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Inverted hot-electron states and negative conductivity in semiconductors

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The feasibility of obtaining inverted hot-electron distribution functions and a negative differential conductivity (NDC) in semiconductors at high frequencies and in strong electric and magnetic fields is considered for the case when a strong scattering mechanism is turned on only above a certain critical electron energy $\epsilon=\epsilon_0$. In pure n-GaAs in a moderate electric field at lattice temperatures $T\ll\hbar\omega_0$, the electron energy ϵ_0 can be identified with the optical phonon energy $\hbar\omega_0$. Numerical calculations of the electron distribution function by the Monte Carlo method show that in this case it should be possible to obtain in crossed electric and magnetic fields a maser type NDC at frequencies $\omega\gtrsim 10^{12}$ Hz near the cyclotron resonance. In a strong electric field, when ϵ_0 is the intervalley transition energy in n-GaAs, the NDC should appear at frequencies of the order of the reciprocal free-flight time of a light-valley electron in the electric field. The conditions for the appearance of NDC at frequencies determined by the time of flight of the electrons in a field E between $\epsilon=0$ and $\epsilon=\epsilon_0=\hbar\omega_0\gg T$ in n-InSb in a magnetic field $\mathbf{H}\parallel\mathbf{E}$ are also considered for the case when the electrons occupy one lower Landau level.

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1. INTRODUCTION

Negative conductivity should be possible in inverted distributions of charged particle systems with a region of energies ε for which the population of the high-energy states is greater than the population of the low-energy states. For "hot" electrons, however, inverted distributions are not possible as a rule. In fact, under typical conditions elastic (or quasielastic) scattering predominates for hot electrons over all other processes. Hence, the distribution function is almost isotropic, and is of the Druyvesteyn type with a maximum at $\varepsilon=0$ for an arbitrary energy dependence of the scattering intensity (see, e.g., [1,2,3]). Under these conditions, inverted hot-electron distributions and the resulting negative conductivity can arise, apparently, only when the electron creation and annihilation (capture and recombination) are substantial[1)] (cf. [4]). In addition, as shown by Rabinovich, [6] there is no inverted hot-electron distribution also when elastic scattering predominates, while the electron-to-lattice energy transfer proceeds via inelastic processes.

In pure semiconductors, however, conditions exist under which strong scattering sets in (is "turned on") only at electron energies above a certain threshold ε_0, and at $\varepsilon<\varepsilon_0$ the scattering is small and the electrons move almost freely under the influence of the electromagnetic fields. Under these conditions the hot-electron distribution can be strongly anisotropic and inverted due to electron recoil or accumulation at $\varepsilon=\varepsilon_0$. The energy ε_0 can be the optical phonon energy $\hbar\omega_0$ (for $\hbar\omega_0\gg T$), the energy ε_h at which the intervalley transfer begins to take place (as for example in n-GaAs), and others. Such conditions are also favorable to the formation of a bulk negative differential conductivity (NDC) of hot electrons at microwave frequencies. The microwave NDC can be of two types.

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The first is microwave NDC of the maser type. Its presence is due to the inversion in the electron distribution. It can take place, for example, at frequencies ω near the cyclotron resonance (CR)[2] ($\omega \approx \omega_c$). The possibility of hot-electron NDC at CR was noted in[14] for the case of electron scattering by optical phonons (at $\varepsilon_0 = \hbar\omega_0 \gg T$) under conditions similar to those discussed by Vosilius and Levinson[15] in connection with the galvanomagnetic hot-electron effects. A detailed analysis of this possibility, based on a Monte Carlo calculation of the electron characteristics, is given in Sec. 2 of the present article. We shall demonstrate that NDC at CR is possible in pure materials (p-Ge and n-GaAs) under these conditions at $\omega > 10^{12}$ Hz and above. The maser-type NDC is apparently also possible in sufficiently intense electric fields ($E \gtrsim 12$–15 kV/cm) in n-GaAs under conditions of an intense electron exchange between the light and heavy valleys of this material. An inversion in the light-valley electron distribution was observed under these conditions in[16]. It will be shown in Sec. 3, on the basis of a model calculation, that this inversion can lead to NDC at frequencies above $(3-5)\times 10^{12}$ Hz.

Secondly, microwave NDC is possible near frequencies determined by the time required to accelerate the electrons in electric and magnetic fields to the energy[3] ε_0. Here the NDC is due to the bunching of the electrons in phase space under the action of the alternating and constant electric fields; it resembles the negative conductivity effects in monotrons and klystrons. We shall consider this possibility in Sec. 4 for the case of n-InSb in a quantizing field $\mathbf{H} \parallel \mathbf{E}$ at cyclotron frequencies[4] $\omega_c \gtrsim \omega_0$. In this case the optical scattering intensity is greater than at $H = 0$, the scatter of the electron flight times is smaller, and microwave NDC becomes possible at frequencies $\omega \sim 2\times 10^{12}$ Hz at impurity concentrations N_I up to 10^{15} cm^{-3}. In addition, electron-flight microwave NDC appears to be also possible in n-GaAs under conditions of intervalley transfer in a strong electric field (i.e., under the same conditions as those for the maser-type microwave NDC mentioned above) at frequencies on the order of the reciprocal time of flight of electrons across the light valley. This microwave NDC is discussed in Sec. 3.

It should be stressed that there is no static NDC (cf. [14]) in the two considered cases of NDC at microwave frequencies—cyclotron-resonance NDC and microwave NDC in n-InSb. This circumstance can be important in the observation or utilization of microwave NDC. These questions are discussed in the Conclusion.

The considered effects can be properly verified at this time only by means of numerical experiments on hot electrons. In particular, Fawcett and others[16] detected precisely in these experiments an energy inversion in the distribution of light-valley electrons in n-GaAs at $E > 10$ kV/cm and $T \leq 300$ K, due to return of the heavy-valley electrons to the light valley, whereas Kurosawa and Maeda[21] discovered an inversion in the hole distribution in p-Ge, due to the interaction with optical phonons[5] at $H \neq 0$. It should be noted, however, that the non-Maxwellian needle-shaped hole distribution in p-Ge at $H = 0$ (which is what turns into the inversion in the hole distribution at $H \neq 0$) was experimentally studied a long time ago.[22] We emphasize also that the considered microwave NDC effects take place under conditions when the interelectron and impurity scatterings are insignificant. These effects are determined by the details of the scattering dynamics and of the hot-electron distribution. They can be therefore analyzed either on the basis of a calculation of idealized situations that admit of an analytical solution of the kinetic equations (cf. [14,17], Sec. 2, and the Appendix), or by numerical methods.

2. INVERTED STATES, NDC AT CR, GALVANOMAGNETIC EFFECTS AND STATIC DIFFERENTIAL CONDUCTIVITY OF HOT ELECTRONS IN SCATTERING BY OPTICAL PHONONS

1. Let us consider the CR differential conductivity of hot electrons in the case when the primary electron scattering mechanism is spontaneous emission of optical phonons. In this case, at an electron energy $\varepsilon < \hbar\omega_0$ (in the "passive" region[15]), the frequency ν of the elastic collisions is low, and the electrons move almost freely under the action of the electromagnetic fields. At the same time, at $\varepsilon > \hbar\omega_0$ (in the "active" region) the electron emits optical phonons rapidly with a characteristic time $\tau_0 \ll 1/\nu$, and returns to the passive region with $\varepsilon \approx 0$.

Figure 1 illustrates the onset of NDC at CR in this case. It shows the electron distribution and their dynamics in the idealized case when the characteristic time $\tau_0 = \nu_0^{-1} = (eE_0/p_0)^{-1}$ of spontaneous emission of the optical phonon tends to zero, where $p_0 = (2m\hbar\omega_0)^{1/2}$ is the electron momentum at $\varepsilon = \varepsilon_0$, m is the effective mass, and E_0 is the characteristic optical-scattering field (see, e.g., [1]). In this case, the electrons land in a small region near $p = 0$ after emitting the optical phonons. As a result, they are concentrated in the vicinity of the trajectory b (the "principal" trajectory[15]). As a consequence of the "switching on" of the strong optical scattering at $p > p_0$, the width of the CR line is greater for electrons on the trajectory c than on the trajectory a.

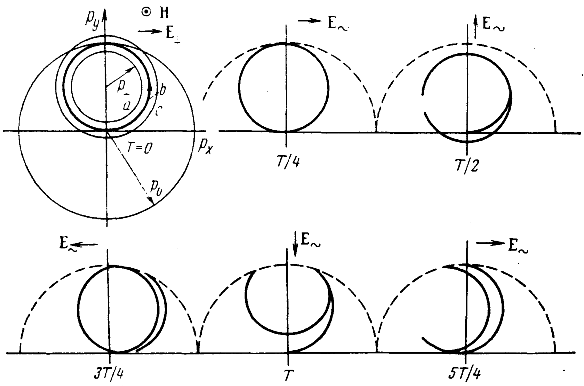

FIG. 1. Schematic representation of the electron distribution in a constant electric field $\mathbf{E}_\perp$ and in a weak alternating electric field $\mathbf{E}_\sim$ at cyclotron resonance for different instances of the cyclotron rotation; $T = 2\pi/\omega_c$ is the cyclotron period, $p_0 = (2m\hbar\omega_0)^{1/2}$, $E_\perp = p_0H/2mc$.

This, together with the energy inversion of the cyclotron rotation[6] (relative to the small population at the center of the phase-trajectories compared with the population of the trajectories $(a-c)$), is the reason for the predominant transitions to the lower Landau levels and for the NDC at CR (cf. [8,10,11]). This also illustrates the electron distribution, shown in the figures, at individual instants of the cyclotron period following application, at $T=0$, of a weak uniform circularly polarized alternating field $\mathbf{E}_\sim$ at CR was turned on. We see that surviving electrons are those in the cyclotron-rotation phases in which the rotation energy is decreased by the alternating field, and for which $\mathbf{E}_\sim\mathbf{j}_\sim<0$, where $\mathbf{j}_\sim$ is the alternating-current density, i.e., NDC at CR.

Under these conditions, however, the inversion relative to the cyclotron-rotation energy appears only at sufficiently low cyclotron frequencies ($\omega_c \ll \nu_0$), since in a strong magnetic field, owing to the relation $E_\perp \approx p_0 H/2mc$, the electrons penetrate deeply in the region $p>p_0$ during the optical-phonon emission time τ_0. As a result, following the emission of the optical phonons, the electrons land in a sufficiently wide region near $p=0$, and the inversion in cyclotron-rotation energy is weakened. The latter fact is confirmed by numerical results[21]: at $\omega_c/\nu_0 \approx 0.5$, the inversion of the distribution of the electrons in the cyclotron-rotation energy is very weak. Moreover, it can be shown that the requirements which the inversion in terms of the cyclotron-rotation energy must satisfy in order that NDC at CR be formed turn out very stringent in this case.

A more favorable situation for the formation of an inversion in the cyclotron-rotation energy and of NDC at CR at large cyclotron frequencies arises already while there is still a weak electric field $\mathbf{E}_\parallel \parallel \mathbf{H}$ in the case shown in Fig. 2. This figure shows the conditions under which the inversion in cyclotron rotation energy is pronounced, and indicates the form of the electron phase trajectories and of their distributions in this case. The electrons "graze" the sphere $\varepsilon=\varepsilon_0$ almost tangentially; their penetration into the region $\varepsilon>\varepsilon_0$ is therefore determined in practice by the value of $E_\parallel$, rather than by $E_\perp$ and H; it is consequently small even at large $E_\perp$ and H. As a result, after emitting the phonons, the electrons land in a small region near $\varepsilon=0$. This is in fact the cause of the tubular electron distribution shown in the figure. The width of the CR line is determined in this case primarily by the electron time of flight in the fields E and H from $p\approx 0$ to $p=p_0$, and it depends substantially on $E_\parallel$. This circumstance facilitates the onset of NDC at CR. Under these conditions, differential conductivity at CR (σ_c) can be obtained by neglecting the contribution of electrons with $p>p_0$ and the scattering at $p<p_0$.

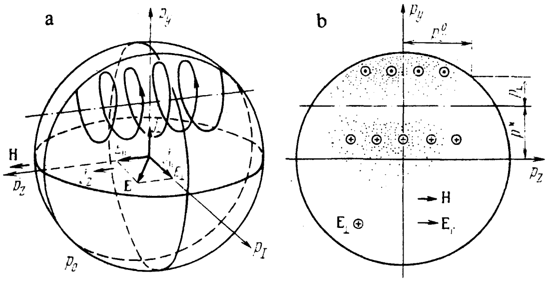

FIG. 2. a) Electron phase trajectories and current components; b) electron distributions at $\nu p_0/e \ll E_\parallel \ll E_0$, $E_\parallel \ll E_\perp \approx p_0 H/2mc$; $p^*=mcE_\perp/H$.

We write down the linearized kinetic equations for the alternating component $f_\sim = f_\sim^0 \exp(i\omega_c t)$ of the distribution function in a homogeneous alternating circularly-polarized cyclotron-frequency field $E_\sim = E_\sim^0 \exp(i\omega_c t)$ at $p<p_0$:

$$i\omega_c f_\sim^0 + \omega_c \frac{\partial f_\sim^0}{\partial \varphi} + eE_\parallel \frac{\partial f_\sim^0}{\partial p_z} = eE_\sim^0 N e^{-i\varphi}\left\{\frac{\partial f_0}{\partial p_\perp} - \frac{i}{p_\perp}\frac{\partial f_0}{\partial \varphi}\right\} + I(\mathbf{p}), \quad (2.1)$$

where

$$p_\perp^2 = p_x^2 + (mcE_\perp/H - p_y)^2, \quad \mathrm{tg}\,\varphi = p_x (mcE_\perp/H - p_y)^{-1}$$

(see Fig. 1); N is the electron concentration, $I(\mathbf{p})$ is the arrival term of the electrons from the region $p>p_0$, and f_0 is a static distribution function such that $\int f_0 d^3p=1$. From (2.1) we obtain

$$\sigma_c = \frac{e}{E_\sim^0 m}\int p_\perp^2 e^{i\varphi} f_\sim^0 dp_\perp d\varphi\, dp_z = (\sigma_c^{f_0} + \sigma_c^I) \leqslant \sigma_c^{f_0}, \quad (2.2a)$$

where $\sigma_c^{f_0}$ is the contribution of the first term in the right hand side of (2.1):

$$\sigma_c^{f_0} = \frac{eN}{E_\parallel m}\iiint\limits_{p<p_0} p_\perp\, dp_\perp\, d\varphi\, dp_z \int\limits_{-p_z^0}^{p_z} \frac{\partial f_0}{\partial p_\perp} dp_z'$$

$$= \frac{eN}{E_\parallel m}\left\{\int\limits_0^{p_0-p^*}\left[2p_\perp p_z^0 - \frac{\partial p_z^0}{\partial p_\perp} p_\perp^2\right] F(p_\perp)\, dp_\perp - 2\int\limits_{p<p_0} p_z f_0 p_\perp\, dp_\perp\, d\varphi\, dp_z\right\}, \quad (2.2b)$$

and σ_c^I is the contribution of $I(\mathbf{p})$. In (2.2b)

$$F(p_\perp) = \int\limits_0^{2\pi}\int\limits_{-p_z^0}^{p_z^0} d\varphi\, dp_z f_0, \quad p^* = mcE_\perp/H,$$

$p_z^0 = [p_0^2 - (mcE_\perp/H + p_\perp)^2]^{1/2}$ (see Fig. 2), and the integration with respect to $p_\perp$ is performed within the limits $\{0, p_0 - mcE_\perp/H\}$, i.e., along helical trajectories whose upper portions (see Fig. 2) are at $p<p_0$. The contribution to the differential conductivity of the trajectories with $p_\perp > (p_0 - mcE_\perp/H)$ at CR is independent of $E_\parallel$ and is small.

The relation $\sigma_c^I<0$ is satisfied in a sufficiently strong magnetic field when, first, the electrons reach the spheres at $\varphi \approx \pi$ and, second, execute several cyclotron rotations at $p>p_0$ before they emit the phonons. The corresponding condition is

$$(eE_\parallel/p_0\omega_c) \ll (E_\parallel/E_0)^{1/2}.$$

In this case, it can be assumed (at both $p<p_0$ and $p>p_0$) that f_0 is independent of φ, and $f_\sim^0 = f_\sim(p_\perp, p_z)e^{-i\varphi}$. This makes it possible to solve comparatively simply the kinetic equation for $p>p_0$, to find $I(\mathbf{p})$, and to estimate its contribution to σ_c (see the Appendix). At $T\neq 0$ and $N_I \neq 0$, Eq. (2.2) apparently overestimates the value of

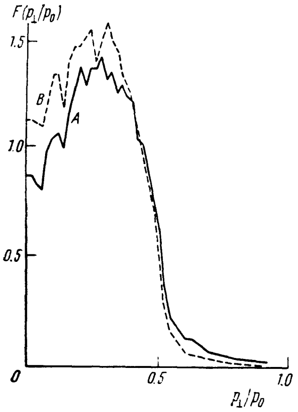

FIG. 3. Electron distribution in the momentum $p_\perp$ of the cyclotron rotation for $\tilde{E}_\perp = \tilde{H}/2$, $\tilde{E}_\parallel = 0.01$, $N_I = 0$, and $T = 0$, where $\tilde{H} = \omega_c/\nu_0$, and $\tilde{E}_{\perp,\parallel} = E_{\perp,\parallel}/E_0$ are dimensionless variables. A) $\tilde{H} = 0.25$; B) $\tilde{H} = 4.0$.

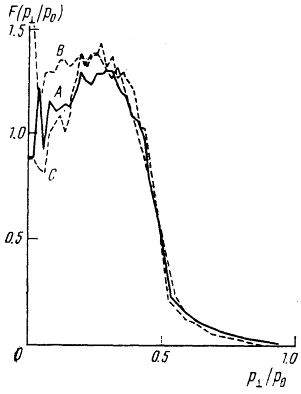

FIG. 4. Electron distribution in the momentum $p_\perp$ of cyclotron rotation in n-GaAs at $\tilde{E}_\perp = \tilde{H}/2$, $\tilde{E}_\parallel = 0.01$ and $\tilde{H} = 0.25$, where $\tilde{H} = \omega_c/\nu_0$ and $\tilde{E}_{\perp,\parallel} = E_{\perp,\parallel}/E_0$ are dimensionless variables. A) $N_I = 0$, $T = 77$ K; B) $N_I = 10^{14}$ cm^{-3}, $T = 0$; C) $N_I = 0$, $T = 0$.

σ_c, since the scattering at $p < p_0$ decreases most of all the contribution made to σ_c by the electrons that are long-lived at $p < p_0$, viz., the electrons with $p_\perp \approx 0$, which make a positive contribution to σ_c.

The results of the calculations of $F(p_\perp)$ by the Monte Carlo method (see, e.g., [16]) are shown in Figs. 3 and 4. Self-scattering, polar optical scattering and impurity scattering (according to Conwell and Weisskopf) were taken into account in the calculations. The electric[7)] and magnetic fields are given in dimensionless variables (for n-GaAs, the field is $E_0 \sim 6$ kV/cm, $\nu_0 \sim 4 \times 10^{12}$ sec^{-1} and $\omega_c = \nu_0$ at $H \sim 14$ kG; for n-InSb the field is $E_0 \sim 500$ V/cm, $\nu_0 = 8 \times 10^{11}$ sec^{-1} and $\omega_c = \nu_0$ at $H = 620$ G; for heavy holes in p-Ge we have $E_0 \sim 10$ kV/cm, $\nu_0 = 2.5 \times 10^{12}$ sec^{-1} and $\omega_c = \nu_0$ at $H = 50$ kG). The number of optical-phonon emissions by the electron in the Monte Carlo simulation is of the order of 20×10^3 for the cases represented in Figs. 3 and 4. The jagged appearance of the curves at low $p_\perp$ reflects the calculation accuracy.

Figure:	3, A	3, B	4, A	4, B	4, C
σ_c^{lo}:	$-28\sigma^*$	$-17\sigma^*$	$-18\sigma^*$	$-18\sigma^*$	$-28\sigma^*$,

where $\sigma^* = \omega_p^2/4\pi\nu_0 = e^2N/m\nu_0$ (ω_p is the plasma frequency).

The inversion in the function $F(p_\perp)$ (i.e., $\partial F(p_\perp)/\partial p_\perp > 0$, which, incidentally is not necessary at $\sigma_c > 0$) and the condition $\sigma_c < 0$ are preserved up to high frequencies ($\omega_c > 10^{13}$ Hz in n-GaAs), and in pure n-GaAs a conductivity $\sigma_c < 0$ is possible also at $T = 77$ K.

2. We now examine the possibility of low-frequency NDC and of instability under the same conditions. On one hand, this instability could suppress the NDC at CR because of the decay of the electronic state into low frequency oscillations. On the other hand, it is of interest in itself. As pointed out by Kurosawa and Maeda, [23] the most favorable for this NDC is the case of differential conductivity in a strong magnetic field in a plane perpendicular to the magnetic field, when the current-voltage characteristics are nonlinear. The differential-conductivity tensor in this plane can be transformed, by proper choice of the coordinate axes, into

$$\sigma_{\alpha',\beta'} = \begin{vmatrix} \sigma_1 & -\sigma_2 \\ \sigma_2 & \sigma_2 \end{vmatrix}, \quad (2.3)$$

where

$$\sigma_{1,2} = {}^1/_2\{(\sigma_{xx}+\sigma_{yy}) \pm [(\sigma_{xx}-\sigma_{yy})^2 + (\sigma_{xy}+\sigma_{yx})^2]^{1/2}, \quad (2.4)$$

and

$$\begin{vmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{vmatrix} = \begin{vmatrix} \partial j_\parallel/\partial E_\perp & -j_\perp/E_\perp \\ \partial j_\perp/\partial E_\perp & j_\parallel/E_\perp \end{vmatrix} \quad (2.5)$$

is the DC tensor in the coordinate system in which $\mathbf{E}_\perp = \{E_\perp, 0\}$.

The condition for low-frequency NDC and for instability is that σ_1 or σ_2 be negative; in this case, one can choose the direction of the electric-field increment $\Delta\mathbf{E}$ such that $\Delta\mathbf{E} \cdot \Delta\mathbf{j} < 0$, where $\Delta\mathbf{j}$ is the current increment. The low frequency instability constitutes instability to the excitation of quasi-electrostatic oscillations of the semiconductor space charge in the magnetic field. These oscillations are described by the equation for the potential $\varphi_\sim$ of the alternating electric field $\mathbf{E}_\sim = -\nabla\varphi_\sim$. In the long-wave limit, the equation for $\varphi_\sim$ is of the form

$$\frac{\partial}{\partial r_{\alpha'}}\left\{\varepsilon_{\alpha'\beta'}\frac{\partial\varphi_\sim}{\partial r_{\beta'}}\right\} = \varepsilon_0\Delta_\perp\varphi_\sim - \frac{4\pi i\sigma_1}{\omega}\frac{\partial^2\varphi_\sim}{\partial x'^2} - \frac{4\pi i\sigma_2}{\omega}\frac{\partial^2\varphi_\sim}{\partial y'^2} = 0. \quad (2.6)$$

Here $\varepsilon_{\alpha\beta} = \varepsilon_0\delta_{\alpha\beta} - 4\pi i\sigma_{\alpha\beta}/\omega$ is the dielectric tensor for the electric field $\mathbf{E}_\sim \perp \mathbf{H}$, $\partial\varphi_\sim/\partial z = 0$, $\mathbf{r} = \{x, y, z\}$. Equa-

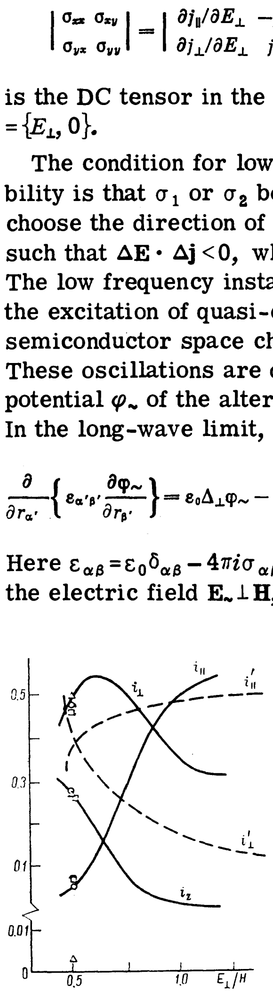

FIG. 5. Dimensionless current components $i_\perp = j_\perp m/eNp_0$, $i_\parallel = j_\parallel m/eNp_0$ and $i_z = j_z m/eNp_0$ as functions of the ratios $\tilde{E}_\perp/\tilde{H}$ in n-GaAs at $\tilde{H} = 0.25$, $\tilde{E}_\parallel = 0.005$, $N_I = 0$ and $T = 0$; $i_\perp'$ and $i_\parallel'$ are from the idealized calculations of[15] for $\tilde{E}_\parallel = 0$, $T = 0$, $N_I = 0$ and $\tau_0 = 1/\nu_0$. o—current components at $\tilde{H} = 0.25$, $N_I = 10^{14}$ cm^{-3}, $T = 0$ and $\tilde{E} = 0.005$; □—current components at $\tilde{H} = 0.25$, $N_I = 0$, $T = 77$ K and $\tilde{E}_\parallel = 0.005$; Δ—current components at $\tilde{H} = 4.0$, $T = 0$, $N_I = 0$ and $\tilde{E}_\parallel = 0.005$.

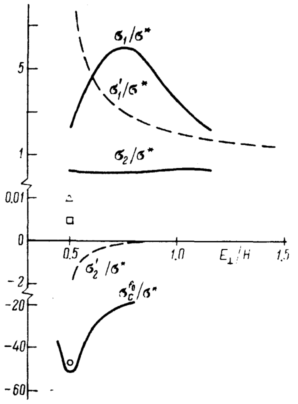

FIG. 6. Dimensionless differential conductivity at CR σ_c^{f0}/σ^* and the dimensionless components σ_1/σ^* and σ_2/σ^* of the differential-conductivity tensor at $\tilde{H}=0.25$; $N_I=0$; $T=0$; $\tilde{E}_\parallel=0.005$; σ_1/σ^*, σ_2/σ^*—determined for idealized calculations at $T=0$, $N_I=0$, $\tau_0=1/\nu_0$ and $\tilde{E}_\parallel=0$; △, □, ○ —values, respectively, of σ_1/σ^*, σ_2/σ^* and σ_c^{f0}/σ^* at $\tilde{H}=4.0$, $T=0$, $N_I=0$, $\tilde{E}_\parallel=0.005$, and $\sigma^*=\omega_p^2/4\pi\nu_0$.

tion (2.6) does not contain antisymmetric off-diagonal components of the tensor $\sigma_{\alpha'\beta'}$; the instability criterion[8)] presented above follows directly from this equation. It can be shown that this instability criterion holds also for electromagnetic perturbations.

Calculations of the current components and of the conductivities $\sigma_{1,2}$ and σ_c^{f0} by the Monte Carlo method are illustrated in Figs. 5 and 6. These show also the calculated current components $j_\perp$ and $j_\parallel$ [3,15] as well as the conductivities σ_1 and σ_2 under the idealized conditions ($\tau_0=0$, $T=0$ and $N_I=0$); the calculated current components and the conductivities differ appreciably from their theoretical values. This indicates in fact the impossibility of using theoretical calculations at $\tau_0=0$ under real conditions. The calculated values obey the inequality $\sigma_{1,2}>0$, with $|\sigma_c|\gg\sigma_{1,2}$. The latter justifies the method used to calculate σ_c. All our attempts to date to obtain low-frequency instability for other parameters under similar conditions have been unsuccessful, although, as can be seen from Fig. 6, an instability occurs under ideal conditions ($\tau_0=0$) (cf. [14]).

We can conclude thus that under the considered conditions the NDC occurs only in the case of CR. In pure n-GaAs ($\mu_{77}\gtrsim 1.5\times10^5$ cm^2/V sec) NDC is also possible at $T\sim 80$ K. The NDC at CR is apparently particularly easy to observe at $T\sim 4$ K in p-Ge, pure samples of which are obtainable.[9)]

3. MICROWAVE NDC OF LIGHT-VALLEY ELECTRONS UNDER CONDITIONS OF INTERVALLEY TRANSFER IN n-GaAs IN STRONG FIELDS

1. We consider now the possibility of NDC at microwave frequencies in n-GaAs in the case of intervalley transfers in an electric field $E\gg E_0$. We have already mentioned that under these conditions numerical calculations by Fawcett and others[16] have revealed an inversion of the light-valley electrons, due to their return from the heavy valley. We shall show below that this inversion, and the electron dynamics under these conditions, could lead in a sufficiently strong electric field to NDC at microwave frequencies much higher than the frequency usually regarded as the frequency limit for intervalley transfer in n-GaAs.

In a strong electric field ($E\gg E_0$) the lifetime of the electrons in the light valley is determined by the free-flight time of the electron in the light valley $\tau_l=(eE/p_h)^{-1}$ (where $\varepsilon_h=p_h^2/2m$ is the energy of the bottom of the heavy valley); it can be shorter than the electron scattering time within the light valley $\tau\approx\tau_0=(eE_0/p_0)^{-1}$ ($p_h\sim 3p_0$ in n-GaAs), and the condition $\tau_l<\tau$ is equivalent to the condition $E>3E_0$). In this case we can neglect in the approximation the electron scattering within the light valley and assume that the electrons that transfer from the heavy to the light valley pass freely through it and return again to the heavy valley (see Fig. 7). An inversion is formed precisely in this situation (see Fig. 6 in[16b]). In this approximation, the transfer of electrons into the heavy valley and their return from it are of importance for the light-valley electrons. Thus, the kinetic equation for the light-valley electron distribution function is in this approximation

$$\frac{\partial f}{\partial t}+e\left\{\mathbf{E}+\frac{1}{mc}[\mathbf{p}\times\mathbf{H}]\right\}\frac{\partial f}{\partial \mathbf{p}}+\nu_h f=J(\mathbf{p}), \qquad (3.1)$$

where ν_h is the frequency of departures to the heavy valley, and $J(\mathbf{p})$ is the source determined by electron transitions from the heavy valley.

We consider for the sake of argument the case $\hbar\omega_0\gg T$ in n-GaAs. Under these conditions, the intervalley transfer is due to the spontaneous emission of optical phonons. As the estimates and the numerical calculations show,[1,16] the energy of heavy-valley electrons does not exceed significantly the energy of the intervalley optical phonon $\hbar\omega^*\approx 0.8\hbar\omega_0$. We can conclude from this that the source J in Eq. (3.1) is contained in the energy interval $\varepsilon_h<\varepsilon<\varepsilon_h-\hbar\omega^*$, where ε_h is the bottom energy of the heavy valley (see Fig. 7). Transitions from the light to the heavy valley start at $\varepsilon>\varepsilon_h+\hbar\omega^*$. Recognizing further that $\varepsilon_h/\hbar\omega_0\approx 10$, that the probability of an intervalley transition is quite high,[1] and that transitions from the heavy to the light valley are almost

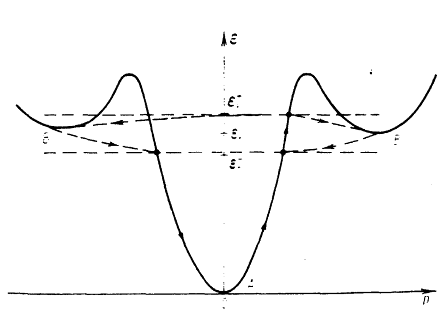

FIG. 7. Dispersion law, dynamics, and scheme of electron transitions in n-GaAs in a strong electric field; A—"light" valley, B—"heavy" valleys, dashed lines—intervalley transitions with phonon emission at an energy $\hbar\omega_0^*$, solid—nearly-free electron motions in the light valley; $\varepsilon_h^+=\varepsilon_h+\hbar\omega^*$, $\varepsilon_h^-=\varepsilon_h-\hbar\omega^*$.

isotropic, we can assume approximately that $J(\mathbf{p}) = B\delta(p^2 - p_h^2)$, and that the range of permissible values of p is $p < p_h$ (inasmuch as the electrons disappear rapidly when $p > p_h$ via transitions to the heavy valley).

Finally, the initial approximate equations for the analysis of the static distribution f_0 in the field $\mathbf{E} = \{0, 0, E\}$, $H = 0$ and of the electron distribution $f_\sim$ in a weak alternating field $E_\sim (f_\sim, E_\sim \propto e^{i\omega t})$ will be[10]

$$\frac{\partial f_0}{\partial p_z} = B_0 \delta(p^2 - p_h^2), \tag{3.2}$$

$$i\omega f_\sim + eE \frac{\partial f_\sim}{\partial p_z} = -e\mathbf{E}_\sim \frac{\partial f_0}{\partial \mathbf{p}} + B_\sim \delta(p^2 - p_h^2). \tag{3.3}$$

Here $p < p_h$, and $B_0\delta(p^2 - p_h^2)$ and $B_\sim\delta(p^2 - p_h^2)$ are sources connected with the electron transitions from the heavy valley. The characteristic transition time from the heavy to the light valley is $\tau_{h-l} \gg \tau_0$.[1] This circumstance decreases effectively the alternating component of the source in the light valley for $\omega \gg (\tau_{h-l})^{-1}$, which will henceforth be neglected, assuming that $\omega \gg (\tau_{h-l})^{-1}$.

We obtain from (3.2) ($p \leq p_h$)

$$f_0 = \frac{N\alpha}{(p_h^2 - p_\perp^2)^{1/2}} \frac{1}{2\pi p_h^2}, \tag{3.4}$$

where $p_\perp = (p_x^2 + p_y^2)^{1/2}$, N is the electron density, and α is the fraction of electrons in the light valley for a given value of E. The latter can be obtained from numerical calculations. Ruch and Fawcett[16a] give the calculated value of f_0 integrated with respect to the angle θ between the momentum and the direction of the electric field, namely $\bar{f}_0$. In our case we obtain from (3.4)

$$\bar{f}_0 = \int \sin\theta \, d\theta \, d\varphi f_0(p, \theta, \varphi) = \frac{2N\alpha}{p p_h^2} \ln \frac{p + p_h}{(p_h^2 - p^2)^{1/2}}. \tag{3.5}$$

The plot of the function (3.5) is given in Fig. 8, along with the plot of the distribution function taken from[16a] for $E = 15$ kV/cm. We see that the functions differ qualitatively only at small ε, because the electrons with $\varepsilon \approx 0$ have sufficient time to experience collisions in their flight. It must be pointed out, however, that in a sufficiently strong field this difference can arise only at low energies whose contribution to the conductivity is small.

Determining $f_\sim$ from (3.3) with $B_\sim = 0$, we obtain expressions for the current $\mathbf{j}_\sim = (e/m) \int \mathbf{p} f_\sim d^3p$ and for the differential conductivity tensor σ_{ik} ($j_{\sim i} = \sigma_{ik} E_{\sim k}$). In solving

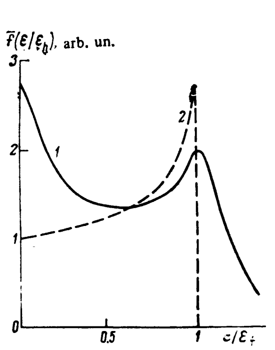

FIG. 8. Light-valley electron distribution function in n-GaAs; 1—numerical calculations of[16a] for $E = 15$ kV/cm and $T = 77$ K; 2—idealized calculation.

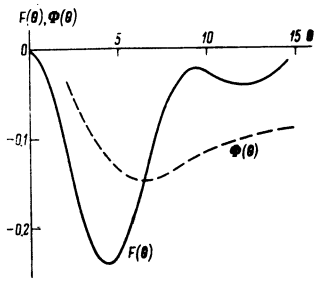

FIG. 9.

Eq. (3.3) one must take into account the fact that the derivative $\partial f_0/\partial p$ contains also a term with $\delta(p^2 - p_h^2)$, connected with the fact that $f_0 = 0$, $p > p_h$ in our approximation. As a result, the differential-conductivity tensor of the light-valley electrons takes the form

$$\sigma_{ik} = \begin{vmatrix} \sigma_\perp & 0 & 0 \\ 0 & \sigma_\perp & 0 \\ 0 & 0 & \sigma_\parallel \end{vmatrix}, \tag{3.6}$$

where

$$\operatorname{Re}\sigma_\parallel = \frac{e^2 N\alpha p_h}{meE}\left(\frac{6}{\theta^3}\sin\theta - \frac{4}{\theta^2} - \frac{2\cos\theta}{\theta^2}\right) = \tilde{\sigma} F(\theta), \tag{3.7}$$

$$\operatorname{Re}\sigma_\perp = \tilde{\sigma}\left(\frac{4}{\theta^3} - \frac{\sin\theta}{\theta} - \frac{3\sin\theta}{\theta^3} - \frac{\cos\theta}{\theta^2}\right) = \tilde{\sigma}\Phi(\theta). \tag{3.8}$$

Here $\theta = 2\omega p_h/eE$, $\tilde{\sigma} = e^2 N\alpha p_h/meE$. Plots of the functions F and Φ are given in Fig. 9. We see that both F and Φ are negative. Thus, there exists NDC at microwave frequencies in the examined model, and the frequencies at which the NDC is a maximum can be very high.

The examined simulated conditions may not correspond to the actual situation for the following reasons: 1) the presence of electron scattering for $p < p_h$; 2) the noninstantaneous electron transition into the heavy valley at $p > p_h$, as a result of which electrons enter the region $p > p_h$; 3) the contribution of the heavy valley to the conductivity. The first reason is of greatest importance. In fact, the noninstantaneous transition into the heavy valley and the appearance of electrons in the electric field in the region $p > p_h$ increase in the time spent by the electrons in the light valley. However, the signs of both $|\operatorname{Re}\sigma_\perp|$ and of $|\operatorname{Re}\sigma_\parallel|$ in (3.6)–(3.8) doe not depend on the time spent by the electrons in the light valley. Hence, this circumstance can change only the values of θ for which $|\operatorname{Re}\sigma_\perp|$ and $|\operatorname{Re}\sigma_\parallel|$ are a maximum. Furthermore, as shown by numerical calculations,[16] for $E > 10$–15 kV/cm the mobility of heavy-valley electrons is $\mu_h \leq 200$ cm²/V sec. On the other hand the effective mobility in the light valley, which enters in Eqs. (3.7) and (3.8), is $\mu_e \approx p_h/mE \sim (7-9)\times 10^3$ cm²/V-sec, i.e., 30–50 times larger. Taking further into account the fact that in the same fields the number of electrons in either valley is substantially the same,[16] we conclude that, at least at frequencies that correspond to the maximum of F or Φ, the contribution of the heavy valley to the total conductivity is insignificant.

Thus, the most significant factor that limits the possibility of using the above described calculations of the differential conductivity in the real cases is the electron scattering in the light valley. The electric field at which this scattering ceases to be significant can only be obtained by numerical methods. Apparently, a field $E \gtrsim 25$ kV/cm at $T \lesssim 80$–100 K is sufficient for the model-dependent calculations to give a reasonable approximation, for even at $E \sim 15$ kV/cm a qualitative difference between the simulated and the actual distribution functions exists only for $\varepsilon \approx 0$ (see Fig. 8).

We see thus that there are sufficient grounds to assume that NDC can take place in n-GaAs in strong electric fields ($E > 12$–15 kV/cm) at frequencies significantly greater than the frequency ceiling of the Gunn effect. We can assume that for $\sigma_\perp$ this NDC is of the maser type, and that for $\sigma_\parallel$ it is of the electron-flight type (see the Introduction). It is quite clear that experiments—numerical ones to begin with—are needed to investigate the differential conductivity under the indicated conditions. The existing numerical calculations of microwave differential conductivity under these conditions[27] were performed only for $\omega < 2\times10^{12}$ Hz in fields $E < 15$ kV/cm at $T = 300$ K, whereas it follows from the above that microwave NDC can be expected at high values of ω and E, and preferably at $T \lesssim 77$–100 K (the influence of the electron scattering in the light valley is diminished as the temperature is lowered, and the inversion in the electron distribution becomes more clearly pronounced[16a]).

In conclusion, we note the following additional facts. The foregoing calculations show that the differential conductivity reaches maximum values at $\theta = \pi$, and it is precisely in this case that the differential conductivity should be the least sensitive to imperfections of the model. However, if we take $E = 15$ kV/cm, we obtain $\omega = 5\times10^{12}$ Hz—a sufficiently high frequency. It is desirable to be able to vary the frequency at which the NDC is a maximum. This can be accomplished in a magnetic field $\mathbf{H} \parallel \mathbf{E}$. In this case, in fact, for an alternating circularly polarized field $\mathbf{E}_\sim \perp \mathbf{H}, \mathbf{E}$ the quantity that determines the value of $\sigma_\perp = \bar{\sigma}\Phi(\theta)$ can be readily seen to be $\theta = \theta_c = 2(\omega \pm \omega_c)p_h/eE$ (where ω_c is the cyclotron frequency, and the sign in θ_c is determined by the direction of rotation of the vector $E_\sim$). Thus, the frequency at which Φ is a maximum is determined by the value of the magnetic field.

4. NDC OF HOT ELECTRONS AT MICROWAVE FREQUENCIES IN A QUANTIZING FIELD H ∥ E FOR THE CASE OF INELASTIC SCATTERING BY OPTICAL PHONONS

We consider now hot electrons under conditions when the fundamental electron scattering mechanism is the spontaneous emission of optical phonons in a (quantizing) magnetic field $\mathbf{H} \parallel \mathbf{E}$ such that $\omega_c > \omega_0$. In this case, one Landau level exists at $p < p_0$, and we can assume that the electrons occupy only this level. These conditions can be realized comparatively simply in n-InSb at $H > 20$–30 kG and $T \sim 4$ K; we confine ourselves to a discussion of this case. In a quantizing field H, the electron density of states increases, so that the probability of spontaneous emission of optical phonons also increases (compared with the case $H = 0$). As a result, the flight resonance of electrons in an electric field,[19] which is due to the cyclic acceleration of the electrons from $p = 0$ to $p = p_0$ and to the subsequent transition from the region $p > p_0$ to the region $p \approx 0$, becomes more strongly pronounced than for $H = 0$. This facilitates the formation of microwave electron-flight NDC[17,14]—NDC at frequencies $\omega \approx n\omega_E$ (where $\omega_E = 2\pi eE/p_0 = 2\pi/\tau_E$, and τ_E is the time in which the electron momentum changes in the electric field from $p = 0$ to $p = p_0$). The electron distribution function f depends in this case only on the longitudinal (with respect to the magnetic field H) momentum of the electron p_z, and it satisfies the one-dimensional kinetic equation.[11] The latter condition makes it relatively easy to consider not only the static characteristics of the electrons, but the differential conductivity at microwave frequencies.

Taking into account, as in Sec. 2, once more only the spontaneous emission of optical phonons and the impurity scattering, we have for the kinetic equations in the static representation ($f = f_0$, $f_0^- = f_0$ at $p > p_0$; $f_0^+ = f_0$ at $|p_z| < p_0$):

$$eE\frac{\partial f_0^-}{\partial p_z} = -\frac{2eE_0M}{[2p_0(p_z-p_0)]^{1/2}}f_0^-, \quad p_z > p_0; \tag{4.1a}$$

$$eE\frac{\partial f_0^+}{\partial p_z} = \frac{eE_0}{p_0}M\left(1+K\frac{p_z}{p_0}\right)f_0^-\left(p_0+\frac{p^2}{2p_0}\right)+\nu_I[f_0^+(-p_z)-f_0^+(p_z)], |p_z| < p_0. \tag{4.1b}$$

The linearized equation for the distribution function $f_\sim (f = f_0 + f_\sim e^{i\omega t})$ in a homogeneous electric field $(E + E_\sim e^{i\omega t})\mathbf{z}_0$ is

$$i\omega f_\sim^- + eE\frac{\partial f_\sim^-}{\partial p_z} = -\frac{2eE_0M}{[2p_0(p_z-p_0)]^{1/2}}f_\sim^- - eE_\sim\frac{\partial f_0^-}{\partial p_z}, \quad |p_z| > p_0; \tag{4.2a}$$

$$i\omega f_\sim^+ + eE\frac{\partial f_\sim^+}{\partial p_z} = -eE_\sim\cdot\frac{\partial f_0^+}{\partial p_z}+\frac{eE_0}{p_0}M\left(1+K\frac{p_z}{p_0}\right)f_\sim^-\left(\frac{p_z^2}{2p_0}+p_0\right)$$
$$+\nu_I[f_\sim^+(-p_z)-f_\sim^+(p_z)], \quad |p_z| < p_0. \tag{4.2b}$$

Here

$$M = e^a|E_i(-a)|, \quad K = \frac{2(1-aM)}{M}, \quad a = \frac{l_H^2p_0^2}{2\hbar^2},$$
$$l_H^2 = \frac{c\hbar}{eH}, \quad \nu_I = \frac{4\pi e^4N_Iml_H^2}{\varepsilon_0^2\hbar^2|p_z|}[e^u\mathrm{Ei}(-u)+u^{-1}], \tag{4.3}$$
$$u = 4p_z^2l_H/\hbar^2 + l_H^2N_I^{2/3}.$$

At $p_z > p_0$, the impurity scattering is neglected in Eqs. (4.1) and (4.2), and only the outflow due to spontaneous emission of optical phonons is taken into account, inasmuch as at $E \ll E_0$ the depth of electron penetration into the region $p > p_0$ is small, and the distribution function decreases rapidly. For the same reason, the arrival and departure terms due to optical transitions are expanded in the small parameters $(p_z - p_0)/p_0$ (for $p_z > p_0$) and p_z/p_0 (for $|p_z| < p_0$), the energy conservation law for emission of optical phonons $p_0^2 + p_z^2 = (p_z')^2$ is written in the form $p_z^2 = 2p_0(p_z' - p_0)$, and the equations for f_0^+ and $f_\sim^+$ contain the values of f_0^- and $f_\sim^-$ at $p_z' = p_0 + p_z^2/2p_0$. The impurity scattering in (4.1)–(4.3) is by ionized impurities (where ε_0 is the lattice dielectric constant). The average interimpurity distance is used as the largest parameter (hence the term $l_H^2N_I^{2/3}$ in the expression for u).

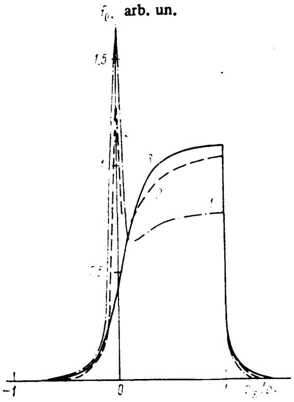

FIG. 10. Electron distribution function at the zero Landau level in n-InSb in a quantizing magnetic field at $H = 50$ kOe, $E = 150$ V/cm, $T = 0$; 1) $N_I = 3\times10^{14}$ cm^{-3}; 2) $N_I = 10^{14}$ cm^{-3}; 3) $N_I = 0$.

Equations (4.1) and (4.2) can be readily solved in the static approach both analytically and numerically in an alternating field. The boundary conditions are as follows: $f_0^+ \to 0$, $p_z \to -p_0$; $f_0^+ \to f_0^-$, $p_z = p_0$ ($\int f_0 dp_z = 1$) $f_\perp^+ \to 0$, $p_z \to -p_0$; $f_\perp^+ = f_\perp^-$, $p_z = p_0$. Figure 10 shows the calculated static distribution function of the electrons (cf. [20]). The influence of impurity scattering on the distribution, and the weak electron penetration into $p > p_0$ are seen here. Figure 11 shows the plot of the differential conductivity $\sigma(\omega) = \sigma_1 + i\sigma_2$. A negative Re$\sigma$ appears only at sufficiently high frequencies. As shown by the calculations, at $\omega\tau_E \approx 2\pi$ the sign of Re$\sigma < 0$ is preserved even at relatively high impurity concentrations (up to $N_I \sim 10^{15}$ for the field values given in Fig. 11).

5. CONCLUSION

We have thus considered microwave NDC cases that are connected in one way or another with inversion in the hot-electron distribution. In two cases there is no static NDC. The microwave differential conductivity in n-InSb in a quantizing field $\mathbf{H} \parallel \mathbf{E}$ was investigated in greatest detail; it was calculated here for a wide range of frequencies. The differential conductivity for electron scattering by optical phonons in a classical field H was investigated in less detail. The static CR differential conductivities were determined, and the latter being calculated from formulas that pertain, strictly speaking, only to the case $N_I = 0$ and $T = 0$. The limiting values N_I and T for which $\sigma_c < 0$ is still valid are therefore unknown. Finally, while the microwave differential conductivity in n-GaAs was calculated in a sufficiently wide range of frequencies higher than that usually regarded as the ceiling frequency of the Gunn effect, a rather idealized model was used for the calculations. Hence the conditions for realization of a similar microwave NDC call for further study. It would be most desirable to perform direct numerical calculations of the differential conductivity in the last two cases for a wide range of N_I and T.

On the experimental level, besides the investigation of differential conductivity at microwave frequencies in n-InSb in a quantizing field $\mathbf{H} \parallel \mathbf{E}$, differential conductivity at CR in n-GaAs for $T \leq 77$ K, and the differential conductivity at frequencies $\omega > 2\times10^{12}$ Hz in n-GaAs in a strong electric ($E > 15$ kV/cm) or possibly a magnetic field, in intervalley transitions, also is of interest for p-Ge at $T = 4$ K. First, pure samples of this material are available. Second, the equal-energy surfaces for heavy holes in this material are corrugated and this leads, as is known, to resonant effects at harmonics of the cyclotron frequency[12] and to the existence of negative-mass regions. The former could lead to the existence of NDC at harmonics of the cyclotron frequency, and under the discussed conditions the latter would produce more favorable conditions for NDC at CR. Third and last, the investigation of the differential conductivity of p-Ge under conditions of CR for light holes is of interest. If the conditions discussed in Sec. 2 are fulfilled for light holes, their lifetime, at $\varepsilon < \hbar\omega_0$, will be determined by $E_\parallel$, whereas the heavy-hole lifetime will be determined by $E_\perp$. As a result, the population of the light-hole band may exceed the population in the equilibrium state.[13] This could cause the contribution of the light holes to prevail over that of the heavy holes to the DC at the CR of the light holes, and to NDC at CR due to light holes.

The presence of microwave NDC must lead to excitation of an electromagnetic instability in the sample. We shall consider the possibility of an electromagnetic instability for microwave NDC using as an example n-InSb in a quantizing field $\mathbf{H} \parallel \mathbf{E}$, for which differential conductivity has been investigated in greatest detail. The electromagnetic waves as well as the space-charge oscillations (longitudinal oscillations or waves) can be unstable. The former are determined by the equation

$$k^2c^2/\omega = \varepsilon(\omega, k) = \varepsilon_0 - 4\pi i\sigma/\omega,$$

and the latter by the condition $\varepsilon = 0$ (here ε_0 is the lattice dielectric constant, k is the wave number, and $\omega = \omega' + i\omega''$ is the frequency). To find the conditions for existence of unstable solutions to these equations ($\omega'' < 0$) we can use the Nyquist criterion with the calculated values of $\sigma(\omega, k=0)$. As a result it turns out that the electromagnetic waves are unstable whenever Re$\sigma < 0$; it can also be easily verified that, for example, as Re$\sigma \to 0$ (Re$\sigma < 0$) the unstable waves are the high frequency electromagnetic waves with those frequencies for which

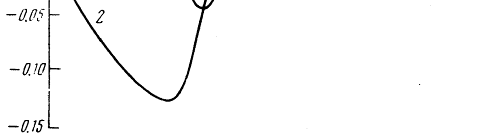

FIG. 11. Dimensionless values of the real and imaginary components of the electron conductivity in n-InSb in a quantizing magnetic field at $H = 50$ kOe, $E = 150$ V/cm, $N_I = 3\times10^{14}$ cm^{-3}; 1) σ_1/σ_0; 2) σ_2/σ_0, where $\sigma_0 = \omega_p^2\tau_E/4\pi$ ($\omega_p^2 = 4\pi e^2N/m$), ($1/\tau_E = eE/p_0$).

$\mathrm{Re}\,\sigma<0$ at $\omega''=0$. As to the longitudinal oscillations, they can be unstable in accordance with the Nyquist criterion only for sufficiently high electron concentrations ($N>N_{cr}$). In the case depicted in Fig. 11, $N_{cr}\sim 2\times10^{14}$ cm^{-3}.

This result must also hold for longitudinal waves. In fact, allowance for the finite wavelength of the alternating electric field ($k\neq 0$) reduces mainly to allowance for the Doppler effect of electrons drifting at a velocity $v\approx p_0/2m$, i.e., to the replacement, in the first approximation, of ω by $\omega-kp_0/2m$ in the dielectric constant $\varepsilon(\omega, k=0)$. Although this substitution does change the frequency regions in which $\mathrm{Re}\,\sigma<0$ ($\mathrm{Im}\,\omega=0$), it does not change the Nyquist diagrams, and consequently, the instability criterion of the longitudinal oscillations. On the other hand, at sufficiently large values of k ($\mathbf{k}\perp\mathbf{H}$) electromagnetic waves cannot be excited because ε is finite and the factor k^2 enters in the dispersion equation of these waves.

Thus, at a sufficiently low electron density, the only unstable electromagnetic waves are those with frequencies for which $\mathrm{Re}\,\sigma<0$ at $\omega''=0$. It appears that an analogous situation takes place also in other cases of microwave NDC without static NDC. The value $N>N_{cr}$ at which excitation of longitudinal oscillations is still possible is quite large, since low values of N_I and consequently of N (since $N_I>N$) are required for the existence of the considered effects. Hence, the actually considered microwave NDC effects without static NDC can lead only to buildup of electromagnetic waves at microwave frequencies.

In conclusion, we note that from the point of view of the effects considered in the present article, other semiconductor materials (besides n-GaAs, n-InSb, and p-Ge) with large coupling to optical phonons, large optical-phonon energy, and small effective mass (in order that NDC at CR be possible at nitrogen temperatures) can also be of interest, for example InP. The currently available samples of this material are not pure enough, however.

We take the opportunity to thank E. M. Gershenzon, P. E. Zil'berman, M. I. Petelin and V. K. Yulpatov for useful discussions.

APPENDIX: CALCULATION OF THE DIFFERENTIAL CONDUCTIVITY AT CYCLOTRON RESONANCE IN ELECTRON SCATTERING BY OPTICAL PHONONS AT $T=0$ AND $N_I=0$

We consider the differential conductivity of hot electrons at CR in a magnetic $\mathbf{H}=\{0,0,H\}$ and an electric field $\mathbf{E}=\{E_\perp, 0, E_\parallel\}$ under conditions when the sole electron-scattering mechanism is spontaneous emission of optical phonons. It is convenient in this case to divide the electron phase space into regions, $p>p_0$ and $p<p_0$. We assume that the electron penetration into the region $p>p_0$ is sufficiently small, so that we need retain in the collision integral of the kinetic equation for the distribution function f only the departure term due to the spontaneous emission of optical phonons, which can moreover be expanded in the small parameter $(p-p_0)/p_0$. At $p<p_0$, only the incoming term $I(\mathbf{p})$ due to transitions from $p>p_0$ is present in the collision integral; this term, in virtue of the weak electron penetration into $p>p_0$, can be expanded in the small parameter p/p_0 at $p<p_0$. Consequently, the equations for $f^-=f(p>p_0)$ and $f^+=f(p<p_0)$ take the form (see, e.g., [1])

$$\frac{\partial f^-}{\partial t}+\mathbf{F}\frac{\partial f^-}{\partial \mathbf{p}}=-\frac{2eE_0}{p_0}\left(\frac{p^2-p_0^2}{p_0^2}\right)^{1/2}f^-=-\nu_{out}f^-, \tag{A.1}$$

$$\frac{\partial f^+}{\partial t}+\mathbf{F}\frac{\partial f^+}{\partial \mathbf{p}}=\frac{eE_0}{\pi p_0^2}\int f_-(p')\,\delta(p'^2-p_0^2-p^2)\,d^3p=I(\mathbf{p}), \tag{A.2}$$

where E_0 is the characteristic field of the optical scattering and $\mathbf{F}$ is the Lorentz force. In this form, the equations hold for polar as well as for deformational optical scattering, inasmuch as at $(p-p_0)/p_0\ll 1$ the probabilities of both scatterings depend on p in like manner.

We note that it was precisely within the framework of Eqs. (A.1) and (A.2) that the microwave NDC was calculated[14] for hot-electron scattering by optical phonons at $T=0$, $N_I=0$ and $H=0$ (cf. also Sec. 4). The hot-electron static characteristics under idealized conditions were first analyzed by Gunn[29] within a framework of equations similar to (A.1) and (A.2).

Differential conductivity at CR can be calculated relatively simply in two cases. The first case is $E_0\to\infty$, when $f^-=0$ and the equation for f^+ is

$$\frac{\partial f^+}{\partial t}+e\left\{\mathbf{E}+\frac{1}{mc}[\mathbf{p}\times\mathbf{H}]\right\}\frac{\partial f^+}{\partial \mathbf{p}}=B\delta(\mathbf{p}), \tag{A.3}$$

where δ is the Dirac function and the constant B is determined from the normalization condition $\int f^+d^3p=1$ (cf. [15]). The second case is that of moderate E_0 and comparatively strong magnetic fields, when it can be assumed that the electrons execute several cyclotron rotations prior to emitting an optical phonon. In this case we can average over the cyclotron rotation in Eqs. (A.1) and (A.2). We consider the second more realistic case. The stationary equations (A.1) and (A.2) take in the variables φ, $p_\perp$, and p_z (see Sec. 2 and Figs. 1 and 2) the form

$$\omega_c\frac{\partial f_0^-}{\partial \varphi}+eE_\parallel\frac{\partial f_0^-}{\partial p_z}=-\nu_{out}(p)f_0^-, \tag{A.4}$$

$$\omega_c\frac{\partial f_0^+}{\partial \varphi}+eE_\parallel\frac{\partial f_0^+}{\partial p_z}=I_0(\mathbf{p}). \tag{A.5}$$

If ω_c is the largest parameter in these equations, then in first-order approximation $\partial f_0/\partial\varphi=0$, i.e., f_0 depends weakly on φ, we have approximately $f_0\approx f_0(p_\perp, p_z)$. The equations for f_0 can be obtained from (A.4) and (A.5) if the latter are averaged over φ (i.e., if they are integrated with respect to φ and divided by 2π). As a result we have

$$eE_\parallel\frac{\partial \bar f_0^-}{\partial p_z}=-\bar\nu(p_\perp, p_z)\bar f_0^-, \tag{A.6}$$

$$eE_\parallel\frac{\partial \bar f_0^+}{\partial p_z}=\bar I_0(p_\perp, p_z), \tag{A.7}$$

where

$$\bar{\nu}=\int_0^{2\pi}\nu_{out}\frac{d\varphi}{2\pi},\quad \bar{I}_0=\int_0^{2\pi}I_0\frac{d\varphi}{2\pi}.$$

Simple expressions for $\bar{\nu}$ and $\bar{I}_0$ can be obtained if one takes into account the fact that in a strong magnetic field and at not too large $E_\parallel$ the electrons with not too small $p_\perp$ have $p>p_0$ during a small fraction of the cyclotron period, and emit an optical phonon at the upper point of the helical trajectories, i.e., at $\varphi\approx\pi$. We have as a result

$$\bar{\nu}=\frac{\nu_0 p_z^0\Delta}{p_0(p^*p_\perp)^{1/2}}=\nu^*(p_\perp)\frac{\Delta}{p_0}, \tag{A.8}$$

$$\bar{I}=\frac{\nu_0}{\pi p_0}\int_0^\infty\frac{p_\perp' dp_\perp'}{(p^*p_\perp')^{1/2}}\int_{p^2/2p_{z0}'}^\infty\frac{f^-(\varphi\approx\pi)d\Delta}{(2p_{z0}'\Delta-p^2)^{1/2}}. \tag{A.9}$$

Here $p^*=mcE_\perp/H$, $\Delta=p_z-p_{z0}'$, and $p_{z0}'=[p_0^2-(p^*+p_\perp')^2]^{1/2}$ is the value of p_z at $\varphi=\pi$ and at the given $p_\perp'$ at which $p=p_0$ (see Fig. 2). Equation (A.9) also takes into account the fact that for sufficiently large p_{z0}' we have $\Delta\ll p_{z0}'$, since the electrons penetrate weakly into $p>p_0$. Equation (A.9) is suitable for the arrival term both in the static case ($\bar{I}_0=\bar{I}(f_0^-)$) and in the general case. The integration with respect to Δ and $p_\perp'$ in Eq. (A.9) is carried out over those values for which the trajectory passes outside the sphere $p_0^2+p^2$; for a given $p_\perp'$ the values of p_z are those for which the expression inside the square root of the denominator in (A.9) is positive, since the corresponding condition $(p_\perp+p^*)^2+p_z^2>p_0^2+p^2$ can be written in the approximate form

$$2p_z^0(p_z-p_z^0)>p^2.$$

The linearized equations for a circularly polarized alternate field at cyclotron resonance are (cf. Sec. 2):

$$i\omega_c f_\sim^-+\omega_c\frac{\partial f_\sim^-}{\partial\varphi}+eE_\parallel\frac{\partial f_\sim^-}{\partial p_z}=-eE_\sim e^{-i\varphi}\frac{\partial f_0^-}{\partial p_\perp}-\nu_{out}f_\sim^-, \tag{A.10}$$

$$i\omega_c f_\sim^++\omega_c\frac{\partial f_\sim^+}{\partial\varphi}+eE_\parallel\frac{\partial f_\sim^+}{\partial p_z}=-eE_\sim e^{-i\varphi}\frac{\partial f_0^+}{\partial p_\perp}+I_\sim(f_\sim^-). \tag{A.11}$$

In a strong magnetic field, in analogy with the static case, we have approximately $f_\sim\approx\bar{f}_\sim e^{-i\varphi}$. Multiplying Eqs. (A.10) and (A.11) by $e^{i\varphi}$ and integrating with respect to φ, we obtain

$$eE_\parallel\frac{\partial\bar{f}_\sim^-}{\partial p_z}=-eE_\sim\frac{\partial f_0^-}{\partial p_\perp}-\bar{\nu}\bar{f}_\sim^-, \tag{A.12}$$

$$eE_\parallel\frac{\partial\bar{f}_\sim^+}{\partial p_z}=-eE_\sim\frac{\partial\bar{f}_0^+}{\partial p_\perp}+\bar{I}_\sim(-\bar{f}_\sim^-). \tag{A.13}$$

Equations (A.12) and (A.13) take into account the fact that electrons leave the region $p>p_0$ at $\varphi\approx\pi$, where $f_\sim^-\approx-\bar{f}_\sim^-$. For the same reason, Eq. (A.10) contains $\bar{\nu}$, since in this case

$$\int e^{i\varphi}\nu_{out}d\varphi\approx\int\nu_{out}d\varphi.$$

Similarly, Eq. (A.13) contains $\bar{I}_\sim(f_\sim^-(\varphi=\pi))=\bar{I}_\sim(-\bar{f}_\sim^-)$, inasmuch as $I_\sim(\mathbf{p})$ is localized at $p\approx 0$, and consequently

$$\int I_\sim e^{i\varphi}d\varphi\approx\int I_\sim d\varphi=2\pi\bar{I}_\sim.$$

In the static case we have

$$\bar{f}_0^-=B_0(p_\perp)\exp(-M\Delta^2), \tag{A.14}$$

where $M=\nu^*/2eE_\parallel p_0$ and $B_0(p_\perp)=f_0^+(p_\perp,p_z^0)$. From (A.12) we obtain in the approximation

$$\frac{\partial\bar{f}_\sim^-}{\partial\Delta}=-\varepsilon\frac{\partial\bar{f}_0^-}{\partial p_\perp}-2M\Delta\bar{f}_\sim^-\approx-\varepsilon\frac{\partial\bar{f}_0^-}{\partial\Delta}\frac{\partial\Delta}{\partial p_\perp}-2M\Delta f_\sim^-, \tag{A.15}$$

where $\varepsilon=E_\sim/E_\parallel$. In the derivation of the last equation we have neglected the derivatives of B_0 and M with respect to $p_\perp$, since the contribution from these terms is smaller in terms of the parameter $M^{-1/2}$ than the contribution from the retained terms.

We obtain from (A.15)

$$\bar{f}_\sim^-=-\varepsilon M\Delta^2B_0\exp(-M\Delta^2)\frac{\partial p_z^0}{\partial p_\perp}+B_\sim\exp(-M\Delta^2),$$
$$B_\sim=\bar{f}_\sim^+(p_\perp,p_z^0),\ B_0=\bar{f}_0^+(p_\perp,p_z^0). \tag{A.16}$$

We shall now determine the differential conductivity at CR. In the considered case of circular polarization we have

$$\sigma_c=\frac{eN}{mE_\sim}\int p_\perp^2e^{i\varphi}f_\sim dp_\perp d\varphi\,dp_z+\frac{EN}{E_\sim}c\frac{E_\perp}{H}\int p_\perp f_\sim dp_\perp d\varphi\,dp_z. \tag{A.17}$$

In virtue of the conservation of the number of particles we have

$$\int p_\perp f_\sim dp_\perp d\varphi\,dp_z=0,$$

so that

$$\sigma_c=\frac{eN}{mE_\sim}\int p_\perp^2e^{i\varphi}f_\sim dp_\perp d\varphi\,dp_z.$$

The contribution of $f_\sim^-$ to σ_c is small in the parameter M^{-1}, and we shall neglect it. As a result we obtain

$$\sigma_c\approx\frac{2\pi eN}{mE_\sim}\int p_\perp^2\bar{f}_\sim^+dp_\perp dp_z. \tag{A.18}$$

Solving Eq. (A.13) with the boundary conditions $\bar{f}_\sim^+=0$, $p_z=-p_z^0$ (inasmuch as at $eE_\parallel>0$, $p_z<0$, $p>p_0$ there are no electrons in our approximation), we obtain

$$\sigma_c\approx-\frac{2\pi eN}{E_\parallel}\int_0^{p_0-p^*}p_\perp^2\int_{-p_z^0}^{p_z}\frac{\partial f_0^+}{\partial p_\perp}dp_z\,dp_\perp\,dp_z'+\frac{2\pi}{E_\parallel}\int_0^{p_0-p^*}p_\perp^2\int_{-p_z^0}^{p_z}\frac{\bar{I}_\sim}{E_\sim}dp_z\,dp_\perp\,dp_z' \tag{A.19}$$

In (A.19) we have taken into account only the contribution of $p_\perp<p_0-p^*$ to σ_c. For electrons with $p_\perp>p_0-p^*$, the upper segments of the trajectories (see Fig. 2) lie entirely outside the sphere $p=p_0$. The lifetime of the electrons along these trajectories is determined by the optical-phonon spontaneous-emission time rather than by the value of $E_\parallel$, and is relatively small. We have therefore neglected the contribution to σ_c from such electrons. Since $I(\mathbf{p})$ is approximately symmetrical with respect to p_z and is localized at $p\approx 0$, we obtain, approximately, integrating by parts,

$$\sigma_c = -\frac{2\pi eN}{E_\parallel}\int_0^{p_0-p^*} p_\perp^2 \int_{-p_z^0}^{p_z} \frac{\partial f_0^+}{\partial p_\perp} dp_z'\, dp_\perp\, dp_z + \frac{\bar{p}_z\bar{p}_\perp}{E_\parallel E_\sim}\int I_\sim d^3p = \sigma_c^{f_0} + \sigma_c^I. \tag{A.19a}$$

Here we returned to the value of $I_\sim$ that was not averaged with respect to φ; $\bar{p}_z$ and $\bar{p}_\perp$ are certain average values of p_z and $p_\perp$, determined by the width of the source. We will not calculate them, since it is important for us to estimate the magnitude and sign of σ_c.

Thus, to determine the contribution of the arrival term to σ_c it is necessary to calculate the integral

$$\int I(f_\sim^-)\, d^3p.$$

Substituting Eq. (A.16) in (A.9) and recognizing that $f_\sim^-(\varphi=\pi) = -\bar{f}_\sim^-$, and

$$B_\sim(p_\perp, p_z^0) = -\varepsilon\int_{-p_z^0}^{p_z^0}\frac{\partial f_0^+}{\partial p_z}dp_z + \frac{1}{2\pi E_\parallel}\int_{-p_z^0}^{p_z} d\varphi\, dp_z I_\sim$$

(see (A.13)), we obtain an integral equation for $I_\sim$:

$$I_\sim(\mathbf{p}) = -\frac{\nu_0}{\pi p_0}\int\frac{dp_\perp' p_\perp'}{(2p_{z0}'p^* p_\perp')^{1/2}}\Bigg\{\Bigg(-\varepsilon\int_{-p_{z0}}^{p_{z0}'}\frac{\partial f_0^+}{\partial p_\perp'}dp_z$$

$$+\frac{1}{2\pi eE_\parallel}\int_{-p_{z0}'}^{p_{z0}} I_\sim d\varphi\, dp_z\Bigg)\int_{p^2/2p_{z0}'}^{\infty}\frac{\exp(-M\Delta^2)\, d\Delta}{(\Delta - p^2/2p_{z0}')^{1/2}}$$

$$-\frac{\partial p_z^0}{\partial p_\perp}\varepsilon f_0(p_\perp', p_{z0}')\int_{p^2/2p_{z0}'}^{\infty}\frac{M\Delta^2\exp(-M\Delta^2)\, d\Delta}{(\Delta^2 - p^2/2p_{z0}')^{1/2}}\Bigg\}. \tag{A.20}$$

From this equation, the expression for $\int I_\sim d^3p$ can be obtained by integrating with respect to p in the right-hand side of (A.20) under the integral sign. The resultant integrals

$$\mathcal{I}_1 = \int_0^\infty p^2 dp \int_{p^2/2p_{z0}'}^{\infty}\frac{\exp(-M\Delta^2)\, d\Delta}{(\Delta - p^2/2p_{z0}')^{1/2}},$$

$$\mathcal{I}_2 = \int_0^\infty p^2 dp \int_{p^2/2p_{z0}'}^{\infty}\frac{M\exp(-M\Delta^2)\Delta^2 d\Delta}{(\Delta - p^2/2p_{z0}')^{1/2}} = -\frac{\partial \mathcal{I}_1}{\partial M}$$

can be calculated, $\mathcal{I}_1 = \mathcal{I}_2 = \pi(2p_{z0}')^{3/2}/8M$ (see[30], pp. 351 and 899).

Finally the equation for σ_c^I is reduced to the form

$$\sigma_c^I = -\frac{eN}{2mE_\parallel}\bar{p}_\perp\bar{p}_z\int dp_z\, dp_\perp\, d\varphi\, f_0^+. \tag{A.21}$$

We see that $\sigma_c^I < 0$ and thus $\sigma_c^{f_0} < 0$ turns out to be the condition sufficient for $\sigma_c < 0$ in the considered case. The condition for which the foregoing analysis is valid can be obtained by comparing the depth of electron penetration into $p > p_0$ along p_z with the spacing of the electron trajectories. It is evident from (A.14) that the penetration depth is $\Delta p_z^* \approx M^{-1/2}$, and the spacing is $\Delta p_z^c = 2\pi eE_\parallel/\omega_c$. From the condition $\Delta p_z^* \gg \Delta p_z^c$ we obtain the condition

$$(eE_\parallel/p_0\omega_c) \ll (E_\parallel/E_0)^{1/4}, \tag{A.22}$$

given in Sec. 2. It is satisfied practically in all the cases discussed in Sec. 2.

1) It is possible that this very situation takes place in a weakly ionized plasma.[5].

2) Semiconductor masers at CR are an old idea and hope.[7–9] Considerable progress was made in gas electronics in the construction and analysis of masers at CR (see, e.g.,[10–13]).

3) This type of microwave NDC was first discussed, as far as we know, by Bonch-Bruevich and El-Sharnubi[17] and by Andronov and Kozlov[14] under conditions of hot-electron scattering by optical phonons, at $\hbar\omega_0 \gg T$, in the absence of magnetic fields. The flights and jumps of the electrons from $\varepsilon = 0$ to $\varepsilon = \varepsilon_0$ in the case of electron scattering by optical phonons was also discussed by Shockley[18]; in-flight resonance under these conditions was discussed, for instance, by Price *et al.*.[19]

4) The static characteristics of hot electrons in a quantizing field $\mathbf{H}\parallel\mathbf{E}$ in scattering by optical phonons were discussed by Magnusson.[20]

5) As we shall see below, in order for the NDC to occur at CR in E and H fields, the required inversion in the electron distribution is in terms of cyclotron-rotation energy rather than electron energy.

6) The conclusion that inversion in the cyclotron-rotation energy is present under similar conditions can be drawn from the numerical results of Kurosawa and Maeda.[21]

7) $E_\perp$ is the total transverse field in the sample, including the Hall field.

8) Shur[24] considered a particular class of low-frequency instabilities, namely the purely electrostatic instability defined by the equation $\varepsilon_{\alpha\beta}E_{\alpha\beta} = 0$. Naturally, the instability criterion obtained in[24] turned out to be more rigid than the criterion of Kurosawa and Maeda[23] presented above.

9) Thus, the conditions for the onset of NDC at CR turn out to be less rigid than the conditions for the NDC due to regions of negative mass in p-Ge.[25,26]

10) We neglect the electrons that come from the heavy valley into the light valley with $p_z > 0$ and immediately return to the heavy valley. The contribution of these electrons to the conductivity is low.

11) Although complications due to the restructuring of the electron spectrum and connected with the quasi-one-dimensional character of its motions can arise under these conditions (see, e.g.,[28]), they should be unimportant in not too weak electric fields, and the kinetic equations should be valid.

12) Resonance effects at harmonics of the cyclotron frequency can can arise at $E_\perp \neq 0$ also in an isotropic but nonparabolic electron band. On the other hand, it is unlikely that these effects are significant in n-GaAs under the considered conditions ($\bar{E}_\perp \approx \bar{H}/2$).

13) The decrease in the population of the light-hole band in p-Ge at $H = 0$ was discussed in[22c,26].

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Dielectric transition in a quasi-one-dimensional system with repulsion

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It is shown that the interaction between electrons located on different chains may lead to a three-dimensional dielectric transition even if the electrons on a single chain repel one another. The relevant interaction constant of electrons on a single chain is obtained. If the converse inequality holds, the ground state of the system is metallic.

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1. INTRODUCTION

Materials possessing a chain structure have been intensively investigated in recent years. We have in mind experiments on complexes on a TCNQ base and complexes of variable valence on a Pt or Ir base and so on. Such quasi-one-dimensional systems should in principle possess a number of interesting properties. Nevertheless, in spite of the large variety of states theoretically possible, almost all the known quasi-one-dimensional systems are dielectrics at low temperatures.

In the present research, we consider one of the possible mechanisms of dielectric transition. As a model, we consider a system of parallel metallic chains arranged in a lattice. The interaction between electrons located on each chain is assumed to be large in comparison with the interaction between electrons on different chains. This interaction can be a direct Coulomb interaction or indirect, through phonons. The interaction through phonons becomes significant at temperatures below the Debye temperature; therefore, in the initial junctions we consider only the Coulomb interaction.

The case of repulsion brought about by the Coulomb interaction is usually considered to be "uninteresting." The fact is that the simplest calculation by the molecular-field method,[1] with neglect of the interaction between neighboring chains, leads to the conclusion that any sort of transition is absent. An exception is the case of just one electron per unit cell. We shall not consider this case. We shall show that allowance for an arbitrarily weak interaction between electrons on different chains leads to a renormalization of the interaction of the electrons on a single chain and, in final analysis, to a dielectric transition. The characteristic feature of this transition is that the renormalized interaction between electrons on different chains becomes of the order of the renormalized interaction on a single chain. This leads to the suppression of purely one-dimensional fluctuations and makes possible the descrip-

