Determination of the electron scattering amplitudes from the high-frequency properties of conductors

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We consider the interaction of a normal non-gyrotropic degenerate conductor with a small (1%) concentration of randomly distributed impurities in transverse electro-magnetic radiation. It is assumed that the wave is incident at an arbitrary angle on the vacuum—conductor interface. The conductor is characterized by longitudinal and transverse dielectric constants ϵ_l and ϵ_t, calculated in the random-phase approximation and containing in the lowest order allowance for the spatial dispersion. The conductor reflection coefficient $R(\theta,\omega)$ is calculated and approximate expressions are obtained for it as a function of the incidence angle θ in the region of frequencies in which the normal skin effect is possible. A connection is established between $R(\theta,\omega)$ and the relaxation times τ_{tr1} and τ_{tr2}. This connection makes it possible to assess the corresponding electron-impurity scattering cross sections. A situation is indicated in which radiation incident on a conductor is transformed into a surface wave. The results are analyzed and means of verifying them experimentally are recommended.

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1. INTRODUCTION

A theoretical study and determination of the electronic properties of solids by optical methods is an interesting and timely problem of solid-state physics. This problem has been the subject of the reviews[1] and of a number of original papers,[2] in which it is shown how, by measuring the reflection coefficient R of the solid and the energy-loss function $\mathrm{Im}\,\varepsilon^{-1}$, and by using the Kramers-Kronig dispersion relations, it is possible to reconstruct the functional form of the real and imaginary parts of the dielectric constant ε, and to determine the effective mass of the electron m^*, the plasma frequency ω_p, the topology of the Fermi surface, and the structure of the energy bands of the investigated medium.

The solution of the problem of the incidence of radiation on a solid (dielectric, conductor, etc.) occupying a half-space can be found in many books (see, e.g.,[3]) and journal articles. In the treatment of isotropic media, use is made of only the transverse dielectric constant ε_t, which characterizes the propagation of transverse electromagnetic waves in the medium. However, if conduction electrons are present in the solid, then propagation of longitudinal waves becomes possible, and their interaction with the material is characterized by ε_l. Naturally, these waves can exist only inside the medium or in a narrow layer in the vicinity of the interface. It is obvious that excitation of longitudinal waves will occur if the wave is incident at a certain angle to the surface.

Inasmuch as in the general case even an isotropic medium is characterized by two macroscopic parameters, ε_t and ε_l, the boundary conditions must be modernized in comparison with the traditional ones and, as a consequence, a difference arises (substantial in some cases) between the reflection coefficients of these media. In addition, allowance for the spatial dispersion alters significantly the picture of the considered phenomena, and sometimes leads to new effects, hitherto not observed or not explained.[4]

In a certain sense, the simplest situation in which the influence of these factors can be observed is the case of interaction of radiation with an isotropic conductor containing impurities. In addition to everything else, it is necessary also to solve correctly the problem of the boundary conditions (or of the supplementary boundary conditions), which is also necessitated by allowance for spatial dispersion and has been so far the subject of numerous discussions and original papers.[5] When considering the problems and tasks connected with the presence of a boundary between media, it is necessary to take into account, besides the short-wave normal volume oscillations, also the possible appearance of long-wave surface states, which are peculiar to spatially inhomogeneous systems.[6] In an ideal metal without allowance for spatial dispersion, ε_t and ε_l are equal, so that when solving the problem there is no point in resolving the total electric field into a longitudinal and a transverse component. By doping the metal with impurities, we introduce microscopic inhomogeneities, which lead to the need for averaging all the macroscopic quantities over distances greatly exceeding the characteristic dimension of the inhomogeneity.

We calculate in this paper the longitudinal and transverse dielectric constants of a normal nonmagnetic nongyrotropic degenerate conductor with impurities, in the random-phase approximation and under normal skin-effect conditions, with allowance for the spatial dispersion, accurate to terms proportional to $\mathbf{k}^2$ inclusive. This increases the order of Maxwell's differential equations and leads to the appearance of "new" solutions for the electromagnetic field. Introduction of boundary conditions in the form $\mathbf{j}\cdot\mathbf{n}=0$ on the interface, where $\mathbf{j}$ is the conduction current-density vector in the metal and $\mathbf{n}$ is the normal to the surface of the conductor, makes it possible to determine uniquely the amplitudes of all the produced waves. Notice should be taken of the fact that the solutions of the field equations for the region occupied by the conductor contain, besides the traditional transverse solution, also a longitudinal component that

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can predominate in certain situations, and does not appear at all without allowance for the spatial dispersion.

We calculate next the reflection coefficient of a semi-infinite conductor $R(\theta, \omega)$ as a function of the incidence angle θ and the frequency ω of the incident radiation. The normal skin-effect regime imposes limitations on the modulus of the wave vector $\mathbf{k}$ in the metal in the form of the inequality

$$|\mathbf{k}|v_0\tau \ll 1, \tag{1}$$

where v_0 is the electron velocity on the Fermi surface, τ is the time between the collisions of the electrons with the impurities. A connection is established between $R(\theta, \omega)$ and the relaxation times τ_{tr1} and τ_{tr2}, which are defined with the aid of the relations

$$\tau_l^{-1} = nmp_0(2\pi)^{-2}\int d\Omega |f(\theta)|^2 \hat{L}\{P_l(\cos\theta)\}, \qquad \tau_{tr\,l}^{-1} = \tau_0^{-1} - \tau_l^{-1}, \quad \tau_0 = \tau, \tag{2}$$

where $\hat{L}\{P_l(\cos\theta)\}$ is a linear combination of Legendre polynomials, for example,

$$\hat{L}\{P_0\} = P_0, \quad \hat{L}\{P_1\} = P_1,$$
$$\hat{L}\{P_2\} = {}^1/_3(2P_2 + P_0), \quad \hat{L}\{P_3\} = {}^1/_5(2P_3 + 3P_1);$$

n is the impurity concentration, m is the electron mass, p_0 is the Fermi momentum, and $f(\theta)$ is the amplitude of the scattering of an electron by an impurity. The reason why R contains only the times τ_{tr1} and τ_{tr2} is the allowance for the spatial dispersion accurate to terms of order $\mathbf{k}^2$. The next term of the expansion in $\mathbf{k}^2$ will contain the times τ_{tr3} and τ_{tr4}. Thus, in the general case R depends on all the τ_{trl} and as a result it becomes possible in principle to reconstruct the modulus of the electron scattering amplitude $|f(\theta)|^2$ from optical experiments. In the case of almost parallel incidence of the radiation of the conducting medium, a situation is indicated, under which $R(\theta, \omega)$ has a minimum, and the incident transverse wave is transformed into a surface wave.

2. FORMULATION OF PROBLEM

A plane electromagnetic wave is incident from vacuum at an angle θ on a conductor occupying the half-space $x > 0$. The wave is polarized in the incidence plane xz and is given by

$$E_0(\mathbf{r}, t) = E_0 \exp\{i(\mathbf{kr} - \omega t)\}, \tag{3}$$

where the real amplitude is $\mathbf{E}_0 = (E_0 \sin\theta;\ 0;\ -E_0\cos\theta)$, the real wave vector is $\mathbf{k} = (\omega c^{-1}\cos\theta;\ 0;\ \omega c^{-1}\sin\theta)$, and ω is the real frequency, so that $\mathbf{k}\cdot\mathbf{E}_0 = 0$ and $|\mathbf{k}| = \omega/c$.

Owing to the presence of the interface, a reflected and refracted wave are produced, likewise polarized in the incidence plane—P-type waves. We represent the field $\mathbf{E}$ as a sum of longitudinal and transverse parts

$$\mathbf{E} = \mathbf{E}_l + \mathbf{E}_t, \tag{4}$$

which, as usual, are distinguished by the relations

$$\operatorname{rot}\mathbf{E}_l = 0, \quad \operatorname{div}\mathbf{E}_t = 0. \tag{5}$$

Maxwell's equations for a conducting nonmagnetic ($\mu = 1$) nongyrotropic medium, neglecting the inertial polarizability of the ions, are written for a monochromatic wave of frequency ω, when account is taken of (4) and (5), in the form

$$\operatorname{rot}\mathbf{E}_t = \frac{i\omega}{c}\mathbf{H}, \quad \operatorname{rot}\mathbf{H} = -\frac{i\omega}{c}(\mathbf{E}_t + \mathbf{E}_l) + \frac{4\pi}{c}\mathbf{j}, \qquad \operatorname{div}\mathbf{E}_l = 4\pi\rho, \quad \operatorname{div}\mathbf{H} = 0. \tag{6}$$

These equations must be supplemented by the material relations that connect the current density $\mathbf{j}$ with $\mathbf{E}_t$ and $\mathbf{E}_l$:

$$\mathbf{j} = \hat{\sigma}_t\mathbf{E}_t + \hat{\sigma}_l\mathbf{E}_l, \tag{7}$$

where $\hat{\sigma}_t$ and $\hat{\sigma}_l$ are the transverse and longitudinal conductivities, the calculation of which for the case of a spatially homogeneous medium is given in the Appendix. Eliminating the magnetic field from the equations in (6), we transform them into

$$\nabla^2\mathbf{E}_t + \omega^2 c^{-2}\mathbf{D} = 0, \quad \operatorname{div}\mathbf{D} = 0, \qquad \operatorname{rot}\mathbf{E}_l = 0, \quad \operatorname{div}\mathbf{E}_t = 0, \tag{8}$$

where $\mathbf{D} = \hat{\varepsilon}_l\mathbf{E}_l + \hat{\varepsilon}_t\mathbf{E}_t$, and the operators $\hat{\varepsilon}_{l,t}$ take the form

$$\hat{\varepsilon}_{l,t}(\mathbf{r}, \omega) = \varepsilon_0(\omega) + \alpha_{l,t}(\omega)c^2\omega^{-2}\nabla^2. \tag{9}$$

The expansion coefficients $\alpha_{l,t}(\omega)$ are defined by the following expressions (see the Appendix)

$$\alpha_l(\Omega) = \frac{\xi^2(6\Omega + i5\eta)}{5(\Omega + i\delta)^2(2\Omega + i3\eta)}, \qquad \alpha_t(\Omega) = \frac{2\xi^2\Omega}{5(\Omega + i\delta)^2(2\Omega + i3\eta)}. \tag{10}$$

where we have introduced the dimensionless quantities

$\Omega = \omega/\omega_p, \quad \xi = v_0/c, \quad \delta = 1/\omega_p\tau_{tr\,1}, \quad \eta = 1/\omega_p\tau_{tr\,2}.$

Without allowance for the spatial dispersion we have

$$\lim_{k/\omega\to 0} \varepsilon_{l,t}(k, \omega) = \varepsilon_l(0, \omega) = \varepsilon_t(0, \omega) = \varepsilon_0(\omega)$$

and we arrive at the usual expression for the dielectric constant $\varepsilon_0(\Omega)$ of a conductor:

$$\varepsilon_0(\Omega) = 1 - 1/\Omega(\Omega + i\delta). \tag{11}$$

The longitudinal and transverse conductivities are connected with $\varepsilon_{l,t}$ by the standard relation

$$\varepsilon_{l,t} = 1 + i4\pi\sigma_{l,t}/\omega. \tag{12}$$

In view of the homogeneity of the space along the z axis, we seek the electric field for all waves in the form

$$\mathbf{E}(x, z, t) = \mathbf{E}(x)\exp\{i\omega c^{-1}(z\sin\theta - ct)\}. \tag{13}$$

Before we proceed to the solution of Eqs. (8), let us dwell on the assumptions that must be made in order to simplify the problem appreciably: 1) The macroscopic quantities are constant in the different regions and change jumpwise on going through the boundary. 2) The system (8) is valid in all of space, including directly on the interface. Unfortunately, at the present time there is no sufficiently reliable procedure by which to estimate the accuracy of these assumptions. However, these simplifications are practically always used in the solution of similar problems. The foregoing assumptions enable us to write down with the aid of (8) the boundary conditions in the form

$$\mathbf{E}_t(\mathrm{I})=\mathbf{E}_t(\mathrm{II}),\quad \mathbf{E}_l(\mathrm{I})=\mathbf{E}_l(\mathrm{II}),\quad \mathbf{D}_x(\mathrm{I})=\mathbf{D}_x(\mathrm{II}), \tag{14}$$

where I and II denote respectively the regions $x>0$ and $x<0$. The boundary conditions $E_{lx}(\mathrm{I})=E_{lx}(\mathrm{II})$ stem from the requirement that the normal component of the conduction current be equal to zero on the interface, and are therefore a consequence of the continuity equation.

3. SOLUTION OF THE BASIC EQUATIONS

From the system (8) we obtain the following solutions:

$$\begin{aligned} E_{lx}^{(1)}(x,\omega)&=-iC_1\exp[\omega c^{-1}x\sin\theta],\\ E_{lz}^{(1)}(x,\omega)&=C_1\exp[\omega c^{-1}x\sin\theta],\\ E_{tx}^{(1)}(x,\omega)&=C_2\exp[-i\omega c^{-1}x\cos\theta]+iC_1\exp[\omega c^{-1}x\sin\theta],\\ E_{tz}^{(1)}(x,\omega)&=C_2\operatorname{ctg}\theta\exp[-i\omega c^{-1}x\cos\theta]-C_1\exp[\omega c^{-1}x\sin\theta], \end{aligned} \tag{15}$$

where $x<0$, vacuum, $\mathbf{E}^{(1)}(x)$ is the reflected wave, and

$$\begin{aligned} E_{lx}^{(2)}(x,\omega)&=iC_3\exp\left[-\frac{\omega}{c}x\sin\theta\right]+\frac{\beta}{\sin\theta}C_4\exp\left[i\frac{\omega}{c}\beta x\right],\\ E_{lz}^{(2)}(x,\omega)&=C_3\exp\left[-\frac{\omega}{c}x\sin\theta\right]+C_4\exp\left[i\frac{\omega}{c}\beta x\right],\\ E_{tx}^{(2)}(x,\omega)&=C_5\exp\left[i\frac{\omega}{c}\gamma x\right]-iC_3\exp\left[-\frac{\omega}{c}x\sin\theta\right],\\ E_{tz}^{(2)}(x,\omega)&=-\frac{\gamma}{\sin\theta}C_5\exp\left[i\frac{\omega}{c}\gamma x\right]-C_3\exp\left[-\frac{\omega}{c}x\sin\theta\right], \end{aligned} \tag{16}$$

where $x>0$, metal, $\mathbf{E}^{(2)}(x)$ is the refracted wave,

$$\beta=\left[\frac{\varepsilon_0(\Omega)}{\alpha_l(\Omega)}-\sin^2\theta\right]^{1/2},\quad \gamma=\left[\frac{\varepsilon_0(\Omega)}{1+\alpha_l(\Omega)}-\sin^2\theta\right]^{1/2},$$

and it is necessary to choose those values of the roots for which $\operatorname{Im}\beta>0$ and $\operatorname{Im}\gamma>0$.

The complex wave amplitudes C_i $(i=1, 2, 3, 4, 5)$ are obtained from the system of the boundary conditions (14)

$$\begin{aligned} C_1&=\frac{i\cos\theta\sin\theta(\tilde\varepsilon-1)(\beta-i\sin\theta)}{\beta(\tilde\varepsilon\cos\theta+\gamma)-(\tilde\varepsilon-1)\sin^2\theta}E_0,\\ C_2&=\left[\frac{\beta(\tilde\varepsilon\cos\theta-\gamma)+(\tilde\varepsilon-1)\sin^2\theta}{\beta(\tilde\varepsilon\cos\theta+\gamma)-(\tilde\varepsilon-1)\sin^2\theta}\right]E_0\sin\theta,\\ C_3&=\frac{i\cos\theta\sin\theta(\tilde\varepsilon-1)(\beta+i\sin\theta)}{\beta(\tilde\varepsilon\cos\theta+\gamma)-(\tilde\varepsilon-1)\sin^2\theta}E_0,\\ C_4&=\frac{2\cos\theta\sin^2\theta(\tilde\varepsilon-1)}{\beta(\tilde\varepsilon\cos\theta+\gamma)-(\tilde\varepsilon-1)\sin^2\theta}E_0,\\ C_5&=\frac{2\beta\cos\theta\sin\theta}{\beta(\tilde\varepsilon\cos\theta+\gamma)-(\tilde\varepsilon-1)\sin^2\theta}E_0, \end{aligned} \tag{17}$$

where $\tilde\varepsilon=\varepsilon_0(\omega)/(1+\alpha_l(\omega))$.

We define the real reflection coefficient of the medium as the ratio of the squares of the moduli of the amplitudes of the electric field of the reflected and incident waves:

$$R=|\mathbf{E}^{(1)}|^2/|\mathbf{E}_0|^2. \tag{18}$$

Using expressions (16) and (14), we obtain the reflection coefficient in the form

$$R(\theta,\Omega)=\left|\frac{\beta(\tilde\varepsilon\cos\theta-\gamma)+(\tilde\varepsilon-1)\sin^2\theta}{\beta(\tilde\varepsilon\cos\theta+\gamma)-(\tilde\varepsilon-1)\sin^2\theta}\right|^2. \tag{19}$$

Neglecting spatial dispersion, we obtain from (19) the usual expression for the reflection coefficient[3]:

$$R=\left|\frac{\varepsilon_0\cos\theta-\gamma}{\varepsilon_0\cos\theta+\gamma}\right|^2$$

where

$$\gamma=(\varepsilon_0-\sin^2\theta)^{1/2},\quad \operatorname{Im}\gamma>0.$$

4. REFLECTION COEFFICIENT OF CONDUCTOR IN LOW-FREQUENCY REGION

According to Maxwell's equations, in the region occupied by the conductor we obtain for the complex wave vector $\mathbf{k}$ two values:

$$k_1^2=\frac{\omega^2}{c^2}\frac{\varepsilon_0(\omega)}{1+\alpha_l(\omega)},\quad k_2^2=\frac{\omega^2}{c^2}\frac{\varepsilon_0(\omega)}{\alpha_l(\omega)}$$

The normal skin-effect condition (1) should be satisfied for $\mathbf{k}_1$ and $\mathbf{k}_2$ simultaneously, defining by the same token the admissible frequency regions:

1) case of comparatively pure metal

$$\Omega\ll(\delta/\xi)^2\eta\ll\delta,\quad \eta\ll\xi; \tag{20}$$

2) case of sufficiently contaminated metal (δ, $\eta\lesssim1$)

$$\Omega\ll\xi\ll\delta,\ \eta. \tag{21}$$

One more frequency region satisfies the foregoing requirements, namely a very highly contaminated metal:

$$\xi\ll\Omega\ll1\ll\delta,\ \eta,$$

but this region is not investigated in the present paper.

The properties of the conductor are characterized by the parameters ξ, δ, and η. In the calculation of the reflection coefficient, only two parameter ratios are significant: δ/ξ and Ω/δ or Ω/η, inasmuch as we always have $\delta\sim\eta$.

In the low-frequency regions, defined by inequalities (20) and (21), the quantities that figure in $R(\theta,\omega)$ can be written approximately for the entire range of variation of θ $(0\leq\theta\leq\pi/2)$ in the form

$$\varepsilon_0\approx\frac{i}{\Omega\delta},\quad \alpha_l\approx-\frac{1}{3}\left(\frac{\xi}{\delta}\right)^2,\quad \alpha_l\approx i\frac{2}{15}\left(\frac{\xi}{\delta}\right)^2\frac{\Omega}{\eta},$$
$$\beta\approx\frac{1}{\xi}\left(\frac{3\delta}{2\Omega}\right)^{1/2}(-1+i),\quad \gamma=\frac{1}{(2\Omega\delta)^{1/2}}(1+i),$$

with $|\alpha_l|\ll1$, $|\varepsilon_0|\gg1$, $|\beta|\gg1$, $|\gamma|\gg1$. We represent here the expression for $R(\theta, \omega)$ in the form

$$R(\theta,\omega)=\frac{(\cos\theta-\chi_0\sin^2\theta)^2+(\cos\theta-\psi_0)(\cos\theta-\psi_0-b\chi_0\sin^2\theta)}{(\cos\theta+\chi_0\sin^2\theta)^2+(\cos\theta+\psi_0)(\cos\theta+\psi_0+b\chi_0\sin^2\theta)}, \quad (22)$$

where

$$\chi_0=\xi\sqrt{\frac{2\Omega}{3\delta}},\quad \psi_0=\sqrt{2\Omega\delta}=a\chi_0,\quad a=\sqrt{3}\frac{\delta}{\xi},\quad b=\frac{8\Omega}{15\eta}.$$

The reflection coefficient in a wide range of variation of the incidence angle $\theta (0 \leqslant \theta < \pi/2)$, as a function of this angle, can be expressed in the form

$$R(\theta,\omega)=1-\frac{2}{\cos\theta}\left[\psi_0+\chi_0\sin^2\theta\left(1+\frac{b}{2}\right)\right]. \quad (23)$$

As a consequence of (22), for the particular case of small angles $\theta \ll 1$ and for a relatively pure metal, just as for a sufficiently contaminated metal, we obtain the well known Hagen-Rubens relation[7] with a correction for the spatial dispersion

$$R(\theta,\omega)\approx1-2\left\{\left(\frac{2\omega}{\omega_p{}^2\tau_{tr\,1}}\right)^{1/2}+\theta^2\left[\left(\frac{\omega}{2\omega_p{}^2\tau_{tr\,1}}\right)^{1/2}+\frac{v_0}{c}\left(\frac{2\omega\tau_{tr1}}{3}\right)^{1/2}\left(1+\frac{4\omega\tau_{tr\,2}}{15}\right)\right]\right\}. \quad (24)$$

It should be noted that in the angle region indicated above, the terms proportional to β in (19) are much larger than the terms proportional to $\sin^2\theta$, so that the corrections that take into account the spatial dispersion are small and produce no singularities whatever in the behavior of the reflection coefficient.

Particular interest attaches to a situation of almost parallel incidence of the radiation on the interface. In this case the terms containing β as a factor in (19) become comparable with the terms containing $\sin^2\theta$, which drop out in the absence of spatial dispersion. The reflection coefficient has therefore substantial anomalies in this region of angles.

For the case of a relatively clean metal we obtain from (22)

$$R(\chi)=1-\frac{2[2(1+a)+b]\chi\chi_0}{(\chi+\chi_0)^2+(\chi+a\chi_0)[\chi+(a+b)\chi_0]}, \quad (25)$$

where $\theta=\pi/2-\chi$ ($\chi \ll 1$). The value $\chi=\chi_0$ characterizes that angle at which the reflection coefficient $R(\chi, \omega)$ becomes minimal and equal to

$$R_{min}(\chi_0)\approx0.2[1-2.4a-2.2b]. \quad (26)$$

From the expression for the field in a conductor (16), taking into account the values of the coefficients (17), we see that at $\chi=\chi_0$ the following amplitude relations are satisfied on the interface:

$$\frac{|E_{lx}^{(2)}|}{|E_{lz}^{(2)}|}\sim\frac{|E_{tx}^{(2)}|}{|E_{tz}^{(2)}|}\sim\frac{|\mathbf{E}_t|^2}{|\mathbf{E}_l|^2}\sim1$$

The total field in the conductor is a superposition of two waves, one of which makes a contribution to the longitudinal field component $\mathbf{E}_l$, and the other to the transverse component $\mathbf{E}_t$, with the different characteristic penetration depths; we designate them $\delta_l=c\chi_0/\omega$ and $\delta_t=c\psi_0/\omega$, respectively. In the considered limiting case of a relatively pure metal $\delta_t/\delta_l\sim\delta/\xi\ll1$, so that at distances comparable with δ_t the total field can be regarded as predominantly longitudinal with a penetration depth

$$\delta_l=c\chi_0/\omega=v_0(2\tau_{tr\,1}/3\omega)^{1/2}. \quad (27)$$

It should be noted that δ_t coincides with the depth of the ordinary skin layer

$$\delta_0=c/(2\pi\omega\sigma)^{1/2}=\sqrt{2}c/\omega_p(\omega\tau_{tr\,1})^{1/2}.$$

In the situation described above, at $\theta=\pi/2-\chi_0$, the transverse electromagnetic radiation incident on the conductor is transformed predominantly into a longitudinal surface wave.

A similar analysis for a sufficiently contaminated metal leads to the following expression for the reflection coefficient:

$$R(\chi)=1-\frac{2[2(1+1/a)+b/a]\psi_0\chi}{(\chi+\psi_0/a)^2+(\chi+\psi_0)[\chi+\psi_0(1+b/a)]} \quad (28)$$

The minimum value of the reflection coefficient, which is equal to

$$R_{min}(\chi_0)\approx0.2\left[1-\frac{2.4}{a}-\frac{0.4b}{a}\right], \quad (29)$$

is reached at $\chi=\psi_0$. The relations for the field amplitudes in the conductor on the interface, as follows from (16) and (17), are

$$\frac{|E_{lx}^{(2)}|}{|E_{lz}^{(2)}|}\sim\frac{|E_{tx}^{(2)}|}{|E_{tz}^{(2)}|}\sim\frac{|\mathbf{E}_t|^2}{|\mathbf{E}_l|^2}\sim1.$$

However, in contrast to the preceding case, for a sufficiently contaminated metal the ratio $\delta_t/\delta_l\sim\delta/\xi\gg1$. Therefore at distances on the order of δ_l the total field in the conductor can be regarded as transverse with a penetration depth $\delta_t=\delta_0$. In this situation, at $\theta=\pi/2-\psi_0$ the external transverse radiation will generate predominantely a transverse surface wave. The qualitative change of the reflection coefficient as a function of the incidence angle is shown in Fig. 1.

In both considered cases, the electromagnetic radiation incident on the surface of the conductor is transformed, under certain conditions, into surface oscillations of the system comprising the interface between the vacuum and the metal. Owing to the presence of impurities near the interface, the incident radiation becomes randomized, followed by separation of the natural surface oscillations of the indicated resonant system from this radiation. The amplitude $\mathbf{E}^{(1)}$ of the reflected wave then becomes minimal. The dispersion equation relating the frequency $\omega=\omega_{sur}$ with the surface wave vector Q_x of these surface waves is

$$\beta\left(\varepsilon\frac{cQ_x}{\omega}-\gamma\right)+\frac{c^2Q_x{}^2}{\omega^2}(\varepsilon-1)=0, \quad (30)$$

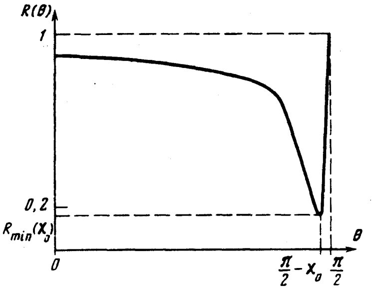

FIG. 1. Dependence of the reflection coefficient on the incidence angle. The minimum corresponds to excitation of surface waves.

where in the general case both ω and Q_z can be complex, $Q_x = (\omega^2/c^2 - Q_z^2)^{1/2}$, while β and γ have the same meaning as before. On the other hand, in particular cases one can obtain the complex function ω_{sur} of the real argument Q_z (the initial-value problem) or, conversely, seek the surface complex wave vector Q_z as a function of a real frequency ω_{sur} (boundary-value problem). In the latter case, since the incident-photon energy and the momentum component tangential to the interface are conserved, we write down the conservation laws in the form

$$\omega_{sur}=\omega_{rad}, \quad \mathrm{Re}\,[Q_z(\omega_{sur})]=k_{z\,rad}. \tag{31}$$

In an ideal metal without impurities, the surface oscillations with non-zero electric field cannot be excited by an external electromagnetic field, since the conservation laws (31) cannot be satisfied simultaneously. For example, at a fixed frequency $\omega = \omega_0$ the tangential components of the momentum are always subject to the inequality

$$\mathrm{Re}[Q_z(\omega_0)]>\frac{\sin\theta}{c}\omega_0. \tag{32}$$

Such waves, however, can be excited by fast charged particles or by electromagnetic radiation if the conductor contains defects, impurities, etc., which alter the conservation laws (31). Therefore, when impurities are introduced into the metal, the surface mode becomes subject to some uncertainty with respect to frequency, and consequently also with respect to momentum, this uncertainty being due to the finite relaxation time τ. When the conservation laws (31) are satisfied, the left-hand side of (30) cannot vanish exactly, and only a minimum of the modulus of this expression can be obtained. Then the modulus of the electric-field amplitude of the reflected wave $|\mathbf{E}^{(1)}|$ also becomes minimal, and with it the reflection coefficient R. The solutions of Eq. (30) for the real part of $Q_z(\omega)$ can be represented in the following forms:

1) for the case of a relatively pure metal

$$\mathrm{Re}\,[Q_z(\omega)]\approx\omega c^{-1}(1-\chi_0^2/2), \tag{33}$$

2) for the case of a sufficiently contaminated metal

$$\mathrm{Re}\,[Q_z(\omega)]\approx\omega c^{-1}(1-\psi_0^2/2). \tag{34}$$

In both cases there exists therefore a finite angle χ_0 or ψ_0, at which the conditions become most favorable for the excitation of a surface mode by external radiation.

5. CONCLUSION

To determine the quantities τ_{tr1} and τ_{tr2} from optical measurements we can use the frequency and angular dependences of $R(\theta, \omega)$ in the region of small angles (see expression (24)). At $\theta = 0$, it is easy to obtain τ_{tr1} from the Hagen-Rubens formula. If $\theta \neq 0$, an additional frequency dependence $\propto \omega^{3/2}$ arises, from which it is easy to determine τ_{tr2}. The same quantity can be obtained also by investigating the behavior of R in the vicinity of the minimal value R_{min} (see relations (26) and (29)). Let us estimate the angle at which the reflection coefficient reaches the minimum connected with the conversion of the incident-wave into a surface wave. For the case of a relatively pure metal, this angle is of the order of $\chi_0 \sim 10^{-3}$ rad. By way of estimates of the quantities that enter in the expression for χ_0, we chose the following values, which agree with inequality (20):

$$\tau_{tr\,1}\sim10^{-13}\ \mathrm{sec}, \quad \omega\sim1.6\cdot10^{10}\ \mathrm{Hz}.$$

For the case of a sufficiently contaminated metal, the critical angle ψ_0 is $\sim 10^{-2}$ rad. The quantities that enter in the expression for ψ_0 were estimated from the inequality (21):

$$\tau_{tr\,1}\sim10^{-15}\ \mathrm{sec}, \quad \omega\sim10^{13}\ \mathrm{Hz}.$$

For the velocity of the electrons on the Fermi surface and for the electron plasma frequency we used the typical values for good conductors such as Cu and Ag, namely $v_0 \sim 10^8$ cm/sec and $\omega_p \sim 1.5 \cdot 10^{16}$ Hz.

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APPENDIX

We present a calculation of the longitudinal and transverse conductivities for a normal degenerate conductor with impurities (for the calculation details see[8], Sec. 39).

We define the conductivity tensor $\sigma_{\alpha\beta}(k, \omega)$ and the susceptibility tensor $Q_{\alpha\beta}(k, \omega)$ by the following relation:

$$j_\alpha(k,\omega)=\sigma_{\alpha\beta}(k,\omega)E_\beta(k,\omega)=-Q_{\alpha\beta}(k,\omega)A_\beta(k,\omega), \tag{35}$$

from which it follows that

$$Q_{\alpha\beta}(k,\omega)=-i\omega\sigma_{\alpha\beta}(k,\omega). \tag{36}$$

The kernel $Q_{\alpha\beta}$ is defined by the expression

$$Q_{\alpha\beta}(\mathbf{k},\omega)=\frac{Ne^2}{m}\delta_{\alpha\beta}-i\frac{2e^2}{m^2}\int p_\alpha'\Pi_\beta(\mathbf{p}_+',\mathbf{p}_-')\frac{dp'\,d\omega'}{(2\pi)^4}, \tag{37}$$
$$\mathbf{p}_\pm=(\mathbf{p}'\pm\mathbf{k}/2,\quad \omega'\pm\omega/2).$$

The integral equation for $\Pi(\mathbf{p}_+, \mathbf{p}_-)$ is of the form

$$\Pi(\mathbf{p}_+,\mathbf{p}_-)=G(\mathbf{p}_+)G(\mathbf{p}_-)\left[\mathbf{p}+\frac{n}{(2\pi)^3}\int|f(\mathbf{p}-\mathbf{p}')|^2\Pi(\mathbf{p}_+',\mathbf{p}_-')\,d\mathbf{p}'\right], \tag{38}$$

where $G(\mathbf{p})=[\omega-\xi+(i/2\tau)\,\mathrm{sign}\,\omega]^{-1}$ is the electron Green's function averaged over the impurity distribution. When solving (38), we confine ourselves to the region of the normal skin effect $|\mathbf{k}|v_0\tau \ll 1$ and expand the quantities $G(\mathbf{p}_+)G(\mathbf{p}_-)$ and $\Pi(\mathbf{p}_+, \mathbf{p}_-)$ in powers of $\mathbf{k}$ accurate to terms $\sim\mathbf{k}^2$ inclusive. Equating the coefficients of equal powers of $\mathbf{k}$ on the right and left-hand sides of (38), we obtain a system of three equations, from which we determine the coefficients of the expansion of the quantity $\Pi_\alpha(\mathbf{p}_+, \mathbf{p}_-)$ in the form

$$\Pi_\alpha(\mathbf{p}_+, \mathbf{p}_-)=\Pi_\alpha+\Pi_\alpha{}^\beta k_\beta+\Pi_\alpha{}^{\beta\gamma}k_\beta k_\gamma. \tag{39}$$

The expressions for the coefficients $\Pi_\alpha^{\beta\nu}$, Π_α^β and Π_α are too complicated to be presented here. Next, substituting (39) in (37) and integrating, we obtain the susceptibility tensor $Q_{\alpha\beta}(k, \omega)$:

$$Q_{\alpha\beta}(k,\omega)=\frac{\omega^2}{4\pi}\left\{\left[1-\varepsilon_0(\omega)+\frac{c^2}{\omega^2}\alpha_t(\omega)k^2\right]\delta_{\alpha\beta}+\frac{c^2}{\omega^2}(\alpha_l-\alpha_t)k_\alpha k_\beta\right\}. \tag{40}$$

Knowing the connection between the susceptibility tensor and the dielectric tensor

$$Q_{\alpha\beta}(k,\omega)=\frac{\omega^2}{4\pi}[\delta_{\alpha\beta}-\varepsilon_{\alpha\beta}(k,\omega)],$$

we obtain the latter in the form

$$\varepsilon_{\alpha\beta}(k,\omega)=\left(\varepsilon_0-\frac{c^4}{\omega^2}\alpha_t k^2\right)\delta_{\alpha\beta}-\frac{c^4}{\omega^2}(\alpha_l-\alpha_t)k_\alpha k_\beta. \tag{41}$$

It is now easy to determine the longitudinal and transverse dielectric constants:

$$\begin{aligned}\varepsilon_l(k,\omega)&=\frac{k_\alpha k_\beta}{k^2}\varepsilon_{\alpha\beta}(k,\omega)=\varepsilon_0-\frac{c^2}{\omega^2}\alpha_l(\omega)k^2,\\ \varepsilon_t(k,\omega)&=\left(\delta_{\alpha\beta}-\frac{k_\alpha k_\beta}{k^2}\right)\varepsilon_{\alpha\beta}(k,\omega)=\varepsilon_0-\frac{c^2}{\omega^2}\alpha_t(\omega)k^2.\end{aligned} \tag{42}$$

Here $\varepsilon_0(\omega)$, $\alpha_l(\omega)$ and $\alpha_t(\omega)$ are defined by formulas (10) and (11).

[1]D. Pines, Elementary Excitations in Solids, Benjamin, 1963. J. Phillips, Optical Spectra of Solids [Russ. transl.], Mir, 1968.

[2]H. Ehrenreich and H. R. Philipp, Phys. Rev. 128, 1622 (1962); D. Beaglehole, Proc. Phys. Soc. (Lond.) 85, 1007 (1965); E. N. Economou, Phys. Rev. 182, 539 (1969).

[3]L. D. Landau and E. M. Lifshitz, Elektrodinamika sploshnykh sred (Electrodynamics of Continuous Media), Fizmatgiz 1959; [Pergamon, 1959]; A. V. Sokolov, Optika metallov (Optics of Metals), Fizmatgiz, 1961; J. A. Stratton, Electromagnetic Theory, McGraw, 1941.

[4]V. P. Silin, Zh. Eksp. Teor. Fiz. 38, 1578 (1960) [Sov. Phys. JETP 11, 1136 (1960)]; 40, 616 (1961) [13, 430 (1961)]; V. P. Silin and E. P. Fetisov, Zh. Eksp. Teor. Fiz. 41, 159 (1961) [Sov. Phys. JETP 14, 115 (1962)]; V. P. Silin and A. A. Rukhadze, Élektromagnitnye svoĭstva plazmy i plazmopodobnykh sred (Electromagnetic Properties of Plasma and Plasmalike Media), Atomizdat, 1961; V. M. Agranovich and V. L. Ginzburg, Kristallooptika s uchetom prostranstvennoĭ dispersii i teoriya eksitonov (Spatial Dispersion in Crystal Optics and the Theory of Excitons) Nauka, 1965 [Wiley, 1966]; V. P. Silin, and E. P. Fetisov, Phys. Status Solidi 39, 49 (1970).

[5]S. I. Pekar, Zh. Eksp. Teor. Fiz. 33, 1022 (1957) [Sov. Phys. JETP 6, 785 (1958)]; J. J. Hopfield, Phys. Rev. 112, 1555 (1958); K. L. Kliewer and R. Fuchs, Phys. Rev. 172, 607 (1968); P. R. Rimbey and G. D. Mahan, Solid State Commun. 15, 35 (1974); Yu. A. Romanov, Radiofizika 7, 242 (1964); 10, 1533 (1968); A. N. Kondratenko, Zh. Tekh. Fiz. 36, 1943 (1966) [Sov. Phys. Tech. Phys. 11, 1448 (1967)]; 42, 743 (1972) [17, 584 (1972)].

[6]R. H. Ritchie, Phys. Rev. 106, 874 (1957); R. A. Ferrell, Phys. Rev. 111, 1214 (1958); V. M. Agranovich, Teoriya eksitonov (Exciton Theory), Nauka, 1968; V. L. Ginzburg and A. A. Rukhadze, Volny v magnitoaktivnoĭ plazme (Waves in a Magnetoactive Plasma), Nauka, 1975.

[7]J. M. Ziman, Principles of the Theory of Solids, Cambridge, 1972.

[8]A. A. Abrikosov, L. P. Gor'kov, and L. E. Dzyaloshinskiĭ, Metody kvantovoĭ teorii polya v statisticheskoĭ fizike (Quantum Field-Theoretical Methods in Statistical Physics), Fizmatgiz, 1962, §39, 2. [Pergamon, 1965].

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Longitudinal oscillations in superconducting alloys

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We study the propagation of sound in superconductors for arbitrary electron mean free paths. We show that in the main approximation in terms of $(s/v)^2$ the usual BCS expression for the absorption of sound is valid. We study the effect of a current on the absorption of sound and collective oscillations in superconductors.

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INTRODUCTION

At the present time there are a large number of papers devoted to a study of sound oscillations in superconductors. We can distinguish three main methods of calculation: the kinetic equation method, a direct calculation of the polarization operator for the phonon Green function, and the linear response method. The last approach is, in fact, very close to the kinetic equation method, but it has the advantage that it is not connected with any restriction on the oscillation frequency or on the electron mean free path. Moreover, it is rather