

# Absorption of Rayleigh sound in metals with several carrier groups

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Absorption of Rayleigh sound in metals with a multiply connected Fermi surface is considered. The absorption coefficient in the absence of an external magnetic field or in a strong (nonquantizing) magnetic field  $\mathbf{H}$  perpendicular to the wave vector  $\mathbf{k}$  is calculated for different orientations of the vector  $\mathbf{H}$  with respect to the sample boundary. In a normal magnetic field, absorption of surface waves is lower than that of volume sound; this decrease is due to departure of nonequilibrium electrons by diffusion from the "skin layer" of the Rayleigh wave. In a magnetic field parallel to the surface, the absorption coefficients of the surface and volume sound are of the same order of magnitude. The dependence of the Rayleigh-wave absorption on the nature of the scattering from the metal boundary is discussed.

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## INTRODUCTION

1. The character of the propagation of sound waves in metals has by now been studied in considerable detail. The absorption and dispersion of volume sound oscillations have been studied especially fully—their dependence on the direction of wave propagation, on the shape of the energy spectrum of the carriers, and on the external conditions in which the system is situated (temperature, magnetic field and so on). In bounded samples, in addition to the volume sound waves, elastic oscillations that are localized near the surface—Rayleigh waves—can also be propagated.<sup>[1]</sup> Their amplitude falls off exponentially with penetration into the interior of the metal, at a distance of the order of the sound wavelength. The singularities of the absorption to which the surface character of the Rayleigh waves leads have been intensively studied theoretically in the recent papers of Grishin and the present authors.<sup>[2-5]</sup> Some of the results (giant quantum oscillations<sup>[4]</sup>) have already found their experimental confirmation.<sup>[6]</sup>

Up to the present time, in all the theoretical papers devoted to the study of surface sound in metals, a model of a metal has been considered in which there is a single group of carriers (or else situations in which the contribution of several groups to the absorption is additive). The Fermi surfaces of many metals (and, in particular, semimetals) are multiply connected. The study of the absorption of Rayleigh sound is therefore of interest in such a situation in which the presence of several groups of carriers can lead to qualitative differences from the results obtained previously.

2. We recall in brief the effect of a multiply connected Fermi surface on the propagation of volume sound oscillations (this question has been considered in detail by Gilinskii and Sultanov<sup>[7]</sup>). The disequilibrium of the electron distribution in the sound-wave field in the presence of several groups of carriers can be of the intra-group or of the intergroup type. The latter represents the departure of the number of particles in each group from the instantaneous equilibrium value in the case of total electric neutrality of the system. The characteristic relaxation times of these disequilibria are gen-

erally different. At sufficiently low temperatures, the intergroup relaxation frequency  $\nu_M$  can turn out to be much smaller than the relaxation frequency of the intra-group disequilibrium. If a sound wave of frequency  $\omega \ll \nu$  propagates in the metal in this case, then, thanks to the rapid intragroup relaxation, the decisive role in sound absorption will be played by the nonequilibrium distribution of carriers between the groups. This non-equilibrium character can be removed in two ways—either by direct transitions between groups with frequency  $\nu_M$ , or by diffusion of the carriers in coordinate space along the direction of sound propagation.

The presence of two relaxation times, one of which (the diffusional) depends on the frequency  $\omega$ , leads to the result that the relative absorption  $\Gamma/\omega$  as a function of frequency has either two maxima at the frequencies  $\nu_M$  and  $\omega_D \equiv s^2/D$  ( $s$  is the sound velocity,  $D$  is the diffusion coefficient along the wave vector  $\mathbf{k}$ ), or one maximum at a frequency  $(\nu_M \omega_D)^{1/2}$ .

The described picture can be observed, of course, only in the case in which the mean free path of the electrons along the wave vector  $\mathbf{k}$  is small in comparison with the sound wavelength. In the absence of an external magnetic field, satisfaction of the condition  $kl \ll 1$  is necessary here ( $l$  is the mean free path). If the sample is placed in a strong (nonquantizing) magnetic field  $\mathbf{H}$ , perpendicular to the vector  $\mathbf{k}$  ( $\Omega \gg \nu$ ,  $kR \ll 1$ ,  $\Omega$  is the cyclotron frequency, and  $R$  is the radius of the orbit of the electrons in the magnetic field), then the range of frequencies in which intergroup disequilibrium appears, turns out to be much broader. This is connected with the fact that even at  $kl \gg 1$  the motion of electrons along the vector  $\mathbf{k}$  is limited by the magnetic field and the intergroup disequilibrium is removed by the two methods mentioned above.

3. The absorption of the Rayleigh wave in a metal with several groups of carriers possesses a number of singularities in comparison with the absorption of volume sound. The differences are due to two circumstances. First, the dynamics of the electrons interacting with the Rayleigh wave depends on the character of their reflection from the surface and, consequently, the absorption

ought to be sensitive to the state of the boundary of the sample. Second, the disequilibrium of the sound wave along the normal to the surface of the metal leads to the appearance of still another relaxation mechanism, due to the escape of nonequilibrium carriers from the "skin layer" of the Rayleigh wave. In the absence of a magnetic field, and also in a magnetic field parallel to the surface of the metal and perpendicular to the vector  $\mathbf{k}$ , the diffusion relaxation time agrees in order of magnitude with the time of escape of the carriers from the skin layer, and the absorption coefficient of the surface sound turns out to be of the same order as that of the volume sound.

Inclination of the vector  $\mathbf{H}$  relative to the surface of the metal changes the relation between the diffusion coefficients in the direction of the wave vector  $\mathbf{k}$  and along the normal to the surface. In particular, if the vector  $\mathbf{H}$  is perpendicular to the boundary of the sample, the diffusion of carriers along  $\mathbf{k}$  is frozen, while the diffusion along the magnetic field is free. In this situation, the nonequilibrium electrons leave the skin layer of the sound wave before they manage to relax by diffusion.

As a result it turns out that, in that range of frequencies in which the absorption of volume sound is determined by the diffusion relaxation time, the Rayleigh sound absorption coefficient depends on the time of escape of the nonequilibrium carriers from the skin layer and becomes smaller than the absorption of volume sound.

## 1. STATEMENT OF THE PROBLEM

The components of the strain tensor of the Rayleigh wave, propagating along the boundary of an elastic half-space, have the form

$$u_{ik}(\mathbf{r}, t) = \sum_a u_{ik}^a(0) \exp[i(kr - \omega t) - \kappa_a x], \quad (1.1)$$

where  $u_{ik}^a(0)$  are the amplitudes of the longitudinal ( $a=l$ ) and transverse ( $a=t$ ) components of the strain tensor at the surface  $x=0$ , and are connected with one another by the boundary conditions, the  $x$  axis is directed into the depth of the metal,  $\kappa_a = (k^2 - \omega^2/s_a)^{1/2}$  is the damping decrement of the sound field along the normal to the surface, and  $s_a$  is the velocity of longitudinal and transverse sound oscillations.

In the present paper, we shall consider only the deformation interaction of electrons with sound. The neglect of the induction part of the interaction in strong magnetic fields is admissible only for semimetals; therefore, all the results are quantitatively valid only in this case. The coefficient of deformation absorption of surface sound is determined by the formula<sup>[2]</sup>

$$\Gamma = \frac{1}{4W} \operatorname{Re} \int_0^\infty dx \sum_a \langle \Lambda_{ik}^a \chi_a \rangle_a \dot{u}_{ik}, \quad (1.2)$$

where  $W$  is the energy density in the Rayleigh wave,  $\Lambda_{ik}^a$  is the deformation potential tensor, and  $\chi_a \delta(\varepsilon - \varepsilon_F)$  is the nonequilibrium part of the electron distribution function. The dot denotes the partial derivative with respect to time, the angular brackets  $\langle \dots \rangle_a$  denote integration

over the portion of the Fermi surface corresponding to the group of carriers with index  $a$ , the asterisk denotes the complex conjugate, and summation is carried out over the vector indices  $i$  and  $k$ .

It is known that the deformation potential tensor in semimetals depends weakly on the momentum. It can also be shown that even in typical metals, with multiply connected Fermi surfaces, in the region of frequencies of most interest to us and the sound absorption under certain conditions should be determined essentially by the constant component of the deformation potential for the group. Therefore, further calculations will be carried out with the tensor  $\Lambda_{ik}^a = \text{const}$ . In this case, the initial formula for the absorption coefficient takes the form

$$\Gamma = \frac{1}{4W} \operatorname{Re} \sum_a Q_a \Lambda_{ik}^a \int_0^\infty dx \tilde{\chi}_a \dot{u}_{ik}, \quad (1.3)$$

$$\tilde{\chi}_a = \langle \chi_a \rangle_a / Q_a,$$

$Q_a = \langle 1 \rangle_a$  is the density of states on the Fermi surface.

The nonequilibrium part of the distribution function is determined from the kinetic equation (see, for example, Ref. 7):

$$\left[ \Omega_a \frac{\partial}{\partial \varphi_a} + v_{xa} \frac{\partial}{\partial x} + i(kv_a - \omega) \right] \chi_a + v_a (\chi_a - \tilde{\chi}_a) + \sum_{b \neq a} v_{ab} (\tilde{\chi}_b - \tilde{\chi}_a) = \Lambda_{lm}^a \dot{u}_{lm}. \quad (1.4)$$

Here  $\varphi_a$  is the azimuthal angle of the electron in the plane perpendicular to the magnetic field,  $v_a$  is the velocity of the electron,  $v_a$  is the frequency of the intra-group relaxation, and  $v_{ab}$  is the frequency of transitions of carriers between the groups  $a$  and  $b$ .

With the aim of simplifying the calculations, we consider a model of a metal with two groups of carriers. Using the equation of electric neutrality of the system

$$\sum_a \tilde{\chi}_a Q_a = 0,$$

we represent the kinetic equation for this case in the following form:

$$\left[ \Omega_a \frac{\partial}{\partial \varphi_a} + v_{xa} \frac{\partial}{\partial x} + i(kv_a - \omega) + v_a \right] \chi_a = g_a(x) = \Lambda_{lm}^a \dot{u}_{lm} + (v_a - v_x) \tilde{\chi}_a, \quad (1.5)$$

$$v_x = v_{ab} Q_b / Q \quad Q = Q_a Q_b / (Q_a + Q_b).$$

Equation (1.5) must be supplemented by the boundary condition on the surface  $x=0$ . The character of the reflection of the electrons from the boundary of the metal can be taken into account by the introduction of the phenomenological specular-reflection probability  $\rho$ . We can represent by its means the boundary condition in the form of the equation

$$\chi_a^\downarrow(0) = \rho \chi_a^\uparrow(0) + (1-\rho) \psi_a. \quad (1.6)$$

The left-hand side contains the nonequilibrium part of the distribution function of electrons reflected from the surface of the metal (the arrow  $\downarrow$ ). It coincides, with probability  $\rho$ , with the distribution function of the par-

ticles traveling to the surface (arrow ↑), and with probability  $1 - \rho$  becomes a constant independent of the momentum. The constant  $\psi_a$ , which renormalizes the chemical potential of the nonspecularly reflected particles, is found from the condition of vanishing of the normal component of the electric current at the boundary  $x=0$ . We note that the condition (1.6) does not take into account the possibility of transfer of carriers from one group to the other upon collision with the surface. Therefore, the current of each group of carriers should vanish separately at the surface. In what follows, we shall omit the number of the group everywhere where this does not cause misunderstanding.

The solution of Eq. (1.5) at an arbitrary orientation of the magnetic field relative to the surface of the metal is rather difficult. For this reason we limit ourselves to consideration of two limiting cases—a magnetic field perpendicular to the surface, and one parallel to it. In the normal magnetic field, the distribution function has a different form for electrons flying to the surface and for electrons flying away from it. The solution of Eq. (1.5) for electrons moving toward the surface is given by the formula

$$\chi^+(x, \varphi) = \int_{-\infty}^{\varphi'} \frac{d\varphi'}{\Omega} g \left[ x + \frac{|v_x|}{\Omega} (\varphi - \varphi') \right] \exp \left( - \int_{\varphi'}^{\varphi} \gamma d\varphi'' \right), \quad (1.7)$$

$$\gamma = [v + i(kv - \omega)]/\Omega.$$

The distribution function of reflected electrons depends on the boundary specularity parameter  $\rho$  and has the following form:

$$\chi^-(x, \varphi) = \int_{\varphi'}^{\varphi} \frac{d\varphi'}{\Omega} g \left[ x - \frac{|v_x|}{\Omega} (\varphi - \varphi') \right] \exp \left( - \int_{\varphi'}^{\varphi} \gamma d\varphi'' \right) + \rho \int_{-\infty}^{\varphi} \frac{d\varphi'}{\Omega} g$$

$$\times \left[ -x + \frac{|v_x|}{\Omega} (\varphi - \varphi') \right] \exp \left( - \int_{\varphi'}^{\varphi} \gamma d\varphi'' \right) + (1 - \rho) \psi \exp \left( - \int_{\varphi'}^{\varphi} \gamma d\varphi' \right), \quad (1.8)$$

$$\psi = \frac{1}{\pi} \int_0^{\varphi} \frac{v_x dv_x}{v^2} \oint d\varphi \int_{-\infty}^{\varphi} \frac{d\varphi'}{\Omega} g \left[ \frac{v_x}{\Omega} (\varphi - \varphi') \right] \exp \left( - \int_{\varphi'}^{\varphi} \gamma d\varphi'' \right).$$

Here  $v$  is the modulus of the electron velocity on the Fermi surface (both surfaces are assumed to be spherical which, however, is not a serious limitation), and  $\varphi_- = \varphi - \Omega x / |v_x|$ .

In the case in which the magnetic field is parallel to the boundary, the solution of Eq. (1.5) can also be obtained at any value of the parameter  $\rho$ .<sup>[5]</sup> The situation in a parallel magnetic field will be discussed below.

The electron distribution function in a normal magnetic field, represented by Eqs. (1.7) and (1.8), is a functional of its mean value over the group  $\bar{\chi}$ , the finding of which is our problem. A rather complicated integral equation is obtained for  $\bar{\chi}$  in the case of arbitrary  $\rho$ . We therefore limit ourselves to the analysis of the limiting cases of specular ( $\rho=1$ ) and diffuse ( $\rho=0$ ) scattering of electrons by the boundary of the metal.

## 2. NORMAL MAGNETIC FIELD. SPECULAR REFLECTION

The electron distribution function in a normal magnetic field (formulas (1.7) and (1.8)) is obtained without any assumptions as to the values of the parameters  $kR$

and  $\nu_M/v$ . Since we are interested in effects connected with the intergroup disequilibrium of the carriers, we assume these parameters to be small in comparison with unity.

1. We introduce first the general formula for the absorption coefficient of the Rayleigh waves. Averaging the distribution function over the Fermi surface and carrying out the corresponding change of variables, we obtain an equation for  $\bar{\chi}$ :

$$\bar{\chi}(x) = \int_{-\infty}^{\infty} dx' [\bar{\chi}(x') + (v - v_M)^{-1} \Lambda_{lm} \dot{u}_{lm}(x')] [P(x-x') + P(x+x')], \quad (2.1)$$

$$P(\xi) = \frac{v - v_M}{4\pi v} \int_0^{\pi} \frac{dv_x}{v_x} \oint d\varphi \exp \left( - \int_{\varphi_-}^{\varphi} \gamma d\varphi' \right), \quad \varphi_- = \varphi - \frac{\Omega |\xi|}{v_x}.$$

Equation (2.1) admits an even continuation of the solution into the region  $x < 0$ . Introducing the functions

$$F(x) = (v - v_M)^{-1} \Lambda_{lm} \dot{u}_{lm}(x), \quad U(x) = \bar{\chi}(x) + F(x), \quad (2.2)$$

we represent this function in the form

$$U(x) = \int_{-\infty}^{\infty} dx' P(x-x') U(x') + F(|x|). \quad (2.3)$$

It is easily solved by means of the Fourier transformation

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dq e^{ixq} f(q). \quad (2.4)$$

For the Fourier transform  $\bar{\chi}$  we obtain the expression

$$\bar{\chi}(q) = \frac{F(q) P(q)}{1 - P(q)}. \quad (2.5)$$

The transform of the function  $F(|x|)$  is easily calculated, since the strain tensor is taken in the unperturbed form (1.1):

$$F(q) = (v - v_M)^{-1} \sum_a \Lambda_{lm} \dot{u}_{lm}^a(0) \frac{2\kappa_a}{q^2 + \kappa_a^2}. \quad (2.6)$$

The asymptotic form of the kernel  $P(q)$  in the limit  $kR \ll 1$  is

$$P(q) = \frac{v - v_M}{qv} \left\{ \operatorname{arctg} \frac{qv}{\bar{v}} - \frac{1}{2} (kR)^2 \left[ \left( 1 + \left( \frac{\bar{v}}{qv} \right)^2 \right) \operatorname{arctg} \frac{qv}{\bar{v}} \right. \right.$$

$$\left. - \frac{\bar{v}}{qv} \right] - \frac{1}{2} \left( \left[ 1 + \left( \frac{\bar{v}_+}{qv} \right)^2 \right] \operatorname{arctg} \frac{qv}{\bar{v}_+} - \frac{\bar{v}_+}{qv} \right)$$

$$\left. - \frac{1}{2} \left( \left[ 1 + \left( \frac{\bar{v}_-}{qv} \right)^2 \right] \operatorname{arctg} \frac{qv}{\bar{v}_-} - \frac{\bar{v}_-}{qv} \right) \right\}, \quad (2.7)$$

$$\bar{v} = v - i\omega, \quad \bar{v}_{\pm} = \bar{v} \pm i\Omega.$$

If we substitute the function  $\bar{\chi}(x)$  in the form of the Fourier integral in Eq. (1.3) and carry out integration over  $x$ , we obtain the surface sound absorption coefficient

$$\Gamma = \sum_a \mathcal{T}_a \sum_{\alpha, \beta} \operatorname{Re} (B_{\alpha}^a B_{\beta}^a) k^2 \kappa_a \kappa_b \frac{v_a}{v_a - v_M}$$

$$\times \frac{1}{\pi} \operatorname{Re} \int_{-\infty}^{\infty} \frac{dq}{(q^2 + \kappa_a^2)(q^2 + \kappa_b^2)} \frac{P_a(q)}{1 - P_a(q)}, \quad (2.8)$$

$$\mathcal{T}_a = \frac{3}{8} \frac{N_a \varepsilon_F k}{\rho_L v_a}, \quad B_{a^*} = \frac{\Lambda_{ik} u_{ik}^*(0)}{k u_x^*(0) (\varepsilon_F (\zeta_a A)^{1/2})}, \quad \zeta_a = \left( \frac{\Lambda^a}{\varepsilon_F} \right)^2.$$

The quantity  $\mathcal{T}_a$  represents the coefficient of collisionless absorption of volume sound by the carrier group  $a$  with concentration  $N_a$  in the absence of a magnetic field,  $\rho_L$  is the density of the metal,  $u_x^*(0)$  is the amplitude of the normal component of the potential mode of the sound field at the boundary,  $\zeta_a$  is the electron-phonon interaction constant,  $\Lambda^a$  is the characteristic value of the deformation potential, the quantity  $A$  is the dimensionless function of the ratio of the sound velocities, [2], and the factors  $B_{a^*}$  characterize the relative contribution of the potential ( $a=l$ ) and vortical ( $a=t$ ) sound modes in the interaction with the electrons.

We shall calculate the integral in Eq. (2.8) by closing the contour of integration in the upper halfplane of the complex variable  $q$ . For this purpose, it is necessary to know the analytical properties of the function  $P(q)$  in this region. It can be shown that at  $\omega \ll \nu$  the function  $[1 - P(q)]^{-1}$  has a simple pole at the point  $q = iK$  in the region specified, in the vicinity of which

$$1 - P(q) = \frac{\nu v^2}{3 \tilde{v}^3} (q^2 + K^2), \quad (2.9)$$

$$K^2 = 3 \frac{\tilde{v}^2}{v^2} \left( \frac{\tilde{v}_M}{v} + \frac{1}{3} \frac{k^2 v^2}{\tilde{v}^2 + \Omega^2} \right), \quad \tilde{v}_M = v_M - i\omega.$$

Moreover, the point  $q_0 = i\tilde{v}/v$  is a logarithmic branch point, in which connection we must draw a corresponding cut in the upper half-plane of  $q$ . The branch points  $q_{\pm} = i\tilde{v}_{\pm}/v$ , which are contained in the coefficient of  $(kR)^2$  in Eq. (2.7), can be disregarded, since the contribution to the absorption from the integrals over the edges of the cuts, made from these points, turns out to be relatively small.

At high frequencies ( $\omega \gg \nu$ ) the function  $[1 - P(q)]^{-1}$  has no pole and it is necessary to take into account only the branch point  $q_0$ .

With account of everything said above, the absorption coefficient can be represented in the form of a sum of three terms:

$$\Gamma = \Gamma_1 + \Gamma_2 + \Gamma_3, \quad (2.10)$$

$$\Gamma_1 = \sum_a \mathcal{T}_a \sum_{\alpha, \beta} B_{\alpha \beta} \frac{k^2}{\zeta_{\beta}^2 - \zeta_{\alpha}^2} \frac{v_a}{v_a - v_M} \operatorname{Re} \left[ \zeta_{\beta} \frac{P_a(i\zeta_{\alpha})}{1 - P_a(i\zeta_{\alpha})} - \zeta_{\alpha} \frac{P_a(i\zeta_{\beta})}{1 - P_a(i\zeta_{\beta})} \right],$$

$$\Gamma_2 = \sum_a \mathcal{T}_a \sum_{\alpha, \beta} B_{\alpha \beta} k^2 \zeta_{\alpha} \zeta_{\beta} \times$$

$$\times \operatorname{Re} \tilde{I}_a \int_1^{\infty} \frac{t^{-1} dt}{(\zeta_{\alpha}^2 \tilde{I}_a^2 - t^2)(\zeta_{\beta}^2 \tilde{I}_a^2 - t^2)} \left[ \left( 1 - \frac{v_a}{2 \tilde{v}_a t} \ln \frac{t+1}{t-1} \right)^2 + \left( \frac{\pi v_a}{2 \tilde{v}_a t} \right)^2 \right]^{-1},$$

$$\Gamma_3 = \sum_a \mathcal{T}_a \sum_{\alpha, \beta} B_{\alpha \beta} \operatorname{Re} \frac{k^2 \zeta_{\alpha} \zeta_{\beta}}{(\zeta_{\alpha}^2 - K_a^2)(\zeta_{\beta}^2 - K_a^2)} \frac{3v_a}{K_a v_a}.$$

The following notation is introduced here:  $\tilde{I}_a = v_a / \tilde{v}_a$ ,  $B_{\alpha \beta}^* = \operatorname{Re}(B_{\alpha}^* B_{\beta}^{*\dagger})$ . The term  $\Gamma_3$ , which represents the contribution of the poles  $q = iK_a$  to the integral of Eq. (2.8), needs to be taken into account only in the case  $\omega \lesssim v_a$ .

2. We now consider the behavior of the absorption coefficient at different values of the parameters. In the

limiting case  $|\zeta \tilde{I}| \ll 1$ , the term  $\Gamma_2$ , which is proportional to  $(\zeta \tilde{I})^4$ , can be neglected. Using Eqs. (2.9), we obtain the following expression for the absorption coefficient:

$$\Gamma = \sum_a \mathcal{T}_a \sum_{\alpha, \beta} B_{\alpha \beta} \operatorname{Re} \frac{k^2}{K_a^2 - \zeta_{\alpha}^2} \frac{3v_a}{K_a v_a} \left( \frac{K_a}{\zeta_{\alpha} + \zeta_{\beta}} - \frac{\zeta_{\alpha}}{K_a + \zeta_{\beta}} \right). \quad (2.11)$$

In the opposite limiting case  $|\zeta \tilde{I}| \gg 1$ , it suffices to retain only the principal term in the expression for  $P(i\zeta)$

$$P(i\zeta) \approx \frac{\nu}{2\zeta v} \ln \frac{1+\zeta \tilde{I}}{1-\zeta \tilde{I}}. \quad (2.12)$$

In the integral contained  $\Gamma_2$ , the fundamental contribution is made by the region  $t \sim |\zeta \tilde{I}|$ . This allows us to simplify the integrand by replacing the last factor by unity. Using (2.12), we get an expression for the sum of the quantities  $\Gamma_1$  and  $\Gamma_2$  that agrees in form with the Rayleigh sound absorption coefficient of a single carrier group [2]:

$$\Gamma_1 + \Gamma_2 = \sum_a \mathcal{T}_a \sum_{\alpha, \beta} B_{\alpha \beta} \frac{k^2}{\zeta_{\beta}^2 - \zeta_{\alpha}^2} \operatorname{Re} \left[ \zeta_{\beta} \ln(1 + \zeta_{\alpha} \tilde{I}_a) - \frac{\zeta_{\alpha}}{\zeta_{\beta}} \ln(1 + \zeta_{\beta} \tilde{I}_a) \right]. \quad (2.13)$$

3. We now analyze the dependence of the absorption coefficient on the frequency. In the region of sufficiently low frequencies, when  $|\zeta \tilde{I}| \ll 1$ , the absorption is determined by Eq. (2.11). If the inequality

$$|K|/\zeta \gg 1, \quad (2.14)$$

is satisfied in this case, the principal contribution to the absorption is made by the first term which (if we neglect the quantity  $\zeta$  in comparison with  $K$ ) is identical in form with the absorption coefficient of volume sound.

As the frequency is increased, the opposite inequality begins to be satisfied

$$|K|/\zeta \ll 1 \quad (2.15)$$

(getting ahead of ourselves, we note that the transition from the inequality (2.14) to (2.15) takes place at frequencies at which we still have  $|\zeta \tilde{I}| \ll 1$ ). The second term in Eq. (2.11) becomes dominant (actually,  $\Gamma_3$ ) and the absorption of surface waves differs essentially from the absorption of volume sound.

In the range of frequencies when  $|\zeta \tilde{I}| \gg 1$ , but  $\omega \ll \nu$ , it is generally necessary to take into account the contribution of (2.13) in addition to  $\Gamma_3$ . In this region,

$$\frac{\Gamma_1 + \Gamma_2}{\Gamma_3} \sim \left| \frac{Kv}{v} \ln \zeta \tilde{I} \right|. \quad (2.16)$$

Since the inequality

$$|Kv/v| \ll 1, \quad (2.17)$$

is satisfied at  $\omega \ll \nu$ , and the quantity  $|\ln \zeta \tilde{I}| \sim 8-9$  even at  $\omega \sim \nu$ , we can then assume that the absorption throughout the range of frequencies in which the conditions (2.15) and (2.17) are satisfied, is determined essentially by the term  $\Gamma_3$ . Thus, the formula (2.11), which rep-

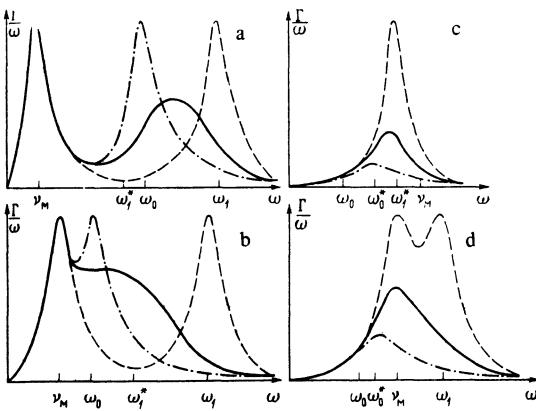


FIG. 1. Frequency dependence of the relative absorption of surface sound in the region  $\omega \ll \nu$  (the frequency scale is logarithmic). Dot-dash line—absorption at  $H=0$ , dashed line—in a magnetic field parallel to the surface of the metal, continuous line—in a normal magnetic field. In the absence of a magnetic field and in the case of a parallel magnetic field, the absorption coefficients of volume and surface sound are of the same order of magnitude. Graphs a and b correspond to the case  $\nu_M < \omega_0$ , c and d—to the case  $\omega_0 < \nu_M$ .

resents the sum of  $\Gamma_1$  and  $\Gamma_3$ , gives a good description of the absorption of Rayleigh sound over the entire range of frequencies  $\omega < \nu$ .

The analytic dependence of the surface sound absorption on the frequency changes substantially or going from the region (2.14) to (2.15). The characteristic frequency at which this change occurs can be different, depending on the ratio of the parameters  $\nu_M$  and  $\omega_0 = 3\nu(s/v)^2$  ( $\omega_0$  is the frequency at which the sound frequency  $\omega$  and the frequency  $k^2 D = \omega^2 v^2 / 3\nu s^2$  of diffusional relaxation of carriers in the absence of a magnetic field become equal). If  $\nu_M < \omega_0$ , then interchange of the inequalities (2.14) and (2.15) takes place in the region of the frequency  $\omega_0$ . If  $\nu_M > \omega_0$ , then the transition occurs at the frequency  $\omega_0^* = (\nu_M \omega_0)^{1/2}$  ( $\omega_0^*$  is the frequency at which the frequencies of relaxation  $\nu_M$  and  $k^2 D$  become equal).

The frequency characteristics of the relative absorption  $\Gamma/\omega$  of volume and surface sound in the low frequency region ( $\omega \ll \nu$ ) are shown in Figure 1 for different ratios of the parameters  $\nu_M$ ,  $\omega_0$ ,  $\omega_0^*$ , and  $\omega_1^*$  ( $\omega_1 = \omega_0[1 + (\Omega/\nu)^2]$ ,  $\omega_1^* = (\nu_M \omega_1)^{1/2}$ ). Figures a and b correspond to the case  $\nu_M < \omega_0$ , while c and d correspond to  $\nu_M > \omega_0$ . The general tendency is for the relative absorption of Rayleigh sound to decrease significantly in the region (2.15) in comparison with the case of volume oscillations.

The character of the dependence of the absorption of Rayleigh and volume sound on the magnetic field is also well investigated. As is seen from Eq. (2.9), in the absence of a magnetic field  $|K|/\kappa \gtrsim 1$ ; therefore, the absorption coefficients of volume and surface sound are of the same order of magnitude (they are represented in the figure by the dot-dash line). In the case  $\nu_M < \omega_0$ , the diffusion maximum of the relative absorption of volume sound is shifted from the point  $\omega_0$  to the point  $\omega_1$  with increase in magnetic field, remaining constant in mag-

nitude ( $(\Gamma/\omega)_{max} \sim \Lambda^2 Q / \rho_L s^2$ ). The change in the Rayleigh sound absorption in this same situation turns out to be more complicated. The diffusional maximum of  $\Gamma/\omega$ , which is first located at the point  $\omega_0$ , shifts to the right, broadens and decreases in magnitude.

At  $\nu_M > \omega_0$ , the picture looks somewhat different (Figs. c and d). In the absence of the magnetic field, the relative absorption of volume sound has a single maximum at the frequency  $\omega_0^*$

$$\left(\frac{\Gamma}{\omega}\right)_{max} \sim \frac{\Lambda^2 Q}{\rho_L s^2} \left(\frac{\omega_0}{\nu_M}\right)^{1/2}.$$

With increase in the magnetic field, the relative absorption increases at its maximum:

$$\left(\frac{\Gamma}{\omega}\right)_{max} \sim \frac{\Lambda^2 Q}{\rho_L s^2} \left(\frac{\omega_1}{\nu_M}\right)^{1/2},$$

and this maximum occurs at  $\omega = \omega_1^*$ . When the frequency  $\omega_1^*$  becomes equal to  $\nu_M$ , the maximum splits in two and thereafter the absorption of the volume sound behaves the same as in the case  $\nu_M < \omega_0$ . The relative damping of the Rayleigh sound, which has a maximum at the frequency  $\omega_0^*$  in the case  $H=0$ , also increases with increase in the field, always remaining smaller than the volume sound absorption. The maximum of  $\Gamma/\omega$  in this case shifts in the direction of high frequencies, lagging somewhat behind the position of the "volume" maximum. At  $\omega_1^* \sim \nu_M$ , the maximal value of the relative absorption of Rayleigh sound saturates and does not change with further increase in the magnetic field. The characteristic feature of the Rayleigh sound absorption in the case  $\nu_M > \omega_0$  is the absence of a diffusion maximum at any value of the magnetic field.

To understand the reason for the significant difference of the absorption of surface sound from the absorption of the volume oscillations, we consider the distribution function of the nonequilibrium carriers in the field of the Rayleigh wave. The Fourier transform of the distribution function (2.5) has the same singularities in the upper halfspace of the variable  $q$  as the integrand in (2.8). Since the inequality  $|\kappa \tilde{l}| \ll 1$  is satisfied at the characteristic frequencies  $\omega_0$  and  $\omega_0^*$ , and also because of the condition (2.17), the contribution of the integral along the edges of the cut drawn from the branch point  $q_0$  can be neglected. As a result, the distribution function is represented by the expression

$$\bar{\chi}(x) = 3 \frac{\nu}{v^2} \sum_{\alpha} \frac{\Lambda_{lm} \dot{u}_{lm}^{\alpha}(0)}{K^2 - \kappa_{\alpha}^2} \left( e^{-\kappa_{\alpha} x} - \frac{\kappa_{\alpha}}{K} e^{-\kappa_{\alpha} x} \right). \quad (2.18)$$

In the range of frequencies in which  $|K| \gg \kappa$ , the second term can be omitted. The situation in this limiting case turns out to be in a certain sense local—the distribution of the nonequilibrium carriers repeats the distribution of the sound field. Upon satisfaction of the condition  $|K| \ll \kappa$  the second term in Eq. (2.18) becomes dominant. The density of the nonequilibrium electrons in this limit decreases exponentially at a distance that is much greater than the thickness of the acoustic skin layer. The quantity  $K$  consequently has the meaning of the reciprocal depth of the diffusional penetration of the non-

equilibrium carriers into the interior of the metal within the relaxation time of the intergroup disequilibrium, either by direct transitions between groups or by diffusion of the carriers along the vector  $\mathbf{k}$ , or as a consequence of the change in phase of the sound wave. All these mechanisms are taken into account in Eq. (2.9) for  $K$ .

Thus, the difference between the coefficient of absorption of Rayleigh sound in the considered geometry and the volume sound absorption coefficient is connected with the fact that, owing to the difference in the value of the diffusion coefficients along and transverse to the magnetic field, under certain conditions the nonequilibrium carriers leave the skin layer of the surface wave without relaxing and cease to participate in the absorption because of their relatively short lifetime in the field of the sound wave.

The described difference between surface sound absorption and volume sound absorption develops under conditions of intergroup disequilibrium of the carriers, i.e., at  $\omega \ll \nu$ . At high frequencies ( $\omega \gg \nu$ ), the disequilibrium of the carriers within the limits of the individual groups should play the decisive role. Actually, the condition  $|Kv/\nu| \gtrsim 1$  is satisfied at  $\omega \gtrsim \nu$ . Because of the relation (2.16), the absorption in this region is determined principally by the term (2.13), i.e., as expected, it is identical with the absorption coefficient of a Rayleigh wave in a metal with a single group of carriers.<sup>[2]</sup>

### 3. NORMAL MAGNETIC FIELD. DIFFUSE REFLECTION

1. In diffuse reflection of electrons from the boundary ( $\rho=0$ ) the equation for the averaged distribution function has the following form:

$$U(x) = \int_0^\infty dx' U(x') [P(x-x') + R(x)R(x')] + F(x),$$

$$R(x) = \frac{(v-v_M)^{1/2}}{2\pi v^{3/2}} \int_0^\infty dv_x \oint d\varphi \exp\left(-\int_{\varphi_-}^{\varphi} \gamma d\varphi'\right), \quad (3.1)$$

$$\varphi_- = \varphi - \Omega x/v_x.$$

The solution of Eq. (3.1) is equivalent to the solution of the set of equations

$$U_{1,2}(x) = \int_0^\infty dx' U_{1,2}(x') P(x-x') + \begin{cases} F(x) \\ R(x) \end{cases}. \quad (3.2)$$

The sought function  $U(x)$  is represented in the form of a linear combination of the functions  $U_1$  and  $U_2$ :

$$U(x) = U_1(x) + A U_2(x),$$

$$A = \left[ 1 - \int_0^\infty dx U_2(x) R(x) \right]^{-1} \int_0^\infty dx U_1(x) R(x). \quad (3.3)$$

Omitting the intermediate steps, we write out the final expression for the absorption coefficient. The quantity  $\Gamma$  is represented in the form of a sum of two terms

$$\Gamma = \Gamma_1 + \Gamma_2, \quad (3.4)$$

$$\Gamma_1 = \sum_a \mathcal{T}_a \sum_{\alpha,\beta} B_{\alpha\beta}^a \frac{k^2}{\kappa_\alpha + \kappa_\beta} \frac{v_a}{v_a - v_M} \operatorname{Re}[N_a^+(i\kappa_\alpha) N_a^+(i\kappa_\beta) - 1],$$

$$\Gamma_2 = \sum_a \mathcal{T}_a \sum_{\alpha,\beta} B_{\alpha\beta}^a \operatorname{Re} \frac{(kL_a)^2}{1-C_a} N_a^+(i\kappa_\alpha) N_a^+(i\kappa_\beta)$$

$$\times \int_1^\infty \frac{dt}{t^2} \frac{N_a^+(it/L_a)}{t + \kappa_a L_a} \int_1^\infty \frac{dt'}{t'^2} \frac{N_a^+(it'/L_a)}{t' + \kappa_a L_a},$$

$$C_a = \frac{v_a - v_M}{v_a} \int_1^\infty \int_1^\infty \frac{dt dt'}{(tt')^2(t+t')} N_a^+\left(\frac{it}{L_a}\right) N_a^+\left(\frac{it'}{L_a}\right).$$

The function  $N^+$ , which enters into (3.4), arises in the solution of the set (3.2) by the Wigner-Hopf method and is of the form

$$N^+(q) = \exp \left\{ \frac{1}{2\pi i} \int_{-\infty}^\infty dq' \frac{\ln[1-P(q')]}{q+q'} \right\}, \quad \operatorname{Im} q' > \operatorname{Im} q. \quad (3.5)$$

2. We now consider the behavior of the absorption (3.4) in different frequency ranges. At low acoustic frequencies ( $\omega \ll \nu$ ) it is convenient to represent the function  $N^+$  in the following form, by using the analytic properties of the kernel  $P(q)$ :

$$N^+(q) \approx \frac{I^{-1}}{K-iq} \exp \left\{ \frac{1}{2} \int_1^\infty \frac{dt}{t^2-1} \ln(t-iqI) \left[ \left(1 - \frac{1}{2t} \ln \frac{t+1}{t-1}\right)^2 + \left(\frac{\pi}{2t}\right)^2 \right]^{-1} \right\}. \quad (3.6)$$

a)  $|\kappa L| \ll 1$ . In this region, the principal term of the asymptotic form of the function  $N^+(i\kappa)$  is

$$N^+(i\kappa) \approx 3^{1/2} \frac{I^{-1}}{K+\kappa}. \quad (3.7)$$

Expanding the quantity  $C_a$  from (3.4) in the small parameter  $Kv/\nu$  and using (3.7), we obtain the following expression for  $\Gamma_1$  and  $\Gamma_2$ :

$$\Gamma_1 = \sum_a \mathcal{T}_a \sum_{\alpha,\beta} B_{\alpha\beta}^a \frac{3k^2 v_a}{(\kappa_\alpha + \kappa_\beta) v_a} \operatorname{Re} \frac{1}{(K_a + \kappa_\alpha)(K_a + \kappa_\beta)}, \quad (3.8)$$

$$\Gamma_2 = \sum_a \mathcal{T}_a \sum_{\alpha,\beta} B_{\alpha\beta}^a \frac{3k^2 v_a}{v_a} \operatorname{Re} \frac{1}{K_a(K_a + \kappa_\alpha)(K_a + \kappa_\beta)}. \quad (3.9)$$

b)  $|\kappa L| \gg 1$ . We make use of the fact that in this limit the principal contribution to the integral of Eq. (3.6) is made by  $t \sim |\kappa L|$ . We then obtain the following asymptotic expansion for  $N^*(i\kappa)$ :

$$N^*(i\kappa) \approx 1 + \frac{1}{2\kappa L} \ln(1+\kappa L). \quad (3.10)$$

With account of Eq. (3.10), the coefficient  $\Gamma_1$  is represented in the form

$$\Gamma_1 = \frac{1}{2} \sum_a \mathcal{T}_a \sum_{\alpha,\beta} B_{\alpha\beta}^a \frac{k}{\kappa_\alpha + \kappa_\beta} \operatorname{Re}[q_\alpha \ln(1+\kappa_\alpha L_a) + q_\beta \ln(1+\kappa_\beta L_a)], \quad (3.11)$$

$q_\alpha = k/\kappa_\alpha$  and agrees with the absorption in the case of a singly connected Fermi surface.<sup>[2]</sup> The coefficient  $\Gamma_2$ , as in the case  $|\kappa L| \ll 1$ , has the form (3.9).

At high frequencies ( $\omega \gg \nu$ ), there is no  $\Gamma_2$  term in the absorption and the coefficient  $\Gamma$  is identical with expression (3.11), which describes the absorption due only to the intragroup disequilibrium of the carriers.

It is not difficult to establish the fact that the Rayleigh sound absorption coefficient, which is represented by the formulas (3.8) and (3.9), is identical in accuracy with the expression (2.11) in the low frequency region, i.e., the absorption in this frequency range does not depend on the character of the reflection of the electrons from the boundary of the metal. The dependence of the absorption coefficient on the specularity parameter  $\rho$  is contained only in the single-band components (2.13) and (3.11), which describe the absorption due to the intra-group disequilibrium, and can appear only in the region of high acoustic frequencies.

We note that the absence of a dependence of the sound absorption in the low-frequency region on the character of the reflection of electrons at the metal boundary is entirely natural. In this case, the dominant role is played by the intergroup disequilibrium of the carriers. The model used by us for electron scattering from the surface (the boundary condition (1.6)) does not take into account the possibility of transfer of the carriers from one group to another. Collisions with the boundaries thus cannot affect the character of the intergroup disequilibrium and, consequently, the value of the absorption coefficient.

#### 4. PARALLEL MAGNETIC FIELD

We now consider a geometry in which the magnetic field is parallel to the metal surface and perpendicular to the direction of propagation of the sound wave. The classification of electron states in a parallel magnetic field is somewhat different than in the case of a normal magnetic field. Those electrons whose orbit centers are located at a depth larger than the turning radius do not collide with the metal boundary. The distribution function of such electrons is also of the same form as in an unbounded metal:

$$\chi(x, \varphi) = \left[ 1 - \exp\left(-\frac{2\pi\bar{v}}{\Omega}\right) \right]^{-1} \int_{\varphi=2\pi}^{\varphi} \frac{dv_H}{\Omega} g\left(x+R \int_{\varphi}^{\varphi'} n_x d\varphi''\right) \exp\left(-\int_{\varphi'}^{\varphi} \gamma d\varphi''\right). \quad (4.1)$$

$n_x$  is the projection of the velocity unit vector along the normal to the metal boundary.

The distribution function of electrons colliding with the boundaries should satisfy the boundary condition (1.6) (see, for example, Ref. 5). However, in the calculation of the absorption, we shall not take the surface electrons into account. For metals with a single group of carriers, this neglect is valid because of the small (of the order of  $kR$ ) relative number of such electrons. In our case, a much larger number of nonequilibrium carriers reach the boundary by diffusion. Since the diffusion coefficients along the vector  $\mathbf{k}$  and along the normal to the surface are equal, however, the depth of the diffusion penetration of the nonequilibrium carriers into the metal does not exceed the thickness of the skin layer of the Rayleigh wave. If this depth is much less than the sound wavelength, then the relative number of electrons interacting with the boundary of the metal is small, as before. If the diffusion penetration is of the same order as the thickness of the skin layer of the wave, then allowance for the boundary, i.e., for the surface elec-

trons, leads only to a certain numerical change in the absorption coefficient in comparison with that obtained below.

Thus, we shall consider a model of an unbounded metal with a "Rayleigh" distribution of the sound field. The equation for the averaged distribution function of the carriers is identical in this case in form with Eq. (2.3), and differs only in the form of the kernel  $P(x - x')$ . We shall not write down explicitly the rather complicated expression for the kernel and give only its Fourier transform  $P(q)$ :

$$P(q) = \frac{v - v_M}{4\pi v [1 - \exp(-2\pi\bar{v}/\Omega)]} \int_{-\pi}^{\pi} dv_H \oint d\varphi \int_{\varphi=2\pi}^{\varphi} \frac{d\varphi'}{\Omega} \exp\left[-\int_{\varphi'}^{\varphi} d\varphi'' (\gamma + iqRn_x)\right], \quad (4.2)$$

$v_H$  is the projection of the velocity in the direction of the magnetic field. In the limit  $kR \ll 1$ , the asymptotic form of this expression is

$$P(q) \approx 1 - \frac{v}{3\bar{v}} \frac{v^2}{\bar{v}^2 + \Omega^2} (q^2 + K_{||}^2), \quad K_{||}^2 = k^2 + \frac{3\bar{v}_M}{v v^2} (\bar{v}^2 + \Omega^2). \quad (4.3)$$

We note that the quantity  $K_{||}$  which has the meaning of the reciprocal depth of the diffusion penetration of the non-equilibrium carriers into the metal, is never smaller than  $k$  in modulus, which confirms the correctness of the assumption made by us.

The absorption coefficient calculated from Eq. (2.8) with the use of (4.3) is represented in the low-frequency region by the expression

$$\Gamma = \sum_a \mathcal{T}_a \sum_{\alpha, \beta} B_{\alpha\beta} \frac{k^2}{\kappa_\alpha + \kappa_\beta} \frac{\Omega_\alpha^2}{v_\alpha v_\beta} \operatorname{Re} \frac{K_{||}^2 + \kappa_\alpha + \kappa_\beta}{K_{||}^2 (K_{||}^2 + \kappa_\alpha) (K_{||}^2 + \kappa_\beta)}. \quad (4.4)$$

Because of the fact that  $|K_{||}| \gtrsim k > \kappa$ , the absorption coefficient (4.4) differs little from the absorption coefficient of volume sound. Actually, formally neglecting the quantities  $\kappa_\alpha$  in comparison with  $K_{||}$ , we obtain an expression that is identical in form with the absorption coefficient of volume oscillations. Allowance for the quantities  $\kappa_\alpha$  leads only to some numerical difference in the coefficient (4.4) from that of volume absorption.

At high acoustic frequencies, the asymptotic form of the absorption coefficient is

$$\Gamma = \sum_a \mathcal{T}_a \sum_{\alpha, \beta} B_{\alpha\beta} \frac{kl_a}{1 + (\omega/v_a)^2}.$$

This expression coincides, as it should, with the absorption coefficient of Rayleigh sound in metals with a single group of carriers.<sup>[3]</sup>

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# Isomorphic phase transitions in CuCl at high pressures

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The  $T$ - $P$  phase diagram of CuCl was investigated in the temperature interval 4.2–800°K and at pressures  $\gtrsim 35$  kbar. Isomorphic phase transitions are observed with small low thermal ( $\Delta Q \leq 0.1$  kcal/mole) and volume ( $\Delta V/V_0 \leq 1.5\%$ ) effects in the dielectric phase. The obtained  $T$ - $P$  diagram of CuCl is explained within the framework of the already existing theoretical concepts.

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## INTRODUCTION

Copper halides with sphalerite structure have a number of anomalous physical properties when compared with the properties of all other tetrahedral compounds (II–VI or III–V compounds and covalent crystals). For example, the CuCl crystal has an anomalously small bulk modulus.<sup>[1,2]</sup> The well known Lyddane–Sachs–Teller (LST) relation does not hold for CuCl.<sup>[2]</sup> The anomalies of the properties of copper halides become most pronounced, however, at high pressures. It is known<sup>[3]</sup> that in covalent crystals and in III–V compounds with small asymmetry of the pseudopotentials, pressure produces a dielectric–metal phase transition with tetragonal symmetry of the  $\beta$ -Sn type. This transition is attributed to the overlap, under pressure, of the maximum of the valence band at the Brillouin  $\Gamma$  point and the bottom of the conduction band in the vicinity of the X point, i.e., under pressure, the indirect energy gap  $E_{Rx} \rightarrow 0$ . In II–VI compounds and in the most ionic of the III–V compounds, a transition to the NaCl structure takes place under pressure. Nonetheless, instead of the expected<sup>[3]</sup> phase transition into the NaCl-type structure under pressure, what is produced in CuCl is first a dielectric–metal transition without change of lattice symmetry, followed by a transition into a structure of the  $\beta$ -Sn type, and only at higher pressure ( $P > 100$  kbar) does a transition take place into a dielectric phase with a structure of the NaCl type.<sup>[2,4,5]</sup>

It was shown<sup>[2]</sup> that all the known anomalies of the properties of CuCl can be explained if one assumes the presence of a small indirect gap between the maximum of the valence band at the point  $\Gamma$  and the minimum of the conduction band at the point X ( $E_{Rx} \approx 0.3$  eV). This model of the band structure is confirmed by the experimental dependence of the resistivity on the temperature,<sup>[6]</sup> by calculations of the band structure, and by measurements of the optical absorption of CuCl crystals.<sup>[7]</sup> Owing to the large ratio of the effective masses

of the holes and electrons, and also because of the hindrance rules for transitions in the vicinity of the X point of the Brillouin zone, the gap  $E_{Rx}$  is difficult to observe by standard optical measurements on thin CuCl films.<sup>[2]</sup> Inasmuch as the experimental binding energies of the direct excitons is  $E^d \approx 0.2$  eV<sup>[8]</sup> such a dielectric state with  $E^d \approx E_{Rx} \ll E_D$  (direct gap between the maximum of the valence band and the minimum of the conduction band at the point  $\Gamma$ ) can be unstable with respect to electron–hole pairing, i.e., with respect to a phase transition into the state of an excitonic dielectric. The physical properties of the excitonic dielectric CuCl should exhibit many anomalies. Owing to the polarization of the electron–hole pairs, the LST relation is violated.<sup>[2]</sup> Owing to the interaction of the exciton (two-particle) and phonon branches in the spectrum of the elementary excitations, the speed of sound decreases sharply,<sup>[9]</sup> and with it the bulk modulus of the CuCl crystal. At an energy  $E = 0.358$  eV,<sup>[7]</sup> an additional absorption band appears and corresponds to single-particle excitations in the exciton system.

Anomalies arise in the short-wave region of the spectrum.<sup>[10]</sup> With the increasing pressure, the indirect gap  $E_{Rx}$  for the nonrestructured electron spectrum decreases. As a result, an insulator–metal transition takes place at  $P = 42$  kbar without a change in the lattice symmetry<sup>[5]</sup> and with a small volume effect. In addition to these experimental observed effects, the theory predicts a number of additional effects, which have not yet been observed in the case of CuCl. Thus, it follows from the theory of Guseinov and Keldysh<sup>[11]</sup> that a phase transition from a dielectric into an excitonic dielectric is of first order with a small volume effect. Consequently, if the CuCl crystal is indeed an excitonic dielectric, then isomorphic phase transitions with very small volume effect  $\Delta V/V_0$  should be observed on the  $T$ - $P$  phase diagram in the dielectric region. To check on this possibility it is necessary to study in detail the  $T$ - $P$  diagrams in a wide range of temperatures and