ated much more slowly, and this leads to a shift of the energy center of the spectrum toward lower frequencies. When $\mathrm{Re}_n \exp(\mu k_0 \gamma t) \gtrsim 1$, the total power within the pedestal is approximately equal to the power in a regular harmonic, i.e., the random modulation index of the signal becomes high. Interactions between the low-frequency noise and the pedestal harmonics, which lead to the broadening of the spectrum mainly in the direction of lower frequencies, then become important.

The authors are indebted to A. I. Saichev for useful suggestions.

[1]B. B. Kadomtsev and V. I. Karpman, Usp. Fiz. Nauk 103, 193 (1974) [Sov. Phys. Usp. 14, 40 (1971)].
[2]V. I. Karpman, Nelineĭnye volny v dispergiruyushchikh sredakh (Nonlinear Waves in Dispersive Media), Nauka, 1975. (English transl. publ. by Pergamon Press, Oxford, 1975).
[3]O. V. Rudenko and S. I. Soluyan, Teoreticheskie osnovy nelineĭnoĭ akustiki (Theoretical Foundations of Nonlinear Acoustics), Nauka, 1975.
[4]L. K. Zarembo, Akust. Zh. 7, 189 (1961) [Sov. Phys. Acoust. 7, 149 (1961)].
[5]O. V. Rudenko and A. S. Chirkin, Dokl. Akad. Nauk SSSR 214, 1045 (1974).
[6]O. V. Rudenko and A. S. Chirkin, Zh. Eksp. Teor. Fiz. 67, 1903 (1974) [Sov. Phys. JETP 40, 945 (1975)].
[7]A. N. Malakhov and A. I. Saichev, Zh. Eksp. Teor. Fiz. 67, 940 (1974) [Sov. Phys. JETP 40, 467 (1975)].
[8]A. I. Saichev, Izv. Vyssh. Uchebn. Zaved. Radiofiz. 17, 1025 (1974).
[9]R. V. Khokhlov, Radiotekh. Elektron. 6, 917 (1961).
[10]B. B. Kadomtsev and V. I. Petviashvili, Dokl. Akad. Nauk SSSR 208, 794 (1973) [Sov. Phys. Dokl. 18, 115 (1973)].
[11]J. Tatsumi and S. Kida, J. Fluid Mech. 55, 659 (1972).
[12]E. N. Pelinovskiĭ and V. E. Fridman, Akust. Zh. 18, 590 (1972) [Sov. Phys. Acoust. 18, 482 (1973)].
[13]O. V. Rudenko and A. S. Chirkin, Dokl. Akad. Nauk SSSR 225, 520 (1975) [Sov. Phys. Dokl. 20, 748 (1975)].
[14]E. Hopf, Comm. Pure Appl. Math. 3, 201 (1951).

Translated by S. Chomet

Interaction of charged particles with strong monochromatic radiation in an inhomogeneous medium

V. M. Arutyunyan and S. G. Oganesyan

Erevan State University

(Submitted June 25, 1976)
Zh. Eksp. Teor. Fiz. 72, 466–470 (February 1977)

Multiphoton processes of stimulated absorption and emission by a charged particle in an external electromagnetic field incident on the interface between two media are considered. The effect can manifest itself in a broadening of the energy spectrum of the particle beam. It is also shown that the stimulated processes lead to modulation of the beam at the frequency of the external field.

PACS numbers: 41.70.+t

1. Charged particles that move uniformly in optically inhomogeneous media can radiate electromagnetic waves. Examples are the transition radiation at the interface between two media, radiation in a layered medium, and diffraction radiation by different screens. These phenomena have been sufficiently well studied both theoretically and experimentally.[1–3]

It is of interest to ascertain the variation of the character of this radiation in the presence of an external electromagnetic field, particularly a laser field. In contrast to ordinary spontaneous emission, this process has a stimulated character: the particle can not only radiate but also absorb quanta of the external field. These two phenomena are of definite interest both from the point of view of acceleration of charged particles by an external electromagnetic field and from the point of view of amplifying electromagnetic radiation by charged particles, as in the case of stimulated Čerenkov radiation.[4,5] In addition, as a result of such a stimulated interaction, the beams are modulated at the external-field frequency or its multiples, which is also of definite interest.

The present paper is devoted to a study of the interaction of an external monochromatic electromagnetic radiation with charged particles crossing the interface between two media, i.e., the analog of stimulated transition radiation.

2. We assume as usual that the particles themselves do not interact with the medium. Let the medium be inhomogeneous along z and let a plane linearly polarized electromagnetic wave of frequency ω propagate in the same direction. We seek the solution of the Klein-Gordon equation

$$-\hbar^2\frac{\partial^2\psi}{\partial t^2}=\left[M^2c^4+c^2\hat{p}_x^2+c^2\hat{p}_z^2+c^2\left(\hat{p}_y-\frac{e}{c}A_y\right)^2\right]\psi \tag{1}$$

in the form

$$\psi=\exp[i\hbar^{-1}(\mathbf{pr}-Et)]\varphi, \tag{2}$$

where φ is a function that varies slowly in comparison with the exponential function. This approximation is valid if the following inequalities are satisfied:

$$|\Delta p| \ll p, \quad |\Delta E| \ll E, \tag{3}$$

where Δp and ΔE are the changes of the momentum and energy of the particle as a result of the external field.

Neglecting the term A_y^2, which is quadratic in the field, and recognizing that $E^2 = M^2c^4 + p^2c^2$, we arrive at the equation

$$\frac{\partial\varphi}{\partial t} + c\beta_z\frac{\partial\varphi}{\partial z} = i\frac{\beta_y e}{\hbar}A_y\varphi. \tag{4}$$

The solution of this equation in terms of the variables $\tau = t$ and $\eta = t - z/c\beta$ is of the form

$$\varphi = \rho_0^{1/2}\exp\left\{i\frac{e\beta_y}{\hbar}\int_{-\infty}^{\infty}A_y(\tau', c\beta_z(\tau'-\eta))\,d\tau'\right\}, \tag{5}$$

where $\rho_0^{1/2}$ is the constant amplitude of the incident wave of the electron at $z = -\infty$, and $c\beta_{z,y} = v_{z,y}$ is the electron velocity.

Let us calculate the phase of the wave function, assuming that the electromagnetic field is monochromatic:

$$\delta = \frac{e\beta_y}{\hbar}\int_{-\infty}^{\infty}A_y\,d\tau' = \frac{2\pi\beta_y e}{\hbar c\beta_z}\left[A\left(\frac{\omega}{c\beta_z}\right)\exp\left(i\omega t - \frac{i\omega z}{c\beta_z}\right) + \text{c.c.}\right]. \tag{6}$$

In the particular case of one boundary we obtain

$$\delta = \frac{\Delta E'}{\hbar\omega}\cos\omega\left(t - \frac{z}{c\beta_z}\right)$$
$$= 4\xi\beta^3\frac{Mc^2}{\hbar\omega}\frac{n_1(n_1-n_2)\sin\theta\cos^2\theta}{(1-n_1{}^2\beta^2\cos^2\theta)(1-\beta n_2\cos\theta)}\cos\omega\left(t - \frac{z}{c\beta_z}\right), \tag{7}$$

where we have introduced the quantity $\Delta E'$, which coincides with the amplitude of the classical energy oscillations; $\xi = eA_0/Mc^2$ is a dimensionless intensity parameter, n_1 and n_2 are the refractive indices of the first and second media, and θ is the angle between the z axis and the direction of the electron velocity. The final form of the particle wave function is

$$\psi = \rho_0^{1/2}\exp\left[\frac{i}{\hbar}(\mathbf{pr} - Et)\right]\sum_{\nu=-\infty}^{+\infty} i^\nu J_\nu\left(\frac{\Delta E'}{\hbar\omega}\right)\exp\left[i\nu\omega\left(t - \frac{z}{c\beta_z}\right)\right]. \tag{8}$$

The relative probability of encountering a particle with energy $E_\nu = E - \nu\hbar\omega$ and momenta $p_{\nu x} = p_x$, $p_{\nu y} = p_y$, $p_{\nu z} = p_z - \nu\hbar\omega/c\beta_z$ is equal to

$$W_\nu = J_\nu^2(\Delta E'/\hbar\omega). \tag{9}$$

If $|\Delta E'| \gg \hbar\omega$, then the main contribution is made by states with $\nu \approx \Delta E'/\hbar\omega$, and consequently

$$E_{\pm|\nu|} \approx E \pm \hbar\omega\frac{\Delta E'}{\hbar\omega} = E \pm \Delta E'.$$

An analogous result is obtained also from the classical equations of motion

$$\Delta E = \Delta E'\sin\frac{\omega z_0}{c\beta_z}, \tag{10}$$

where z_0 is the initial coordinate of the particle.

If we are dealing with a particle beam whose dimensions along the z axis exceed $c\beta_z/\omega = \beta_z\lambda/2\pi$, then it follows from (10) that an initially monochromatic beam will spread out with equal probability in the region $2\Delta E'$. An analysis of expression (9), confirming this result, will also make it possible to take quantum effects into account. Let $\Delta E'/\hbar\omega = \nu_0 \gg 1$. In the region $\nu \sim \nu_0$ we introduce the notation $m = \nu_0 - \nu \ll \nu_0$. Then $W_m \equiv W_{\nu_0-m} = J^2_{\nu_0-m}(\nu_0)$. The argument and the index of the Bessel function differ by an amount

$$\varepsilon = \frac{\nu_0}{\nu_0 - m} = 1 + \frac{m}{\nu_0} \sim 1,$$

so that this function can be expressed in terms of an Airy function:

$$J_{\nu_0-m}(\nu_0) = J_{\nu_0-m}(\varepsilon(\nu_0 - m)) \approx \frac{1}{\sqrt{\pi}}\frac{2^{1/3}}{\nu_0^{1/3}}\Phi\left(-\frac{m2^{1/3}}{\nu_0^{1/3}}\right). \tag{11}$$

The probability of encountering the state m is

$$W_m = \frac{1}{\pi}\left(\frac{2}{\nu_0}\right)^{2/3}\Phi^2\left(-\frac{2^{1/3}m}{\nu_0^{1/3}}\right). \tag{12}$$

This function has a maximum at $m = 0$:

$$W_0 = (2/\nu_0)^{2/3}/\pi. \tag{13}$$

The width of the peak is of the order of $m \approx (\nu_0/2)^{1/2}$. Consequently, the number of electrons under the peak is $\sim \nu_0^{-1/2}$. At $\nu \ll \nu_0$ we get the usual asymptotic expression $J_\nu^2(\nu_0) \approx 1/\nu_0$. Finally, in the region $\nu \approx \nu_0$ the probability decreases exponentially.

The region $|\nu| \lesssim \nu_0$ corresponds to the classical equally-probable spreading of a beam of electrons; the bulk of the particles is concentrated in this region. The region of the peaks has a pure quantum character. Substituting $\Delta E = \nu\hbar\omega$ in (3), we obtain the region where the derived formulas are valid:

$$|\nu| \ll E/\hbar\omega. \tag{14}$$

Since $\nu_0 \lesssim |\nu|$, we obtain from (14) the following restriction on the field

$$\xi \ll 1. \tag{15}$$

3. We proceed now to consider effects connected with the allowance for the quantum corrections in the energy and momentum conservation laws.

As a result of stimulated interaction of the electromagnetic radiation with the particle beam, the latter is modulated at the laser frequency and its multiples. On striking the screen, the beam radiates in analogy with Schwarz's experiment.[6] We shall seek the wave function of the particle by perturbation theory, taking only single-photon processes into account. Representing this function in the form

$$\psi = \rho_0^{1/2}\exp\left(\frac{i}{\hbar}\mathbf{pr} - Et\frac{i}{\hbar}\right)\left[1 + \varphi_1\exp\left(i\omega t - i\frac{p_z}{\hbar}z\right)\right.$$
$$\left. + \varphi_2\exp\left(-i\omega t - i\frac{p_z}{\hbar}z\right)\right], \tag{16}$$

we obtain from (1)

$$\varphi_{1,2}=-\frac{i}{2\gamma_{1,2}}\Big[\exp(i\gamma_{1,2}z)\int_{-\infty}^{+\infty}\exp(-i\gamma_{1,2}z')\mu_{1,2}(z')dz'$$
$$-\exp(-i\gamma_{1,2}z)\int_{-\infty}^{+\infty}\exp(i\gamma_{1,2}z')\mu_{1,2}(z')dz'\Big], \tag{17}$$

where

$$\gamma_1=\frac{p_z}{\hbar}\left[1-\frac{2\hbar\omega}{v_z p_z}+\left(\frac{\hbar\omega}{cp_z}\right)^2\right]^{1/2},\quad \mu_1=-\frac{2p_y}{c\hbar^2}eA_y\exp\left(\frac{i}{\hbar}p_z z-i\omega t\right),$$
$$\gamma_2=\frac{p_z}{\hbar}\left[1+\frac{2\hbar\omega}{v_z p_z}+\left(\frac{\hbar\omega}{cp_z}\right)^2\right]^{1/2},\quad \mu_2=-\frac{2p_y}{c\hbar^2}eA_y^*\exp\left(\frac{i}{\hbar}p_z z+i\omega t\right). \tag{18}$$

In the case of the inequality

$$c^2p_z^2\ll 2E\hbar\omega-(\hbar\omega)^2, \tag{19}$$

the particle may be captured by the interface, for in this case γ_1 becomes complex. We shall not consider this case here, assuming that the inverse inequality is satisfied,

$$c^2p_z^2\gg\pm 2E\hbar\omega-(\hbar\omega)^2. \tag{20}$$

If $p_z/\hbar\gg\omega/c\beta_z$ (or $\cos^2\theta\gg\hbar\omega/E\beta^2$), then by calculating, for example φ_1 in (17), we can neglect the second integral, since such terms correspond to the Fourier component of the field $A_y(z,t)$ at $q=2p_z/\hbar$, which is a small purely quantum effect. Knowing the wave function, we can calculate the particle density:

$$\rho=|\psi|^2=\rho_0\left[1+2\frac{\Delta E'}{\hbar\omega}\sin\left(\frac{\hbar\omega^2(1-\beta_z^2)z}{2cE\beta_z^3}\right)\cos\omega\left(t-\frac{z}{c\beta_z}\right)\right], \tag{21}$$

where $\Delta E'$ is defined in (7). The spatial-modulation length is $4\pi cE\beta_z^3/\hbar\omega^2(1-\beta_z^2)$.

If $p_z\hbar\gtrsim 2\omega/c\beta_z$ or if $\pi/2-\theta\approx(2\hbar\omega/E\beta^2)^{1/2}$, i.e., if the particles "graze" along the boundary, both integrals must be taken into account in (17). The particle density is then

$$\rho=\rho_0\left[1+R_1\sin\frac{\hbar\omega^2z}{2cE\beta_z^3}\cos\omega\left(t-\frac{z}{c\beta_z}\right)+R_2\sin\left(\frac{2p_z}{\hbar}z\right)\cos\omega\left(t+\frac{z}{c\beta_z}\right)\right], \tag{22}$$

where $R_1=2\Delta E'/\hbar\omega$, and

$$R_2=-4\xi\left(\frac{\hbar\omega}{Mc^2}\right)^2\frac{1}{\beta^3}\frac{\sin\theta}{\cos^4\theta}n_1(n_1-n_2). \tag{23}$$

It is obvious that $R_1\sim R_2\sim\xi$, and the depth of modulation is large for strong fields. The beam is spatially modulated with a period $\pi\hbar/p_z=(c^2\hbar\pi^2/E\omega)^{1/2}$.

The results obtained by perturbation theory are valid if

$$\varphi_{1,2}\ll 1. \tag{24}$$

The modulation depth is $R_{1,2}\sim\varphi_{1,2}$. From the inequality (24) follows a restriction on the value of the field. In the first case, which is of practical interest, we have

$$\xi\ll\hbar\omega/E.$$

4. Let us examine the possibility of experimentally observing and using our results. If the wavelength is in microns and the wave power density Q is in W/cm^2, we obtain $\xi=10^{-9}\lambda\sqrt{Q}$.

It follows from (5)–(9) that $\Delta E/E\sim\xi$ (for ultrarelativistic electrons we have $\theta\sim Mc^2/E$). Regardless of the initial particle energy, to change this energy by 0.1% we must have $\xi\sim10^{-3}$, corresponding, for example, to a power density 10^{12} W/cm^2 for a neodymium-glass laser or 10^{10} W/cm^2 for a CO_2 laser.

Let the energy of the initial beam be 100 MeV, and then the number of absorbed photons is $\nu_0=\xi E/\hbar\omega\approx10^6$. Consequently, $\nu_0^{-1/3}=10^{-2}$, i.e., one percent of the electrons acquire the maximum possible energy, and this can be easily revealed by the energy broadening of the beam, if its initial width is less than $\Delta E'$. Although the effect is small on one boundary, it can be increased by using layered media. Calculations show that $\Delta E'$ increases in proportion to the number N of the boundaries crossed. The number of electrons having the maximum energy, however, decreases in this case like $N^{-1/3}$.

The inverse effect, stimulated emission, seems to us of greater interest from the point of view of amplifying the initial radiation. Within a unit time the electron beam loses a power $Q\sim\rho_0 c\beta_z\Delta E$, which amounts to 5×10^{12} W/cm^2 for a 100-MeV beam with particle density 10^{15} cm^{-3}, if the particle energy loss is one percent of the initial energy.

To observe the modulation effect in experiment, the particle beam must satisfy a number of the requirements obtained in Schwarz's studies; we shall not dwell on them here. We present numerical estimates of the fields, when the depth of modulation reaches 10%. In the nonrelativistic case, assuming that all the angular dependences make a contribution on the order of unity, $v_0/c=10^{-1}$, $\hbar\omega=1$ eV, we obtain $E_0=5\times10^5$ V/cm. In the case of moderate energies $E\sim Mc^2$ and θ not close to zero or $\pi/2$, we have $E_0=6\times10^2$ V/cm. Finally, in the ultrarelativistic case $\theta\sim Mc^2/E$, $E=10^9$ eV and $n_1=1$, $E_0=0.6$ V/cm.

If a layered medium is used, the effect increases: $R_1\propto N$. We note also the possibility of modulation at higher harmonics. The depth of modulation is of the order of $J_\nu(R_1)$, and the period of the spatial modulation decreases by a factor ν^2.

The authors are grateful to the participants of G. M. Garibyan's seminar for a useful discussion of the results.

[1]V. L. Ginzburg and I. M. Frank, Zh. Eksp. Teor. Fiz. **16**, 15 (1946).
[2]G. M. Garibyan, Makroskopicheskaya teoriya perekhodnogo izlucheniya (Macroscopic Theory of Transition Radiation), EFI **27**, 73 (1973).
[3]M. L. Ter-Mikaelyan, Vliyanie sredy na élektromagnitnye protsessy pri vysokikh energiyakh (Effect of Medium on Electromagnetic Processes at High Energies), Erevan, Izd. Akad. Nauk Arm. SSR, 1961.
[4]V. M. Arutyunyan and G. K. Avetisyan, Kvantovaya Elecktronika **7**, 34 (1972).
[5]V. M. Arutyunyan and G. K. Avetisyan, Zh. Eksp. Teor. Fiz. **62**, 1639 (1972) [Sov. Phys. JETP **35**, 854 (1972)].
[6]H. Schwarz and H. Hora, Appl. Phys. Lett. **15**, 349 (1969).

Translated by J. G. Adashko