Excitation of dopplerons and Gantmakher-Kaner waves in metals

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A theory is constructed of Doppler-shifted cyclotron resonance in compensated metals in the case of diffuse scattering of electrons for the surface. The distribution of the radio frequency field at large distances from the surface is determined, and the surface impedance of a plate is calculated. It is shown that in the case of diffuse reflection the doppleron and the Gantmakher-Kaner wave are excited much more strongly in the region of strong magnetic fields than in the case of specular reflection. The behavior of the surface resistance of cadmium plates in a magnetic field H ∥ [001] was investigated. Besides the doppleron oscillations, Gantmakher-Kaner oscillations due to electrons and holes were observed. The conclusions of the theory for the case of diffuse carrier reflection are in good agreement with the experimental results.

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1. INTRODUCTION

It was shown earlier[1–2] that the character of the reflection of the electrons from the surface of a metal can greatly influence the spatial distribution of the radio-frequency field in a plate, and consequently the impedance of the plate. Impedance oscillations due to Doppler-shifted cyclotron resonance (DSCR) differ strongly in the case of diffuse and specular reflection of the electrons. This difference is due to two circumstances. First, the doppleron wave[3] whose penetration causes the oscillations is more strongly excited in the case of diffuse reflection. Second, when the transmitted wave is reflected from the second surface of the plate, in contrast to the specular case, in the diffuse case there is produced a skin layer whose electric field is much stronger than the electric field of the transmitted doppleron. As a result, the oscillations of the plate impedance as functions of the constant magnetic field turn out to be much stronger than would be expected by starting from the field amplitude in a semi-infinite metal. The experimental data offer evidence that in most cases the electrons in the metals are reflected diffusely and not specularly. Therefore the calculations of the impedance under the assumption of specular reflection of the electrons, which are quite simple, do not correspond to the real situation. At the same time, the analysis of DSCR in the case of diffuse reflection is a much more complicated task. The increase of the impedance oscillations was demonstrated by using a compensated metal as a simple model,[1,2] in which the electron Fermi surface takes the shape of a parabolic lens, and the whole Fermi surface takes the shape of a right cylinder parallel to the rotation axis of the lens. The constant magnetic field H and the normal to the surface of the plate are directed along the same axis. In this model there are no non-local effects in the hole part of the conductivity, the displacements of all the electrons during one cyclotron period are the same, and the electrons take part simultaneously in the DSCR. The resonance is therefore pronounced most strongly—the electron part of the surface has a singularity of the pole type. The absence of branch points in the non-local conductivity leads to the absence of Gantmakher-Kaner "waves,"[4] whose existence is due to dispersion of the displacements of the resonant electrons, which always exist in real metals. In real metals, the DSCR is less strongly pronounced and the singularity of the conductivity is either of the square-root or logarithmic type. The greater part of the electromagnetic energy is transferred to the Gantmakher-Kaner wave and is dissipated near the surface, while the doppleron excitation makes an additional small contribution to the impedance. The dependence of the amplitude of the doppleron oscillations on the magnetic field is in this case substantially different than for a parabolic lens. It is clear from the foregoing that it is of interest to consider more realistic Fermi surfaces, for which the resonant singularity is not of the pole but of the root or logarithmic type.

The present paper is devoted to a theoretical and experimental study of Gantmakher-Kaner oscillations (GKO) and doppleron oscillations. In the theoretical part we consider two models of a compensated metal. In one of them the electron Fermi surface is a corrugated cylinder (model I), and in the other—the lens described earlier[5] (model II). The hole Fermi surface has in both cases the shape of a right cylinder parallel to the axis of rotation of the electron surface. The first model is quite general and reveals the main regularities in the oscillating part of the impedance of such metals as tungsten and molybdenum. The lens model, as shown in[5,6], is true to the true electron Fermi surface of cadmium.

The experimental part is devoted to the oscillations of the impedance of a cadmium plate. The main purpose was to observe the GKO. The point is that although oscillations similar to GKO[4] have been observed in many metals,[7–16] in most cases they turn out to be due to doppleron excitation. As applied to cadmium,[17] copper,[18] silver,[19] indium,[20] aluminum,[21] and tungsten this was demonstrated by using circular polarization of the exciting field. In anisotropic metals, doppleron oscillations should be much stronger than the GKO. Precision measurements are therefore necessary for the registration and identification of GKO. Our mea-

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surements of the impedance of cadmium have revealed, besides the doppleron oscillations, also oscillations in polarizations opposite to those of the dopplerons. It is shown that these oscillations have all the properties of GKO. The dependence of their amplitude on the magnetic field is investigated and the results are compared with calculation.

2. THEORY

We are interested in the region of fields and frequencies defined by the inequalities $\omega \ll \nu \ll \omega_c$, where ω is the frequency of the exciting field, ν is the frequency of the collisions of the electrons with the phonons and impurities, and ω_c is the cyclotron frequency. The problem still remains quite complicated. We confine ourselves therefore to the study of a thick plate, with a thickness d satisfying the inequalities $\mathrm{Im}\, k_{s,D}\, d > 1$, where k_s and k_D are the wave vectors of the skin-effect and doppleron components of the field. In addition, we assume the constant field H to be strong enough to be able to neglect the non-local interaction of the components propagating in opposite directions. In this case, to calculate the impedance of the plate under antisymmetrical excitation it suffices to know the impedance of the semi-infinite metal and the distribution of the field in it at large distances from the surface.[2] This is already a much simpler problem and reduces to a solution of an integral-differential equation of the Wiener-Hopf type.

We shall obtain below the impedance of a semi-infinite metal and the distribution of the field at large distances for the described models, we calculate the impedance of the plate under antisymmetrical excitation, and show that the regularities obtained earlier[2] with the aid of qualitative reasoning is valid also for these models.

a) Corrugated-cylinder model. The distribution of the field for circular polarizations $E_\pm(\zeta) = E_x \pm iE_y$ and the impedance $Z_\pm^\infty$ of a semi-infinite metal are defined by the expressions[23]

$$e_\pm(\zeta) = \frac{E_\pm(\zeta)}{E_\pm(0)} = \frac{1}{2\pi i}\int\limits_{C_\pm} \frac{ds}{s}\exp[is\zeta(1\pm i\gamma) + U_\pm(s)], \tag{1}$$

$$U_\pm(s) = \frac{1}{2\pi i}\int\limits_{C'_\pm} \frac{dq}{q-s}\ln\frac{D_\pm(q)}{q^2}, \tag{2}$$

$$\frac{cZ_\pm^\infty}{4\pi q_0} = \frac{iE_\pm(0)}{E'_\pm(0)} = \left(\frac{i}{2\pi}\int\limits_{C'_\pm} dq \ln\frac{D_\pm(q)}{q^2}\right)^{-1}, \tag{3}$$

$$D_\pm(q) = q^2 - \frac{4\pi i\omega u^2}{(1\pm i\gamma)^2 c^2 (2\pi)^2}\sigma_\pm(q), \quad q = \frac{ku}{2\pi(1\pm i\gamma)}. \tag{4}$$

Here

$$D_\pm(q) = 0 \tag{5}$$

is the dispersion equation that determines the natural modes propagating in the indinite metal along a constant magnetic field H, and

$$\sigma_\pm(q) = \pm i\frac{nec}{H}\left[\frac{2}{(2\pi\hbar)^3 n(1\pm i\gamma)}\int dp_z S(p_z) \Big/ \left(1 - \frac{qc}{eHu}\frac{\partial S}{\partial p_z}\right) - 1\right] \tag{6}$$

is the non-local conductivity for circular polarizations of the radio-frequency field. The integration contours

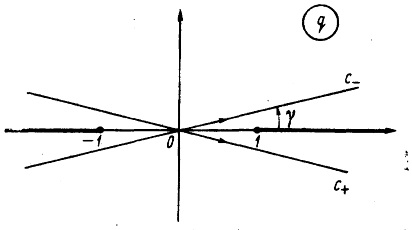

FIG. 1.

$C_\pm$ and the cuts in the complex plane are shown in Fig. 1, the contours $C'_\pm$ being shifted relative to $C_\pm$ downward by an infinitesimally small amount. In Eqs. (1)–(6) we use the following notation: k—wave vector, u—extremal displacement of the electrons during the cyclotron period

$$u = 2\pi\left(\frac{v_z}{\omega_c}\right)_{ext} = \frac{c}{eH}\left|\frac{\partial S}{\partial p_z}\right|_{ext},$$

where $S(p_z)$ is the area of the intersection of the Fermi surface with the plane p_z = const, p_z is the component of the electron momentum along the field $\mathbf{H}$, $\gamma = \nu/\omega_c$, n is the electron or hole density, $\zeta = 2\pi z/u$ is a dimensionless coordinate, $L = 2\pi d/u$, and $q_0 = \omega u/2\pi c$.

We consider first the simpler model I. The area of the section of the electron Fermi surface is given by[7]

$$S_e(p_z) = \frac{(2\pi\hbar)^3 n}{p_0} + \frac{p_0 eH}{\pi c}u\cos\frac{p_z}{p_0}\pi, \quad |p_z| \leqslant p_0, \tag{7}$$

where the dimension p_0 of the Brillouin zone, the density n, and the extremal shift u are parameters of the model, and the area of the section of the hole cylinder is constant and equal to

$$S_h = \frac{(2\pi\hbar)^3}{4p_0}n, \quad |p_z| \leqslant p_0. \tag{8}$$

In this model, the non-local conductivity (6) takes the form

$$\sigma_\pm(q) = \pm i\frac{nec}{H(1\pm i\gamma)}[(1-q^2)^{-1/2} - (1\pm i\gamma)], \tag{9}$$

and the dispersion equation is

$$D_\pm(q) = q^2 \pm \xi_\pm[(1-q^2)^{-1/2} - (1\pm i\gamma)] = 0, \tag{10}$$

where

$$\xi_\pm = \frac{4\pi\omega nec}{c^2 H(1\pm i\gamma)^3}\left(\frac{u}{2\pi}\right)^2 \equiv \xi\frac{1}{(1\pm i\gamma)^3}. \tag{11}$$

The position of the threshold of the doppleron wave that exists in the minus polarization is determined by the condition $\xi = 2$. In the region of strong fields, far from threshold, where

$$\xi \ll 1, \tag{12}$$

Eq. (10) for minus polarization has two solutions, the reduced wave vectors of which are equal to

$$q_s = k_s u/2\pi = (i\gamma\xi)^{1/2}, \quad q_D = k_D u/2\pi = -1 + \xi^2/2 + i\gamma, \tag{13}$$

and for the plus polarization it has one solution

$$q_*=(i\gamma\xi)^{1/3}. \tag{14}$$

Let us evaluate the integral (2). To this end we integrate by parts and then deform the integration contour in the upper half-plane in such a way that it encircles the cut. As a result, Eq. (2) is determined by the contribution of the poles and by the integral along the edge of the cut

$$U_+(s)=-\ln\frac{s-q_*}{1-s}+V_+(s), \tag{15}$$

$$U_-(s)=-\ln\frac{(s-q_*)(s-q_D)}{1-s}+V_-(s), \tag{16}$$

$$V_\pm(s)=\ln(\pm1-s)^{1/2}\mp\frac{\xi_\pm}{\pi}\int_1^\infty\frac{q\,dq\ln(\pm q-s)}{(q^2-1)^{1/2}}\frac{3q^2-2\mp\xi_\pm}{(q^2\mp\xi_\pm)^2(q^2-1)+\xi_\pm^2} \tag{17}$$

If the condition (12) is satisfied, the main contribution to the integral is made by the region of values of q close to unity $1(q-1\ll1)$. Taking this into account and introducing a new integration variable $z=(q^2-1)^{1/2}$, we can reduce (17) to the form

$$V_\pm(s)\approx\ln(\pm1-s)^{1/2}\mp\frac{\xi_\pm}{\pi}\int_0^\infty\frac{dz}{z^2+\xi_\pm^2}\ln\left(\pm1-s\pm\frac{z^2}{2}\right).$$

This integral can be calculated, and we obtain as a result

$$V_+(s)=-\ln\left[1+\frac{\xi_+}{2(1-s)^{1/2}}\right],\quad V_-(s)=\ln\left[-1-s+i\frac{\xi_-}{\sqrt2}(-1-s)^{1/2}\right]. \tag{18}$$

Let us calculate the integral (1) with the aid of (15), (16), and (18). To this end we again represent it in the form of a sum of the contributions of the poles and of the integral along the edge of the cut

$$e_+(\zeta)=\frac{1}{2\pi i}\int_{C_+}\frac{ds}{s-q_*}\frac{[2(1-s)]^{1/2}e^{i\zeta s(1+i\gamma)}}{[2(1-s)]^{1/2}+\xi_+}=\left(1-\frac{\xi_+}{\sqrt2}\right)e^{i\zeta q_*}+e_{GK}(\zeta), \tag{19}$$

$$e_-(\zeta)=\frac{1}{2\pi i}\int_{C_-}\frac{ds}{s-q_*}\frac{[2(-1-s)]^{1/2}e^{i\zeta s(1-i\gamma)}}{[2(-1-s)]^{1/2}-i\xi_-}=(1+\xi_-/\sqrt2)e^{i\zeta q_*}-\xi^2e^{iq_D\zeta}-e_{GK}(\zeta),$$

$$e_{GK}(\zeta)=\frac{\xi_+}{\pi\sqrt2}\int_1^\infty\frac{ds}{s}\frac{(s-1)^{1/2}e^{is\zeta}}{s-1+\xi_+^2/2}. \tag{20}$$

Using (12) and the inequality $\zeta\gg1$, we obtain

$$e_{GK}(\zeta)=\frac{\xi^2}{2\pi}e^{i\zeta}\int_0^\infty y^{1/2}dy\frac{\exp(iy\zeta\xi^2/2)}{y+1}. \tag{21}$$

Calculating this integral by the saddle-point method we obtain

$$e_{GK}(\zeta)=\begin{cases}(2\pi\zeta)^{-1/2}\xi e^{i(\zeta+\pi/4)(1+i\gamma)} & \zeta\xi^2\ll1,\\ (2\pi\zeta)^{-1/2}(\xi\zeta)^{-1}e^{i(\zeta+3\pi/4)(1+i\gamma)} & \zeta\xi^2\gg1.\end{cases} \tag{22}$$

The integral in (3), which determines the impedance of a semi-infinite metal, can be calculated in analogy with (2). The only difference is that in the case of minus polarization, after integrating by parts, one more pole appears at the point $q=-1$, which coincides with the start of the cut. For both polarizations, the main contribution is determined by the integral along the edges of the cut. In the field region defined by the condition (12) we obtain

$$\frac{cZ_\pm^\infty}{4\pi q_0}=\frac{1}{q_*+\xi/\pi}. \tag{23}$$

The analogous quantities in the case of specular reflection are given by[6]

$$e_\pm(\zeta)i\frac{E(0)}{E'(0)}=\frac{1}{\pi i}\int_{C_\pm'}\frac{e^{is\zeta(1\pm i\gamma)}}{D_\pm(s)}ds, \tag{24}$$

$$\frac{cZ_\pm^\infty}{4\pi q_0}=\frac{1}{\pi i}\int_{C_\pm'}\frac{ds}{D_\pm(s)} \tag{25}$$

The integrals (24) and (25) can be evaluated for model I in the field region (12), and as a result we get

$$\frac{cZ_\pm^\infty}{4\pi q_0}\approx\frac{1}{q_*}, \tag{26a}$$

$$i\frac{E_+(\zeta)}{E_+'(0)}=\frac{e^{iq_*\zeta}}{q_*}+2e_{GK}(\zeta), \tag{26b}$$

$$i\frac{E_-(\zeta)}{E_-'(0)}=\frac{e^{iq_*\zeta}}{q_*}+2\xi^2e^{iq_D\zeta}+2e_{GK}^*(\zeta). \tag{26c}$$

We note that Eq. (4) of [2] is satisfied in our model for the asymptotic expressions of both the doppleron and the GK components, namely

$$\left(\frac{E_D}{E_*}\right)^{sp}\Big/\left(\frac{E_D}{E_*}\right)^{dif}=-2q_*,\qquad\left(\frac{E_{GK}^\pm}{E_*^\pm}\right)^{sp}\Big/\left(\frac{E_{GK}^\pm}{E_*^\pm}\right)^{dif}=\pm2q_*. \tag{27}$$

Thus, it follows from (16), (20), (23), and (26) that in the case of diffuse reflection both the doppleron and the GKO are excited $\pi/2\xi$ times more strongly than in the case of specular reflection. In accordance with formula (8) of[2], the impedance of a thick plate under antisymmetrical excitation, in the field region (12), is given by the expression

$$Z=2Z^\infty\left(1+i\frac{cZ^\infty}{4\pi q_0}\frac{E'(L)}{E(0)}\right), \tag{28}$$

where $E(L)$ is the field in the semi-infinite metal at a depth L. Using (23), (19), (20), and (28) we obtain the oscillating part of the impedance

$$\Delta Z_+=-\frac{8\pi q_0}{c}\frac{1}{(q_*+\xi/\pi)^2}e_{GK}(L),$$
$$\Delta Z_-=-\frac{8\pi q_0}{c}\frac{1}{(q_*+\xi/\pi)^2}[e_{GK}^*(L)+\xi^2e^{iq_DL}]. \tag{29}$$

It was shown earlier[2] that the oscillating part of the impedance of the plate in the case of specular reflection of the electrons is determined by the formula

$$\Delta Z=-\frac{16\pi q_0}{c}i\frac{E(L)}{E'(0)}. \tag{30}$$

It follows from (30) and (26c) that with increasing magnetic field the amplitude of the doppleron oscillations of the impedance decreases like H^{-7}. On the other hand, the amplitude of the Gantmakher-Kaner oscillations varies nonmonotonically. According to (30), (26c), and (22), it increases in proportion to $H^{1/2}$, reaches a maximum at $\xi^2L\sim1$, and then decreases like $H^{-9/2}$.

The oscillating part of the impedance, in the case of diffuse reflection, differs from the corresponding quantity in the case of specular reflection by an additional factor

$$
{}^1/_4 (q_s+\xi/\pi)^{-2}, \tag{31}
$$

the value of which is much larger than unity. In the field region where $\gamma \ll \xi$, the first term in the round brackets is small in comparison with the second, and the quantity (31) varies with the field like H^6. In stronger fields $\xi \ll \gamma$ the situation is reversed and (31) increases in proportion to H^4. Accordingly, the amplitude of the doppleron oscillations in the diffuse case decreases like H^{-1} in the region $\gamma \ll \xi \ll 1$, and then like H^{-3}. On the other hand, the amplitude of the GKO in the strong-field region decreases in proportion to $H^{-1/2}$.

b) Lens model. We proceed now to the study of model II. The area of the interaction of the electron lens and the plane p_z = const is given by the expression[4]

$$
S_e(p_z)=\frac{3}{5}\frac{(2\pi\hbar)^3 n}{p_1}\left(1-\frac{|p_z|}{p_1}\right)\left[1-\frac{1}{3}\left(1-\frac{|p_z|}{p_1}\right)^2\right], \quad |p_z|\leqslant p_1, \tag{32}
$$

and while the intersection with the hole cylinder is give by (8). At $p_1\hbar^{-1}=0.28$ Å^{-1} and at a density n equal to the electron density is cadmium, formula (32) describes very well the sections of the true lens in cadmium. By calculating the conductivity, we can write down the dispersion equation in the form

$$
D_\pm(q)=q^2\pm\xi_\pm\left[\ln\frac{1+q}{1-q}-\frac{1}{2q}\ln(1-q^2)\right]-(1\pm i\Gamma)=0. \tag{33}
$$

In this model we still took into account the hole-collision frequency $\gamma_h=\nu_h/\omega_c$, and $\Gamma=\gamma_e+\gamma_h$. The doppleron threshold corresponds to the value $\xi=30/11$. The equation for the plus polarization has only a skin-effect root, given in the field region (12) by

$$
q_s=(i\Gamma\xi)^{1/3}. \tag{34a}
$$

The equation corresponding to minus polarization has two roots.

$$
q_s=(i\Gamma\xi)^{1/3}, \quad q_D=-1+e^{-1/a}+i\gamma_e, \tag{34b}
$$

where we have introduced the new quantities $a_\pm=\frac{3}{5}\xi_\pm$ and $a=\frac{3}{5}\xi$. The calculation of the integral (2) for the model II is carried out in analogy with the calculation for model I. After calculating the integral by parts and deforming the contour, we obtain expressions (13) and (14), in which the function $V_\pm(s)$ is now defined by the formula

$$
V_\pm(s)=\mp\frac{2}{5}\xi_\pm\int_1^\infty\frac{dq}{q^3}\ln(\pm q-s)
$$
$$
\times\left\{q^2(3q+2)\pm\xi_\pm\left[\frac{3}{5}\frac{1+2q}{1-q^2}-(1+q)+\frac{2}{5q}\ln(1+q)\right]\right\}
$$
$$
\times\left\{\left[q^2\pm\xi_\pm\left[\frac{2}{5q}\left(\ln\frac{q+1}{q-1}-\frac{1}{2q}\ln(q^2-1)\right)-1\right]\right]^2+\left[\frac{\pi\xi_\pm}{5q^2}(2q+1)\right]^2\right\}^{-1}. \tag{35}
$$

At small values of ξ and a, the main contribution to this integral is made by the region of values of q close to unity. We can therefore introduce a new variable $z=q-1$ and expand the numerator and denominator of the integrand in powers of z, retaining only the principal terms. In this case, however, the integral begins to diverge in the region $z\to\infty$, which makes a negligible contribution to the initial integral. To eliminate this divergence, we divide the expression under the logarithm sign by $(1+z)$. The addition of this factor does not change the behavior of the integrand at $z\ll 1$ and at the same time ensures smallness of the contribution from the region $z>1$. As a result, the expressions for $V_\pm(s)$ reduced to the form

$$
V_-=\pi i+W_-, \quad V_+=W_+,
$$
$$
W_\pm(s)=a_\pm^2\int_0^\infty\frac{dz}{z}\ln\left(\frac{1+z\mp s}{1+z}\right)[(1\mp a_\pm\ln z)^2+(\pi a_\pm)^2]^{-1}. \tag{36}
$$

The expression for $V_+(s)$ can be obtained from $V_-(s)$ by subtracting πi and replacing a_- by a_+ and s by $-s$. We therefore calculate $V_-(s)$. We calculate the integral by parts and assume henceforth that $\Gamma\to 0$,

$$
V_-(s)=\pi i+\frac{1}{\pi}\int_0^\infty\ln\left(\frac{1+z+s}{1+z}\right)d\left(\operatorname{arctg}\frac{1+a\ln z}{\pi a}\right)
$$
$$
=\pi i+\frac{1}{2}\ln(1+s)-\frac{1}{\pi}\int_0^\infty\operatorname{arctg}\left(\frac{1+a\ln z}{\pi a}\right)\left(\frac{1}{1+z+s}-\frac{1}{1+z}\right)dz,
$$

and then change over to integration with respect to the variable t, which is connected with z by the relation $z=\exp(\pi t-1/a)$:

$$
V_-(s)=\pi i+\frac{1}{2}\ln(1+s)-\lim_{A\to\infty}\int_{-\infty}^{A}\operatorname{arctg}t\left(\frac{1}{(1+s)e^{1/a-\pi t}+1}-\frac{1}{e^{1/a-\pi t}+1}\right)dt.
$$

The integral of each of the terms will be calculated by parts:

$$
\int_{-\infty}^{A}\frac{\operatorname{arctg}t\,dt}{e^{\pi(t_0-t)}+1}=f(A)-\int_{-\infty}^{A}f(t)\frac{\pi e^{\pi(t_0-t)}\,dt}{[1+e^{\pi(t_0-t)}]^2};
$$

We have introduced here the function $f(t)=t\tan^{-1}t-\ln(1+t^2)^{1/2}$ and have left out the exponentially small terms, recalling that $A\to\infty$. For the first term, t_0 is determined by the expression

$$
e^{\pi t_0}=(1+s)e^{1/a}, \quad t_0=\frac{1+a\ln(1+s)}{\pi a}.
$$

As a result we get

$$
V_-(s)=\pi i+{}^1/_2\ln(1+s)+\int_{-\infty}^{\infty}dt\,f(t)\left\{\frac{\pi e^{\pi(1/\pi a-t)}}{[1+e^{\pi(1/\pi a-t)}]^2}-\frac{\pi e^{\pi(t_0-t)}}{[1+e^{\pi(t_0-t)}]^2}\right\}.
$$

We expand the function $f(t)$ and integrate term by term. For all t_0, with the exception of $|t_0|<1$, we can confine ourselves to a single term

$$
V_-(s)=\pi i+\frac{1}{2}\ln(1+s)+\frac{1+a\ln(1+s)}{\pi a}\operatorname{arctg}\frac{1+a\ln(1+s)}{\pi a}
$$
$$
-\frac{1}{\pi a}\operatorname{arctg}\frac{1}{\pi a}-\frac{1}{2}\ln\frac{(\pi a)^2+[a\ln(1+s)]^2}{(\pi a)^2+1}. \tag{37}
$$

At the point t_0, expression (37) is equal to $\pi i+1-\ln\pi a-1/a$, whereas an accurate calculation yields $\pi i+\ln\pi-\ln\pi a-1/a$. We shall not make expression (37) more complicated, recalling that it must be readjusted in the vicinity of the point $t_0=0$, i.e., near the doppleron pole. In accordance with the remark made above concerning $V_+(s)$, we obtain

$$V_+(s)=-\ln[1-a\ln(1-s)]. \tag{38}$$

The field distribution in a semi-infinite metal is obtained in the same manner as for model I:

$$e_+(\zeta)=e^{iq_s\zeta}+ae^{i\zeta}\int\limits_0^\infty \frac{dx\, e^{i\zeta x}}{(\pi a)^2+(1+a\ln x)^2}=e^{iq_s\zeta}+e_{GK}(\zeta). \tag{39}$$

Calculating the integral in (39) by the saddle-point method, we obtain

$$e_{GK}(\zeta)=\begin{cases} iae^{i\zeta(1+i\gamma_e)}/\zeta, & a\ln\zeta\ll 1\\ i\pi^{1/2}e^{i\zeta(1+i\gamma_e)}/a\zeta\ln^{5/2}\zeta, & a\ln\zeta\gg 1\end{cases}. \tag{40}$$

For the minus polarization we obtain analogous

$$e_-(\zeta)=e^{iq_s\zeta}-e_{GK}^*(\zeta)-Be^{-1/a}e^{iq_D\zeta}/a, \tag{41}$$

where the last term represents the contribution of the doppleron pole. The coefficient B in (41), which turns out to equal e/π when (37) is substituted in (1), must be set equal to unity when account is made of the remark that follows formula (37).

According to (3), the impedance of a semi-infinite metal is defined by the expression

$$\left(\frac{cZ_\pm}{4\pi q_0}\right)^{-1}=\sum_{i(\pm)} q_i+\frac{2}{10}\xi_\pm\int\limits_1 dq\, q^{-2}\Big\{q^2(3q+2)$$
$$\leqslant \xi_\pm\Big[\frac{3}{5}\frac{1+2q}{1-q^2}-(1+q)(1\pm i\Gamma)+\frac{2}{5q}\ln(1+q)\Big]\Big\}$$
$$\times\Big\{[D_\pm(q)]^2+\Big[\frac{\pi\xi_\pm}{5q^2}(1+2q)\Big]^2\Big\}^{-1}, \tag{42}$$

where the first term is the sum of the roots of the dispersion equation. In the field region (12), the expression becomes simpler:

$$\left(\frac{cZ_\pm}{4\pi q_0}\right)^{-1}\approx\sum_{i(\pm)} q_i+\frac{2}{3}a_\pm\int\limits_1^\infty\frac{3q+2}{q^4}dq\mp a_\pm^2\int\limits_0^\infty\frac{dz}{z\{[1+3z\mp a_\pm\ln z]^2+(\pi a_\pm)^2\}}$$
$$=\sum_{i(\pm)} q_i+\frac{13}{9}a_\pm\mp a_\pm\int\limits_{-\infty}^\infty\frac{dy}{[(1\pm y)+3e^{-y/a}]^2+(\pi a)^2}$$
$$=\sum_{i(\pm)} q_i+\frac{13}{9}a_\pm\mp a_\pm\int\limits_0^\infty\frac{dy}{(1\pm y)^2+(\pi a)^2}.$$

Omitting the small term $1+q_D$, we obtain

$$\frac{cZ_\pm}{4\pi q_0}\approx\left(q_s+\frac{4}{9}a\right)^{-1}. \tag{43}$$

In the case of specular reflection, the analogous quantities can also be calculated:

$$\frac{cZ_\pm}{4\pi q_0}=\frac{1}{q_s} \tag{44a}$$

$$i\frac{E_+(\zeta)}{E_+'(0)}=\frac{e^{iq_s\zeta}}{q_s}+2e_{GK}(\zeta), \tag{44b}$$

$$i\frac{E_-(\zeta)}{E_-'(0)}=\frac{e^{iq_s\zeta}}{q_s}+2e_{GK}^*(\zeta)+2e^{iq_D\zeta}\frac{e^{-1/a}}{a}. \tag{44c}$$

It is easily seen that relations (27) and the conclusions that follow from them are valid also for model II.

The oscillating part of the impedance of a thick plate takes in the diffusion case, in accordance with (28), the form

$$\Delta Z_+=-\frac{8\pi q_0}{c}\left(q_s+\frac{4}{15}\xi\right)^{-2}e_{GK}(L), \tag{45a}$$

$$\Delta Z_-=-\frac{8\pi q_0}{c}\left(q_s+\frac{4}{15}\xi\right)^{-2}\left[e_{GK}^*(L)+\frac{5e^{-5/3\xi}}{3\xi}e^{iq_DL}\right], \tag{45b}$$

where $e_{GK}(L)$ is defined by relations (39) and (40).

These expressions differ from the corresponding formulas for the case of specular reflection by the large factor

$$\frac{1}{4}\left(q_s+\frac{4\xi}{15}\right)^{-2}, \tag{46}$$

which is analogous to the factor (31) in model I. Recognizing that $\xi\sim H^{-3}$ and $q_0\sim H^{-1}$, we can express the impedance-oscillation amplitudes (45) as functions of $x=H/H_L$, where H_L is the magnetic field corresponding to the doppleron threshold ($\xi=30/11$). Thus, for example, the amplitude of the doppleron oscillations in (45b) takes the form

$$|\Delta Z_D|=\frac{6\pi\omega u_L}{c^2}\left(\frac{11}{12}\right)^3\left|1+\frac{1+i}{8}(165\Gamma_L)^{1/2}x\right|^{-2}x^8\exp\left(-\frac{11}{18}x^3\right), \tag{47}$$

where u_L and Γ_L are the values of u and Γ at $H=H_L$. The expression for the GKO amplitude can be similarly expressed in terms of x.

3. EXPERIMENT

One of the purposes of the present experimental investigation was to observe GKO in cadmium and to study the dependence of their amplitude on the magnetic field. It follows from the theory that the amplitude of the GKO due to the electrons of the lens is much smaller than the maximum amplitude of the electron doppleron. The region of the magnetic field where the amplitude of the GK oscillations has a maximum lies inside the region where the electron doppleron exists. The GKO can be observed by investigating the surface impedance of a plate in a positive circular polarization, in which there is no electron doppleron.

We investigated the surface impedance of single-crystal plates of cadmium in the frequency interval 13–950 kHz and at temperatures 1.3–4.2 °K in a constant magnetic field perpendicular to the surface of the plate. The investigated sample was placed in crossed flat inductance coils, one of which was connected in a parallel resonant circuit of a simple amplitude bridge. The second (external) coil was used to produce on the *sample surface* fields with circular polarization, in a manner described earlier.[17] The coil dimensions were 2–3 times larger than the sample dimensions, thus ensuring sufficient homogeneity of the exciting field. The constant magnetic field was produced either by a superconducting solenoid

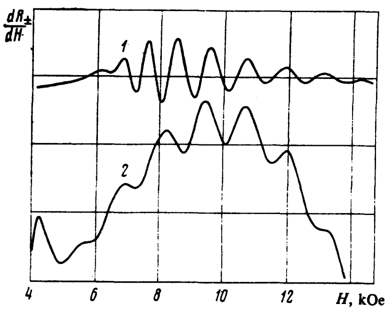

FIG. 2. Dependence of the derivatives dR_+/dH on the magnetic field for a cadmium sample $d = 0.43$ mm thick; H ∥ [0001]; $\omega/2\pi = 0.95$ MHz; $T = 4.2$ °K.

or by an electromagnet. In either case, the field was determined with accuracy not worse than 0.5%. We used a traditional modulation procedure to obtain plots of the derivatives of the surface resistance dR/dH and d^2R/dH^2 with an $X-Y$ potentiometer.

The cadmium plates were cut by the electric-spark method from a single crystal with resistivity ratio $\rho_{300}/\rho_{4.2K} \approx 3.10^4$. The hexagonal axis [0001] of the crystal was parallel to the normal to the plate.

Figure 2 shows typical plots of the derivative dR/dH in minus polarization (curve 1) and in plus polarization (curve 2). Curve 2 was plotted with a gain 25 times larger than curve 1. The oscillations of dR/dH in minus polarization are due to excitation in the plate of an electron doppleron, the properties of which were investigated in detail.[3,5,6,17] Characteristic features of doppleron oscillations are a strong dependence of their amplitude on the magnetic field and a dispersion of the period, which is most noticeable in the region of weaker fields, as well as a fully defined circular polarization. In strong fields, the period of the oscillations of the electron doppleron is determined by electrons with extremal value of $\partial S/\partial p_z$. The change of the doppleron-oscillation amplitude in a magnetic field depends substantially on the electron mean free path and on the sample thickness. Figure 3 shows a plot of dR_-/dH for a sample 0.6 mm thick. From a comparison of curve 1 on curve 2 and the curve on Fig. 3 it is seen that the amplitude of the oscillations for a thick sample decreases more strongly than for a thin one. The dependence of the wave form of the envelope of the doppleron oscillations on the mean free path is observed from measurements of the amplitude at different temperatures.

In plus polarization in a magnetic field $H = 4.45$ kOe, there is a sharp maximum of dR/dH, due to the threshold of the hole doppleron.[5] No oscillations connected with the passage of this doppleron were observed at $T = 4.2$ K. Above the threshold, small oscillations are distinctly seen, with an amplitude that first increases, reaches a maximum in fields 9–11 kOe, and then decreases slowly. The position of the maximum of the amplitude relative to the magnetic field changes with frequency approximately like $\omega^{1/3}$. Similar oscillations in plus polarization were observed earlier.[18] It was suggested in[16] that these oscillations are due to the electron doppleron that cannot be fully suppressed as a result of the insufficient homogeneity of the RF field. As seen from a comparison of the curves in Fig. 1, the presence of oscillations in plus polarization cannot be attributed to the inaccuracy of the choice of the circular polarization of the exciting field. This is evidenced, for example, by the difference in the number of oscillations on curves 1 and 2 in the field interval 6–14 kOe.

The plots on Fig. 2 were obtained at 4.2 °K. When the temperature is decreased to 1.3 °K, the amplitude of the oscillations in plus polarization increases by 2–3 times, i.e., by approximately the same factor as the amplitude of the oscillations of the electron doppleron in strong fields. This, however, did not improve the conditions for the observation of the oscillations, for when the temperature is lowered the amplitude of the doppleron oscillations in minus polarization increases by more than one order of magnitude near the threshold. This imposes much more stringent requirements on the correct choice of the circular polarization. In addition, at temperatures lower than 3 °K in plus polarization, oscillations of a hole doppleron are observed and greatly complicate the general picture. With decreasing frequency, the number of the oscillations observed in plus polarization decreases. On the other hand, an increase of the frequency leads to a relatively stronger increase of the amplitude of the electron doppleron. Therefore the oscillations in plus polarizations are most distinctly seen in the frequency region 0.2–1.0 MHz.

Analysis has shown that, within the limits of the experimental error (3–5%), the observed oscillations are periodic, in the magnetic field with a period inversely proportional to the sample thickness. The main source of the error in the determination of the period for two neighboring extrema is connected with the strong change of the smooth part of the derivative dR/dH. Assuming the period to be constant over eleven sufficiently distinct maxima, we determine the average value of the period of the oscillations for a sample (430 ± 10) μ thick, namely 1.47 kOe. The obtained value of the period is close to the limiting period of the oscillations of the electron doppleron in strong fields.

Investigations have shown that in a wide frequency interval the position of the extrema of the oscillations relative to the magnetic field in plus polarization is independent of frequency, within the limits of the measurement accuracy, and is determined only by the sample thickness. Figure 4 shows the positions of the extrema of the oscillations in both polarizations at different fre-

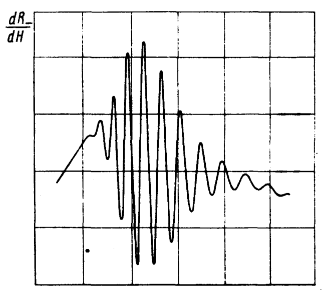

FIG. 3. Plot of the derivative dR_-/dH; $d = 0.6$ mm; H ∥ [0001]; $\omega/2\pi = 0.95$ MHz; $T = 4.2$ °K.

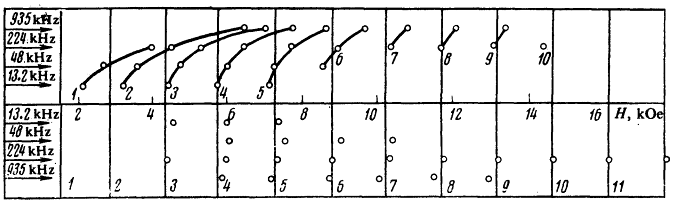

FIG. 4. Frequency dependence of the positions of the extrema of the oscillations of $dR_\pm/dH$ with respect to the magnetic field. The upper part of the figure corresponds to minus polarization and the lower to plus polarization. Sample thickness $d=0.43$ mm; $T=4.2$ °K.

quencies. From an examination of the upper part of Fig. 4, which pertains to the electron doppleron, it is seen that with increasing frequency the positions of the oscillation maxima shift noticeably towards stronger fields. The maxima of the oscillations in plus polarization partly shift relative to the magnetic field in the investigated frequency interval.

When the magnetic field is inclined 3–5° to the [001] direction, the amplitude of the electron doppleron decreases noticeably as a result of the appearance of magnetic Landau damping.[6] The picture of the oscillations in plus polarization for an oblique field is practically the same, moreover, the amplitude of the extrema even increases somewhat in this case.

We have attempted to observe these oscillations in minus polarization. We did not succeed, however, in establishing the presence of small oscillations against the background of the electron doppleron, and the question of the existence of such oscillations in minus polarization remains open.

The foregoing results of the experimental investigation give grounds for stating that the observed oscillations in plus polarization have all the properties of GKO and are due to excitation of GKO in the plate. From the period of the GK oscillations we determined the value of $(\partial S/\partial p_z)_{ext}$ for the lens electrons, namely $\hbar\cdot 9.6$ Å^{-1}, in good agreement with the earlier results.[3–5]

Besides the GKO connected with the lens electrons, oscillations were observed in weak fields, which were periodic in the magnetic field and also possessed GKO properties. Examples of plots of d^2R/dH^2 are shown in Fig. 5. The oscillations are observed in a magnetic field both above and below the threshold of the hole doppleron that exists in plus polarization. As seen from the figure, the considered oscillations exist only in minus polarization, or at least, have in this polarization a much larger amplitude than in the opposite polarization. The polarization of the oscillations is strongly distinguishable in the region of fields below the threshold of the whole doppleron. It is difficult to draw any conclusion concerning the polarization in strong fields, inasmuch as the relatively large oscillations of the whole doppleron are observed in plus polarization.

4. DISCUSSION

We begin with a comparison of the threshold and experimental results concerning the amplitude of the doppleron oscillations. Let us compare the dependence of the amplitude of the electron doppleron on the field H with the results of the calculation for the case of specular and diffuse reflection. In Fig. 3 the doppleron oscillations have a maximum at $H=8.5$ kOe. This field corresponds to $x=H/H_L=1.5$, where $H_L=5.7$ kOe is the calculated value of the threshold field at $\omega/2\pi=0.95$ MHz. With increasing field, the amplitude of the oscillations dR_-/dH falls off and decreases by a factor 25 at $H\approx 14$ kOe ($x=2.4$). The theory constructed by us for the case of diffuse reflection is valid, strictly speaking, only in the region of fields corresponding to small a. At values $x=1.5$, however, we have $a=0.5$. Nonetheless one should expect the theory to describe the behavior of the doppleron amplitude approximately in this region, too. From (47) it is seen that in the theory the doppleron amplitude also has a maximum. Calculation shows that both the magnitude and the position of this maximum depend strongly on the carrier mean free paths. At an electron mean free path $1=0.5$ mm and $\gamma_h=2\gamma_e$, the maximum is located approximately in the same place as on the experimental curve. In this case, when x changes from 1.5 to 2.4, the amplitude of the oscillations of dR_-/dH decreases by a factor of 30, in agreement with experiment. On the other hand, in the case of specular reflection the amplitude of the oscillations should decrease by an approximate factor of 300.

We proceed now to discuss the GKO observed in plus polarization. On curve 2 of Fig. 2, the maximum of the oscillations is located at $H\approx 10$ kOe ($x\approx 1.75$). When the field is increased to 14 kOe, the amplitude of the oscillations decreases by approximately 30%. Calculation shows that in the case of diffuse reflection the GKO amplitude has a rather broad maximum situated at $x\approx 1.85$. When x is increased to 2.4, the amplitude decreases by 10%. In the case of specular reflection, when x changes from 1.8 to 2.4, the GKO amplitude decreases by a factor of 4. Thus, the theory with diffuse reflection is in much better agreement with experiment than the theory with specular reflection.

Finally, let us compare the GKO amplitude with the amplitude of the doppleron oscillations. Measurements show that the ratio of the amplitudes of the oscillations

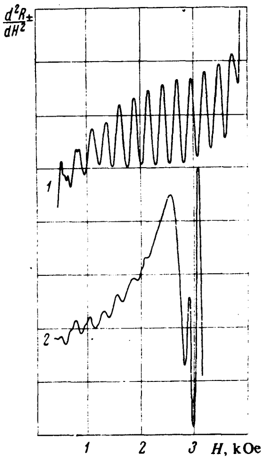

FIG. 5. Plots of $d^2R_\pm/dH^2$ against H at identical gains: $d=0.60$ mm, $\omega/2\pi=0.24$ MHz; $T=1.6$ °K. Curve 1—minus polarization, 2—plus polarization.

on curves 1 and 2 of Fig. 2 in a field 11 kOe ($\varkappa \approx 2$) is equal to 14. The theoretical ratio of the amplitude of the doppleron oscillations to the GKO amplitude in the same field is 9. We do not compare the results on the GKO in minus polarization and due to holes. The reason is that the theory constructed above does not take into account non-local effects connected with the holes.

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Cross relaxation between the Zeeman and spin-spin degrees of freedom in a solid

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A theory is presented of the establishment of a single spin temperature under the influence of a nonsecular dipole-dipole interaction $\mathscr{H}''_d$, in higher orders of perturbation theory. It is shown that when the cross relaxation processes are considered it is necessary to exclude in each succeeding approximation the secular contribution from the perturbation $\mathscr{H}''_d$ and to redefine correspondingly both the Zeeman subsystem and the subsystem of the spin-spin interactions. It is pointed out that the ideas presently advanced in the literature, concerning the unification of the Zeeman subsystem with $\mathscr{H}''_d$, are inconsistent.

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The dipole-pool concept, advanced by Provotorov,[1] has permitted considerable progress to be made in research on magnetic resonance.[2] The gist of this concept is that in strong constant magnetic fields $H_0 \gg H_L$ (H_L is the field due to the dipole–dipole (dd) interaction of the given spin with the environment) the populations of the aggregates of the levels which result from the lifting of the degeneracy of the Zeeman levels of the spin system by the secular part $\mathscr{H}'_d$ of the dd interaction are determined by a temperature β_d^{-1} that is in general different from the Zeeman temperature β_z^{-}. It is the interaction $\mathscr{H}'_d$, regarded as a thermodynamic subsystem with temperature β_d^{-1}, which is in fact called the dipole pool.

The Provotorov two-temperature model has a clear physical foundation. $\mathscr{H}'_d$ conserves the Zeeman energy