tained at smaller $\chi_\perp$, when the influence of scattering by the residual gas becomes important. Then the empirical function $D[\psi(\alpha, \chi_\perp)]$ is a generalization of the coefficient of diffusion to the case of small $\chi_\perp$.

If in (9) we average μ^2 and take into account the fact that $2\psi R_e/\rho = a_0 + a_1 y$, $y = \sin^2\alpha$, then the solution to Eq. (9) with the obvious boundary condition $n(\mu_c, t) = 0$ will be

$$n(y,t)=\sum_{m=0}^{\infty} c_m \exp\left(-k_m^2 t+\frac{a_1 y}{2}\right) J_1\left\{\lambda_m \exp\left[\frac{a_1(y-y_c)}{2}\right]\right\},$$

$$k_m^2 = {}^1/_4 a_1^2 \lambda_m^2 \bar{y}^2 D(y_c),$$

where J_1 is the Bessel function of the first order. The escape rate (i.e., the counting rate) is determined by the function $(\partial n/\partial y)_{y=y_c}$, and its decreasing part is approximately described by the equation

$$j(y_c, t) \approx A \exp(-k_1^2 t), \quad k_1^2 \approx 3.68 a_1^2 \bar{y}^2 D(y_c).$$

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The change in the anisotropy of the Fermi surface in the p-type semiconducting alloy $Bi_{0.9}Sb_{0.1}$ upon going over into the gapless state under the action of pressure

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The oscillatory and galvanomagnetic effects arising in the p-type semiconducting alloy $Bi_{0.9}Sb_{0.1}$ on going over into the gapless state (GS) under pressure at liquid-helium temperatures have been investigated. As the GS was approached in samples of the alloy with "light"-hole concentrations $\sim 10^{15}$ cm^{-3}, Shubnikov oscillations were observed in the entire angle range as the magnetic field was rotated in the binary-bisector and bisector-trigonal planes, and this allowed the complete reestablishment of the shape of the hole Fermi surface. It is shown that, in the first approximation, the hole Fermi surfaces at the L point in the investigated alloy are highly anisotropic ellipsoids, the anisotropy of the ellipsoids increasing appreciably in the transition into the GS. The data obtained are discussed on the basis of the Abrikosov theory of the band spectrum of materials of the Bi type.

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1. INTRODUCTION

A characteristic property of the energy spectrum of Bi and the alloys $Bi_{1-x}Sb_x$ ($x<0.2$) is the smallness of the direct gap ε_{gL} at the L point of the reduced Brillouin zone. The band structures of Bi and Sb at the L point are mutually inverted[1–5]; in the $Bi_{1-x}Sb_x$ alloys the inversion of the terms at the L point is removed as x is increased, as a result of which a gapless state (GS: $\varepsilon_{gL} \approx 0$) is realized at some $x = x_0$. The most probable value of x_0 is roughly 0.02.[6]

The smallness of the gap ε_{gL} leads to highly nonparabolic electron and hole spectra at the L points in Bi and the alloys $Bi_{1-x}Sb_x$. Several models for the energy spectrum of the carriers at L have been coexisting right up to the present time.[7–9]

In the coordinate system fixed to the ellipsoid at L the Lax dispersion law has the form[7]

$$\sum_i p_i^2/2m_i = \varepsilon(1+\varepsilon/\varepsilon_{gL}), \quad (1)$$

where the m_i are the masses at the bottom of the band; these masses satisfy the relation

$$m_i/m_0 = (1+2|M_i|^2/\varepsilon_{gL})^{-1}, \quad (2)$$

where the M_i are the Kane matrix elements.

It is known from experiment that the electron masses in the directions of the short semiaxes (x, y) of the ellipsoids are much smaller than the free-electron mass,[10–12] from which it follows that $m_{x,y}/m_0 \approx \varepsilon_{gL}/2|M_{x,y}|^2$. Lax assumed that in the direction, z, of elongation of the electron ellipsoids the mass m_z at the bottom of the band depends similarly on the gap ε_{gL}.

According to the Lax model (see (1) and[7]), the electron and hole constant-energy surfaces at L are strictly

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ellipsoidal, the electron and hole spectra are mirror spectra, and the anisotropy of the Fermi surface at L is determined by the anisotropy of the corresponding matrix elements M_i and depends neither on the gap ε_{gL} nor on the Fermi energy ε_F. The Lax spectrum is not sensitive to the sign of the gap ε_{gL}, so that the dispersion laws for the carriers both in the case of the inverted spectrum and in the case of the normal spectrum are identical.

Cohen[8] has suggested that the electron and hole masses at the bottom of the band in the direction of the short semiaxes, $m_{x,y}$, are determined by the gap ε_{gL}, while the masses, m_z, in the direction of elongation do not depend on ε_{gL} and are determined by the distances to more remote bands. In the general case the electron and hole masses, m_z, in the direction of elongation do not coincide, so that the corresponding spectra in the z direction are not mirror spectra.

The constant-energy surfaces in the case of the Cohen spectrum are not strictly ellipsoidal. The anisotropy of the quasiellipsoids now depends on both ε_F and ε_{gL}, and should increase sharply as the latter quantities decrease.

The most detailed theoretical analysis of the electron and hole spectra at the L point in Bi-type materials has been carried out by Abrikosov[9] on the basis of the theory developed in[13]. In the general case the dispersion law for the electrons and holes at L has the form[9]

$$[\Omega-\Omega_0-p_z^2/2m_1]^2=[\gamma-\gamma_0+p_z^2/2m_2]^2+(v_xp_x)^2+(v_yp_y)^2. \tag{3}$$

Here the directions of the x and y axes coincide with the directions of the short semiaxes of the quasiellipsoids (the x axis is parallel to the binary axis), while the z axis coincides with the direction of elongation. The quantities m_1, m_2, v_x, and v_y are constants of the spectrum; v_x and v_y, which have the dimension of velocity, are analogs of the Kane matrix elements M_x and M_y (see (2)).

The character of the spectrum is determined by the magnitude and sign of the parameter $\gamma-\gamma_0$: the case $\gamma-\gamma_0>0$ corresponds to the normal spectrum, the case $\gamma-\gamma_0<0$ corresponds to the inverted spectrum, and GS is realized at $\gamma-\gamma_0=0$. If we measure the energy from the bottom of the conduction band at L, then from (3) we obtain for the electron spectrum in the $\gamma-\gamma_0>0$ case the expression

$$\frac{(v_xp_x)^2}{\varepsilon_{gL}}+\frac{(v_yp_y)^2}{\varepsilon_{gL}}=\left(\varepsilon-\frac{p_z^2}{2m^*}\right)\left(1+\frac{\varepsilon}{\varepsilon_{gL}}+\frac{p_z^2}{2m^{**}\varepsilon_{gL}}\right), \tag{4}$$

where $m^*=(1/m_1+1/m_2)^{-1}$ is the electron mass in the direction of elongation and $m^{**}=(1/m_2-1/m_1)^{-1}$ is the hole mass in the direction of elongation (according to Abrikosov's calculations,[9] $m_2<m_1$, so that $m^{**}>m^*$). The expression (4) formally coincides with the analogous Cohen relation for nonmirror spectra.[8]

It follows from (3) that, for $\gamma-\gamma_0>0$, the electron and hole dispersion laws in the z direction ($p_x=p_y=0$) are parabolic. At the same time, the spectrum in the direction of the short semiaxes (x, y) of the Fermi surface is highly nonparabolic, and retains the Lax character. The sign of the gap parameter $(\gamma-\gamma_0)$ does not play a role in the latter ($p_z=0$) case (see (3)). From (4) we obtain for the electron spectrum at L in the $p_z=0$ case the expression

$$(v_xp_x)^2/\varepsilon_{gL}+(v_yp_y)^2/\varepsilon_{gL}=\varepsilon(1+\varepsilon/\varepsilon_{gL}). \tag{5}$$

The small electron and hole cyclotron masses $m_c^z(\varepsilon_F)$ at the Fermi level ($\mathbf{H}\parallel z$) at L depend linearly on the quantity $\varepsilon_{gL}+2\varepsilon_F$:

$$m_c^z(\varepsilon_F)=\frac{\varepsilon_{gL}}{2v_xv_y}\left(1+\frac{2\varepsilon_F}{\varepsilon_{gL}}\right)=\frac{\varepsilon_{gL}+2\varepsilon_F}{2v_xv_y}. \tag{6}$$

It follows from[9] that in the case of the normal spectrum ($\gamma-\gamma_0>0$), which is realized in the semiconducting alloys $Bi_{1-x}Sb_x$,[1-6] the anisotropy of the electron and hole Fermi surfaces at L increases with decreasing ε_{gL} and ε_F.

The disagreement between the Lax and Abrikosov models turns out to be greatest in the GS. In the first case the linear dispersion law, $\varepsilon\propto p$, is realized in all the directions in the GS; in the second case the spectrum remains parabolic in the direction of elongation: $\varepsilon\propto p^2$ (see (3)).

In spite of the abundance of experimental papers, the choice between the models for the spectrum at the L point in Bi-type materials was not finally made until very recently.[12,14-20] New possibilities opened up for the experimental verification of two models following the discovery of pressure-induced transitions into the GS in the alloys $Bi_{1-x}Sb_x$.[3,5,6,21] In[5,21] it was discovered that the transition of the "pure" semiconducting $Bi_{1-x}Sb_x$ alloys into the GS under pressure is accompanied by a sharp increase in the anisotropy of the mobility of the carriers at liquid-helium temperatures, which can be related with the drawing out of the constant-energy surfaces into "spikes" as $\varepsilon_{gL}\to 0$.

In the present paper we investigate at liquid-helium temperatures the oscillatory and galvanomagnetic effects in the p-type semiconducting alloy $Bi_{0.9}Sb_{0.1}$ with a hole-impurity concentration ~ 10^{15} cm^{-3} on undergoing a pressure-induced transition into the GS.[22] A sharp increase has been observed in the anisotropy of the hole Fermi surface at L as $\varepsilon_{gL}\to 0$. The effect is in qualitative and quantitative agreement with the Abrikosov[9] and Cohen[8] theories, and cannot be explained on the basis of the Lax model.[7]

2. THE MEASUREMENT PROCEDURE. SAMPLES

Quasihydrostatic pressures of up to 10 kbar were produced, using a procedure which is a modification of Itskevich's technique.[23] The pressure booster, which was made from a heat-treated beryllium bronze of the BRB-2 brand, had a working channel of diameter ~3.9 mm.[24] As the pressure-transmitting medium, we used a mixture consisting of 50% pentane, 25% kerosene, and 25% transformer oil. The pressure in the working channel was measured by an induction method from the shift in the superconducting-transition temperature of a tin sensing element.[25]

The samples, in the form of rectangular parallelepipeds ($0.8\times 0.8\times 3.0$ mm) were cut with the aid of an electro-erosion device from a monocrystalline stock of the $Bi_{1-x}Sb_x$ ($x=0.1$) alloy prepared by the zone-leveling method. The present alloy was kindly made available to us by D. V. Gitsu (Institute of Applied Physics of the Moldavian Acad. of Sci. in Kishinev). The measurement of the components of the galvanomagnetic tensor in weak magnetic fields ($\omega\tau \ll 1$) was carried out, using a compensation method. The Shubnikov-de Haas (SdH) effect was investigated in a setup that allowed the recording of $\rho(H)$ and $\partial\rho(H)/\partial H$ in fields of up to 13 kOe.

3. EXPERIMENTAL RESULTS

The determination in weak fields ($\omega\tau \ll 1$) of the galvanomagnetic tensor components of the p-type alloy $Bi_{0.9}Sb_{0.1}$ was carried out in the pressure range 1 bar $\leq p \leq 10$ kbar at $T=4.2$ K. The pressure dependences of the resistivities ρ_{11} and ρ_{33}, the transverse magnetoresistance coefficients ρ_{1133}/ρ_{11} and ρ_{3311}/ρ_{33}, and the Hall coefficients R_{123} and R_{231} (the indices 1,2, 3 denote respectively the binary, bisectrix, and trigonal directions; the designations of the galvanomagnetic-tensor components were taken from[26]) were measured.

The weak-magnetic-field region, determined by the condition $\omega\tau \ll 1$, narrows down drastically upon going over into the GS under pressure (up to 5–10 Oe for $\varepsilon_{gL} \approx 0$). In order to exclude possible errors in the measurement of the tensor components, we carefully measured at each pressure the field dependences of the Hall emf, $U_{\rm Hall}$, and the magnetoresistance, $\Delta\rho(H) = \rho(H) - \rho(0)$. The Hall and transversemagnetoresistance coefficients were determined along the linear sections of the dependences $U_{\rm Hall}=F(H)$ and $\Delta\rho = f(H^2)$. The measuring current through the samples was set so as to exclude superheating effects.

It was found that the resistivities ρ_{11} and ρ_{33} at $T=4.2$ K decrease reversibly with increasing pressure p and go through a minimum at $p_{\rm GS}=7.5\pm 0.5$ kbar (Fig. 1). In[3,5,6] it was demonstrated that the minimum in $\rho(p)$ corresponds to the minimum of the thermal gap and arises because of the transition of the alloy into the GS as a result of the inversion of the L_a and L_s terms under the pressure $p=p_{\rm GS}$.

The two independent components, R_{231} and R_{123}, of the Hall tensor at $T=4.2$ K are positive in the entire pressure range, which indicates that the investigated alloy is a p-type alloy. The Hall-coefficient anisotropy R_{231}/R_{123} at $T=4.2$ K increases sharply with increasing pressure, and reaches a maximum at $p=p_{\rm GS}$ (Fig. 1).

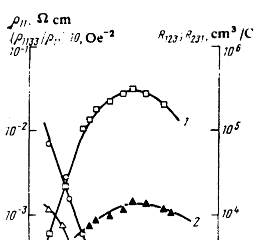

FIG. 1. The pressure dependence at $T=4.2$ K of the galvanomagnetic-tensor components of the p-type $Bi_{0.9}Sb_{0.1}$ alloy: 1) the transverse-magnetoresistance coefficient ρ_{1133}/ρ_{11}; 2) the Hall coefficient R_{231}; 3) the resistivity ρ_{11}; 4) the Hall coefficient R_{123}.

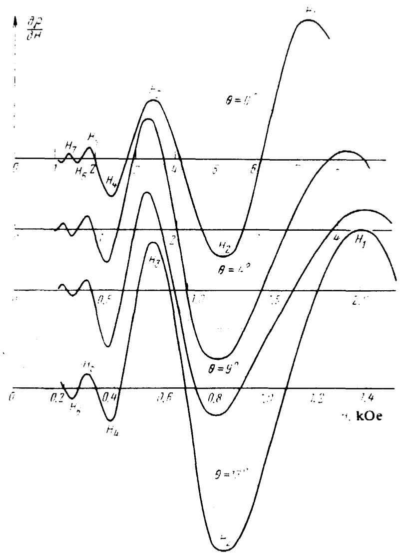

FIG. 2. Field dependences of $\partial\rho(H)/\partial H$ as the magnetic field is rotated in the binary-bisector at $p=3$ kbar; the angle θ is measured from the direction of the field to the binary axis; $T=2$ K.

The value, P, of the integrated hole concentration at L at liquid-helium temperatures can be roughly estimated from the relation $R_{231}\approx 1/ePc$, and is $P\approx 1\times 10^{15}$ cm^{-3}.

The transition into the GS of the investigated alloy is accompanied by a sharp increase in the magnetoresistance-tensor components at $T=4.2$ K, which is connected with increases in the hole mobilities at L as $\varepsilon_{gL}\to 0$ (Fig. 1). The magnitude of the hole mobility in the GS can be estimated from the condition $\mu H_c \sim 1$, under which the dependence $\Delta\rho = f(H^2)$ deviates from linearity. Assuming that $H_c \approx 10$ Oe in the GS, we obtain for the mean hole mobility for $\varepsilon_{gL}=0$ the value $\mu_{\rm GS}\sim 10^7$ cm^2/V-sec.

The SdH oscillations were investigated as the magnetic field was rotated in the binary-bisector and bisector-trigonal planes. At $p \geq 1.5$ kbar the Shubnikov oscillations were observed in the entire angle range, but their amplitude was small, and the recording of the oscillation curves presented great difficulties. We succeeded at pressures $p<6$ kbar in recording only the dependence $\partial\rho(H)/\partial H$, whose monotonic component was canceled out with the aid of an analog computer. An example of the Shubnikov oscillations during the rotation of the magnetic field in the binary-bisector plane ($p=33$ kbar, $T=2$ K) is shown in Fig. 2. In the vicinity of the GS the SdH-oscillation amplitude increased, which enabled us to record the curves $\rho(H)$; in some cases the elimination of the monotonic component was not necessary.

The Shubnikov oscillations in the investigated alloy exhibited even at $p = 1.5$ kbar a strong angular dependence, which is explained by the considerable anisotropy of the hole Fermi surface at L. This dependence was especially critical for directions of the field **H** near the binary axis, which required the determination of the binary direction to within at least 0.3°. The indicated accuracy was attained through comparison of the phases of the "high-frequency" oscillations connected with the large cross-sections of the Fermi surface.

It should be noted that, as the pressure increased, the number of extrema in the oscillatory curves increased, owing to the appearance of oscillations in weak fields (the magnetic-field range in which the condition $\omega\tau \gg 1$ was fulfilled broadened as the GS was approached).

In detecting the Shubnikov oscillations, we paid special attention to the recording of the curves $\partial\rho(H)/\partial H$ in fields of intensities far from the quantum limit, since only in the latter case can we with confidence use for the computation of the cross-sections the relation $\Delta(1/H) = eh/cS_{\rm extr}$ (see Fig. 2).

Because of the high anisotropy of the Fermi surface at L, the essential information about the angular dependence of the cross-sections when the field is rotated in the binary-bisector plane is contained in the angle range $-15° < \theta < 15°$ near the binary direction (θ is the angle between the field **H** and the binary axis). The interpretation of the oscillation curves then offers no difficulty, since the oscillations from the large and small cross-sections are fieldwise well separated. The measurements showed that, as the GS is approached, the angular dependence of the frequency of the oscillations near the binary direction becomes more critical, which is a consequence of the increase in the anisotropy of the hole Fermi surface at L as $\varepsilon_{gL} \to 0$.

For **H** parallel to the binary axis, the fields, H_i, of the extrema in the curves shift toward the region of higher fields as the pressure p is increased (Fig. 3a). At the same time, for angles θ exceeding several degrees, the fields, H_i, of the extrema decrease with pressure (Fig. 3b).

The increase of the anisotropy of the Fermi surface with pressure can be judged from the angular dependences, obtained for different pressures (Fig. 4), of the fields, H_i, of the extrema in the $\partial\rho(H)/\partial H$ curves. The continuous curves in Fig. 4 are ellipses; the anisotropy of the ellipses increases appreciably on going over into the GS.

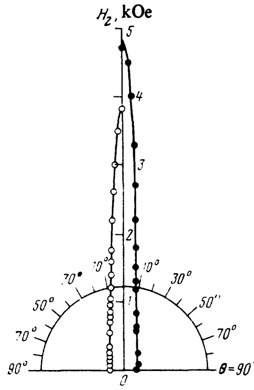

FIG. 4. The angular dependences of the field, H_2, of the penultimate extremum in the $\partial\rho(H)/\partial H$ curves as the field was rotated in the binary-bisector plane in a sample of the p-type $Bi_{0.9}Sb_{0.1}$ alloy; $T = 2$ K; ○) $p = 1.5$ kbar, ●) $p = 2.7$ kbar; the continuous curves are ellipses.

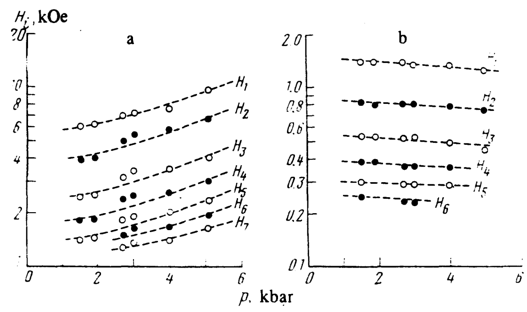

FIG. 3. Pressure dependences of the fields of the extrema in the $\partial\rho(H)/\partial H$ curves: a) for $\theta = 0°$ (the field **H** is parallel to the binary axis), b) for $\theta = 13°$; $T = 2$ K (H_1 is the last extremum in $\partial\rho(H)/\partial H$).

One circumstance that makes the interpretation of the oscillatory curves difficult at high pressures should be noted. At $p > 5$ kbar the amplitude of the second harmonic in fields close to the quantum limit increased sharply. Near the GS in this interval of fields we observed the fourth harmonic and found the first harmonic to be virtually completely suppressed.

A similar effect was earlier observed by us in the investigation of the effect of pressure on the spectrum of pure Bi. A doubling and a quadrupling of the SdH-oscillation frequency arise when the sample is placed across the channel, the effect being most strongly pronounced in the **H** ∥ **J** case. An example of the SdH oscillations for pure Bi under pressure is shown in Fig. 5. All the three curves were recorded in equivalent directions (**H** was parallel to the binary axis); the curve 1 corresponds to **H** ⊥ **J**, the curves 2 and 3 to the cases ∢ (**H**, **J**) = ± 30°. It can clearly be seen from Fig. 5 that the oscillation frequency doubles in the last two cases. The causes of this effect are at present not clear.

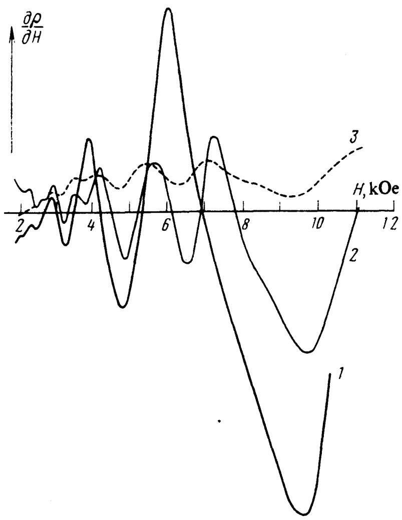

FIG. 5. SdH oscillations in $\partial\rho(H)/\partial H$ for pure Bi under pressure ($p = 2.3$ kbar) in the case when **H** is parallel to the binary axis: 1) **H** ⊥ **J**, 2) ∢ (**H**, **J**) = 30°, 3) ∢ (**H**, **J**) = −30°.

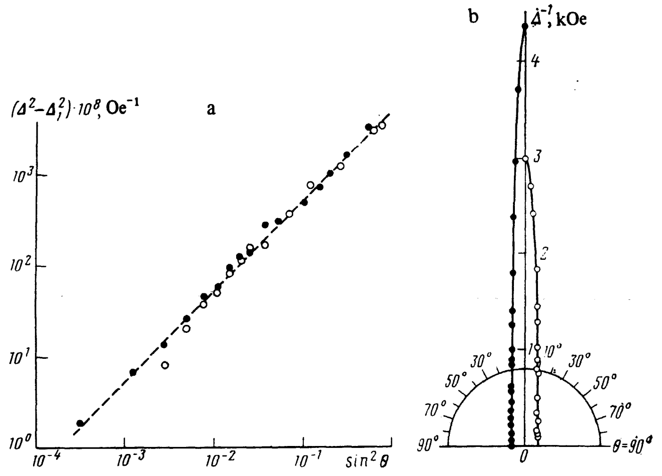

FIG. 6. The angular dependences of a) the quantity $\Delta^2-\Delta_1^2$ and b) the inverse period $\Delta^{-1}\propto S_{\text{extr}}$ as the magnetic field is rotated in the binary-bisector plane in the p-type $Bi_{0.9}Sb_{0.1}$ alloy at different pressures: ○) $p=1.5$ kbar, ●) $p=3.0$ kbar; $T=2$ K.

4. DISCUSSION OF THE RESULTS

The investigations of the SdH and galvanomagnetic effects in a weak magnetic field that have been carried out in the present paper on the semiconducting alloy $Bi_{0.9}Sb_{0.1}$ in a broad pressure range allowed us to determine the sign of the charge carriers at liquid-helium temperatures, to reestablish the shape of the Fermi surface, and to determine the character of the change in the shape of the constant-energy surfaces on going over into the GS under pressure. The positive sign of the Hall-tensor components R_{231} and R_{123} in the entire pressure range indicates that charge is transported in the investigated alloy at liquid-helium temperatures by holes. The form of the angular dependences of the SdH-oscillation frequency allows us to infer that these oscillations are determined by the hole Fermi surfaces at L.

1. In the case of an ellipsoidal shape of the Fermi surface at L the angular dependence of the SdH-oscillation period when the field is rotated in the binary-bisector plane for one ellipsoid has the form

$$\Delta^2(\theta)=\Delta_1^2\cos^2\theta+\Delta_2^2\sin^2\theta=\Delta_1^2+(\Delta_2^2-\Delta_1^2)\sin^2\theta, \qquad (7)$$

where Δ_1 and Δ_2 are the periods of the SdH oscillations for **H** parallel respectively to the binary axis and the bisectrix.

The calculations showed that the dependence of the quantity $\Delta^2-\Delta_1^2$ on $\sin^2\theta$ in the investigated alloy is linear at all pressures (Fig. 6a), from which it follows that the shape of the hole Fermi surface at L is close to the ellipsoidal shape in the entire pressure range.

As can be seen from the angular dependences of the inverse period, $\Delta^{-1}\propto S_{\text{extr}}$, of the SdH oscillations (Fig. 6b and Table I), the transition of the investigated alloy into the GS is accompanied by an appreciable increase in the anisotropy of the Fermi surface. At the same time, the angle of inclination of the ellipsoids to the basal plane does not vary with pressure, and is equal to $\varphi=(5.5\pm0.5)°$. This value agrees well with the data obtained for the hole ellipsoids at the L point in the semiconducting $Bi_{1-x}Sb_x$ alloys under atmospheric pressure.[27-30] Notice that, in Bi, the angle of inclination of the electron ellipsoids to the basal plane has virtually the same value.[10]

The results obtained in the present investigation cannot be explained on the basis of the Lax model, but are in qualitative agreement with the predictions of the Abrikosov theory.[9]

It should be noted that, according to the Abrikosov model,[9] for $\gamma-\gamma_0>0$ the Fermi surfaces at L are ellipsoidal only in the case when the Fermi energy ε_F is much smaller that the gap ε_{gL} (i.e., when the carriers are located at the very bottom of the band). When the hole Fermi energy ε is measured from the top of the valence band at L (downwards along the energy scale) and the conditions $\gamma-\gamma_0>0$ and $\varepsilon_F\ll\varepsilon_{gL}$ are fulfilled, we obtain from the relation (3) that

$$\varepsilon\approx(v_xp_x)^2/\varepsilon_{gL}+(v_yp_y)^2/\varepsilon_{gL}+p_z^2/2m^{**}. \qquad (8)$$

It follows from[9] that the expression (8) remains valid in a relatively narrow pressure range even in the case of a low current-carrier concentration (in the investigated alloy the integrated hole concentration $P\propto10^{15}$ cm^{-3}). When the transition is made into the GS, the condition $\varepsilon_F\ll\varepsilon_{gL}$ is violated, which should lead to the appearance of deviations of the shape of the Fermi surface from the ellipsoidal shape.[9] These deviations were not observed in the present experiment. The latter circumstance is possibly due to the limited accuracy achieved in the determination of the SdH-oscillation periods.

2. The hole concentrations per ellipsoid at L was computed from the formula

$$P_1\ [\text{cm}^{-3}]=5.76\cdot10^9(\Delta_1^{-1}\Delta_2^{-1}\Delta_3^{-1})^{1/2},$$

where Δ_1^{-1}, Δ_2^{-1}, and Δ_3^{-1} [erg] are the inverse oscillation periods corresponding to the three principal cross-sections of the ellipsoid at L. The values of these quantities were taken from the inverse periods' angular dependences obtained as the field was rotated in the binary-bisector and bisector-trigonal planes. It was observed that the integrated hole concentration $P_L=3P_1$ increases by roughly a factor of two when the pressure is increased from atmospheric pressure to $p_{GS}=7.5$ kbar,

TABLE I. Dependence on pressure of the inverse periods of the SdH oscillations corresponding to the three principal cross-sections of the hole Fermi surface at the L point in the investigated alloy.

p, kbar	$\Delta_{min}^{-1}\sim S_{min}$, kOe	$\Delta_{mid}^{-1}\sim S_{mid}$, kOe	$\Delta_{max}^{-1}\sim S_{max}$, kOe	p, kbar	$\Delta_{min}^{-1}\sim S_{min}$, kOe	$\Delta_{mid}^{-1}\sim S_{mid}$, kOe	$\Delta_{max}^{-1}\sim S_{max}$,
1.5	0.145	–	3.0	4.0	0.150	3.30	4.65
1.9	0.155	–	3.2	5.1	0.150	4.20	5.4
2.7	0.160	–	4.0	6.0	0.145	4.20	5.6
3.0	0.155	–	4.35				

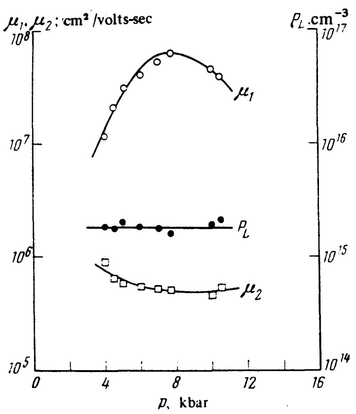

FIG. 7. The pressure dependences of the mobility-tensor components μ_1 and μ_2 and of the hole concentration P_L at $T=4.2$ K according to the data of the galvanomagnetic measurements.

and attains the value $P_L = 1.2\times10^{15}$ cm^{-3} in the GS. The growth of the hole concentration at L with pressure is apparently caused by the flow of holes into the L-extrema from the T-extrema, which, in the $Bi_{0.9}Sb_{0.1}$ alloy, is located in close proximity to the L-extrema.[3-6,21,27] For acceptor-impurity concentrations $\sim10^{15}$ cm^{-3}, it is possible for a T-extremum with "heavy" holes to develop a density-of-states "tail" that overlaps the L-extrema in energy.

3. The hole concentration at L (P_L) and the hole-mobility tensor components μ_i in the vicinity of the GS at $T=4.2$ K were computed from the galvanomagnetic coefficients under the assumption that the contribution to the conductivity from the "heavy" holes at T is negligibly small. In the case when the Fermi surface of the current carriers consists of three ellipsoids at L, the carrier gas is degenerate, the relaxation time can be represented in tensorial form (the Herring-Vogt approximation), and for the galvanomagnetic tensor components ρ_{11}, $\rho_{123} = -R_{123}$, and ρ_{1133} the following relations are valid[26]:

$$\rho_{11}=\frac{2}{eP_L}\frac{1}{\mu_1+\mu_2},\quad -\rho_{123}=\frac{4}{eP_L}\frac{\mu_1\mu_2}{(\mu_1+\mu_2)^2},$$

$$\frac{\rho_{1133}}{\rho_{11}}=\mu_1\mu_2-\frac{4(\mu_1\mu_2)^2}{(\mu_1+\mu_2)^2},$$

where the μ_i are the components of the mobility tensor in the coordinate system fixed to the lattice (1 denotes the binary axis, 2 the bisectrix, and 3 the trigonal axis). For Bi and the $Bi_{1-x}Sb_x$ alloys the inequality $\mu_1 \gg \mu_2$ is usually fulfilled, so that the above-presented relations can be simplified:

$$\rho_{11}\approx\frac{2}{eP_L}\frac{1}{\mu_1},\quad -\rho_{123}\approx\frac{4}{eP_L}\frac{\mu_2}{\mu_1},\quad \frac{\rho_{1133}}{\rho_{11}}\approx\mu_1\mu_2.$$

The results of the computation of the pressure dependences of P_L and μ_i at $T=4.2$ K are presented in Fig. 7. The mobility, μ_1, in the direction of a short semiaxis (x) of the ellipsoid at L increases appreciably when the pressure is increased, and attains a maximum in the GS ($p_{GS} = 7.5\pm0.5$ kbar). At the same time, the mobility μ_2, which is virtually equal to the mobility in the direction of elongation of the ellipsoid, even decreases slightly during the transition into the GS (Fig. 7). The mobility anisotropy μ_1/μ_2 thus increases as $\varepsilon_{gL}\to0$, which agrees with the data obtained in[5]. The results of the present work allow us to conclude that the growth of the anisotropy in the carrier mobility at L during the transition into the GS is primarily connected with the increase in the anisotropy of the Fermi surface (Fig. 6b).

The hole-concentration value $P_L\approx1.8\times10^{15}$ cm^{-3}, computed from the galvanomagnetic coefficients (Fig. 7), turned out to be too high as compared to the P_L value determined in the GS with the aid of the SdH effect ($P_L = 1.2\times10^{15}$ cm^{-3}). This is possibly due to the fact that the contribution to the conductivity of the "heavy" holes at the T point was not taken into account in the computation of P_L from the galvanomagnetic-tensor components.

4. For a quantitative comparison of the data obtained in the present work with Abrikosov's theoretical model,[9], it is necessary in the first place to determine from experiment the values of the parameters v_x and v_y entering into the Abrikosov dispersion law (3). This is most easily done by using the experimentally obtained dependence of the small cyclotron mass $m_c^*(\varepsilon_F)$ at the Fermi level (see (7)) on $\varepsilon_{gL}+2\varepsilon_F$ in the n- and p-type semiconducting $Bi_{1-x}Sb_x$ alloys. Such a dependence, constructed from the data of[27-32], is shown in Fig. 8. The continuous straight line in this figure was constructed from the formula (6) for the case $v_xv_y = 0.7\times10^{16}$ cm^2/sec^2. The product v_xv_y can be split by using the relation[9] $S_{max}/S_{mid} = v_x/v_y$, where S_{max} is the maximal principal cross-section of the Fermi surface at L and S_{mid} is the intermediate principal cross-section. According to the data of the present work, S_{max}/S_{mid} is equal on the average to 1.4; hence for v_x and v_y we obtain: $v_x = 1.0\times10^8$ cm/sec and $v_y = 0.7\times10^8$ cm/sec. The error made in the determination of the quantities v_x and v_y is equal to $\sim20\%$.

The values of the hole Fermi energy in the investigated alloy at different pressures were computed from the minimal principal cross-section S_{min} ($\mathbf{H}\parallel\mathbf{Z}$) with the aid of the relation

$$S_{min}=\pi\varepsilon_F(\varepsilon_{gL}+\varepsilon_F)/v_xv_y,$$

which follows from (5). The value of the gap ε_{gL} at each pressure was determined from the formula ε_{gL}

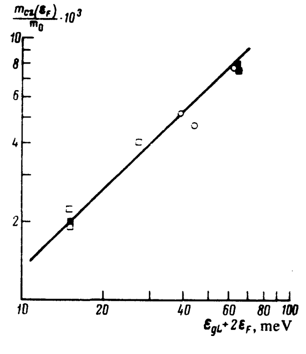

FIG. 8. The $\varepsilon_{gL}+2\varepsilon_F$ dependence of the small cyclotron mass at the Fermi level ($\mathbf{H}\parallel z$) in the n- and p-type semiconducting $Bi_{1-x}Sb_x$ alloys according to the data of different papers: ●)[23], ○)[24], □)[27], ■)[25,28].

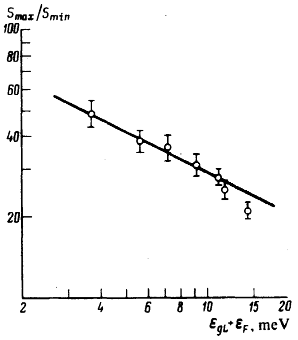

FIG. 9. The $\varepsilon_{gL}+\varepsilon_F$ dependence of the anisotropy of the Fermi surface in the p-type $Bi_{0.9}Sb_{0.1}$ alloy. The continuous line was constructed on the basis of the Abrikosov model under the condition that $v_x=1\times10^8$ cm/sec and $m^{**}=0.75m_0$.

$=(\partial\varepsilon_{gL}/\partial p)(p-p_{GS})$, where $\partial\varepsilon_{gL}/\partial p=-2.2\times10^{-6}$ eV/bar[6] and $p_{GS}=7.5$ kbar. The computation of $\varepsilon_F(p)$ showed that, as the pressure is increased from atmospheric pressure to p_{GS}, the hole Fermi energy at L increases from 1 to 3.3 meV.

In the pressure range where $\varepsilon_F \ll \varepsilon_{gL}$ and where a dispersion law of the form (8) is valid, the ratio of the maximal cross-section of the ellipsoid at L to the minimal cross-section, a ratio which characterizes the anisotropy of the Fermi surface, is equal to

$$S_{max}/S_{min}=p_{zF}/p_{xF}=v_x(2m^{**}/\varepsilon_{gL})^{1/2}. \quad (9)$$

As the GS is approached, the relation (9) is not fulfilled because of the violation of the condition $\varepsilon_F \ll \varepsilon_{gL}$. If we take into account the circumstance that the ellipsoidal nature of the Fermi surface in the investigated alloy is preserved in the transition into the GS, then it is expedient to use for S_{max}/S_{min} the approximate relation obtained for the ellipsoidal Fermi surface under the condition that the spectrum is parabolic in the direction of elongation and of the form (5) in the directions of the short semiaxes, i.e., the relation

$$S_{max}/S_{min}=v_x[2m^{**}/(\varepsilon_{gL}+\varepsilon_F)]^{1/2}. \quad (10)$$

The relation (10) is apparently fulfilled well in a broader pressure range than the relation (9). In the GS the discrepancy between the S_{max}/S_{min} values computed from the exact formulas of[9] and from the approximate relation (10) does not, however, exceed 10–15%, which is comparable to the experimental errors in the determination of S_{max}/S_{min}.

The dependence, obtained experimentally in the present work, of S_{max}/S_{min} on $\varepsilon_{gL}+\varepsilon_F$ in the investigated alloy is shown in Fig. 9. The continuous line in the figure was constructed from the formula (10) under the condition that $v_x=1\times10^8$ cm/sec and $m^{**}=0.75m_0$.

Thus, the results of the present work allow us to make an unequivocal choice from among the existing models for the spectrum at the L point in materials of the Bi type: the Abrikosov model.[9] The detailed verification of the theory developed in[9] and the determination of the parameters of the Abrikosov spectrum with a greater accuracy than has been done in the present work require additional investigations. Reliable results can, apparently, be obtained only when both the gap ε_{gL} and the Fermi energy (in the valence and conduction bands) are varied within broad limits.

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