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Gravitational antenna with SQUID as sensor

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A gravitational antenna with quantum magnetometer (SQUID) as sensor is considered. The factors that restrict the sensitivity of the SQUID in two regimes, without and with hysteresis, are analyzed. Expressions are obtained for the minimal detectable force connected with the intensity of gravitational radiation. Requirements are also formulated for the case of a sensor on an antenna with mechanical transformation of displacements.

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§1. INTRODUCTION

Negative results in experimental searches for gravitational waves[1,2] are stimulating the creation of a second generation of antennas with parameters that are close to the theoretical requirements.[3-9] For a resonance gravitational detector at the frequency $\omega_\mu \approx 10^4$ and linear dimension $l_g \approx 10^2$ cm the required sensitivity level is characterized by fluctuation variations of the vibration amplitude equal to $\Delta x_0 \approx 10^{-17}$ cm.[6,10] Concrete programs are underway to achieve this level by a high mechanical quality factor $Q_\mu \approx 10^{10}$ [3,11] or by lowering the temperature of the gravitational detector to $T_\mu \approx 3\cdot 10^{-3}$ °K.[12] The main experimental difficulties are connected with measuring the small amplitude of the mechanical vibrations. It was suggested in[13,14] that a quantum magnetometer, a so-called SQUID,[1)] should be used.

The aim of the present paper is to analyze the SQUID as a sensing element of a gravitational antenna. For comparison, we shall use the characteristics of an antenna with capacitative parametric transducer,[10,11] which in the optimal regime has the sensitivity

$$(F_0)_{min} \approx \frac{3\sqrt{\pi}}{\hat{\tau}}\left(m\varkappa T\frac{\omega_\mu}{\omega}\right)^{1/2}; \qquad (1)$$

here, m is the equivalent mass of the gravitational detector[9]; F_0 is the amplitude of the "external force", which is related to the flux density I_g of the gravitational radiation by $F_0 = m\omega_\mu l_g(8\pi G c^{-3} I_g)^{1/2}$; T is the temperature of the transducer; ω is the pumping frequency; and τ is the duration of the radiation pulse.

§2. SUPERCONDUCTING DISPLACEMENT TRANSFORMER

On the antenna proposed in[13,14] the SQUID is coupled to the gravitational detector by means of a mechanical displacement transformer. The idea is due to Lavrent'ev[15] and reduces to a model of coupled oscillators: the gravitational detector with mass M and pendulum of small mass m fixed to it. At the natural frequency, the amplitude of the pendulum is $(M/m)^{1/2}$ times greater than the amplitude of the forced vibrations of the gravitational detector. If $Q_{tr} \geqslant Q_\mu$ (Q_μ and Q_{tr} are the Q factors of the detector and the pendulum), then for $T_\mu = T_{tr}$ the inherent fluctuations of the pendu-

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lum do not exceed the displacements due to the thermal noise of the detector.[15] On the antenna proposed in in[13,14], the role of the pendulum is played by a niobium diaphragm fixed to the end of the detector. The condition $Q_{tr} \geqslant Q_{\mu}$ is satisfied if the diaphragm is in the superconducting state. It is claimed in[13,14] that the "amplification" of the vibration amplitude as a result of the transformation will ease the detection condition. This conclusion is reached by a one-sided calculation of the signal-to-noise ratio without allowance for the reaction of the measuring device. We shall show below that the solution of the self-consistent problem leads to more stringent requirements on the sensor, which will make it difficult to use a displacement transformer.

Schematically, a gravitational detector with displacement transformer and SQUID as sensing element is shown in Fig. 1. The inductances (L_1, L_2, L_3) with the superconducting current $\mathcal{J}_0$ in the loop (L_1, L_2) form a null-type measuring circuit. In the equilibrium position, the gaps between the mass m of the transformer and the coils L_1 and L_2 are the same, equal to d, and there is no current in L_3. Displacements of m produce a current in L_3 and accordingly a magnetic flux in the SQUID ring L_s. The operation of the SQUID as a displacement sensor is analyzed below. The only important thing here is to allow for the inherent fluctuations of the SQUID, which we do phenomenologically, introducing a fluctuation flux Φ_{fl} in the ring with spectral density $(\overline{\Phi_F^2})^{1/2} \approx (10^{-4} - 10^{-5})\, \Phi_0\, [\mathrm{Hz}^{-1/2}]$ (Φ_F is the Fourier transform of the fluctuation flux). We ignore the damping in the transformer.

Setting $L_1 = L_0(1 + x/d)$, $L_2 = L_0(1 - x/d)$, $x = x_2 - x_1 + \mathrm{const}$, we obtain for the electromagnetic part of the energy (without fluctuations, to accuracy $(x/d)^2$)

$$E_{em} \approx \frac{\Phi_{13}^2}{L_0}\left(1 + \frac{x}{d}\right) - \Phi_{13}\mathcal{J}_3 \frac{x}{d} + L_3 \mathcal{J}_3^2, \tag{2}$$

where Φ_{12} and Φ_{13} are the magnetic fluxes in the loops (L_1, L_2) and (L_1, L_3); we have used the additional conditions $\Phi_{12} - 2\Phi_{13} = 0$, $L_0 = 2L_3$, $\Phi_{13} = L_0 \mathcal{J}_0$. We introduce the dimensionless notation

$$y_{1,2} = \frac{x_{1,2}}{d}, \quad i_3 = \frac{\mathcal{J}_3}{\mathcal{J}_0}, \quad \beta = \frac{\Phi_{13}\mathcal{J}_0}{m\omega_1^2 d^2}, \quad \alpha = \frac{m}{M},$$

$$\lambda = \frac{\Phi_{13}^2}{2L_3}\frac{1}{m\omega_2^2 d^2}, \quad \varphi_{fl} = \frac{\Phi_{fl}}{\Phi_{13}}, \quad \Lambda = \frac{1+2\lambda}{1+\lambda}.$$

For simplicity, as in[14,15], we assume fulfillment of the relation $\omega_1^2 = \omega_2^2(1 + \lambda) = n_0^2$ (electromagnetic tuning of the diaphragm frequency).[14] The equations describing the antenna (Fig. 1) have the form

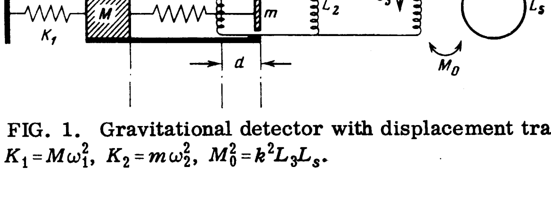

FIG. 1. Gravitational detector with displacement transformer; $K_1 = M\omega_1^2$, $K_2 = m\omega_2^2$, $M_0^2 = k^2 L_3 L_s$.

$$\begin{aligned} &\ddot{y}_1 + Q_\mu^{-1}\dot{y}_1 + (1+\alpha\Lambda)y_1 + \alpha\Lambda y_2 + \alpha\beta i_3 = f_{sig}(\tau), \\ &-\Lambda y_1 + \ddot{y}_2 + \Lambda y_2 - \beta i_3 = f_{sig}(\tau), \\ &-y_1 + y_2 - (1 - k^2/2)\, i_3 = -k(L_3/L_s)^{1/2}\varphi_{fl}(\tau). \end{aligned} \tag{3}$$

In the system (3), the differentiation is with respect to the dimensionless time $\tau = n_0 t$; $f_{sig}(\tau) = F_0/n_0^2 d$ is the force equivalent to the effect of the gravitational wave. The mechanical fluctuation force in (3) is omitted since our aim is to find restrictions on the sensitivity due to electric fluctuations. The result is given by analyzing the signal-to-noise ratio in the coordinate i_3. In an optimal arrangement, this ratio is estimated by means of the expression[16,17]

$$\mu \approx \pi^{-1}\int_{-\infty}^{\infty} d\nu\, |i_{sig}(\nu)|^2 N^{-1}(\nu),$$

where $i_{sig}(\nu)$ is the spectrum of the signal component of the current i_3; $N(\nu)$ is the spectral intensity of the fluctation component, and $\nu = \omega/n_0$ is the dimensionless frequency. We shall assume that the short signal of duration $\tau_{sig} = n_0\tau$, $\tau_{sig} \ll Q_\mu$, has spectrum concentrated in the region $\nu_c - \tau_c^{-1} \leqslant \nu \leqslant \nu_c + \tau_c^{-1}$ with constant spectral density f_ν. Then from (3) we obtain ($k < 1$, $\alpha \ll 1$)

$$\mu \approx \frac{1}{\pi}\int_{\nu_{sig}-\Delta\nu}^{\nu_{sig}+\Delta\nu} d\nu \frac{(1-\Lambda)^2 |f_\nu|^2}{k^2 (L_3/L_s)\varphi_\nu^2}\left\{\left[(1-\nu^2)(\Lambda-\nu^2) - \alpha\nu^2\Lambda\right]^2 + \frac{\nu^2(\Lambda-\nu^2)^2}{Q_\mu^2}\right\}^{-1}. \tag{4}$$

We investigate the case of strong electromagnetic coupling $\lambda \gg 1$ (on the Stanford antenna[14] $\lambda \approx 20$); then $\Lambda \approx 2$. Equation (4) gives the value of μ for an observation time longer than the relaxation time of the gravitational detector. We are interested in the case when the time of observation is limited by the duration of the pulse, and then Q_μ in (4) must be replaced by $\tau_{sig} = n_0\hat{\tau}$. Note also that the natural frequencies of the system (3) are $\nu_\pm^2 \approx 1 \pm \sqrt{\alpha}$. The damping frequencies, i.e., the frequencies at which the effect of the electric fluctuations of the SQUID is minimal (the first brackets in the denominator of (4) vanish) are equal to $\nu_1 \approx 1 - 2\alpha$ and $\nu_2 \approx 2(1 + 2\alpha)$. Under these conditions, for pulses whose duration does not exceed the beat period: $\tau_{sig} \lesssim \tau_0 = \pi\alpha^{-1/2}$ (the equation $\tau_{sig} = \tau_0$ corresponds to the maximum of the signal response), Eq. (4) admits a simple estimate. For $\nu_{sig} = 1$, $\nu_{sig} \approx \nu_\pm \sim \nu_1$, we obtain

$$\mu \lesssim |f_\nu|^2 \tau_{sig} / 2\pi k^2 (L_3/L_s)\varphi_\nu^2. \tag{5}$$

Setting $\mu = 1$, $f_\nu \approx f_{sig}\tau_{sig}/2$ we have in dimensional form for the minimal detectable amplitudes of the acceleration and displacement the estimates

$$(F_0)_{min} \geqslant \frac{2\sqrt{2\pi}}{\hat{\tau}}\left(\frac{W_{fl}}{\alpha M}\right)^{1/2}, \quad (\Delta x_1)_{min} \approx \frac{F_0\hat{\tau}}{2n_0} \geqslant \sqrt{2\pi}\left(\frac{W_{fl}}{\alpha M n_0^2}\right)^{1/2}, \tag{6}$$

where $W_{fl} = k^2\Phi_F^2/L_s\hat{\tau}$ is the energy of the fluctuations in the band $\hat{\tau}^{-1}$.

The form of Eqs. (6) clearly demonstrates the role of the displacement transformer. If $\alpha = 1$, Eqs. (6) give an estimate of the sensitivity for an antenna in which the SQUID directly measures the displacement of the detector when a strong coupling $\lambda \gg 1$ is maintained (for

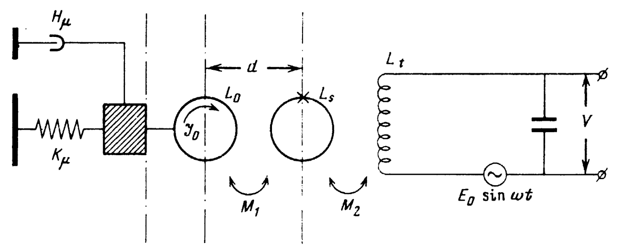

FIG. 2. Gravitational detector with SQUID; $K_\mu = m\omega_\mu^2$.

example, by means of the parameters $\mathcal{J}_0$ and d). The transformation worsens the sensitivity if W_{fl} remains constant. If the advantage of the transformer associated with the increase in the transformation coefficient is to be realized, there must be the same reduction in the intensity of the sensor fluctuations. Thus, the trantransformation does not ease the requirements on the sensor noise but makes them more stringent.

It is the aim of the following sections to analyze the fluctuation properties of a SQUID used as a displacement sensor in various regimes of operation.

3. GRAVITATIONAL ANTENNA WITH SQUID

Figure 2 shows schematically a gravitational detector with equivalent mass m, frequency ω_μ, and SQUID as sensing element. In the general case, the detector is coupled to the SQUID through the loop with current $\mathcal{J}_0$, which is secured to the detector and has coefficient of mutual inductance M_1. An rf tank circuit with pumping $E = E_0 \sin\omega t$ forms part of the quantum magnetometer; the coefficient of mutual inductance is $M_2^2 = k^2 L_t L_s$. The vibrations of the gravitational detector modulate the mutual inductance M_1 in such a way that $M_1 = M_0(1 + x/d)$, where d is the initial gap between the coupling loop and the SQUID. In practical schemes, the constant component of the flux $M_0\mathcal{J}_0$ through the SQUID ring can be compensated (as this is done in Fig. 1), and in calculations it can therefore be omitted. We denote by I_0 the critical current of the weak link in the SQUID ring.

We now give the brief details on the physics of SQUID needed for the following calculations. Two main regimes of operation are possible.

A. Regime without hysteresis (Hansma Regime [18])

The critical flux $\Phi_c \approx I_0 L_s$ at which the superconductivity of the weak link fails is appreciably less than the flux quantum $\Phi_0 = h/2e = 2.10^{-15}$ Wb. This regime is characterized by the dimensionless parameter $l = 2\pi I_0 L_s/\Phi_0 \ll 1$. In this case, the rf circuit and the SQUID ring form a parametric transducer with variable inductance controlled by the motion of the mechanical degree of freedom. The equations of the electrical part[19] have the form

$$li = \varphi_e + \varphi_t - \varphi, \qquad (7a)$$

$$q\dot{\varphi} + l\sin\varphi + \varphi = \varphi_e + \varphi_t - li_{fl}, \qquad (7b)$$

$$\ddot{\varphi}_t + Q^{-1}\varphi_t + (1 - 2\xi_0)\varphi_t = \varepsilon\cos\tau + \varepsilon_{fl} + k^2 l\ddot{i}, \qquad (7b)$$

where the differentiation is with respect to the dimensionless time $\tau = \omega t$. The current in the SQUID ring is $i = \mathcal{J}/I_0$; φ is the difference of the quantum-mechanical phase across the junction due to the internal magnetic flux Φ of the ring: $\varphi = 2\pi\Phi/\Phi_0$; the flux of the tank circuit, $\varphi_t = 2\pi\Phi_t/\Phi_0$, and the flux of the coupling loop, $\varphi_e = 2\pi\Phi_e/\Phi_0$, are normalized similarly. The flux of the tank circuit is related to the output voltage by

$$V = (\Phi_0/2\pi)\gamma_0\varphi_t, \quad \gamma_0 = (\omega/k)(L_t/L_s)^{1/2}.$$

The parameter $q = \Omega l = \omega L_s/R < 1$ is the Q factor of the SQUID, where R is the normal resistance of the Josephson junction, $\Omega = \omega/\omega_0$, $\omega_0 = (2\pi/\Phi_0)V_0 = 2\pi I_0 R/\Phi_0$ is the characteristic frequency of the junction. The difference between the frequency ω and the eigenfrequency ω_t of the tank circuit is characterized by $\xi = (\omega - \omega_t)/\omega_t \ll 1$; $Q \gg 1$ is the Q factor of the tank circuit; $\varepsilon = E_0(2\pi/\Phi_0\gamma_0)$ is the dimensionless amplitude of the pumping.

There are two noise sources. First, i_{fl}, the fluctuation current in the SQUID ring; following Danilov and Likharev, [19] we shall assume that $\langle i_{fl}\rangle = 0$, and that the correlation function satisfies

$$\langle i_{fl}(\tau_1) i_{fl}(\tau_2)\rangle = 2\Gamma\Omega\delta(\tau_1 - \tau_2) = S_i\delta(\tau_1 - \tau_2),$$
$$\Gamma = 2\pi\varkappa T/I_0\Phi_0$$

(T is the temperature of the junction). Second, we have ε_{fl}, the fluctuation voltage in the tank circuit. For it, $\langle\varepsilon_{fl}\rangle = 0$, and

$$\langle\varepsilon_{fl}(\tau_1)\varepsilon_{fl}(\tau_2)\rangle = 2\Gamma_t k^2 l Q^{-1}\delta(\tau_1 - \tau_2),$$
$$\Gamma_t = 2\pi\varkappa T_t/I_0\Phi_0$$

(T_t is the temperature of the tank circuit). For the regime without hysteresis, there is a unique dependence of the internal flux φ in the SQUID ring on the external flux, $\varphi_{ex} = \varphi_e + \varphi_t$, which is shown in Fig. 3 by the dashed curve.

B. Regime with hysteresis (the Zimmerman-Silver Regime [20])

The condition $\Phi_{cr} > \Phi_0$ or $l \gg 1$ is satisfied. If the rf flux exceeds the critical value, the function $\varphi = \varphi(\varphi_{ex})$ ceases to be single-valued (the continuous curve in Fig.

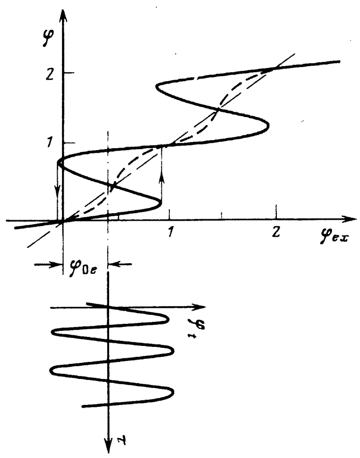

FIG. 3. Internal magnetic flux in the SQUID as a function of the external flux; $\varphi = 2\pi\Phi/\Phi_0$, $\varphi_{ex} = 2\pi\Phi_{ex}/\Phi_0$.

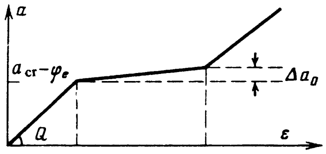

FIG. 4. Amplitude of tank circuit $a = \max\varphi_t$ as a function of the external pumping in the regime with hysteresis.

3) and there are jumps in φ associated with transitions of the SQUID ring to a different quantum state. In the case of harmonic pumping, the hystersis energy losses during the transitions between neighboring states are $W_0 \approx I_0\Phi_0$ per cycle. These losses decrease the amplitude of the forced oscillations of the circuit, reducing its Q. The dependence of the amplitude of the tank circuit on the pumping is shown in Fig. 4. The steep section has slope $\partial a/\partial\varepsilon = Q$ and corresponds to the absence of jumps in φ. The slowly rising section, the so-called plateau, is the working section. The amplitude is here close to the critical value $a = a_{cr} \sim \varphi_{cr}$. It is important that on the plateau there are hysteresis losses, and that the frequency of the jumps and, therefore, the magnitude of the losses, increase from the beginning until the end of the plateau.

Slow changes of the signal flux φ_e through the SQUID change the effective critical value of the flux $(\varphi_{cr} - \varphi_e)$ and, therefore, the amplitude of the circuit a. The plateau sinks and rises, following φ_e. For small changes (within π), the dependence of a and φ_e is linear, and the optimal coupling condition is $k^2Q \sim 1$.[21]

In the absence of thermal fluctuations, the jump in φ occurs exactly when the external flux reaches the critical value φ_{cr}. The thermal fluctuations in the SQUID ring mean that the jump time becomes random with mean $\varphi_{cr'} < \varphi_{cr}$ and variance $(\Delta\varphi_{cr})^2$. The stochastic nature of the transition to the neighboring quantum state is the origin of the specific inherent noise of the SQUID. The probability p of there being no jump is determined by the following expression, which is obtained in[22]:

$$p(u_m) = \exp\left\{-Y \int_{-\infty}^{\infty} (u_m^2 + z^2)^{1/2} \exp[-(u_m^2 + z^2)^{3/2}]\, dz\right\}, \tag{8}$$

$$Y \approx 0.2(\omega_0/\omega)\gamma^{2/3}(l^2-1)^{5/?}l^{-1/3}a^{-1/2}, \quad \gamma = 2\pi\varkappa T/I_0\Phi_0.$$

Here, a is the amplitude of the external flux through the SQUID: on the plateau $a \sim a_{cr} \sim l$. The variable u_m is related to the difference $a_{cr} - a$ by

$$u_m = [(3/4\sqrt{2})\, l\,(l^2-1)^{1/4}\gamma]^{-2/3}(a_{cr} - a)\ . \tag{9}$$

For the magnetometer in this regime, the conditions $\omega_0/\omega \gg 1$ and $\gamma \ll 1$ are also typical. Equation (8) is valid at not too small $Y \gtrsim 10$. The form of the function (8) is that of a step. The transition from zero to unity occurs at some value $\bar{u}_m \sim 1$, which increases with increasing Y. The width of the step, Δu_m, depends weakly on Y: for $10 \leqslant Y \leqslant 10^3$ we have on the average $\Delta u_m \approx 3$. This regime is described in more detail in[21–23].

Let us return to the antenna scheme (Fig. 2) and write down one further equation for the mechanical degree of freedom:

$$\ddot{y} + Q_\mu^{-1}\nu_\mu\dot{y} + \nu_\mu^2 y = \nu_\mu^2(f_{sig}(\tau) + \lambda i). \tag{10}$$

Here $y = x/d$, $\nu_\mu = \omega_\mu/\omega$; the coupling coefficient is $\lambda = M_0\mathcal{I}_0 I_0/m\omega_\mu^2 d^2$. The relations (7)–(10) make it possible to solve the problem of the limiting sensitivity of the gravitational antenna with SQUID.

§4. REGIME WITHOUT HYSTERESIS ($l \ll 1$)

We transform Eqs. (7) and (10). To the right and left of the SQUID ring there are systems with very different frequencies $\omega_\mu \ll \omega$, and therefore we need only separate two narrow bands near ω_μ and ω from the complete spectrum of frequencies for the current in the SQUID ring. We represent the phase by a sum of a slow and a fast component:

$$\varphi = \varphi_{sl} + \varphi_f\,|\dot{\varphi}_{sl}/\varphi_{sl}| \ll 1,\ \ \varphi_f = z\cos(\tau + \psi),$$
$$|\dot{z}/z| \ll 1,\ \ |\dot{\psi}| \ll 1.$$

The first of Eqs. (7) gives

$$\varphi_{sl} + l\sin\varphi_{sl}\,J_0(z) \approx \varphi_e - l i_{fl}^{sl},$$

$$q\varphi_f + \left(1 + l\cos\varphi_{sl}\,\frac{2J_1(z)}{z}\right)\varphi_f \approx \varphi_t - l i_{fl}^{f}, \tag{11}$$

where $J_0(z)$ and $J_1(z)$ are Bessel functions; i_{fl}^{sl} and i_{fl}^{f} are independent fluctuations with spectra concentrated respectively, in the region of the frequencies ω_μ and ω. We specify the form of the oscillations in the tank circuit:

$$\varphi_t \approx a\cos(\tau + \theta), \quad |\dot{a}/a| \ll 1, \quad |\dot{\theta}| \ll 1.$$

From the second equation (11) in the first order in l we find

$$\varphi_f \approx z\cos(\tau + \theta - \operatorname{arctg} q)$$
$$- l\,\frac{2J_1(z)}{z}\cos\varphi_{sl}\left[\frac{a}{1+q^2}\cos(\tau + \theta - 2\operatorname{arctg} q)\right] - \frac{l}{(1+q^2)^{1/2}}\, i_{fl}^{f}, \tag{12}$$
$$z = a/(1+q^2)^{1/2}.$$

Into the right-hand side of Eq. (7c) we must substitute $l\ddot{i} \approx \ddot{\varphi}_t - \ddot{\varphi}_f \approx -(\varphi_t - \varphi_f)$. After standard transformations we can arrive at truncated equations for the amplitude and the phase of the oscillations in the tank circuit:

$$\dot{a} + \delta(a\varphi_{sl})a + {}^1\!/_2\varepsilon\sin\theta + n_a = 0, \quad \dot{\theta} + \xi(a, \varphi_{sl})a + {}^1\!/_2\varepsilon\cos\theta + n_\theta = 0. \tag{13}$$

The new dampings and frequency difference are

$$2\delta(a\varphi_{sl}) = Q^{-1} + \frac{k^2}{1+q^2}\left[q - l\,\frac{2J_1(z)}{z}\cos\varphi_{sl}\,\frac{2q}{1+q^2}\right],$$
$$2\xi(a, \varphi_{sl}) = 2\xi_0 + \frac{k^2}{1+q^2}\left[-q^2 - l\,\frac{2J_1(z)}{z}\cos\varphi_{sl}\,\frac{1-q}{1+q^2}\right]. \tag{14}$$

In (13), n_a and n_θ are independent fluctuations with uniform spectrum

$$S_n \approx \frac{k^4 l^2}{\pi}\,\frac{\eta}{1+q^2}\,\Gamma\Omega, \quad \eta = 1 + \frac{\Gamma_t}{\Gamma}\,\frac{1+q^2}{qQk^2}. \tag{15}$$

In Eq. (10), it is necessary to take into account only the slow component $i_{sl} \approx (\varphi_e - \varphi_{sl})/l$ of the SQUID current. The last step in the transformations is to linearize Eqs. (13) in the neighborhood of the working point. Writing

$\varphi_{s1} = \varphi_{0s1} + \tilde{\varphi}$, $|\tilde{\varphi}| \ll \varphi_{0s1}$ (φ_{0s1} is the constant bias in the SQUID), $a = a_0 + \tilde{a}$, $|\tilde{a}| \ll a_0$, $\theta = \theta_0 + \tilde{\theta}$, $\tilde{\theta} \ll \theta_0$, we obtain from (10) and (13) in the limit $Q_\mu \to \infty$:

$$\ddot{y} + \nu_\mu^2(1 - \lambda^2 C_0/l)\, y + \nu_\mu^2(\lambda/l)\,\tilde{\varphi} = \nu_\mu^2 f_{sig}(\tau),$$
$$-\lambda C_0 y + \tilde{\varphi} + B\tilde{a} = -l i_{fl}^{s1}, \tag{16}$$
$$C_a\tilde{\varphi} + \tilde{a} + \beta\tilde{a} - \xi a_0\tilde{\theta} = n_a, \quad C_\theta\tilde{\varphi} + \alpha\tilde{a} + a_0(\dot{\tilde{\theta}} + \delta\tilde{\theta}) = n_\theta .$$

In the system (16), the parameters ξ, δ, $\beta = \partial[a\delta(a, \varphi_{s1})]/\partial a$, $\alpha = \partial[a\xi(a, \varphi_{s1})]/\partial a$ are taken at the point (a_0, φ_{0s1}). The values of the remaining constants are

$$B = l \sin\varphi_{0sig} J_1(z_0)\,(1 + q^2)^{-1/2},$$
$$C_0 = 2\pi m(\omega_\mu d)^2/I_0\Phi_0;$$
$$C_a = \frac{\partial}{\partial\varphi_{s1}}[a\delta(a, \varphi_{s1})]_{a_0, \varphi_{0s1}} = \frac{2k^2 l J_1(z_0)\sin\varphi_{0s1}}{(1+q^2)^{3/2}} q = bq,$$
$$C_\theta = \frac{\partial}{\partial\varphi_{s1}}[a\xi(a, \varphi_{s1})]_{a_0, \varphi_{0s1}} = \frac{b(1-q^2)}{2} . \tag{17}$$

The solution of (16) makes it possible to find the signal and noise response of the antenna and estimate the sensitivity determined by the noise of the SQUID. We give the solution for an amplitude detector. By means of a Fourier transformation, we obtain for the spectral intensity of the fluctuation of the amplitude

$$|\tilde{a}(\nu)|_{fl}^2 \approx \frac{a_0^2}{|\mathrm{Det}(\nu)|^2}\{(\nu_\mu^2 - \nu^2)^2[\xi^2 + \delta^2 + \nu^2]S_n + \nu_\mu^4(\lambda^2 C_0)^2[(\xi C_\theta + \delta C_a)^2 + C_a^2\nu^2]S_i + O(l^3)\}. \tag{18}$$

The square of the spectral component of the signal response is

$$|\tilde{a}(\nu)|_{sig}^2 \approx \frac{a_0^2}{|\mathrm{Det}(\nu)|^2}\{(\xi C_\theta + \delta C_a)^2 + C_a^2\nu^2\} \times (\lambda C_0\nu_\mu^2)^2 |f_{sig}(\nu)|^2 . \tag{19}$$

The signal-to-noise ratio after an optimal linear filter with $\nu_c \sim \nu_\mu$ and a signal shape which is the same as in §2 is

$$\mu = \frac{1}{\pi}\int_{\nu_\mu - \Delta\nu}^{\nu_\mu + \Delta\nu} \frac{|\tilde{a}(\nu)|_{sig}^2}{|\tilde{a}(\nu)|_{fl}^2} d\nu \leqslant \frac{2}{\pi}\frac{\lambda^2 |f_{sig}(\nu_\mu)|^2 \Delta\nu}{\Gamma\Omega[(2\Delta\nu/\nu_\mu C_0)^2\eta\Psi_0 + 2\lambda^4]}, \tag{20}$$
$$\Psi_0 = \frac{(1+q^2)^2}{\pi J_1^2(z_0)\sin^2\varphi_{0s1}} \frac{\xi^2 + \delta^2 + \nu_\mu^2}{[\xi(1-q^2) + 2q\delta]^2 + 4q^2\nu_\mu^2} .$$

By minimizing Ψ_0, we determine the best regime of operation of the SQUID as a magnetometer.[23] The optimal values $\varphi_{0s1} = \pi/2$, $J_1(z_0) \approx 0.58$, $\xi = \delta(1-q)^2/1$ ($\nu_\mu \ll \delta$) give

$$\Psi_0 \approx \pi^{-1} J_1^{-2}(z_0) \approx 1.$$

Then, from (20), setting $\mu = 1$, we obtain

$$f_{sig}^2 \geqslant 2\pi\Gamma\Omega\tau_{sig}^{-1}[(2/\nu_\mu\tau_{sig}C_0)^2\eta\lambda^{-2} + 2\lambda]. \tag{21}$$

1) We investigate the case when the fluctuations in the SQUID ring are decisive, i.e., $\eta = 1$. This occurs for $\Gamma \gg \Gamma_t$ or for sufficiently large Q. From (21) there follows the existence of optimal coupling $\lambda_{opt}^2 \simeq \sqrt{2}(C_0\nu_\mu\tau_{sig})^{-1}$, which makes possible the following smallest detectable force $f_{sig}^2 \geqslant 2\sqrt{2}\pi(C_0\nu_\mu\tau_{sig}^2)^{-1}\Gamma\Omega$, or, in dimensional form,

$$(F_0)_{min} \gtrsim (4/\hat{\tau})\,(mkT\omega_\mu/\omega_0)^{1/2}. \tag{22}$$

A feature of (22) compared with (1) is that the pumping frequency ω has been replaced by the characteristic frequency ω_0 of the junction.

2) The opposite limiting case, when the noise of the tank circuit dominates the noise of the SQUID ring, corresponds to $\Gamma \to 0$. In (21) there remains only the first term, which shows that an increase of the coupling λ must increase monotonically the sensitivity. The level of the output signal falls. Keeping it at a maximum determines the optimal matched coupling. In this case, for $\xi \approx \delta \approx Q^{-1} \gg \nu_\mu$, we have $\lambda_{opt}^2 \approx 4\xi(C_0 C_\theta\nu_\mu\tau_{sig})^{-1}$, and the signal transfer coefficient is $(\tilde{a}/f_{sig})^2 \approx lC_0\nu_\mu\tau_{sig}(k^2 Q)$. Substitution of the optimal coupling into (21) for $\Gamma_t \gg \Gamma$ gives the second limiting estimate of the sensitivity:

$$(F_0)_{min} \geqslant 2\hat{\tau}^{-1}(m\varkappa T_t\omega_\mu/\omega_0)^{1/2}. \tag{23}$$

An analogous relation can be obtained for a phase detector. Equations (21)–(23) solve the problem of the limiting sensitivity of an antenna with SQUID in the regime without hysteresis.

§5. REGIME WITH HYSTERESIS ($l \gg 1$)

There is no complete analytic description of this regime in the literature. For our purposes, we construct a model in the region of the plateau, introducing phenomenologically various parameters for which an estimate can be taken from the solution of corresponding physical problems.

As before, we assume that the oscillations in the tank circuit are nearly harmonic: $\varphi_t = a\cos(\tau + \theta) = a\cos\psi$. We ignore the distortion in the shape due to the rapid changes of the amplitude $\sim k^2$ at the times of the jumps in the phase of φ because they are small compared with the mean amplitude $a \sim l$. We restrict ourselves to the condition of slow variation of the external flux φ_e: $\nu_\mu \ll Q^{-1}$; in this approximation, the amplitude in the tank circuit follows φ_e instantaneously. Substituting the profile of the oscillations into the third equation of the system (7), we obtain

$$[Q^{-1}a - k^2 l i_{sin}]^2 + [2\xi_0 a - k^2 l i_{cos}]^2 = (\varepsilon + \varepsilon_{cos})^2 + \varepsilon_{sin}^2,$$
$$i_{sin,\,cos} = \frac{1}{\pi}\int_0^{2\pi} i\begin{pmatrix}\sin\psi\\ \cos\psi\end{pmatrix} d\psi. \tag{24}$$

Here, ε_{cos} and ε_{sin} are the quadratic components at the pumping frequency of the fluctuation ems in the tank circuit.

We represent the phase φ as the sum of a constant smooth (nonhysteresis) part and an impulsive process $\eta(\tau)$, which describes the jumps to a neighboring quantum state (we do not consider the possibility of a hop through a state):

$$\varphi \approx \varphi_{0s1} + \frac{a}{1+l}\cos\psi + \eta(\tau).$$

The process has the form of a random sequence of pulses:

$$\eta(\tau)=\sum_i \alpha_i \eta_0 f(\psi), \tag{25}$$

where $f(\psi)=1$ for $2\pi n-\Delta_1<\psi<(2n+1)\pi-\Delta_2$ and $f(\psi)=0$ outside this interval; α_i is a random variable that takes the value 1 if a jump occurs and 0 otherwise; η_0 is the pulse amplitude; $f(\psi)$ is a function of the profile: a pulse arises near the maximum of φ_t; Δ_1 and Δ_2 are possible deviations from the exact extremal values of ψ. For sufficiently large l, we have Δ_1, $\Delta_2 \ll \pi$, the pulse duration is $\tau_0\sim\pi$ and the amplitude $\eta_0\sim 2\pi$.

The rf frequency part of the current is related to $\eta(\tau)$ by Eq. (7a):

$$i=(\varphi_t-\varphi)/l=[a\cos\psi-\eta(\tau)]l^{-1},$$

which enables us to find the Fourier components needed for (24): $i_{\sin}\approx-\eta_{\sin}l^{-1}$, $i_{\cos}\approx al^{-1}$. The random Fourier component $\eta_{\sin}$ can be expressed conveniently by separating the mean value $\langle\eta_{\sin}\rangle$ and the fluctuation $\tilde{\eta}_{\sin}$ with zero mean: $\eta_{\sin}=\langle\eta_{\sin}\rangle+\tilde{\eta}_{\sin}$. Choosing the initial frequency difference $\xi_0=k^2/2$ and taking into account the smallness of the fluctuations in the tank circuit compared with the pumping: $\varepsilon\gg|\varepsilon_{\cos}, \varepsilon_{\sin}|$, we obtain from (24)

$$Q^{-1}a+k^2\langle\eta_{\sin}\rangle+k^2\tilde{\eta}_{\sin}\approx\varepsilon+\varepsilon_{\cos}. \tag{26}$$

This is the basic equation for describing the processes on the plateau. It must be augmented by specifying the statistical characteristics of $\tilde{\eta}_{\sin}$. A random sequence analogous to (25) has been analyzed in the literature.[24] Under the assumption that the individual pulses arises independently, $\tilde{\eta}_{\sin}$ has a uniform energy spectrum $S_\eta(\nu)$ at frequencies low compared with the pumping frequency:

$$S_\eta(\nu)=(\omega\tau_0\eta_0)^2p(1-p), \quad \langle\eta_{\sin}\rangle=\omega\tau_0\eta_0(1-p). \tag{27}$$

In (27), we have omitted the details that take into account the random nature of the position of the leading edge and the random nature of the duration of the pulses, but we have retained the main effect—the random nature of the very fact of the occurrence or not of a pulse.

Equation (8) determines the probability p as a function of the amplitude in the tank circuit. In the range of variation of p from unity to zero, we adopt a linear approximation of the function $p(a)$, introducing the parameter Δa_0, which is the width of this region, an estimate of which we take from (8) and (9)

$$1-p=\frac{a-a_{\rm cr}+\varphi_e+\Delta a_0}{\Delta a_0};$$
$$|a_{\rm cr}-\varphi_e|\leqslant a<|a_{\rm cr}-\varphi_e|+\Delta a_0, \tag{28}$$

$\varphi_e=\varphi_{e0}+\varphi_y$; φ_{e0} is the constant external bias, $\varphi_y=\lambda C_0 y$ is the signal flux through the SQUID ring; see (16).

We substitute (27) and (28) into (26) and, in addition, we set $a=a_0+\tilde{a}$, where a_0 is the mean value of the amplitude on the plateau, $\tilde{a}$ is its variation under the influence of the signal and the fluctuations ($a_0\gg\tilde{a}$ but $\tilde{a}_{\rm fl}\gg k^2$, $\varphi_y\sim a\sim\varepsilon_{\cos}$). Collecting terms of the same order, we obtain the following from (26):

a) the equation for the mean value (structure of the plateau)

$$a_0=Q_{\rm eff}\left[\varepsilon+(a_{\rm cr}-\varphi_{e0}-\Delta a_0)\frac{k^2\omega\tau_0\eta_0}{\Delta a_0}\right],$$
$$Q_{\rm eff}=Q\left[1+Qk^2\frac{\omega\tau_0\eta_0}{\Delta a_0}\right]^{-1}; \tag{29}$$

b) equation for the variation of the amplitude,

$$\tilde{a}=Q_{\rm eff}\left[\varepsilon_{\cos}-k^2\tilde{\eta}_{\sin}-k^2\frac{\omega\tau_0\eta_0}{\Delta a_0}\varphi_y\right]. \tag{30}$$

These characteristics of the random process $\tilde{\eta}_{\sin}$ are taken at the point (a_0, φ_{0e}). It can be seen from (29) that with increasing pumping the amplitude of the tank circuit on the plateau increases (Fig. 4), but much more slowly than in the absence of hysteresis losses. Since $Qk^2\sim1$, $\Delta a_0<1$ [see (9)], the effective Q factor or the slope of the plateau are proportional to $\Delta a_0/k^2\omega\tau_0\eta_0$; $k^2\omega\tau_0\eta_0$ is the length of the plateau, and Δa_0 is the height difference across the plateau.

Thus, the slope of the plateau contains information about the intensity of the inherent noise of the SQUID. The origin of the plateau in (29) corresponds to $\varepsilon=0$ and the amplitude to $a_0\approx a_{\rm cr}-\varphi_{e0}$.

For the problem of the sensitivity of a gravitational antenna, it is sufficient to take Eqs. (30) in conjunction with Eq. (10), in which it is now necessary to take into account the low frequency components of the current:

$$i_{\rm sl}\approx(\varphi_y-\eta_{\rm sl})l^{-1}=(\varphi_y-\langle\eta_{\rm sl}\rangle-\tilde{\eta}_{\rm sl})l^{-1}.$$

It follows from[24] that the low-frequency component $\tilde{\eta}_{\rm sl}$ of the random process $\eta(\tau)$ has the same spectral density (27) as $\tilde{\eta}_{\sin}$, and they are completely correlated (this permits us in what follows to make no distinction between them); for the mean value, we have

$$\langle\eta_{\rm sl}\rangle=(1-p)\omega\tau_0\eta_0=[(a_0-a_{\rm cr}-\varphi_{e0})+\varphi_y+\tilde{a}](\omega\tau_0\eta_0/\Delta a_0).$$

Taking into account these remarks, we obtain from (10) and (30)

$$\ddot{y}+Q_\mu^{-1}\nu_\mu\dot{y}+\nu_\mu^2(1+\lambda^2C_0\beta/l)y+\nu_\mu^2(\beta/l)\tilde{a}\approx\nu_\mu^2[f_c-(\lambda/l)\tilde{\eta}_{\sin}],$$
$$\lambda C_0k^2\beta y+Q_{\rm eff}^{-1}\tilde{a}\approx[\varepsilon_{\cos}-k^2\tilde{\eta}_{\sin}]. \tag{31}$$

Here $\beta=\omega\tau_0\eta_0/\Delta a_0\gg1$, and the value of the moments of $\tilde{\eta}_{\sin}$ is taken at the point (a_0, φ_{e0}). Solving (31), we find for the spectral density of the fluctuations of the amplitude in the circuit

$$|\tilde{a}(\nu)|^2_{\rm fl}=\frac{1}{|{\rm Det}(\nu)|^2}\left\{k^4\left[(\nu_\mu^2-\nu^2)^2+\frac{1}{Q_\mu^2}\nu_\mu^2\nu^2\right]S_\eta(\nu)+\left[\left[\nu_\mu^2\left(1+\frac{\lambda^2C_0\beta}{l}\right)-\nu^2\right]^2+\frac{\nu_\mu^2\nu^2}{Q_\mu^2}\right]|\varepsilon_{\rm fl}(\nu)|^2\right\} \tag{32}$$

and for the square of the spectral component of the signal,

$$|\tilde{a}(\nu)|^2_{\rm sig}=\frac{\nu_\mu^4k^4}{|{\rm Det}(\nu)|^2}(\lambda C_0\beta)^2|f_{\rm sig}(\nu)|^2. \tag{33}$$

As in §4, we consider two limiting cases.

1) The noise of the circuit is negligibly small: $|\varepsilon_{f1}(\nu)|^2 \sim \Gamma_t/Q \to 0$. The sensitivity of the antenna is determined solely by the inherent noise of the SQUID. For $\nu \approx \nu_\mu$, as can be seen from (33) and (32), the sensitivity increases with increasing λ. This is due to the correlation between the random processes $\tilde{\eta}_{s1}$ and $\tilde{\eta}_{\sin}$. The optimization of λ is here associated with a maximum of the transfer coefficient: $\lambda^2_{opt} \approx (2/\nu_\mu \tau_{sig})(l/C_0\beta)$; then $(a_{sig}/f_{sig})^2 \approx k^4\beta^3 C_0 l \nu_\mu \tau_{sig}$. Calculation of the signal-to-noise ratio by means of the same rules as in the previous sections, gives

$$\mu \lesssim \lambda^2 f^2_{sig} C_0^2 \tau_{sig} (\nu_\mu \tau_{sig})^2 / 4\Delta a_0^2 p(1-p).$$

For $\mu = 1$ and $\lambda = \lambda_{opt}$, for the middle of the plateau ($p = \frac{1}{2}$) we obtain in dimensional form

$$(F_0)_{min} \geqslant \frac{2}{\hat{\tau}} \left(m \frac{\Phi_0^2}{2L_s} \frac{\omega_\mu}{\omega} \Delta a_0 \right)^{1/2}, \tag{34a}$$

from which it can be seen that for the inherent fluctuations of the SQUID the characteristic energy is that of the quantum of the magnetic flux in the ring $\Phi_0^2/2L_s \gg \varkappa T$; the last inequality is the condition of operation of quantum magnetometers.

2) In the opposite case, the noise of the tank circuit is predominant: $S_\eta(\nu) \gg |\varepsilon_{f1}(\nu)|^2$ or $\varkappa T_t \gg \Phi_0^2/2L_s$. The signal-to-noise ratio is here also restricted only by the condition of maximizing the signal at the output. Optimal coupling corresponds to the same λ_{opt} as in the preceding example, and the estimate for the minimal detectable force is

$$(F_0)_{min} \geqslant \frac{2}{\hat{\tau}} \left(m\varkappa T_t \frac{\omega_\mu}{\omega} \Delta a_0 \right)^{1/2}. \tag{35a}$$

In (34a) and (35a) the important thing is the dependence on the parameter Δa_0, which determines the slope of the plateau. Note that in the absence of thermal fluctuations in the ring there is a regular slope of the plateau $\sim k^2 \approx Q^{-1}$; our calculations, as in the theory of Kurkijarvi and Webb, [22] are valid when $\Delta a_0 \gg k^2$. An estimate from (8) and (9) gives

$$\Delta a_0 = \Delta u_m [(3/4\sqrt{2}) l (l^2-1)^{1/4} \gamma]^{2/3};$$

for $l \gg 1$ and $\Delta u_m \approx 3$ we have $\Delta a_0 \approx 2l\gamma^{2/3}$. For the purpose of comparison, it is convenient to rewrite (34a) and (35a), substituting Δa_0 and introducing the temperature of the Josephson junction:

$$(F_0)_{min} \geqslant \frac{4\pi}{\hat{\tau}} \left[m\varkappa T \frac{\omega_\mu}{\omega} \frac{1}{\gamma^{1/3}} \right]^{1/2}, \tag{34b}$$

$$(F_0)_{min} \geqslant \frac{4\sqrt{\pi}}{\hat{\tau}} \left[m\varkappa T \frac{\omega_\mu}{\omega} \frac{\varkappa T_t}{\Phi_0^2/2L_s} \frac{1}{\gamma^{1/3}} \right]^{1/2} \tag{35b}$$

or $\gamma \ll 1$, $\varkappa T_t \gg \Phi_0^2/2L_s$.

From our model, one can also readily obtain estimates of the limiting sensitivity of a magnetometer based on a SQUID in the hysteresis regime. Equation (30) for given $\varphi_y = \varphi_e$ and $\varepsilon_{\cos} = 0$ gives

$$\Delta a_{min} = (\Delta \varphi_e)_{min} = \Delta a_0 [p(1-p)\tau^{-1}]^{1/2}.$$

In the middle of the plateau ($p = \frac{1}{2}$) in dimensional form we have for the detectable flux

$$\Delta \Phi_{min} \approx \frac{\Phi_0}{4\pi} \frac{\Delta u_m}{(\omega\tau)^{1/2}} \left[\frac{3}{4\sqrt{2}} l (l^2-1)^{1/4} \gamma \right]^{2/3}.$$

For $\Delta u_m \approx 3$, $L_s \approx 10^{-9}$, $L_s I_0 \sim \Phi_0$, $(\omega/2\pi) \sim 20$ MHz, $T \sim 4$ °K, we obtain $\Delta\Phi_{min} \approx 2 \cdot 10^{-5}\, \Phi_0\, \text{Hz}^{-1}$, which agrees with the computer calculation in[22].

§6. DISCUSSION OF RESULTS

Equations (22), (23) and (34), (35) enable us to estimate the possibilities of SQUIDS as displacement sensors for the second generation of gravitational antenna. All of these equations contain the characteristic parametric factor ω_μ/ω, but they have other features that indicate a reduction of the sensitivity compared with (1).

We recall that all the calculations have been made under the assumption that the Brownian noise of the gravitational detector is small. On the background of this noise, the smallest force that is detectable is $(F_\mu)_{min} \approx (m\varkappa T_\mu \omega_\mu / Q_\mu \tau)^{1/2}$. Our equations are valid for $(F_0)_{min} \geqslant (F_\mu)_{min}$. For the program of a dielectric detector[11] such a situation arises when $Q_\mu \geqslant (T_\mu/T_e)(\omega/\omega_\mu)\omega_\mu \hat{\tau}$ (T_e is the temperature of the electric degree of freedom). For $T_\mu = T_e$, $\omega/\omega_\mu = 10^5$, $\omega_\mu = 10^4$, 10^{-3} sec $\leqslant \tau \leqslant 1$ sec we obtain $Q_\mu \geqslant 10^6 - 10^9$. For the program of a supercooled detector, the same conditions arise when $T_\mu \leqslant T_e(\omega_\mu/\omega) \times (Q_\mu/\omega_\mu \hat{\tau})$. Substitution of $T_e = 2$ °K, $Q_\mu = 10^6$ gives $T_\mu \leqslant 1 - 10^{-3}$ °K. Thus, we are dealing with the ranges of Q_μ and T_μ values that occur in the plans of both programs.

It is sensible to regulate the coupling of the detector to the SQUID by means of the current $\mathcal{I}_0$ in the coupling loop and also by the geometry L_0, d of the loop. As is shown in §2, an increase in the electromechanical coupling by means of the displacement transformer does not improve the sensitivity. The contribution of the electric fluctuation energy to the mechanical degree of freedom increases by the same amount as the coupling constant. There remains the same gap between the magnitude of the signal and noise as there was before transformation.

Example. On the antenna of[13,14], the energy of the electric fluctuations of the SQUID is $W_{f1} \sim 10^{-22}$ erg/Hz. If the energy of the mechanical vibrations of the detector were completely converted into electric form, then an amplitude $\Delta x_0 \sim 10^{-18}$ cm ($M \sim 3 \cdot 10^5$ g, $\omega_\mu \sim 10^4$ Hz) would be amenable to measurement. In practice, the losses on conversion are about two orders of magnitude. Raising the coupling coefficient to unity by transformation of the displacements would require $\alpha^{-1} = (M/m) \sim 10^4$; at the same time, the sensitivity remains at the level $\Delta x_{min} \sim \Delta x_0 \sqrt{\alpha} \sim 10^{-16}$ cm.

The limiting sensitivity of an antenna with capacitative transducer is given by Eq. (1), whence for the minimal displacement we obtain

$$\Delta x_{min} \sim F_0 \hat{\tau} / m\omega_\mu \approx 3\sqrt{\pi} (\varkappa T_e / m\omega_\mu \omega)^{1/2} \sim 10^{-17} \text{ cm}$$

($\omega=10^{10}$ Hz, $T_e=2$ °K, $m=10^6$, $\omega_\mu=10^4$ rad/sec). For the SQUID in the hysteresis regime, we take the following typical parameters[23]: $I_0\sim10^{-4}$ A, $T\sim2$ °K, $\omega\approx10^9$, $\omega_0\approx10^9$–10^{10}. Then $\gamma=2\pi\ T/I_0\Phi_0\approx10^{-3}$, and it is obvious that (34b) and (35b) predict a sensitivity worse than (1). In the best case, when the inherent noise of the SQUID is predominant, we obtain from (34b)

$$(\Delta x)_{min}\sim\bar{F}_0\hat{\tau}/m\omega_\mu\approx10^{-16}\text{ cm}$$

which exceeds by an order of magnitude the level of the Brownian noise of the gravitational detector with the parameters of[14].

A SQUID in the regime without hystersis has a sensitivity near that of the capacitative transducer. However, here the existence of the limiting sensitivity (22), which does not depend on the pumping frequency, is important; it replaces the critical frequency of the Josephson junction. The physical origin of this effect is the simultaneous presence of the two independent noise bands generated by the resistance R of the junction around ω_μ and ω. We emphasize that the need to bear in mind the rf component of the junction noise, which is far from the frequency of the signal, is a negative feature of this device. As an example of the parameters of the regime without hysteresis, let us take $L_s\approx5\cdot10^{11}$, $V_0=RI_0\sim10^{-3}$–10^{-4} V, and then for $l<1$ we require $I_0<10^{-5}$ A, and the critical frequency reaches $\omega_0\approx10^{12}$ rad/sec. Substitution of this frequency into (22) is not reasonable, since here we already have $\hbar\omega>\varkappa T$. Our calculations are valid up to frequencies $\omega_0\sim\omega\sim5\cdot10^{10}$ rad/sec, and here the sensitivity of the regime without hysteresis is at the same level as for the capacitative transducer. However, in magnetometers this regime has not yet been achieved experimentally because of the difficulty of obtaining stable junctions with small critical currents.

Thus, a SQUID on a gravitational antenna is capable of providing a sensitivity that at best approaches that of an antenna with a capacitative parametric transducer.

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[1)]The word SQUID derives from the initial letters of the English words Superconducing Quantum Interference Device. It so happens that the Russian transliteration skvid can be regarded as an acronym of the Russian words sverkhprovodyashchiĭ kvantovyĭ interferentsionnyĭ datchik.

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