where $G(\mathbf{p}) = [\omega - \xi + (i/2\tau)\,\mathrm{sign}\omega]^{-1}$ is the electron Green's function averaged over the impurity distribution. When solving (38), we confine ourselves to the region of the normal skin effect $|\mathbf{k}|v_0\tau \ll 1$ and expand the quantities $G(\mathbf{p}_+)G(\mathbf{p}_-)$ and $\Pi(\mathbf{p}_+, \mathbf{p}_-)$ in powers of $\mathbf{k}$ accurate to terms $\sim\mathbf{k}^2$ inclusive. Equating the coefficients of equal powers of $\mathbf{k}$ on the right and left-hand sides of (38), we obtain a system of three equations, from which we determine the coefficients of the expansion of the quantity $\Pi_\alpha(\mathbf{p}_+, \mathbf{p}_-)$ in the form

$$\Pi_\alpha(\mathbf{p}_+, \mathbf{p}_-) = \Pi_\alpha + \Pi_\alpha^{\beta} k_\beta + \Pi_\alpha^{\beta\gamma} k_\beta k_\gamma. \tag{39}$$

The expressions for the coefficients $\Pi_\alpha^{\beta\nu}$, Π_α^{β} and Π_α are too complicated to be presented here. Next, substituting (39) in (37) and integrating, we obtain the susceptibility tensor $Q_{\alpha\beta}(k, \omega)$:

$$Q_{\alpha\beta}(k,\omega) = \frac{\omega^2}{4\pi}\left\{\left[1 - \varepsilon_0(\omega) + \frac{c^2}{\omega^2}\alpha_t(\omega)k^2\right]\delta_{\alpha\beta} + \frac{c^2}{\omega^2}(\alpha_l - \alpha_t)k_\alpha k_\beta\right\}. \tag{40}$$

Knowing the connection between the susceptibility tensor and the dielectric tensor

$$Q_{\alpha\beta}(k,\omega) = \frac{\omega^2}{4\pi}[\delta_{\alpha\beta} - \varepsilon_{\alpha\beta}(k,\omega)],$$

we obtain the latter in the form

$$\varepsilon_{\alpha\beta}(k,\omega) = \left(\varepsilon_0 - \frac{c^4}{\omega^2}\alpha_t k^2\right)\delta_{\alpha\beta} - \frac{c^4}{\omega^2}(\alpha_l - \alpha_t)k_\alpha k_\beta. \tag{41}$$

It is now easy to determine the longitudinal and transverse dielectric constants:

$$\begin{gathered}\varepsilon_l(k,\omega) = \frac{k_\alpha k_\beta}{k^2}\varepsilon_{\alpha\beta}(k,\omega) = \varepsilon_0 - \frac{c^2}{\omega^2}\alpha_l(\omega)k^2, \\ \varepsilon_t(k,\omega) = \left(\delta_{\alpha\beta} - \frac{k_\alpha k_\beta}{k^2}\right)\varepsilon_{\alpha\beta}(k,\omega) = \varepsilon_0 - \frac{c^2}{\omega^2}\alpha_t(\omega)k^2.\end{gathered} \tag{42}$$

Here $\varepsilon_0(\omega)$, $\alpha_l(\omega)$ and $\alpha_t(\omega)$ are defined by formulas (10) and (11).

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Longitudinal oscillations in superconducting alloys

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We study the propagation of sound in superconductors for arbitrary electron mean free paths. We show that in the main approximation in terms of $(s/v)^2$ the usual BCS expression for the absorption of sound is valid. We study the effect of a current on the absorption of sound and collective oscillations in superconductors.

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INTRODUCTION

At the present time there are a large number of papers devoted to a study of sound oscillations in superconductors. We can distinguish three main methods of calculation: the kinetic equation method, a direct calculation of the polarization operator for the phonon Green function, and the linear response method. The last approach is, in fact, very close to the kinetic equation method, but it has the advantage that it is not connected with any restriction on the oscillation frequency or on the electron mean free path. Moreover, it is rather

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cumbersome to detect several general relations in the kinetic equation method as the normal excitations and the superconducting part in it are, in fact, separated. Gal'perin and Kozub[1] obtained an expression for the polarization operator and showed that the effect of impurities on the absorption of sound can be described by a single parameter kl. This last statement is not altogether exact and there exists, even though weakly expressed, a dispersion at frequencies $\omega \sim Dk^2$, where D is the diffusion coefficient. In a recent paper, Vardanyan and Lisitsyn[2] studied the absorption of sound by the linear response method. The expression for the damping of sound obtained by them has a strong dispersion for $\omega \sim Dk^2$ and another number of unusual properties.

We shall show below that allowance for the force due to the transfer of momentum from the electron system to the impurities removes these anomalies and we obtain for the sound absorption, in the main approximation, the usual BCS expression.[3] The dispersion at a frequency $\omega \sim Dk^2$ is contained only in the next terms in k^2.

In a paper by Tsuneto[4] the change in phase was, in fact, not taken into account and at the same time he neglected the contribution to the absorption arising from the "friction" force. Taking the change of phase consistently into account, but neglecting the "friction" force in the equation for the motion of the ions would lead to the results in the paper by Vardanyan and Lisitsyn.[2] In what follows we consider the effect of a current on the propagation of sound and collective oscillations in a superconductor.

1. BASIC EQUATIONS

To find the function $\omega(k)$ when sound propagates we use the set of equations for Green functions integrated over the energy variable.[5] If we linearize these equations with respect to the external perturbation $\mathbf{u}$, where $\mathbf{u}$ is the ion displacement vector, these equations are of the form

$$i\mathbf{kv}G^1+\tau_z\omega_+G^1-\omega G^1\tau_z+(-ie\mathbf{vA}_1\tau_z-i\hat{\Delta}_1+ie\varphi+in\Sigma^1)G^0(\omega)$$
$$-G^0(\omega_+)(-ie\mathbf{vA}_1\tau_z-i\hat{\Delta}_1+ie\varphi+in\Sigma^1)+(-ie\mathbf{vA}_0\tau_z-i\hat{\Delta}_0$$
$$+in\Sigma^0(\omega_+))G^1-G^1(-ie\mathbf{vA}_0\tau_z-i\hat{\Delta}_0+in\Sigma^0(\omega))=0. \quad (1)$$

The linear response Green function G^1 satisfies the normalization condition

$$G^1G^0(\omega)+G^0(\omega_+)G^1=0. \quad (2)$$

In Eq. (1) n is the impurity density; $\mathbf{A}_0$ is the static vector potential; φ is the scalar potential; τ_z is a Pauli matrix; $\mathbf{A}_1$, $\hat{\Delta}_1$, and Σ^1 are, respectively, the corrections to the vector potential, the order parameter, and the self-energy part; $\omega_+ = \omega + \omega_0$, and ω_0 is the frequency of the external field.

We shall assume that when there are sound oscillations the impurities are completely dragged along by the lattice. The expression for the self-energy part Σ in that case takes in the Born approximation the form

$$\Sigma_\mathbf{p}(\tau,\tau')=-2^{-1}iv\int d\Omega_{\mathbf{p}_1}\sigma_{\mathbf{pp}_1}\{G_{\mathbf{p}_1}(\tau,\tau')$$
$$-i(\mathbf{p}-\mathbf{p}_1)(G_{\mathbf{p}_1}(\tau,\tau')\mathbf{u}(\tau')-\mathbf{u}(\tau)G_{\mathbf{p}_1}(\tau,\tau'))\}, \quad (3)$$

where $\sigma_{\mathbf{pp}_1}$ is the cross section for scattering by impurities. Equation (3) differs from the corresponding expression for Σ in the paper by Larkin and the author[5] by the last term which takes into account the dragging along of the impurities when there are sound oscillations present.

When there are sound oscillations present there appears a scalar potential φ and also a correction to the phase of the order parameter. When there is an external current present, the absolute magnitude of the order parameter changes. The corresponding changes are connected with the Green function G^1 through the relations[5]

$$\frac{ipv}{3}\mathbf{ku}+e\varphi=-\frac{i\pi}{2}T\sum_\omega\langle g_1+g_2\rangle,$$
$$\frac{pv}{3}\omega_0\mathbf{ku}=\frac{i\pi}{2m}T\sum_\omega\langle\mathbf{kp}(g_1-g_2)\rangle, \quad (4)$$
$$T\sum_\omega\left\{(\Delta_1{}^1+\Delta_2{}^1)\left\langle\frac{\beta_++\beta}{2\Delta}\right\rangle-\left\langle\frac{\beta_+-\beta}{\alpha_+-\alpha}(g_1{}^1-g_2{}^1)\right\rangle\right\}=0,$$

where the $\langle\ldots\rangle$ sign indicates averaging over the angles of the vector $\mathbf{p}$. We have chosen the Green functions G^0 and G^1 in the form

$$G^0=\begin{pmatrix}\alpha & -i\beta\\ i\beta & -\alpha\end{pmatrix},\quad G^1=\begin{pmatrix}g_1 & F_1\\ -F_2 & g_2\end{pmatrix},\quad \hat{\Delta}_1=\begin{pmatrix}0 & \Delta_1{}^1\\ -\Delta_2{}^1 & 0\end{pmatrix}. \quad (5)$$

The last of Eqs. (4) and also Eq. (5) for the function G^0 are written down, assuming that the order parameter $\hat{\Delta}_0$ is chosen to be real.

The zeroth Green functions α, β satisfy the condition

$$\alpha^2+\beta^2=1. \quad (6)$$

Substituting Eq. (5) for the function G^1 into Eq. (2) we get

$$F_1=\frac{i}{\alpha_+-\alpha}(\beta_+g_2+\beta g_1),\quad F_2=-\frac{i}{\alpha_+-\alpha}(\beta g_2-\beta_+g_1). \quad (7)$$

To obtain the function $\omega(k)$ it is necessary to complete the set of Eqs. (1), (4) with the equation of motion of the ions. In the jelly model this equation has the form

$$M\ddot{\mathbf{u}}=e\mathbf{E}+\mathbf{F}^{imp},\quad \mathbf{E}=-\nabla\varphi-\frac{\partial\mathbf{A}_1}{\partial t}, \quad (8)$$

where M is the ion mass per unit charge. The force $\mathbf{F}^{imp}$ in Eq. (8) is connected with the transfer of momentum from the electron system to the impurities. Using the expression for the total electron momentum

$$\mathbf{P}_{el}=-\frac{imp}{2\pi}T\sum_\omega\langle\mathbf{p}(g_1-g_2)\rangle \quad (9)$$

and using the fact that the impurities occur in the electron Hamiltonian in the form of the term

$$\sum_a V(\mathbf{r}-\mathbf{r}_a)\psi^+\psi,$$

we get for the force $\mathbf{F}^{imp}$ the expression

$$\mathbf{F}^{imp}=-\frac{mpn}{2\pi N}\mathrm{Sp}\,T\sum_\omega\langle\mathbf{p}(\Sigma G-G\Sigma)\rangle, \quad (10)$$

where $N=p^3/3\pi^2$ is the number of electrons per unit volume.

Using the set of Eqs. (1) we bring Eq. (10) for the force $\mathbf{F}^{imp}$ to the form

$$\mathbf{F}^{imp}=\frac{\pi}{2}T\sum_{\omega}\mathbf{k}\langle g_1+g_2\rangle+\omega_0 e\mathbf{A}_1-\frac{imp\omega_0}{2\pi N}T\sum_{\omega}\langle\mathbf{p}(g_1-g_2)\rangle+\frac{mp}{2\pi N}T\sum_{\omega}\left\langle\left(\mathbf{p}(\mathbf{kv})-\frac{pv}{3}\mathbf{k}\right)(g_1+g_2)\right\rangle. \quad (11)$$

Taking the spatial derivative of the first of Eqs. (4) we get

$$e\mathbf{E}+\frac{pv}{3}\mathbf{k}(\mathbf{uk})=-\frac{\pi}{2}T\sum_{\omega}\mathbf{k}\langle g_1+g_2\rangle-\omega_0 e\mathbf{A}_1. \quad (12)$$

The first two terms in Eq. (11) for the force $\mathbf{F}^{imp}$ are exactly equal to the right-hand side of Eq. (12) and they therefore cancel exactly in Eq. (8) for the motion of the ions. But it is just the right-hand side of Eq. (12) which in the paper by Vardanyan and Lisitsyn[2] determined the sound absorption. The third term on the right-hand side of Eq. (11) leads for longitudinal sound oscillations to a renormalization of the ion mass. Multiplying Eq. (8) by $\mathbf{k}$ and using Eqs. (11) and (12) for $\mathbf{F}^{imp}$ and $\mathbf{E}$ we get

$$\omega_0^2(M+m)\mathbf{ku}=-\frac{pv}{3}k^2\mathbf{ku}+\frac{mp}{2\pi N}T\sum_{\omega}\left\langle\left((\mathbf{kp})(\mathbf{kv})-\frac{pv}{3}k^2\right)(g_1+g_2)\right\rangle. \quad (13)$$

To obtain the dispersion relation it is necessary to analytically continue with respect to ω_0 and make the substitution $\omega_0\to -i\omega$ in the right-hand side of Eq. (13). The Green function g_1+g_2 can be expressed in terms of the displacement vector $\mathbf{u}$ by using Eqs. (1), (3), (4).

2. SOUND OSCILLATIONS WHEN THERE IS NO CURRENT

When there is no current the zeroth Green functions α and β equal

$$\alpha(\omega)=\frac{\omega}{(\omega^2+\Delta^2)^{1/2}},\quad \beta(\omega)=\frac{\Delta}{(\omega^2+\Delta^2)^{1/2}}. \quad (14)$$

To simplify the calculations we assume in what follows that the scattering by impurities is isotropic and we choose a special gauge in which $\mathbf{A}_1=0$. One can easily solve the set of Eqs. (1), (2), (3) and we find for the Green functions the expressions

$$\begin{aligned}g_1+g_2&=\frac{1}{\tau}\left(B+\frac{1}{\tau}\right)\frac{\langle g_1+g_2\rangle}{W}+\frac{\omega_0(\alpha_+-\alpha)}{BW}\left(B+\frac{1}{\tau}\right)\left[2ie\varphi-\frac{\beta_++\beta}{\alpha_+-\alpha}(\Delta_1^1-\Delta_2^1)\right]-\frac{(\mathbf{kv})(\mathbf{pu})}{W}\frac{2\omega_0(\alpha_+-\alpha)}{B\tau},\\ g_1-g_2&=-\frac{i\mathbf{kv}B}{\omega_0\tau W}\langle g_1+g_2\rangle-\frac{i\mathbf{kv}(\alpha_+-\alpha)}{W}\left[2ie\varphi-\frac{\beta_++\beta}{\alpha_+-\alpha}(\Delta_1^1-\Delta_2^1)\right]-\frac{2i\mathbf{pu}(\alpha_+-\alpha)}{\tau W}\left(B+\frac{1}{\tau}\right),\end{aligned} \quad (15)$$

$$\alpha=\alpha(\omega),\quad \beta=\beta(\omega),\quad \alpha_+=\alpha(\omega+\omega_0),\quad \beta_+=\beta(\omega+\omega_0),$$
$$B=(\omega^2+\Delta^2)^{1/2}+[(\omega+\omega_0)^2+\Delta^2]^{1/2},\qquad W=(\mathbf{kv})^2+(B+1/\tau)^2.$$

It is convenient to introduce instead of the scalar potential φ a new potential Φ which is connected with φ through the relation

$$\Phi=e\varphi+{}^1/_3 ipv\mathbf{ku}. \quad (16)$$

We write the Green function $\langle g_1+g_2\rangle$ in the form

$$\langle g_1+g_2\rangle=C_1\Phi+C_2(\Delta_1^1-\Delta_2^1)+{}^1/_3 pv\mathbf{ku}C_3. \quad (17)$$

One easily finds expressions for the coefficients $C_{1,2,3}$ by averaging the first of Eqs. (15), and we shall not write them down. Using Eq. (17) we get from Eqs. (4), (5) expressions for $\Delta_1^1-\Delta_2^1$ and the potential Φ which for longitudinal oscillations is proportional to $\mathbf{k}\cdot\mathbf{u}$:

$$\Delta_1^1-\Delta_2^1=\Gamma_1\Phi+{}^1/_3 pv\mathbf{ku}\Gamma_2,\quad \Phi={}^1/_3\varkappa ipv\mathbf{ku}; \quad (18)$$

where

$$\varkappa=-\frac{\pi}{2}T\sum_{\omega}(C_3+C_2\Gamma_2)\Big/\left[1+\frac{i\pi}{2}T\sum_{\omega}(C_1+\Gamma_1C_2)\right], \quad (19)$$

$$\Gamma_1=2iT\sum_{\omega}\left\langle\frac{(\mathbf{kv})^2}{W}\right\rangle\frac{\alpha_+-\alpha}{W_1}\Big/T\sum_{\omega}\left\langle\frac{(\mathbf{kv})^2}{W}\right\rangle\frac{\beta_++\beta}{W_1}, \quad (20a)$$

$$\Gamma_2=2\left\{-\omega_0+\pi T\sum_{\omega}\left[\left\langle\frac{(\mathbf{kv})^2}{W}\right\rangle\frac{1}{W_1}\left(1-\frac{3}{\tau^2pv(\mathbf{ku})}\left\langle\frac{(\mathbf{kv})(\mathbf{pu})}{W}\right\rangle\right)+\frac{3}{pv(\mathbf{ku})}\left(B+\frac{1}{\tau}\right)\left\langle\frac{(\mathbf{kv})(\mathbf{pu})}{W}\right\rangle\right](\alpha_+-\alpha)\right\}\Big/\pi T\sum_{\omega}\left\langle\frac{(\mathbf{kv})^2}{W}\right\rangle\frac{\beta_++\beta}{W_1}, \quad (20b)$$

$$W_1=1-\frac{1}{\tau}\left(B+\frac{1}{\tau}\right)\left\langle\frac{1}{W}\right\rangle.$$

Substituting Eq. (15) for the Green function g_1+g_2 into the dispersion Eq. (13) and expressing the scalar potential and the quantity $\Delta_1^1-\Delta_2^1$ in terms of $\mathbf{k}\cdot\mathbf{u}$ through Eqs. (16) and (18) we find

$$\begin{aligned}-\left[\omega_0^2(M+m)+\frac{pv}{3}k^2\right]\mathbf{ku}&=\frac{\omega_0m^2p}{\pi N}T\sum_{\omega}\frac{\alpha_+-\alpha}{B\tau}\times\left\langle\frac{(\mathbf{kv})^2-k^2v^2/3}{W}\left((\mathbf{kv})(\mathbf{pu})-\frac{pv}{3}\mathbf{ku}\right)(1+B\tau)\right\rangle\\&+\frac{pv}{3}\mathbf{ku}\frac{m^2p}{\pi N}T\sum_{\omega}\frac{\omega_0}{B}\frac{1}{W_1}\left(B+\frac{1}{\tau}\right)\left\langle\frac{(\mathbf{kv})^2-k^2v^2/3}{W}\right\rangle\\&\times\left\{\varkappa(\alpha_+-\alpha)+\frac{\alpha_+-\alpha}{\tau^2}\left[\left\langle\frac{(\mathbf{kv})(\mathbf{pu})}{W}\right\rangle\frac{3}{pv(\mathbf{ku})}-(1+B\tau)\left\langle\frac{1}{W}\right\rangle\right]+\frac{\beta_++\beta}{2}(i\varkappa\Gamma_1+\Gamma_2)\right\}.\end{aligned} \quad (21)$$

The analytical continuation of sums such as (21) is accomplished by standard methods[6] and reduces to the substitution

$$\begin{aligned}\pi T\sum_{\omega}f(\omega,\omega+\omega_0)=-\frac{i}{4}\Bigg\{&\int_{-\infty}^{\infty}d\varepsilon\,\mathrm{th}\frac{\varepsilon}{2T}f(-i\varepsilon+\nu,-i(\varepsilon+\omega)+\nu)\\&-\int_{-\infty}^{\infty}d\varepsilon\,\mathrm{th}\frac{\varepsilon+\omega}{2T}f(-i\varepsilon-\nu,-i(\varepsilon+\omega)-\nu)\\&+\int_{-\infty}^{\infty}d\varepsilon\left(\mathrm{th}\frac{\varepsilon+\omega}{2T}-\mathrm{th}\frac{\varepsilon}{2T}\right)f(-i\varepsilon-\nu,-i(\varepsilon+\omega)+\nu)\Bigg\},\end{aligned} \quad (22)$$

$$\omega_0\to -i\omega,\quad \nu\to +0.$$

The contribution of the second term on the right-hand side of Eq. (21) to the damping of the sound becomes of the order of the contribution from the first term only when $kv\tau\gtrsim 1$. For such values of the momentum we can in the expression

$$W_1=1-\frac{1}{\tau}\left(B+\frac{1}{\tau}\right)\left\langle\frac{1}{W}\right\rangle$$

put $\omega = 0$ when integrating over the anomalous region (third term in Eq. (22)). The quantities $\varkappa$ and $\Gamma_{1,2}$ are small when $kv\tau \gtrsim 1$ and in the anomalous region $\omega_0(\alpha_+ - \alpha_-) = 2B$ when $|\varepsilon| > \Delta$, $\omega \ll \Delta$; using these remarks we can write the dispersion equation in the main approximation in $(s/v)^2$ (s is the sound velocity) in the form

$$\omega^2(M+m) = \frac{pv}{3}k^2\left[1 - i\omega\tau\left(1 - \operatorname{th}\frac{\Delta}{2T}\right)\gamma_N(kv\tau)\right], \qquad \omega \ll \Delta. \tag{23}$$

where

$$\gamma_N(x) = \frac{\operatorname{arctg} x}{x - \operatorname{arctg} x} - \frac{3}{x^2}. \tag{24}$$

It follows from Eq. (21) that the absorption depends only on the ratio ω/Dk^2 in the terms which are smaller than the terms retained in Eq. (23) by a factor $(s/v)^2$. We note that the damping of the sound given by Eq. (23) is the same as the corresponding expression from the paper by Tsuneto.[4]

3. INFLUENCE OF A CURRENT ON THE DAMPING OF SOUND OSCILLATIONS

When there is a superconducting current the zeroth Green functions lose an important property—that they are even under the substitution $\mathbf{v} \to -\mathbf{v}$. As a result of this there appears dispersion in the sound absorption when $\omega\tau$ or $(kv\tau)^2$ become of roder $(evA/\Delta)^2$. The scale of this dispersion is, generally speaking, of the order $(j/j_c)^2$ (j_c is the critical pair breaking current). We note that when there is no current the dispersion at $\omega \sim Dk^2$ has the scale $(s/v)^2$. We restrict ourselves in what follows to the low-frequency, weak-current case:

$$\omega \ll \Delta, \quad kv\tau \ll 1, \quad j \ll j_c. \tag{25}$$

We assume also that there exists in the superconductor a uniform current state which is determined by the static vector potential $\mathbf{A}$ which is independent of the coordinates. The results obtained below are thus applicable for films with a thickness less than the penetration depth. When $k\lambda \gg 1$ (λ is the penetration depth of a static magnetic field) they can be used for bulk samples.

We consider first of all the very simple case of low temperatures and large electron mean free paths:

$$kv\tau \ll T/evA \ll 1, \quad \tau\Delta \gg (\Delta/evA)^{1/2}. \tag{26}$$

When condition (26) is satisfied the main contribution to the absorption comes from a narrow energy region near threshold. From Eqs. (1) and (2) we have in the main approximation in this region of energies

$$g_1 + g_2 = -\left((\mathbf{kv})(\mathbf{pu}) - \frac{pv}{3}\mathbf{ku}\right)\frac{(\alpha_+ - \alpha)^2}{\Delta(\beta_+ + \beta)}. \tag{27}$$

The zeroth Green function G^0 satisfies the equation[5]

$$[(\omega - ie\mathbf{vA})\tau_z - i\hat{\Delta}_0 + in\Sigma^0, G^0] = 0, \qquad (G^0)^2 = 1, \tag{28}$$

where Σ^0 is given by Eq. (3) for $\mathbf{u} = 0$, and $[\ldots, \ldots]$ is a commutator.

Close to threshold we get from Eqs. (28)

$$\beta_+ = i\left(\frac{\Delta}{2}\right)^{1/2}(\varepsilon - \varepsilon_0 - evAt + i\gamma)^{-1/2}, \quad \gamma = \frac{2^{1/2}(\varepsilon - \varepsilon_0)^{3/2}}{12\tau evA\Delta^{1/2}},$$
$$\beta = \beta_+^*, \quad \varepsilon_0 = \Delta - evA + \frac{1}{3\tau}\left(\frac{evA}{\Delta}\right)^{1/2}, \quad t = 1 - \frac{\mathbf{vA}}{vA}, \quad \varepsilon > \varepsilon_0. \tag{29}$$

Substituting expression (27) for $g_1 + g_2$ into the dispersion Eq. (13) and using Eqs. (22) and (29) we get

$$\omega^2(M+m) = \frac{pv}{3}k^2\left[1 - 36i\omega\tau\left(\frac{(\mathbf{kA})^2}{k^2A^2} - \frac{1}{3}\right)^2\exp\left(-\frac{\varepsilon_0}{T}\right)\right], \tag{30}$$
$$kv\tau \ll T/evA \ll 1, \quad \tau\Delta \gg (\Delta/evA)^{1/2}, \quad \omega \ll evA.$$

In pure superconductors the switching on of the current leads to a strong diminution of the gap in the excitation spectrum. At low temperatures sound absorption in regions small in size but with a small gap in the excitation spectrum (e.g., a layer of the order of the penetration depth in a bulk superconductor) may thus turn out to be important. We note that Eq. (30) is applicable up to values $evA \sim \Delta$.

For currents much below the critical the connection between the vector potential $\mathbf{A}$ and the current density $\mathbf{j}$ has the form

$$\mathbf{j} = -\frac{2e^2p^2v\Delta^2}{3\pi}\mathbf{A}T\sum_{\omega>0}\frac{1}{(\omega^2+\Delta^2)((\omega^2+\Delta^2)^{1/2} + 1/2\tau)}. \tag{31}$$

Equations (29) and (30) are applicable for arbitrary film thickness if there is specular reflection at the walls. When the reflection is diffuse there appears an additional restriction on the film thickness $d > \xi$ (ξ is the correlation length of the superconductor). As $d < \lambda$ both these conditions can be satisfied only for type II superconductors. At low temperatures $T \ll \Delta$ and for a long mean free path $\tau\Delta \gg 1$ we get from Eq. (31)

$$\mathbf{j} = -\frac{e^2N}{m}\mathbf{A}, \tag{32}$$

where $N = p^3/3\pi^2$ is the total number of electrons per unit volume.

We turn now to a consideration of the second limiting case:

$$evA \ll T, \quad evA\tau \ll 1, \quad kv\tau \ll 1. \tag{33}$$

When conditions (33) are satisfied the zeroth Green functions α and β can be expanded in powers of the vector potential

$$\alpha(\omega) = \frac{\omega}{(\omega^2+\Delta^2)^{1/2}} + \alpha_1 + \alpha_2,$$
$$\beta(\omega) = \frac{\Delta}{(\omega^2+\Delta^2)^{1/2}} - \frac{\omega}{\Delta}(\alpha_1+\alpha_2) - \frac{(\omega^2+\Delta^2)^{3/2}}{2\Delta^3}\alpha_1^2,$$
$$\alpha_1 = -\frac{ie(\mathbf{vA})\Delta^2}{(\omega^2+\Delta^2)[(\omega^2+\Delta^2)^{1/2} + 1/2\tau]},$$
$$\alpha_2 = \frac{\omega\Delta^2e^2}{(\omega^2+\Delta^2)^{3/2}[(\omega^2+\Delta^2)^{1/2} + 1/2\tau]^2}\left\{\frac{v^2A^2}{2}\left(1 + \frac{1}{3\tau(\omega^2+\Delta^2)^{1/2}}\right) + \frac{3}{2}\left((\mathbf{vA})^2 - \frac{v^2A^2}{3}\right)\right\}. \tag{34}$$

In what follows we restrict ourselves to the case of a sufficiently high current

$$(\omega\tau, Dk^2\tau) \ll (evA/\Delta)^2. \tag{35}$$

In the range of frequencies bounded by condition (35) the dispersion Eq. (13) can by means of straightforward transformations and using the set of Eqs. (1) and (2) be brought to the form

$$\omega_0^2(M+m)\mathbf{ku} = -\frac{pv}{3}k^2\mathbf{ku} + \frac{mp}{2\pi N}T\sum_\omega \Big\langle \Big[(\mathbf{kv})(\mathbf{kp}) - \frac{pv}{3}k^2\Big] (\alpha_+-\alpha)^2 \Big\{-4\Big[(\mathbf{kv})(\mathbf{pu}) - \frac{pv}{3}\mathbf{ku}\Big] + \frac{pv}{3}\mathbf{ku}\Big[\frac{1}{\tau}\Big(1+\frac{\beta_++\beta}{\alpha_+-\alpha}\Big\langle\frac{\beta_++\beta}{\alpha_+-\alpha}\Big\rangle\Big)(C_1+\Gamma_2C_2) - 2\Gamma_2\frac{\beta_++\beta}{\alpha_+-\alpha}\Big]\Big\}$$
$$\times\{4\Delta(\beta_++\beta)+\tau^{-1}[(\alpha_+-\alpha)\langle\alpha_+-\alpha\rangle+(\beta_++\beta)\langle\beta_++\beta\rangle]\}^{-1}\Big\rangle. \tag{36}$$

The quantities $C_{1,2,3}(\mathbf{A})$ and $\Gamma_{1,2}(\mathbf{A})$ are given by Eqs. (17) and (18).

The change in the absolute magnitude of the order parameter Δ turns out to be unimportant. The terms with Γ_2 in Eq. (36) might give a renormalization of the sound velocity. However, calculations show that there does not occur a renormalization of the sound velocity and the terms of the form ω^2A^2 give zero. After simple calculations we find for the coefficient C_3 in the anomalous region under conditions (35) the expression

$$C_3=18\tau\Big(1+\frac{1}{2\tau^2(\varepsilon^2-\Delta^2+1/4\tau^2)}\Big)\Big\langle\frac{(\mathbf{vA})^2}{v^2A^2}\Big(\frac{(\mathbf{kv})(\mathbf{pu})}{pv(\mathbf{ku})}-\frac{1}{3}\Big)\Big\rangle, \quad \varepsilon>\Delta. \tag{37}$$

Using what we have said above we get from Eqs. (34), (36), (37)

$$\omega^2(M+m)=\frac{pv}{3}k^2\Big\{1-\frac{4i\omega\tau}{5}\Big(1-\operatorname{th}\frac{\Delta(A)}{2T}\Big) + i\omega\tau\Delta^2e^2v^2A^2\int_\Delta^\infty d\varepsilon\frac{\partial\operatorname{th}(\varepsilon/2T)}{\partial\varepsilon}\frac{1}{(\varepsilon^2-\Delta^2)(\varepsilon^2-\Delta^2+1/4\tau^2)}$$
$$\times\Big[\frac{4}{15}-\Big(1+\frac{1}{2\tau^2(\varepsilon^2-\Delta^2+1/4\tau^2)}\Big)\Big(\frac{4}{105}-\frac{4}{35}\frac{(\mathbf{kA})^2}{k^2A^2} + \frac{3}{25}\Big(\frac{(\mathbf{Ak})^2}{A^2k^2}-\frac{1}{3}\Big)^2\Big(1+\frac{1}{2\tau^2(\varepsilon^2-\Delta^2+1/4\tau^2)}\Big)\Big)\Big]\Big\}. \tag{38}$$

The absolute magnitude of the order parameter $\Delta(A)$ is, when there is a current, given by the expression

$$\Delta(A)=\Delta_0+\delta\Delta, \tag{39}$$
$$\delta\Delta=-\frac{\Delta e^2v^2A^2}{3}T\sum_\omega\frac{1}{(\omega^2+\Delta^2)^{1/2}[(\omega^2+\Delta^2)^{1/2}+1/2\tau]}\times\Big(\frac{\omega^2}{(\omega^2+\Delta^2)^{1/2}}-\frac{\Delta^2}{2[(\omega^2+\Delta^2)^{1/2}+1/2\tau]}\Big)\Big\{T\sum_\omega\frac{\Delta^2}{(\omega^2+\Delta^2)^{3/2}}\Big\}^{-1}.$$

The integral in Eq. (38) diverges logarithmically at the lower limit and it must be cut off at $\varepsilon-\Delta\sim evA$. For superconductors in which $\tau\Delta\ll1$ we can neglect the contribution from the integral in Eq. (38) and the dispersion law is given by Eq. (23) in which we must substitute $\Delta(A)$ for Δ. In the opposite limiting case $\tau\Delta\gg1$ the contribution from the integral term at not too low temperatures $T>\Delta(\tau\Delta)^{-2}$ becomes the main one and the absorption decreases when the current is switched on. When the temperature is further lowered (in the range $T<\Delta(\tau\Delta)^{-2}$) the main feature is the narrowing of the gap in the excitation spectrum and the dispersion law is again given by Eq. (23). In that case the sound absorption increases when the current is switched on.

4. COLLECTIVE OSCILLATIONS IN SUPERCONDUCTORS

Collective oscillations in superconductors were considered in a paper by Schmid and Schön[7] and also in a paper by Artemenko and Volkov.[8] However, in both those papers the damping was found insufficiently exactly. We study also the effect of a current on the propagation of collective oscillations. The spectrum of the collective oscillations is determined by the poles $\varkappa$ (Eq. (14)). The change in the absolute magnitude of the order parameter turns out to be unimportant. After simple calculations the equation for the spectrum of the collective oscillations is brought to the form

$$\frac{Dk^2}{\omega_0}T\sum_\omega\frac{\alpha_+-\alpha}{B_1}+\Big[2\Delta+\omega_0\Big(T\sum_\omega\frac{\alpha_+-\alpha}{(1+B\tau)B_1}\Big)\Big(T\sum_\omega\frac{\beta_++\beta}{(1+B\tau)B_1}\Big)^{-1}\Big]\times\Big[T\sum_\omega\frac{\beta_++\beta}{BB_1}+\frac{1}{2\omega_0}T\sum_\omega\frac{\tilde{A}^2}{\Delta}\frac{\alpha_+-\alpha}{B_1}\Big]=0, \tag{40}$$
$$B_1=B+Dk^2+\tilde{A}^2,$$

where the functions α, β are given by Eq. (14),

$$B=(\omega^2+\Delta^2)^{1/2}+[(\omega+\omega_0)^2+\Delta^2]^{1/2},$$

$$\tilde{A}^2=\begin{cases}0 & \text{in normal regions}\\ \dfrac{\Delta^2e^2v^2A^2}{3\tau(\varepsilon^2-\Delta^2)(\varepsilon^2-\Delta^2+1/4\tau^2)} & \text{in the anomalous region}\end{cases} \tag{41}$$

Collective oscillations exist only close to T_c in the frequency range $\omega\gg Dk^2$. It is clear from the answer that the sums in Eq. (40) can be expanded in terms of $\tilde{A}^2$. Taking the above remarks into account we get for the spectrum of the collective oscillations the expression

$$\omega^2=2\Delta\eta Dk^2\Big\{1-\frac{i\pi\Delta^2\eta}{2T\omega}-\frac{i\omega}{\pi\Delta\eta}I_1-\frac{2ie^2v^2A^2}{3\pi\omega\tau\Delta^2}I_2\Big\}, \tag{42}$$

where $D=v^2\tau/3$,

$$\eta=1-\frac{8T\tau}{\pi}\Big[\psi\Big(\frac{1}{2}+\frac{1}{4\pi T\tau}\Big)-\psi\Big(\frac{1}{2}\Big)\Big],$$

ψ is the psi-function;

$$I_1=\int_1^\infty\frac{dx}{(x^2-1)^{1/2}[(x+\omega/\Delta)^2-1]^{1/2}}\Big(1+\frac{1}{1+4\tau^2\Delta^2(x^2-1)}\Big)-1,$$
$$I_2=\int_1^\infty\frac{dx}{x^2(x^2-1+1/4\tau^2\Delta^2)} = \frac{1}{a^2}\begin{cases}\dfrac{1}{2a}\ln\dfrac{1+a}{1-a}-1, & 2\tau\Delta>1\\ 1-\dfrac{1}{a}\Big(\dfrac{\pi}{2}-\operatorname{arctg}\dfrac{1}{a}\Big), & 2\tau\Delta<1\end{cases} \tag{43}$$
$$a=(|1-1/4\tau^2\Delta^2|)^{1/2}, \quad \omega\ll\Delta.$$

In the limiting cases the integral I_1 is equal to

$$I_1 = \begin{cases} \frac{1}{2}\ln(8\Delta/\omega) - 1, & \tau\Delta \gg 1 \\ \ln(8\Delta/\omega) - 1, & \tau\Delta \ll 1 \end{cases}$$

It follows from Eq. (42) that when the current density increases the damping of the collective oscillations grows fast and for a current density $j/j_c \sim (\Delta/T)^{1/2}$ in pure superconductors ($\tau\Delta \gg 1$) and for $j/j_c \sim 1$ in dirty superconductors ($T\tau \ll 1$) the oscillations disappear completely.

CONCLUSION

We have studied sound propagation in superconductors for arbitrary electron mean free paths. We have shown that taking into account the force connected with the transfer of momentum from the electron system to the impurities removes the anomaly in the absorption found in Vardanyan and Lisitsyn's paper[2] and that as a result one obtains in the given approximation in $(s/v)^2$ the usual BSC expression[3] for the sound absorption. We studied the effect of a current on the propagation of sound waves. When there is a current present there occurs additional dispersion for $kv\tau \sim j/j_c$ with a scale $(j/j_c)^2$ which can easily be made considerably larger than the small parameter $(s/v)^2$. For relatively pure superconductors $\tau\Delta \gg 1$ at low temperatures and for sufficiently large current densities the sound absorption increases exponentially. This growth of the absorption is connected with the diminution of the gap. Turning on the current changes not only the gap in the excitation spectrum but also the form of the Green functions.

As a result an interesting effect appears—in sufficiently pure superconductors ($\tau\Delta \gg 1$) turning on a current leads in a wide range of temperatures $T > \Delta(\tau\Delta)^{-2}$ to a decrease in the sound absorption.

We studied the effect of the current on the collective oscillations in superconductors. The presence of a current leads to a fast growth of the damping and at current densities of the order of critical there are no oscillations.

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Quantum electromagnetic processes in condensed media and natural decay in van der Waals crystals

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Quantum dynamical equations taking the variation of the populations and natural (radiative) decay into account are derived for an electromagnetic field in matter on the basis of the mean commutator approximation for the molecular transition operators. The magnitude of the decay and the corresponding natural absorption line in the crystal are calculated, and the possibility of explaining the experimental data on absorption in noble gas crystals on this basis is discussed.

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1. INTRODUCTION

In electrodynamical calculations, two basic approximations are used to simplify the exact system of equations for the system of molecules (atoms) and electromagnetic field: the semiclassical approximation widely applied in quantum electronics,[1] and the quantum approximation of the theory of excitons,[2,3] which utilizes simplification of the commutation relations for the second-quantized operators of the medium under the assumption of constancy of the populations. The semiclassical approach naturally includes the dynamics of the populations and the nonlinear effects associated with it; however, a quantum description of the field is adequate for a description of spontaneous processes. Here a new approach to an approximate description of the medium's operators is developed, allowing us also to describe on a strict quantum level the dynamics of the populations within the framework of equations which are linear in all operator variables. This approach has an

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