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Spectrum of hydrogen plasma at the series limit

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A theory of the spectral intensity distribution near the series limit is proposed. It is based on the inclusion of subbarrier ionization of atoms from excited Stark sublevels in the statistical microfield of the plasma. It is found that spectral lines disappear when the ionization probability exceeds the radiative transition probability by two or three orders of magnitude. The transmission of the potential barrier is then still much less than unity and this means that perturbation theory can be used to calculate the line emission, and the sum of the oscillator strengths remains constant during the transformation of lines into the continuum. The latter results can be used to calculate the photocapture spectrum under the disappearing lines. The experimental results are compared with the predictions of simplified models of the spectrum near the series limit.

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Experiment shows (see[1–4] and elsewhere) that the spectral lines of hydrogen (and of other gases) become increasingly broad, overlap, and gradually merge into the continuum as the series limit is approached. The transition region, i.e., the region between the line and the continuous spectra, expands with increasing plasma density, and eventually covers the entire series. There is no satisfactory theory of this type of spectrum at present.[5–7] On the other hand, such a theory would be useful, firstly, because this spectral region may play an important role in radiative heat transfer and thus determine the conditions for the production and maintenance of plasma. Secondly, plasma emission, including emission at the series limit, is an important source of information in contactless diagnostics. Another important application is the development of a primary intensity standard based on the electric arc in hydrogen.[8] Finally, comparison of theoretical predictions with precision measurements would yield further information about the interaction between plasma particles.

The field ionization of excited atoms in the electric field of the charged plasma particles must be taken into account when the radiation at the series limit is calculated.[9] The external field (the plasma microfield) is then comparable with the internal atomic field experienced by an optical electron. This throws some doubt[7] on the validity of standard perturbation theory in this case. It is shown below that such reservations are unjustified, at least for the spectral lines. Insofar as the continuum is concerned, on the other hand, it is possible to construct a satisfactory model based on the constancy of the sum of oscillator strengths.

In this paper, which is a continuation of[10,11], a method is reported for taking into account the effect of the statistical microfield of plasma on spectral-line profiles,

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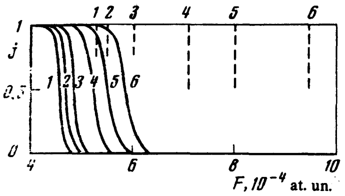

FIG. 1. Dilution coefficient for the Stark components $(nn_1n_2m)-(n'n_1'n_2'm')$ of H_β:

1–(4300)–(2100);
2–(4111)–(2001);
3–(4120)–(2100);
4–(4210)–(2010);
5–(4201)–(2001);
6–(4300)–(2100).

Broken lines represent fields corresponding to the removal of the upper sublevel.

the attenuation of their intensity as a result of field ionization, and the development of a recombinational continuum beyond the series limit.

1. FIELD IONIZATION OF EXCITED STATES

Spontaneous ionization of an atom becomes possible when an external electric field is applied to it. Using the results of the generalized WKB method,[12] and neglecting the "centrifugal" energy of the electron,[13] we find that the probability of a subbarrier transition in a uniform statistical field is given by (in atomic units)

$$S_\alpha(F)=\{8aF^{-1/2}[K(t)-E(t)][1+e^{2\varphi}]\}^{-1},$$
$$\varphi={}^1/_3aF^{1/2}[(a^2+b^2)E(q)-2b^2K(q)],\quad t=b/a, \tag{1}$$
$$\left.\begin{matrix}a^2\\ b^2\end{matrix}\right\}=F^{-1}[|E_\alpha|\pm(E_\alpha{}^2-4Z_{2\alpha}F)^{1/2}],\qquad q=(1-t^2)^{1/2}.$$

In these expression, F is the field strength, E_α is the energy of a Stark sublevel characterized by the parabolic quantum numbers nn_1n_2m, $Z_{2\alpha}$ is the separation parameter in the Schrödinger equation, and $K(x)$ and $E(x)$ are the complete elliptic integrals of the first and second kind, respectively. In contrast to the expression given in[13,14], the formula given by (1) is valid even when the thickness of the potential barrier is small.

Field ionization and radiative processes compete with one another. The decay of the initial state α occurs either via a transition to a state β with the emission of a photon of energy $\hbar\omega_{\alpha\beta}$, or through a radiationless subbarrier transition. The solution of the corresponding quantum-mechanical problem yields the following expression for the spectral-line attenuation coefficient[13,15]

$$j_{\alpha\beta}(F)=A_{\alpha\beta}(F)/[A_{\alpha\beta}(F)+S_\alpha(F)], \tag{2}$$

where $A_{\alpha\beta}$ is the radiative transmission probability. The spectral line will practically disappear when $S_\alpha \sim 100A_{\alpha\beta}\sim 10^{-7}$ atomic units. The ionization probability is then still smaller than unity by several orders of magnitude, the potential barrier is broad, and the atomic energy level is quasi-stationary.[14] It is precisely this that enables us to use standard perturbation theory to determine the matrix elements of the optical transitions and the energies of Stark sublevels as well as their populations in the usual way, right up to the complete attenuation of the spectral lines.

Further increase in the external field after the disappearance of the spectral line is accompanied by the removal of the energy level. This happens when the field ionization probability is of the order of unity. The corresponding critical field F_α is close to the ionizing field deduced from classical mechanics when the potential barrier thickness is zero [$a=b$ in (1)]. We then have

$$F_\alpha=E_\alpha{}^2/4Z_{2\alpha}. \tag{3}$$

Let us estimate the order of magnitude of the critical field $F_{\alpha\beta}$, which will quench the emission of the line spectrum. When the barrier transmission is small ($b\ll a$), Eq. (1) yields (see[13])

$$S_\alpha\sim\exp(-2^{5/2}|E_\alpha|^{3/2}/3F). \tag{4}$$

The field strengths will be determined from the condition $j_{\alpha\beta}=0.5$, i.e., $S_\alpha=A_{\alpha\beta}$, and this gives

$$F_{\alpha\beta}\sim2^{5/2}|E_\alpha|^{3/2}/3\ln A_{\alpha\beta}{}^{-1}. \tag{5}$$

Using the properties of the exponential (4) together with (5), we have

$$\Delta F/F_{\alpha\beta}\sim1/\ln A_{\alpha\beta}{}^{-1}\ll1. \tag{6}$$

Hence, it follows that the line attenuation coefficient (2) falls from unity to zero within a narrow range of variation of the field near the critical value. It is clear from (5) that the value of $F_{\alpha\beta}$ is a slowly-varying function of the radiative transition probability and decreases with increasing level number ($\sim n^{-3}$).

Figure 1 illustrates the steplike form of the attenuation coefficient j. The critical field quenching the emission increases monotonically from the red to the violet component of the line. Stark sublevels are removed when the field is higher by 10–50% (we recall that the barrier penetration then increases by several orders of magnitude).

In the case of field ionization of the atom, a photoionization continuum is produced as a result of the leakage of the wave function through the potential barrier. The associated level spread is then small: $\Delta E_\alpha\approx\hbar S_\alpha$ and, to the same accuracy, the sum of the oscillator strengths for all the possible transitions from the given quantum state is constant[14] even for the transformation of spectral lines into the continuum. The small value of the ratio $\Delta E_\alpha/E_\alpha$, and the absence of an effect of the external field on optical transitions between excited states, ensure that the increase in oscillator strengths for photoionization is equal to their reduction for bound-bound transitions, and occurs in the same spectral interval. The transformation of lines into a continuum occurs when the ionization time is of the order of the radiative transition time $A_{\alpha\beta}^{-1}\sim S_\alpha^{-1}(F_{\alpha\beta})$. When $F>F_{\alpha\beta}$,

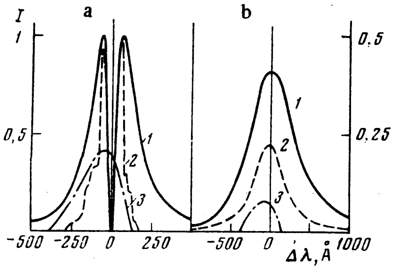

FIG. 2. H_β profile corresponding to the effect of ions (a) and ions and electrons (b). Curve 1—without dilution, 2—with field ionization, 3—the same according to the model in[9]; $P=50$ atm, $T=25\,000\,°K$, $N_e=6.3\times10^{18}$ cm^{-3}. The unit of intensity is $I=5.7\times10^8$ erg/Å · cm^3 · sr · sec.

the oscillator strength of a given line is fully realized in the continuum.

In plasma, the external electric field is, in effect, produced by the ions, and the characteristic time for a change in this field is much greater than the "orbital period" of an optical electron. The Coulomb fields due to the nearest electrons are subject to rapid variation and do not produce an effective reduction in the potential barrier.[16]

2. LINE EMISSION

The spectral-line profile due to broadening in the quasistatic uniform field of plasma ions and including the field ionization effect can be written in the form

$$I^i_{nn'}(\omega)\,d\omega=\sum_{\alpha,\beta} j_{\alpha\beta}(F)\,\hbar\omega_{\alpha\beta}A_{\alpha\beta}(F)\,N_\alpha(F)\,W(F)\left(\frac{\partial F}{\partial\omega}\right)\,d\omega. \tag{7}$$

In this expression, α, β are the Stark sublevels of the upper (n) and lower (n') quantum states, N_α is the sublevel population (Boltzmann population in equilibrium plasma), and F is the low-frequency component of the plasma microfield with a distribution function $W(F)$.1) Equation (7) differs from the usual expression[17] by the presence of the intensity attenuation coefficient due to subbarrier transitions. For the combined effect of ions and electrons (the latter are taken into account in the collision approximation with a half-width $\gamma_{\alpha\beta}$), we have[17]

$$I_{nn'}(\omega)=\sum_{\alpha,\beta}\int\limits_0^\infty j_{\alpha\beta}(F)\,\hbar\omega_{\alpha\beta}A_{\alpha\beta}(F)\,N_\alpha(F)\,W(F)\frac{\gamma_{\alpha\beta}}{2\pi}[(\omega-\omega_{\alpha\beta})^2+(\gamma_{\alpha\beta}/2)^2]^{-1}\,dF. \tag{8}$$

When we calculate the Balmer series emission, we shall confine our attention to the approximation which is linear in the field F,[14,18] but this is unimportant insofar as the magnitude of the attenuation coefficient and the general appearance of the spectrum are concerned. The microfield function is taken from[19] and the broadening of Stark components by electrons is assumed to be the same[20,21]:

$$\begin{gathered}\gamma_{\alpha\beta}\approx\gamma_{nn'}=16N_e v_e\rho_0^2[0.09+\ln(d/\rho_0)],\\ d=(kT/4\pi e^2N_e)^{1/2},\quad \rho_0={}^2/_3 v_e^2(\hbar/m)^2B_{nn'},\\ B_{n,\,n'=2}={}^9/_4(n^4-9n^2+12),\end{gathered} \tag{9}$$

where N_e, T, and v_e are, respectively, the density, temperature, and mean thermal velocity of electrons, and d and ρ_0 are the Debye and Weisskopf lengths. The summation and integration in (8) can be interchanged in view of (9), and this enables us to write

$$I_{nn'}(\omega)=(\gamma_{nn'}/2\pi)\int\limits_{-\infty}^{\infty}I^i_{nn'}(\omega)\,[(\omega-\omega')^2+(\gamma_{nn'}/2)^2]^{-1}\,d\omega', \tag{10}$$

where $I^i_{nn'}(\omega)$ is given by (7).

Figure 2 gives an example of a calculation of a partially dissolved H_β in equilibrium hydrogen plasma. It is clear that field ionization leads to a substantial change in the shape and width of the isolated line. It is important, however, to note that, in the resultant spectrum of the plasma, the observed lines are only slightly diluted (see Fig. 4 below) and this deformation of the profile is less important for such lines.

The theory of dilution of spectral lines in plasma was first considered in[9], but in a more approximate formulation. In the model used in[9], the spectral line in a field F was approximated by a rectangle, the width of which was equal to the separation between the extreme components in the linear Stark effect, and the area under the line was taken to be equal to the line intensity. In accordance with the effective (for the entire line) attenuation coefficient, a fraction of the rectangle is cut off on the long-wave side, and the truncated rectangles are averaged over the microfield distribution. Broadening by electrons is taken into account by doubling the concentration of ions when the field strength is calculated. This approach leads, among other things, to a large shift of the line toward the violet and, when the effect of ions and electrons is added, to a large dilution of the line (see Fig. 2).

3. PHOTORECOMBINATION

To determine the photocapture intensity beyond the series limit, we use the constancy of the sum of oscillator strengths. The inclusion of the transfer of oscillator strengths to bound-free transitions from each Stark component appears to be too difficult. We shall calculate below the loss of oscillator strength in a given spectral interval for the line series as a whole. The photorecombination intensity was obtained on the assumption of an equal increase for bound-free transitions in the same spectral interval.[5]

At a frequency ω, the attenuation coefficient for the line emission, averaged over the series, can be defined in accordance with (2) as follows:

$$\begin{gathered}j_{kn'}(F)=A_{kn'}/[A_{kn'}+S_k(F)],\\ A_{kn'}=(16\alpha_0^3/3\sqrt{3}\pi)\,[n'k^5(k^2-n'^2)]^{-1},\\ k=[2(1/2n'^2-\omega)]^{-1/2},\end{gathered} \tag{11}$$

where $A_{kn'}$ is the average radiative transition probability,[22] α_0 is the fine-structure constant, and $S_k(F)$ is the spontaneous ionization probability. To calculate the last quantity, we must substitute $E_\alpha=-1/2k^2$ in (1); the method of specifying $Z_{2\alpha}$ is indicated below.

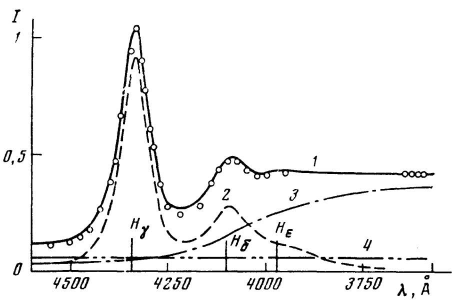

FIG. 3. Fragment of the spectrum of hydrogen plasma: 1—resultant intensity, 2—spectral lines, 3—recombinational continuum ($n'=2$), 4—bremsstrahlung and recombinational continuum ($n'=3$–8). Points—experiment in[1]; $P=1$ atm, $T=16\,250\,^\circ$K, $N_e=1.7\times10^{17}$ cm^{-3}, $I=1.5\times10^6$ erg/Å·cm^3·sr·sec.

In accordance with the foregoing, the photoionization cross section at frequencies between ω and $\omega+d\omega$ in a field F is given by

$$\sigma_{n'}(\omega, F)=\sigma_{n'}^{0}(\omega)\,[1-j_{kn'}(F)], \tag{12}$$

where $\sigma^0_{n'}(\omega)$ is the photoabsorption cross section for a level n' (bound–bound transition), which is numerically equal to the photoionization cross section continued analytically across the series limit.[22] Having averaged (12) over the plasma microfield distribution, we obtain

$$\sigma_{n'}(\omega)=\sigma_{n'}^{0}(\omega)\int_0^\infty[1-j_{kn'}(F)]\,W(F)\,dF. \tag{13}$$

Transforming to the intensity, we finally have

$$I_{n'}(\omega)=I_{n'}^{0}(\omega)\int_0^\infty[1-j_{kn'}(F)]\,W(F)\,dF, \tag{14}$$

where $I^0_{n'}(\omega)$ is the photorecombination intensity continued analytically across the series limits. According to (14), in which there is no restrictions on the minimum quantum energy, the intensity of the recombinational continuum decreases smoothly to zero as we pass through the series limit in the direction of lower frequencies, whereas, beyond the series limit (where $j\equiv0$), the intensity remains unaltered. In view of the steplike behavior of the attenuation coefficient near $j_{kn'}=0.5$, we can rewrite (14) in the form

$$I_{n'}(\omega)=I_{n'}^{0}(\omega)\int_{F_{kn'}}^\infty W(F)\,dF, \tag{15}$$

where $F_{kn'}$ is found from the condition $S_k(F_{kn'})=A_{kn'}$.

The remaining undetermined parameter $Z_{2\alpha}$ is obtained by demanding that the resultant intensity of the spectral lines and the recombinational continuum at the frequency ω_* of the last line [see (23)] be equal to $I^0_{n'}(\omega_*)$.

4. COMPARISON WITH EXPERIMENT

To check on the above model, we have carried out a comparison with the precision measurements reported by Behringer.[1] Behringer investigated the plasma in the electric arc in hydrogen at atmospheric pressure. The temperature and density of the plasma particles were measured by different methods and it was found that local thermodynamic equilibrium prevailed at temperatures $T\approx16\,500\,^\circ$K. A comparison between theory and experiment is shown in Fig. 3. It is clear that complete agreement between theoretical and experimental spectra is achieved. Figure 3 also shows the spectral components. As the series limit is approached, the intensity of lines (due to subbarrier ionization) is found to vanish. The resulting recombinational continuum ($n'=2$) is found to be weakened in the opposite direction as the line dilution is reduced. The combined operation of these two mechanisms is, in fact, responsible for the continuous transition from the line spectrum to the continuous spectrum.

5. SIMPLIFIED MODEL

Calculation of the spectrum from the above formulas becomes laborious when there is a large number of undissolved lines. We shall therefore now describe an approximate scheme based on the following principle (see (see[23]). The spectral-line profiles $I^0_{nn'}(\omega)$ are calculated under the combined operation of plasma ions and electrons, initially without taking field ionization into account. The necessary extensive tabulations and graphs for hydrogen can be found in the literature (see[6,7] and elsewhere). Next, the true line intensity $I_{nn'}(\omega)$, corrected for dilution, is found by multiplying $I^0_{nn'}(\omega)$ by the effective attenuation coefficient $j_{n'}(\omega)$:

$$I_{nn'}(\omega)=j_{n'}(\omega)\,I^0_{nn'}(\omega). \tag{16}$$

The recombinational continuum is determined from the following formula [see (15)]:

$$I_{n'}(\omega)=[1-j_{n'}(\omega)]\,I_{n'}^{0}(\omega). \tag{17}$$

The attenuation coefficient is taken to be

$$\begin{gathered} j_{n'}(\omega)=\int_0^{F_{kn'}} W(F)\,dF,\\ F_{kn'}=E_k^2/4=1/16k^4,\quad k\in[n',\ \infty],\\ k=[2(1/2n'^2-\omega)]^{-1/2}. \end{gathered} \tag{18}$$

The critical field $F_{kn'}$ in (18) is close in value to the field that cuts off the lower Stark sublevel [see (3) with $Z_{2\alpha}\approx1$] and is numerically equal to the field for which the top of the potential barrier of the atom in the direction of the field coincides with the level k.[24] Analogous formulas for the recombinational continuum are given in[25–27] and for spectral lines in[27,28].

In practice, the calculation of the lines ends at a finite value of n [see (23)]. The intensity of highly overlapped lines, which is not taken into account in this procedure, must be compensated by an equivalent contribution of the recombinational continuum. This can be done by transforming $F_{kn'}$ in (18) in the course of the photorecombination calculations, as follows:

$$F_{kn'}=c/16k^4,\quad k\in[n',\ n_*], \tag{19}$$

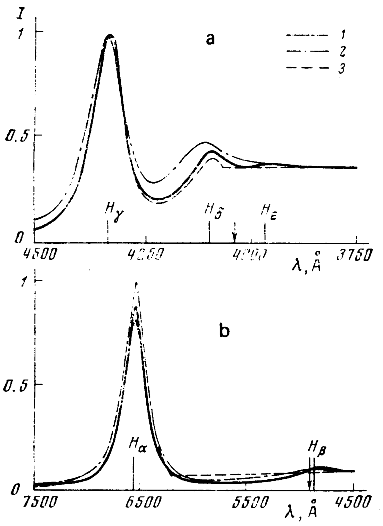

FIG. 4. Balmer series in equilibrium plasma: 1—detailed calculation, 2—simplified model, 3—method from[31]. Arrow shows the series limit shifted according to[30]. a) $P = 1$ atm, $T = 16\,250\,°$K, $N_e = 1.7\times10^{17}$ cm^{-3}, $I = 1.5\times10^6$ erg/Å·cm^3·sr·sec; b) $P = 50$ atm, $T = 25\,500\,°$K, $N_e = 6.3\times10^{18}$ cm^{-3}, $I = 3\times10^9$ erg/Å·cm^3·sr·sec.

where c is determined by analogy with the procedure used for $Z_{2\alpha}$ in Sec. 3. This balancing of the spectrum improves the agreement with detailed calculations.

Figure 4 shows examples of calculations based on the simplified model and compares them with the more rigorous calculations. The approximate scheme yields a satisfactory picture of the spectrum as a whole, despite the fact that the individual lines are shifted asymmetrically toward longer wavelengths. The attenuation coefficient $\bar{j}$ for a line as a whole is, in this scheme, close to that obtained by more detailed calculations (see Table I).

Figure 4 indicates the series limit shifted in accordance with the data in[29,30]. The number of observed lines estimated in[30] is practically the same as that obtained from the model put forward here. However, if, in the calculation of the spectrum, we confine our attention only to these lines (without dilution), we have an intensity discontinuity across the shifted series limit at which the recombinational continuum starts. It was suggested in[31] that this discontinuity could be removed by extrapolating the photocapture intensity curve until it cuts one of the last lines. This procedure for constructing the spectrum (Fig. 4) is satisfactory in the case of a large number of undiluted lines, but it is less reliable when the plasma density is high, and this may complicate diagnostics based on the actual lines present. Moreover, the question as to which radiative processes are responsible for the continuous transition from lines to the continuum remains open in[31].

Guendel[28] has measured the dilution coefficient in an electric arc for a number of spectral lines of argon as a function of the concentration of charged plasma particles. To achieve agreement between theoretical dilution (18) and the observed values, he was forced to introduce a factor ~0.1 on the right-hand side of the expression for $F_{kn'}$. This indicates that the critical fields that transform the argon lines into the continuum are anomalously low. This may be the explanation of the fact reported in[32] that the measured continuum intensity in the case of argon is much higher than predicted by calculation.

6. RENORMALIZATION OF THE ATOMIC PARTITION FUNCTION

The ionic microfield of the plasma determines both the line broadening and the weakening of the lines through subbarrier ionization. The same mechanisms restricts the number of bound states and the electronic partition function Σ_a of the atom. Any model for restricting the latter can be described by the formula

$$\Sigma_a = \sum_{n=1}^{\infty} j_n q_n \exp[-(U+E_n)/kT], \qquad (20)$$

where j_n is the correction factor ensuring the convergence of the series, U is the ionization energy, and q_n is the statistical weight of the unperturbed state of the atom. Below, we give a definition of j_n based on the inclusion of the statistical plasma microfield and, in this sense, providing the same approach as the above model of emission at the series limit.

We shall suppose, for simplicity, that the critical field F_n for which a level n is removed is equal to the corresponding value for an average Stark sublevel ($Z_{2\alpha} = 0.5$).[33] Using (3), we have

$$F_n = E_n^2/2 = 1/8n^4. \qquad (21)$$

The correction to the statistical weight of a level in plasma is[23,27,33–35]

$$j_n = \int_0^{F_n} W(F)\,dF. \qquad (22)$$

The quantity j_n tends to zero rapidly and monotonically as the principal quantum number increases.

The effective number n_* of levels realized in the plasma can be estimated from (21) by replacing F_n with the "normal" plasma field F_0:

TABLE I. Dilution coefficient for Balmer lines. *

Line	$\bar{j}$	
	$P=1$ atm	$P=50$ atm
H_α	1/1	0.80/0.80
H_β	1/1	0.40/0.42
H_γ	0.88/0.94	0.18/0.18
H_δ	0.75/0.65	
H_ε	0.45/0.40	
H_φ	0.17/0.16	

*The table lists values of the ratio of the detailed calculations to the simplified model. The parameters are the same as in Fig. 4.

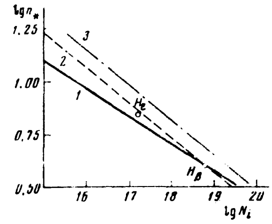

FIG. 5. Dependence of the number of the limiting level on ion density in plasma: 1—Inglis–Teller model, 2—equation (18) with $F_{kn'} = F_0$, 3—estimate based on (23), o—tabulated lines with dilution ~0.5.

$$n_* = (1/8F_0)^{1/4}, \quad F_0 = 2.6031 N_i^{2/3}, \tag{23}$$

where N_i is the concentration of ions. We note that the thermodynamic reduction in the ionization energy of an atom in plasma is not equal to E_{n_*}. The composition and pressure of the plasma were calculated by the balancing method.[36]

A comparison between (23) and the Inglis-Teller model[29] is shown in Fig. 5. Estimates based on (18) correspond to the removal of the lower Stark sublevel and should approach the result obtained from the removal of the intermediate Stark components n_*. The latter corresponds to an overall line dilution of ~0.5, and this is confirmed by the disposition of the points on the graph. It is clear from the figure that the overlap of lines according to[29] occurs somewhat in advance of the dilution process, and the removal of levels lags behind the above effects.

7. DETERMINATION OF THE MICROFIELD FUNCTION

The plasma microfield distribution is at present determined theoretically or by the Monte Carlo method. The correctness of the function $W(F)$ for a quasi-ideal plasma (see, for example,[19]) is indirectly confirmed by the agreement between the calculated and measured profiles of isolated spectral lines. We know of no attempts to determine $W(F)$ from experimental data.

The function $W(F)$ can, at least in principle, be determined from the profile of spectral lines, but this gives rise to considerable difficulties. The line profile (8) is determined by a large number of field-dependent Stark components, and by electron broadening. Moreover, in the case of the quasi-ideal plasma, which is the most difficult from the theoretical point of view, the spectral lines are substantially diluted.

To overcome the above difficulties, we propose to determine $W(F)$ from the recombinational continuum beyond the series limit (see[37]). In fact, we have from (15)

$$W(F) = -\frac{\partial \hat{I}_{n'}}{\partial \omega}\frac{\partial \omega}{\partial F}, \quad \hat{I}_{n'}(\omega) = \frac{I_{n'}(\omega)}{I_{n'}^{0}(\omega)}. \tag{24}$$

We first use a trial function $W_0(F)$ to calculate the spectral lines, and then subtract from them the resultant experimental spectrum. This yields the recombinational continuum beyond the series limit, the derivative of which, (24), yields $W_1(F)$ for the next approximation. The iteration process is repeated when the difference between W_1 and W_0 is large.

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APPENDIX

The momentum of an electron in terms of the parabolic coordinate η is given by

$$g_\alpha(\eta) = \left[\frac{E_\alpha}{2} + \frac{Z_{2\alpha}}{\eta} + \frac{F\eta}{4} - \frac{(m^2-1)}{\eta^2}\right]^{1/2}. \tag{A.1}$$

The ionization probability is determined by the product of the transmission of the potential barrier and the frequency with which the electron collides with the barrier:

$$S_\alpha(F) = D_\alpha(F)/T_\alpha(F), \tag{A.2}$$

where

$$T_\alpha(F) = 2\int_{\eta_1}^{\eta_2} d\eta/g_\alpha(\eta) \tag{A.3}$$

is the period of oscillations between the turning points η_1 and η_2. According to[12], the transmission is given by

$$D_\alpha(F) = \left[1 + \exp\left\{2\int_{\eta_2}^{\eta_3} |g_\alpha(\eta)|\, d\eta\right\}\right]^{-1}. \tag{A.4}$$

The formula is valid for $n^2 \gg 1$ and any thickness of the potential barrier, including $\eta_3 = \eta_2$. When this inequality is satisfied, we can neglect the last term in (A.1) and hence $\eta_1 = 0$. Using the parameters a, b defined in (1), and assuming that

$$\eta = t^2, \quad \eta_2 = b^2, \quad \eta_3 = a^2,$$

we obtain the following expression for the integrals in (A.3) and (A.4):

$$\int_0^{\eta_2} \frac{d\eta}{g_\alpha(\eta)} = \frac{4}{\sqrt{F}}\int_0^b \frac{t^2 dt}{[(a^2-t^2)(b^2-t^2)]^{1/2}} = \frac{4a}{\sqrt{F}}[K(t) - E(t)],$$

$$\int_{\eta_2}^{\eta_3} |g_\alpha(\eta)|\, d\eta = \sqrt{F}\int_b^a [(a^2-t^2)(t^2-b^2)]^{1/2} dt = \frac{a\sqrt{F}}{3}[(a^2+b^2)E(q) - 2b^2K(q)].$$

Substitution of these expressions in (A.2)–(A.4) leads to (1).

[1])It is assumed in the derivation of (7) that the microfield is quasistatic with respect to the ionization process, i.e., $S_\alpha(F) \gg v_i N_i^{1/3}$, where N_i, v_i are, respectively, the density and mean thermal velocity of ions. The validity of this assumption is justified by the following considerations. The region ΔF for which $S_\alpha \sim A_{\alpha\beta}$ is very small, according to (6). When $F < F_{\alpha\beta} - \Delta F$, we have $S_\alpha \ll A_{\alpha\beta}$, and violation of the

static condition is unimportant. When $F \approx 1.2F_{\alpha\beta} - 1.3F_{\alpha\beta}$, the Stark sublevels are removed (Fig. 1). The static condition is then satisfied with a margin of 3–5 orders of magnitude. Hence, one would expect the above condition to be violated for $\Delta F \sim 0.1F_{\alpha\beta}$. The fact that $\Delta F/F_0$ is small [F_0 is the "normal" field of the distribution $W(F)$] indicates that the region of violation of the static condition is small in relation to the entire profile given by (7). When $F_{\alpha\beta} \gg F_0$, this region is large but the attenuation effect itself is then small. For the example of H_β, which is considered below with the parameter values shown in Fig. 2, we have $\Delta F/F_0 \sim 0.05$.

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