Interaction of charged particles with strong monochromatic radiation in an inhomogeneous medium

V. M. Arutyunyan and S. G. Oganesyan

Erivan State University

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Multiphoton processes of stimulated absorption and emission by a charged particle in an external electromagnetic field incident on the interface between two media are considered. The effect can manifest itself in a broadening of the energy spectrum of the particle beam. It is also shown that the stimulated processes lead to modulation of the beam at the frequency of the external field.

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1. Charged particles that move uniformly in optically inhomogeneous media can radiate electromagnetic waves. Examples are the transition radiation at the interface between two media, radiation in a layered medium, and diffraction radiation by different screens. These phenomena have been sufficiently well studied both theoretically and experimentally.\[1\]

It is of interest to ascertain the variation of the character of this radiation in the presence of an external electromagnetic field, particularly a laser field. In contrast to ordinary spontaneous emission, this process has a stimulated character: the particle can not only radiate but also absorb quanta of the external field. These two phenomena are of definite interest both from the point of view of acceleration of charged particles by an external electromagnetic field and from the point of view of amplifying electromagnetic radiation by charged particles, as in the case of stimulated Čerenkov radiation.\[2\] In addition, as a result of such a stimulated interaction, the beams are modulated at the external-field frequency or its multiples, which is also of definite interest.

The present paper is devoted to a study of the interaction of an external monochromatic electromagnetic radiation with charged particles crossing the interface between two media, i.e., the analog of stimulated transition radiation.

2. We assume as usual that the particles themselves do not interact with the medium. Let the medium be inhomogeneous along \( z \) and let a plane linearly polarized electromagnetic wave of frequency \( \omega \), propagating in the same direction. We seek the solution of the Klein-Gordon equation

\[
\frac{\partial^2 \varphi}{\partial t^2} - \nabla^2 \varphi - \left[ \omega^2 \epsilon_0 + \epsilon_0 \gamma^2 \right] \varphi - \left[ \epsilon_0 \frac{\partial \varphi}{\partial x} \right] = 0
\]

in the form

\[
\varphi = \exp \left( \alpha \sqrt{\frac{\omega}{\omega_c}} \right) \chi,
\]

where \( \chi \) is a function that varies slowly in comparison with the exponential function. This approximation is valid if the following inequalities are satisfied:

\[ \alpha \ll 1, \quad \frac{\omega}{\omega_c} \ll 1, \quad \frac{\omega_c}{\omega} \ll 1. \]
where $\Delta p$ and $\Delta E$ are the changes of the momentum and energy of the particle as a result of the external field.

Neglecting the term $A_1$, which is quadratic in the field, and recognizing that $E^2 + M^2 \varepsilon^4 + p_x^2$, we arrive at the equation

$$\frac{\partial^2 \psi}{\partial t^2} + \frac{\partial \psi}{\partial x} \frac{\partial \psi}{\partial z} = \frac{\partial \psi}{\partial x} \frac{\partial \psi}{\partial z} + \frac{\partial \psi}{\partial z} \frac{\partial \psi}{\partial z} + \frac{\partial \psi}{\partial x} \frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial z} \frac{\partial \psi}{\partial z}.$$  

The solution of this equation in terms of the variables $x = t - q/c$ and $q = t - z/v$ is of the form

$$\psi = \psi_0 \exp \left[ \frac{\Delta E}{\Delta t} \right] \exp \left( \frac{\partial \psi}{\partial x} \frac{\partial \psi}{\partial z} + \frac{\partial \psi}{\partial z} \frac{\partial \psi}{\partial z} + \frac{\partial \psi}{\partial x} \frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial z} \frac{\partial \psi}{\partial z} \right).$$  

The relative probability of encountering a particle with energy $E' = E + \Delta E$ and momenta $p_x = p_x + \Delta p_x$, $p_y = p_y + \Delta p_y$, $p_z = p_z + \Delta p_z$ is equal to

$$W = \frac{1}{\Delta E} \exp \left( \frac{\Delta E}{\Delta t} \right) \exp \left( \frac{\partial \psi}{\partial x} \frac{\partial \psi}{\partial z} + \frac{\partial \psi}{\partial z} \frac{\partial \psi}{\partial z} + \frac{\partial \psi}{\partial x} \frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial z} \frac{\partial \psi}{\partial z} \right).$$  

Analogous result is obtained also from the classical equations of motion

$$\Delta p = M \varepsilon \frac{\partial E}{\partial x} \frac{\partial E}{\partial z}$$

where $x_0$ is the initial coordinate of the particle.

If we are dealing with a particle beam whose dimension along the $z$ axis exceed $c \beta_j / u = \beta_j \lambda / 2\pi$, then it follows from (10) that an initially monochromatic beam will spread out with equal probability in the region $\Delta E'$. An analysis of expression (9), confirming this result, will also make it possible to take quantum effects into account. Let $\Delta E'/\Delta E = \tilde{\nu} > 1$. In the region $\tilde{\nu} < \nu$ we introduce the notation $m = \nu - \tilde{\nu}$. Then $W = \tilde{W} - \tilde{W}_0^m (\nu)$. The argument and the index of the Bessel function differ by an amount

$$x = \frac{x}{\nu} = 1 + \frac{m}{\nu}$$

so that this function can be expressed in terms of an Airy function:

$$W = \frac{1}{\tilde{W} - \tilde{W}_0^m (\nu)} \exp \left( \frac{m}{\nu} \right).$$

The probability of encountering the state $m$ is

$$W = \frac{1}{\tilde{W} - \tilde{W}_0^m (\nu)} \exp \left( \frac{m}{\nu} \right).$$

The width of the peak is of the order of $m \equiv \nu \tilde{\nu}$. Consequently, the number of electrons under the peak is

$$W \equiv \nu \tilde{\nu} \exp \left( \frac{m}{\nu} \right).$$

The region $\tilde{\nu} < \nu$ corresponds to the classical equally-probable spreading of a beam of electrons; the bulk of the particles is concentrated in this region. The region of the peaks has a pure quantum character. Substituting $\Delta E = \nu \tilde{\nu}$ in (3), we obtain the region where the derived formulas are valid:

$$|\nu| < \nu \tilde{\nu}.$$  

3. We proceed now to consider effects connected with the allowance for the quantum corrections in the energy and momentum conservation laws.

As a result of stimulated interaction of the electromagnetic radiation with the particle beam, the latter is modulated at the laser frequency and its multiples. On striking the screen, the beam radiates in analogy with Schwarz’s experiment. We shall seek the wave function of the particle by perturbation theory, taking only single-photon processes into account. Representing this function in the form

$$\psi = \psi_0 \exp \left( \frac{\Delta E}{\Delta t} \right) \left[ 1 + \Phi \exp \left( \frac{\partial \psi}{\partial x} \frac{\partial \psi}{\partial z} + \frac{\partial \psi}{\partial z} \frac{\partial \psi}{\partial z} + \frac{\partial \psi}{\partial x} \frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial z} \frac{\partial \psi}{\partial z} \right)$$

V. M. Arutyunyan and S. G. Oganesyan 246
we can calculate the particle density:

$$\Phi_{\text{abs}} = \frac{1}{2\pi} \exp(-\gamma_i x) \int \exp(-\gamma_i x) \exp(i\phi)/\sqrt{2\pi} \exp(-i\phi).$$

where

$$\gamma_i = \frac{1}{E} \sqrt{\frac{1}{2} \frac{m_e}{c^2} \left( \frac{e}{\gamma_i} \right)^2} - \frac{1}{E} \left( \frac{e}{\gamma_i} \right)^2 \exp \left( \frac{1}{\sqrt{2} \gamma_i x} \right),$$

$$\Phi = \frac{1}{E} \sqrt{\frac{1}{2} \frac{m_e}{c^2} \left( \frac{e}{\Phi} \right)^2} - \frac{1}{E} \left( \frac{e}{\Phi} \right)^2 \exp \left( \frac{1}{\sqrt{2} \Phi x} \right).$$

In the case of the inequality

$$\gamma_i > \Delta E' = \gamma_i x$$

the particle may be captured by the interface, for in this case \( \gamma_i \) becomes complex. We shall not consider this case here, assuming that the inverse inequality is satisfied.

$$\gamma_i > \Delta E' = \gamma_i x.$$  

The modulation is large for strong fields. The beam is spatially modulated with a period

$$\gamma_i = \frac{1}{E} \sqrt{\frac{1}{2} \frac{m_e}{c^2} \left( \frac{e}{\gamma_i} \right)^2} - \frac{1}{E} \left( \frac{e}{\gamma_i} \right)^2 \exp \left( \frac{1}{\sqrt{2} \gamma_i x} \right).$$

It follows from (5)–(9) that \( \Delta E/E - 1 \) (for ultrarelativistic electrons we have \( \delta = Mc^2/E \)). Regardless of the initial particle energy, to change this energy by 0.1% we must have \( \Delta E' > 10^5 \), corresponding, for example, to a power density \( 10^{13} \text{W/cm}^2 \) for a neodymium-glass laser or \( 10^{14} \text{W/cm}^2 \) for a CO\(_2\) laser.

Let the energy of the initial beam be 100 MeV, and then the number of absorbed photons is \( \text{w/cm}^2 = 10^9 \). Consequently, \( \text{w/cm}^2 = 10^9 \), i.e., one percent of the electrons acquire the maximum possible energy, and this can be easily revealed by the energy broadening of the beam, if its initial width is less than \( \Delta E' \). Although the effect is small on one boundary, it can be increased by using layered media. Calculations show that \( \Delta E' \) increases in proportion to the number \( N \) of the boundaries crossed. The number of electrons having the maximum energy, however, decreases in this case like \( N^{-1/2} \).

The inverse effect, stimulated emission, seems to us of greater interest from the point of view of amplifying the initial radiation. Within a unit time the electron beam loses a power \( \Delta E' = \gamma_i \Delta E, \) which amounts to \( 5 \times 10^{11} \text{W/cm}^2 \) for a 100-MeV beam with particle density \( 10^{10} \text{cm}^{-3} \), if the particle energy loss is one percent of the initial energy.

To observe the modulation effect in experiment, the particle beam must satisfy a number of the requirements obtained in Schwarz' studies; we shall not dwell on them here. We present numerical estimates of the fields, when the depth of modulation reaches \( 10^5 \). In the relativistic case, assuming that all the angular dependences make a contribution on the order of unity, \( v_i/c \approx 0.1 \), we obtain \( E = 5 \times 10^9 \text{V/cm} \). In the case of moderate energies \( E < Mc^2 \) and \( \delta \) not close to zero or \( \pi/2 \), we have \( E = 6 \times 10^9 \text{V/cm} \). Finally, in the ultrarelativistic case \( \delta > Mc^2/E \), we have \( E = 6 \times 10^9 \text{V/cm} \) and \( n_i = 1, \ E = 0.6 \text{V/cm} \).

If a layered medium is used, the effect increases: \( R_i \propto N \). We note also the possibility of modulation at higher harmonics. The depth of modulation is of the order of \( \Delta x \). The period of the spatial modulation decreases by a factor \( N \).

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