Dipole magnetic interaction in plane Heisenberg magnetic substances

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The thermodynamics of the two-dimensional Heisenberg model with the magnetic dipole interaction taken into account is considered. The existence of two effects is demonstrated: the magnet becomes effectively planar and a spontaneous magnetic moment arises. The longitudinal susceptibility, which turns out to be logarithmically large as the field tends to zero, is also calculated.

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1. INTRODUCTION

Recently magnets with layered structure, in which the interplanar exchange integral $\mathcal{J}'$ is 3 to 6 times smaller than the intraplanar exchange integral $\mathcal{J}$, have been extensively investigated. These system can be first regarded as collections of planar magnets, and the interaction between planes is next taken into consideration. As was shown in[1], a planar magnet with a purely isotropic exchange interaction cannot have a nonvanishing spontaneous magnetic moment at a temperature $T \neq 0$, which is a special case of the general theorem concerning the impossibility of spontaneous symmetry breaking in a degenerate planar system with a local interaction.[2] The formal reason for this is the divergence of the integral that determines the mean square of the transverse fluctuation of the spin at small momenta:

$$\langle S_\perp^2 \rangle \sim T \int \frac{d^2k}{\mathcal{J}k^2}.$$

Two-dimensional magnets with two-component spin (planar) were investigated in detail by Berezinskiĭ,[3] who proved the existence of a phase transition in which rigidity of the system ρ_s with respect to slow rotation of the magnetic moment appears.

Various small perturbations of planar systems were investigated in[4] from the point of view of scale invariance. The two-dimensional isotropic Heisenberg model was investigated by Polyakov.[5] It was observed that the effective temperature increases with increasing size of the fluctuations. Although an exact proof does not exist at the present time, it is nevertheless very probable that neither long-range order nor rigidity exists in this case, and a phase transition is not present (also see[6]).

The model of a two-dimensional Heisenberg magnet with allowance for the magnetic dipole interaction is investigated in the presented work. The magnetic dipole interaction is a long-range effect and leads to the appearance of a term linear in the momentum in the fluctuation energy. As a result, the mean square fluctuations of the spin becomes finite and the formation of a nonvanishing order parameter becomes possible. The important small parameter of the problem is the ratio μ^2 of the intensities of the dipole and exchange interactions:

$$\mu^2 \sim \left(\frac{\mu_B^2}{a^3}\right) / \mathcal{J},$$

where μ_B is the Bohr magneton and a is the lattice constant.

The dipole interaction of the spin in a plane leads to an "easy plane" type of anisotropy, as a consequence of which the magnet becomes effectively two-dimensional over distances of the order of μ^{-1}, but the suppression of fluctuations not emerging from the plane takes place over substantially greater distances of the order of μ^{-2}. This allows one to divide the problem into two parts. In the first part the renormalization-group method[6,7] is applied in order to reduce the system to a collection of "block-spins" associated with regions of size μ^{-1} forming an effectively two-dimensional system. This problem was solved by Khokhlachev,[8] who found that the temperature and the parameter characterizing the dipole interaction at large distances vary significantly in comparison with their "bare" values. In the second stage the problem of a two-dimensional planar system is considered with allowance for the dipole interaction, where the scale invariance of the planar system[4] makes it possible to find the exact dependence of the spontaneous magnetic moment $|\langle \mathbf{m} \rangle|$ on the parameter μ and to determine the equation of state correct to terms of order $O(T^2)$.

2. REDUCTION TO A PLANAR MAGNET

The Hamiltonian of the system has the form

$$\mathscr{H} = \frac{1}{2} \sum_{\mathbf{x},\mathbf{a}} (\mathbf{S}(\mathbf{x}) - \mathbf{S}(\mathbf{x}+\mathbf{a}))^2 - g\mu_B \mathbf{H} \sum_{\mathbf{x}} \mathbf{S}(\mathbf{x}) + (g\mu_B)^2 \sum_{\mathbf{x},\mathbf{x}'} \frac{\mathbf{S}(\mathbf{x})\mathbf{S}(\mathbf{x}') - 3(\mathbf{S}(\mathbf{x})\mathbf{n})(\mathbf{S}(\mathbf{x}')\mathbf{n})}{|\mathbf{x}-\mathbf{x}'|^3}. \quad (1)$$

Here $\mathbf{x}$ and $\mathbf{x}'$ denote vectors to the lattice sites of the two-dimensional lattice (for simplicity we assume it to be a square lattice), $\mathbf{a}$ is the basis vectors of the lattice, $\mathbf{n}$ is a unit vector in the direction $\mathbf{x} - \mathbf{x}'$, and H is the magnetic field. The system of units is adopted in which the exchange integral $\mathcal{J}$, the lattice constant a, and the length S of the spin vector are equal to unity. The spin is assumed to be classical. The latter assumption does not reduce the generality of the investigation since in the present problem, as will be evident from what fol-

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lows, the spins associated with large regions are important and these can be regarded as classical quantities. Let us take as the starting point the conjecture that distances which are large in comparison with the lattice constant are important for the magnetic properties of the system. This conjecture will be validated by the following calculation. It allows us to change in the Hamiltonian (1) from a summation to an integration:

$$\mathcal{H}=\frac{1}{2}\int(\nabla\mathbf{S})^2d^2x-\mathbf{h}\int\mathbf{S}(\mathbf{x})\,d^2x$$
$$+\mu^2\iint\frac{\mathbf{S}(\mathbf{x})\mathbf{S}(\mathbf{x}')-3(\mathbf{n}\mathbf{S}(\mathbf{x}))(\mathbf{n}\mathbf{S}(\mathbf{x}'))}{|\mathbf{x}-\mathbf{x}'|^3}d^2x\,d^2x', \qquad (2)$$

where $\mathbf{h}=g\mu_B\mathbf{H}$ and $\mu=g\mu_B$.

Since the vectors $\mathbf{n}$ lie in the plane of the lattice, the behavior of the components of the moment $\mathbf{S}$ perpendicular to the plane (φ) and lying in the plane ($\mathbf{m}$) will be substantially different. Let us assume that the component φ is small. Then the components φ and $\mathbf{m}$ separate in the Hamiltonian (2). For the φ component we obtain the Hamiltonian

$$\mathcal{H}_\varphi=\frac{1}{2}\int(\nabla\varphi)^2d^2x+\frac{3\mu^2}{2}\int\varphi^2d^2x\int\frac{d^2x'}{|\mathbf{x}-\mathbf{x}'|^3}$$
$$+\frac{\mu^2}{2}\iint\frac{(\varphi-\varphi')^2}{|\mathbf{x}-\mathbf{x}'|^3}d^2x\,d^2x'.$$

A divergent integral, whose value depends on the method of cutoff, appears in the second term of the Hamiltonian $\mathcal{H}_\varphi$. It is natural to replace this integral by the corresponding sum over the lattice, which gives the value 9 for it. If the first two terms are kept in the Hamiltonian $\mathcal{H}_\varphi$, it follows that there is an exponential decrease of the correlation function $\langle\varphi(\mathbf{x})\varphi(\mathbf{x}')\rangle$ with a correlation radius $\lambda^{-1/2}$ where $\lambda=27\mu^2$. This allows one to estimate the relative size of the third term in $\mathcal{H}_\varphi$ (in comparison with the first two terms). Namely, by assuming $|\mathbf{x}-\mathbf{x}'|\sim\mu^{-1}$, $(\varphi-\varphi')^2\sim\varphi^2$, we find that this ratio $\sim\mu$. Thus, the third term in $\mathcal{H}_\varphi$ can be neglected.

It will be shown below that the correlation functions of the $\mathbf{m}$ component decrease according to a power law. Therefore, the spin averaged over distances larger than μ^{-1} essentially lies in a plane.

However, the interaction of the $\mathbf{m}$ component with the fluctuations of the transverse component φ with scales smaller than μ^{-1} substantially alters the coefficient of the Hamiltonian for the $\mathbf{m}$ component.

As was shown by Polyakov,[5] the temperature and correlation functions of the isotropic Heisenberg model in a plane are essentially renormalized during the transition to large spatial scales. In our case this renormalization cuts off at a distance of the order of $\lambda^{-1/2}$, the distance at which the transverse component φ "dies out." The renormalization of the temperature and the anisotropy parameter for this case are contained in Khokhlachev's article[8]:

$$T_1=T\Big/\left(1-\frac{T}{4\pi}\ln\frac{1}{\lambda}\right), \qquad (3)$$

$$\lambda_1=\lambda\left(1-\frac{T}{4\pi}\ln\frac{1}{\lambda}\right). \qquad (4)$$

The renormalized value of the dipole interaction parameter μ is obtained in the following way. Following Khokhlachev,[8] let us write down

$$\mathbf{S}=Z_r\mathbf{S}_r+\psi. \qquad (5)$$

Here $Z_r\mathbf{S}_r$ is the average spin in the enlarged region of size r, ψ are the short wavelength fluctuations inside this region, and $\mathbf{S}_r^2=1$. Let us substitute (5) into the Hamiltonian (1) and average it over the rapid fluctuations ψ. In this connection the dipole interaction term preserves its form even for the spins of the enlarged region; however, the coefficient μ^2 is changed. Omitting the simple calculations, we present the final renormalization group equation for μ_r^2:

$$\frac{\partial\ln\mu_r^2}{\partial\xi}=-\frac{T_r}{2\pi}=-T/2\pi\left(1-\frac{T}{2\pi}\xi\right),\quad \xi=\ln r. \qquad (6)$$

The solution of Eq. (6) is

$$\mu_1^2=\mu^2\left(1-\frac{T}{4\pi}\ln\frac{1}{\lambda}\right). \qquad (7)$$

There is no renormalization of the magnetic field for a three-component spin.

Thus, we have reduced the problem of a three-component Heisenberg magnet to the problem of a two-component (planar) magnet with renormalized values of the temperature and of the dipole–dipole interaction constant. We note that the Hamiltonian of the component in the plane contains a short-range term of the form

$$\int[{}^1/_2(\nabla m)^2+\text{const}\cdot m^2(\mathbf{x})]\,d^2x,$$

where $m(\mathbf{x})$ denotes the modulus of the vector $\mathbf{m}(\mathbf{x})$. Therefore, the correlators of the modulus decrease rapidly, just as the correlators of the transverse components. Only the rotation angles of the vectors are correlated at large distances.

3. PLANAR MAGNET WITH A DIPOLE-DIPOLE INTERACTION

Let us further investigate the system in the region of distances greater than $\lambda^{-1/2}$, that is, greater than the correlation radius of the component $\varphi(\mathbf{x})$. Here the spin can be regarded as lying in the plane. The Hamiltonian has the form

$$\mathcal{H}=\frac{1}{2}\int(\nabla\mathbf{m}(\mathbf{x}))^2d^2x-\frac{\mu_1^2}{2}\iint\frac{\mathbf{m}(\mathbf{x})\mathbf{m}(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3}d^2x\,d^2x'$$
$$-\int\mathbf{h}\mathbf{m}(\mathbf{x})\,d^2x+\frac{3\mu_1^2}{2}\iint\frac{m_a(\mathbf{x})m_b(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3}(\delta_{ab}-2n_an_b)\,d^2x\,d^2x', \qquad (8)$$

where $\mathbf{m}(\mathbf{x})$ is a unit vector. Here the second and fourth terms arose, respectively, from the isotropic and traceless parts of the dipole–dipole interaction of two-dimensional spins. As will be shown in the Appendix, the last term in (8) is not essential for calculation of the average magnetic moment with the exception of the region of very low temperatures, where the magnetic moment saturates. In the region where the value of the magnetic moment does not coincide with the saturation value of the moment, the indicated term can be neglected. The value of the spontaneous magnetic moment is determined accurate to a certain function of the temperature with the aid of considerations based on scale invariance.[4,7] As follows from the form of the Hamiltonian (8), the scaling dimension of the parameter μ_1^2 is given by $\Delta_{\mu_1^2} = 1 - 2\Delta$, where $\Delta = T_1/4\pi\rho_s(T_1)$ is the scaling dimension of the order parameter. The spontaneous moment is given by

$$|\langle \mathbf{m} \rangle| \sim (\mu_1^2)^{\Delta/(1-2\Delta)}. \tag{9}$$

Equation (9) is meaningful at $\Delta < 1/2$. For $\Delta > 1/2$ the dipole–dipole interaction cannot establish long range order. The coefficient in Eq. (9) can be approximately obtained by a variational method.[7]

Our further goal consists in the calculation of the dependence of the magnetic moment on the field h and on the dipole–dipole interaction μ_1. In this connection we shall confine our attention to the case of low temperatures, $T_1 \ll 1$. To this end we introduce the angular variable ω_x: $\mathbf{m}(\mathbf{x}) = (\cos\omega_x, \sin\omega_x)$. In contrast to the usual planar model, the field ω_x is not a free field. The interaction contains an infinite number of vertices originating from the exapnsion of the terms $h\cos\omega_x$ and $\mu_1^2\cos(\omega_x - \omega_{x'})$ in powers of ω_x.

Let us apply the method proposed by Polyakov in order to utilize the small quantity T_1 in explicit form: let us change from the variable ω_x to the variable $\alpha_x = \omega_x/\sqrt{T_1}$. The Hamiltonian (8) takes the form

$$\frac{\mathscr{H}}{T_1} = \frac{1}{2}\int (\nabla\alpha_x)^2 d^2x - \frac{\mu_1^2}{2T_1}\iint \frac{\cos[T_1^{1/2}(\alpha_x - \alpha_{x'})]}{|\mathbf{x}-\mathbf{x}'|^3} d^2x\, d^2x' - \frac{h}{T_1}\int \cos(T_1^{1/2}\alpha_x)\, d^2x. \tag{10}$$

Let us isolate the quadratic terms from the Hamiltonian $\mathscr{H}$:

$$\frac{\mathscr{H}_0}{T_1} = \frac{1}{2}\int (\nabla\alpha_x)^2 d^2x + \frac{\mu_1^2}{2}\iint \frac{(\alpha_x - \alpha_{x'})^2}{|\mathbf{x}-\mathbf{x}'|^3} d^2x\, d^2x' + \frac{h}{2}\int \alpha_x^2\, d^2x.$$

We shall regard the remaining part $\mathscr{H} - \mathscr{H}_0$ as the interaction Hamiltonian. The Hamiltonian $\mathscr{H}_0$ corresponds to the "bare" correlator

$$G_0(\mathbf{k}) = \langle |\alpha_{\mathbf{k}}|^2 \rangle = \frac{1}{k^2 + R^{-1}k + h}$$

$$R = \frac{1}{4\pi\mu_1^2}.$$

Averages of the form $\langle (\omega_x - \omega_{x'})^2 \rangle$ are represented by a summation of graphs, among which the loop graphs a (see Fig. 1) play the most important role, but an infinite number of other graphs exist, for example, graphs b through d. The complete summation of all these graphs is apparently impossible. Therefore, we shall confine our attention to the case of low temperatures, $T_1 \ll 1$, when the loop graphs a make a considerably larger contribution than all of the remaining graphs. For example, the graph in a with two loops is proportional to $T_1^2 \ln^2(1/\mu_1)$ whereas graph b is proportional to T_1^2. It is convenient to carry out the summation of the loop graphs, leading to an essential renormalization of the parameters R and h by the renormalization group method.

Omitting simple calculations, we present the equations of the renormalization group:

$$\partial T_1/\partial\xi = 0, \quad \partial \ln h_r/\partial\xi = -\Delta,$$
$$\partial \ln R_r/\partial\xi = 2\Delta, \quad \partial \ln Z_r/\partial\xi = -\Delta, \tag{11}$$

where Z_r is defined in Eq. (5) and $\xi = \ln(r\sqrt{\lambda})$.

It is necessary to integrate Eqs. (11) with respect to ξ with the initial conditions $R_r|_{\xi=0} = R$, $h_r|_{\xi=0} = h$ from zero up to a certain value $\xi_c = \ln(r_c\sqrt{\lambda})$ determined by the values of R and h. From Eqs. (11) it follows that

$$R_{r_c} = RZ^{-2}, \quad h_{r_c} = hZ. \tag{12}$$

Here r_c is related to Z by the equation: $Z = (r_c\sqrt{\lambda})^{-\Delta}$.

The renormalized value $\langle \mathbf{m}_{r_c} \rangle$ is equal in modulus to unity since upon increasing the dimensions of the cell over which the spin is summed the fluctuations are weakened, and the renormalization procedure which we have adopted reduces the vector $\mathbf{m}$ in each cell to unit length. By definition we have: $\langle \mathbf{m}_{r_c} \rangle = Z^{-1}\langle \mathbf{m} \rangle$. Hence it follows that $|\langle \mathbf{m} \rangle| = Z$. At the same time the average value of the magnetic moment is determined by the formula

$$|\langle \mathbf{m} \rangle| = \langle \cos\omega_x \rangle.$$

For a free field this would give

$$|\langle \mathbf{m} \rangle| = \exp\left(-\frac{1}{2}\langle \omega_x^2 \rangle\right) = \exp\left[-\frac{T_1}{2}\int \frac{d^2k}{(2\pi)^2} G_0(\mathbf{k})\right]. \tag{13}$$

In our problem the field ω_x is not free, and the equation $\langle \cos\omega_x \rangle = \exp[-(1/2)\langle \omega_x^2 \rangle]$ is, strictly speaking, incorrect. However, the graphs violating this equation are of order $O(T^3)$ and are discarded in our approximation. Therefore, the interaction is taken into account only by the replacement of the parameters in $G(\mathbf{k})$ by the renormalized parameters.

After substitution of (12) into (13), we obtain the following equation for the determination of $Z(R, h)$:

$$\frac{1}{\Delta}\ln Z = \frac{1}{2}\ln[hZ(2e)^{-2}] - \frac{1}{2(1-4hR^2Z^{-3})^{1/2}}\ln\frac{1-(1-4hR^2Z^{-3})^{1/2}}{1+(1-4hR^2Z^{-3})^{1/2}}. \tag{14}$$

Considerations of scale invariance show that Z is of the form

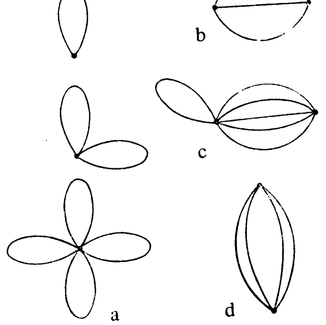

FIG. 1.

$$Z(R,h)=(2eR)^{-\Delta/(1-2\Delta)}F(hR^{(2-\Delta)/(1-2\Delta)}). \quad (15)$$

For the function $F(x)$ of the scale invariant $x = hR^{(2-\Delta)/(1-2\Delta)}$ we obtain the equation

$$\ln\left(\frac{F^{(2-\Delta)/\Delta}}{x}\right)=-\frac{1}{(1-4xF^{-3})^{1/2}}\ln\frac{1-(1-4xF^{-3})^{1/2}}{1+(1-4xF^{-3})^{1/2}}. \quad (16)$$

The solution of Eq. (16) in the region of weak fields $xF^{-3} \ll 1$ is given by

$$F(x)=1-\Delta x\ln x. \quad (17)$$

Thus, the longitudinal susceptibility in a zero field turns out to be logarithmically divergent. For fields which are strong in comparison with the dipole interaction (but substantially smaller than the saturation field $h \sim e^{-1/T}$),

$$F(x)=x^{\Delta/(2-\Delta)},$$

which gives the correct behavior of the moment $Z \sim h^{\Delta/(2-\Delta)}$.

A phase transition associated with the appearance of a "superfluid density" occurs at a temperature T_{c1}. In our system a second phase transition associated with the appearance of long range order may occur at a temperature $T_{c2}<T_{c1}$. Such a phenomenon in a two-dimensional planar weakly anisotropic magnetic substance was investigated in[4], where a variational approach gives the value $\Delta=1/\pi$ at the transition point T_{c1}. From this point of view T_{c2} should have coincided with T_{c1}. However, the accuracy of the variational approach is unknown. At the same time experimental data exist on layered magnetic substances,[9] from which one can, as shown in[4], calculate Δ as the ratio of the susceptibilities $\chi_{\parallel}/\chi_{\perp}$. Judging from the indicated experimental data, the value of Δ near the transition point T_{c1} is close to unity. From this point of view the possibility of a second phase transition at $T_{c2}<T_{c1}$ appears quite plausible to us. A phase transition involving the appearance of a magnetic moment was observed in the $NiCl_2$—graphite system[10] by Karimov and Novikov. The value of Δ at the transition point T_{c2}, observed by these authors, was equal to ~ 0.6.

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APPENDIX

Let us show that the last term of the Hamiltonian (6) is important only at very low temperatures. For this purpose let us represent it in the form

$$\mathscr{H}'=-\frac{3\mu_1^2}{2}\iint d^2x\,d^2x'\,\frac{\cos(\omega_x+\omega_{x'}-2\Phi)}{|\mathbf{x}-\mathbf{x}'|^3}, \quad (A.1)$$

where Φ is the azimuthal angle of the vector $\mathbf{x}-\mathbf{x}'$.

The homogeneous distribution $\omega_x=\text{const}$ does not give any contribution to $\mathscr{H}'$; therefore, let us explicitly isolate the dependence on the difference between the angles $\omega_x-\omega_{x'}$:

$$\mathscr{H}'=-\frac{3\mu_1^2}{2}\iint d^2x\,d^2y\,\frac{\cos 2\omega_x\cos(\omega_x-\omega_{x+y})}{y^3}. \quad (A.2)$$

To the approximation of zero order in T_1, the term (A.2) gives a correction

$$\varepsilon'(\mathbf{k})=R^{-1}k\cos 2\theta,$$

to the energy fluctuation with momentum $\mathbf{k}$, where θ denotes the angle between the direction of $\mathbf{k}$ and the spontaneous moment. However, due to the fact that (A.2) contains the factor $\cos 2\omega_x$, the renormalized value of this term is substantially larger than the scalar term $R^{-1}k$:

$$\partial\ln R_r'/\partial\xi=6\Delta,\quad R_r'|_{\xi=0}=R. \quad (A.3)$$

The relative contribution to the renormalized correlation function turns out to be of the order of Z^4:

$$G^{-1}(\mathbf{k})=k^2+R^{-1}Z^2k+R^{-1}Z^6k\cos 2\theta+hZ. \quad (A.4)$$

In order to evaluate the contribution to the spontaneous moment, we substitute (A.4) into (11) and we shall assume the last term in (A.4) to be small. Then a nonvanishing contribution from it appears only in second order, and the relative correction to the moment turns out to be of order Z^8, which is not small only at very low temperatures, when $Z\approx 1$. The spectrum of the spin waves becomes significantly anisotropic in this region.

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