T = 3/2$, the theoretical estimates are close to the experimental values, we may conclude that the $\Delta T = 1/2$ rule has received a natural explanation within the framework of the above scheme.

$K \to 3\pi$ decays can be considered similarly. The corresponding amplitudes with $\Delta T = 1/2$ and $\Delta T = 3/2$ can be expressed in terms of products of current matrix elements and densities, and can be evaluated. Moreover, it turns out that, if we take into account the consequences of the PCAC hypothesis for these matrix elements, the resulting amplitudes for the $K \to 3\pi$ decays will also satisfy the current algebra. Since, on the other hand, the current algebra completely determines the amplitudes for the $K \to 3\pi$ decays in terms of the amplitudes for the $K \to 2\pi$ decays, this type of calculation does not lead to additional predictions.

It is, however, important to emphasize that current algebra and the valence quark model are mutually consistent, which is not clear *a priori*. Similarly, in the case of Λ, Ξ decays, the PCAC hypthesis leads to the vanishing of amplitudes with $\Delta T = 3/2$ in the limit as $q_\pi = 0$, and the explicit expressions obtained in the valence quark approximation satisfy this condition.

7. MATRIX ELEMENTS OF THE OPERATOR T

The operator T appears in the expansion of the effective Hamiltonian with an appreciable coefficient when the interaction between heavy quarks and light quarks contains right-handed currents. Since the operator T contains the gluon field, its matrix elements depend on the admixture of gluons in ordinary hadrons. The valence quark approximation corresponds to the neglect of this admixture[6)] and this, in turn, leads to small matrix elements of the operator T.

We shall now give an estimate which does not depend on the assumption that the gluon admixture is small. This estimate is based on the analogy between Figs. 8 and 9. The first of these describes the contribution of the operator T to the $K_s \to \pi^+\pi^-$ decay. The second describes the strong decay of a hypothetic scalar meson into two pions. If we suppose that the matrix elements are similar, the entire difference reduces to a difference between the constants in one of the vertices: in Fig. 8, the constant at the T vertex is $G\sqrt{2}\sin\theta_c \times \sin\varphi g m_c m/16\pi^2$ and, in Fig. 9, the corresponding constant is $g/2$. We can therefore express the contribution of T to the width $\Gamma_T(K_s \to \pi^+\pi^-)$ in terms of the width Γ_{str} for strong decay:

$$\frac{\Gamma_T}{\Gamma_{str}} \sim \left(\frac{G\sqrt{2}}{8\pi^2}\sin\theta_C\sin\varphi m_c m\right)^2. \qquad (69)$$

Substituting $\Gamma_{str} = 300$ MeV and $m = m_\rho$, we obtain

$$M_T(K_S^0 \to \pi^+\pi^-) \sim 0.5Gm_K^2 m_\pi \sin\varphi. \qquad (70)$$

When $\sin\varphi \sim 1$, this estimate yields a result that is comparable with the experimental value. However, when $\sin\varphi \sim 1$, the mass difference between the K_L and K_s mesons becomes unacceptably large. The corresponding estimates,[25] assuming that the mass of the strange quark is $m_s = 150$ MeV, lead to the restriction $\sin\varphi \lesssim 0.1$. When this is taken into account, the amplitude given by (70) is found to be small.

We therefore conclude that the operator T provides a small contribution to the observed decays.

8. CONCLUSIONS

We must now formulate the point of view which we have developed as a result of the present work. The

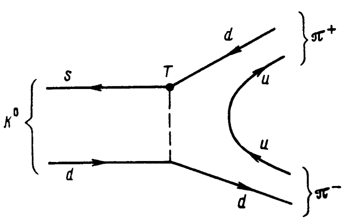

FIG. 8.

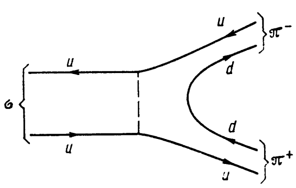

FIG. 9

Hamiltonian for weak nonleptonic interactions with $\Delta S = 1$ is the sum of seven operators. Four of them are contained by the bare Hamiltonian and have the largest coefficients. They include the operator with isospin 3/2. The three operators that satisfy the $\Delta T = 1/2$ rule are found to appear when annihilation diagrams are taken into account and are, therefore, present with small coefficients. They include the operator T, which appears with an appreciable coefficient only in models with right-handed currents.

To estimate the matrix elements, we use the valence quark approximation. It turns out that the contribution of the operators in the bare Hamiltonian is insufficient for the description of nonleptonic decays. The operator O_5 is very important in hyperon decays and predominates in K-meson decays. It contains both left- and right-handed components of quark fields. In particular, for the s and p waves in hyperon decays, we have

$$\frac{\sqrt{3}B(\Lambda_-^0) - B(\Sigma_0^+) + \sqrt{3}B(\Xi_-^-)}{\sqrt{3}A(\Lambda_-^0) - A(\Sigma_0^+) + \sqrt{3}A(\Xi_-^-)} = g_A \frac{m_\Xi + m_\Sigma}{m_\Xi - m_\Sigma}. \qquad (71)$$

and this is in agreement with experiment.

The combinations of amplitudes in (71) are determined by the matrix elements of the operator O_5, and their absolute magnitudes are in agreement with theoretical estimates provided the quark masses are small. Moreover, we can determine the signs of the contributions of the operator O_1. These signs are also in agreement with experiment. In the case of K-meson decays, the theoretical estimates for the amplitude with $\Delta T = 1/2$ also yield a reasonable value.

Estimates of the matrix elements of the operator T, which is connected with right-handed currents in the interaction between light and heavy quarks are rather approximate. However, since, for the constant corresponding to the right-handed currents, there is a strong limitation that follows from the mass difference between K_L and K_S mesons, it would appear that the operator T is unimportant in nonleptonic decays.

Further verification of this scheme may involve an accurate determination of the amplitude with $\Delta T = 3/2$ in hyperon decays. All the transitions with $\Delta T = 3/2$ in hyperon and K-meson decays can be expressed theoretically in terms of a single parameter. Experimental data on K decays can be used to determine this parameter, and the result turns out to be close to the independent theoretical estimates and is certainly in agreement with it in sign.

We may, therefore, conclude that there is relatively weighty evidence for the validity of the hypothesis in which the $\Delta T = 1/2$ rule is connected with the small bare mass of quarks.

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[1])We shall assume that the field c in (10) corresponds to the state of the quark with a definite "mass." In general, the field c could be a linear combination of several states with definite mass. This would give rise to definite complications, largely concerned with the contribution of the operator T. We cannot reproduce here the general expressions because they are both rather unwieldy and not essentially different from the case considered here. The reader can easily introduce the necessary modification for each particular form of the current.

[2])We are assuming that the masses of heavy quarks are of the same order. If, on the other hand, the ratio of heavy-quark masses is large, a larger number of regions has to be considered.

[3])$\Delta S = -1$ in hyperon decays and the Hermitian conjugate operators $O_i^\dagger$ appear.

[4])Our choice of signs of the amplitudes and states differs from that adopted in the tables in[15]. Bearing in mind further generalization to the case of $SU(3)$ symmetry, we note that, in accordance with our notation, the octets of baryons and antibaryons have the same form, whereas negative signs are frequently introduced elsewhere in the antibaryon octet. The meson octets have the same form as the baryon octet with the obvious substitution $\Sigma^+ \to \pi^+$, $p \to K^+$, ... The weak-interaction amplitudes are obtained by a standard procedure by multiplying the corresponding $SU(3)$ matrices, including that corresponding to the "spurion."

[5])More precise values can be obtained with the aid of the Lee-Sugawara relationship $2\Xi_-^- + \Lambda_-^0 - \sqrt{3}\Sigma_0^+ = 0$, which can be used to eliminate Ξ_-^- from (40). In that case, $M(\Xi^- \to \Sigma^-\pi_0) = \frac{1}{2}\sqrt{3}\Lambda_-^0 + \frac{1}{2}\Sigma_0^+$. Instead of (46), we then have 27.4 ± 2.5, and -0.54 ± 0.04 instead of (47).

[6])The fact that the gluon admixture is small is indicated by the success of the quark classification of hadronic states. If the π and K mesons were to contain a large number of gluons, the spectrum of low-lying resonances would be much richer. We note that a large admixture of gluons is frequently said to follow from data on deep inelasticity. This, however, is based on the parton model and neglects the quark-gluon interaction. Moreover, the inclusion of this interaction within the framework of the asymptotically free theory can be used to obtain a satisfactory description of experimental data in the valence quark model.[24]

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Emission of soft photons and electron form factors in the two-dimensional approximation of quantum electrodynamics

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A method is developed for the calculation of the cross section for the emission of soft photons in an arbitrary strong scattering process in a strong magnetic field, when the electrons are on the Landau ground level. The form factors of the electrons are calculated in the two-dimensional approximation of quantum electrodynamics, and the corresponding elastic-scattering cross section is calculated with allowance for the radiative corrections. It is shown that there is no infrared divergence in the summary cross section.

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INTRODUCTION

The discovery of pulsars (neutron stars) which presumably have strong magnetic fields has stimulated interest in the investigation of electrodynamic processes in such fields. An essentially new circumstance that can influence the evolution of similar objects[1] is the suppression of the transverse degrees of freedom of the charged particles (we shall deal here with electrons and positrons). This is possible if the characteristic electron energies ε, which are connected with the field induction B by the relation, are given by

$$\varepsilon^2-m^2<m^2(2B/B_0),\quad B_0=m^2/e=4.41\cdot10^{13}\ \text{G},\tag{1}$$

when the electron is on the Landau ground level. In a number of cases the result is that the quantum electrodynamics degenerates into a practically two-dimensional theory in the sense of the character of the "motion" of the electron (along the field plus a temporal coordinate). This greatly simplifies the calculations, since the electron wave function and the corresponding "two-dimensional" representation of the electron Green's function[2] have an exceedingly simple form in comparison with the usual case (the z axis is directed opposite to the field):

$$\psi(x)=\frac{(\gamma/\pi)^{1/4}}{(2\varepsilon L_2L_3)^{1/2}}\exp\left\{-\frac{\xi^2}{2}+i(p_2x_2+p_3x_3)\right\}u(p),\tag{2a}$$

$$\xi=(x_1\sqrt{\gamma}-p_2/\sqrt{\gamma}),\quad \gamma=|eB|,\quad p=(\varepsilon,\ p_3);$$

$$G(x,y)=\varphi(x,y)G(x-y),\quad \varphi(x,y)=\exp\{{}^1/_2i\gamma(x_1+y_1)(x_2-y_2)\};\tag{3a}$$

$$G(x-y)=-\frac{\gamma}{(2\pi)^3}\frac{1+i\gamma_1\gamma_2}{2}\exp\left\{-\frac{\gamma}{4}[(x_1-y_1)^2+(x_2-y_2)^2]\right\}\times\int d^2p\, e^{ip(x-y)}\frac{\hat p+m}{p^2-m^2}.\tag{3b}$$

All the scalar products here and below are two-dimensional (0, 3), and the spinor $u(p)$ satisfies the equations

$$(\hat p-m)u(p)=0,\quad {}^1/_2(1+i\gamma_1\gamma_2)u(p)=u(p),\quad \bar u(p)u(p)=2m,\tag{2b}$$

with a density matrix

$$\rho={}^1/_2(\hat p+m),\tag{2c}$$

and with no summation or averaging over the electron spin states in the square of the matrix element, since the spin projection in the ground state is fixed [Eq. (2b)].

We have shown earlier that in the case of loop diagram the matter reduces to a calculation of the Feynman integrals with respect to the two-dimensional momentum of the loop, and in addition to (1) it is necessary to satisfy the condition $B\gg B_0$.[1,2] On the other hand, if there is no excitation of the vacuum, the problem likewise degenerates to a two-dimensional one, in which the electron is simply nonrelativistic, in accord with (1).

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