Quantum oscillations of the magnetic moment in a nonequilibrium system with electron-hole pairing

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Phenomena connected with the coherence of the phase of the wave function of an electron-hole pair in a non-equilibrium semiconductor with electron-hole pairing are considered. It is shown that when a voltage is applied to a semiconductor-insulator-semiconductor structure in which optical pumping is produced by an external source, oscillations of the magnetic moment are produced in the quasistationary state when the conditions needed for the realization of electron-hole pairing are realized. The frequency of these oscillations is $=2V$, where $V=V_r-V_l$, $V_r=(1/2)D_r+\mu_r$. $V_l=(1/2)D_l+\mu_l$, D_l and D_r are the widths of the forbidden bands of the left-hand and right-hand semiconductors, respectively, while μ_l and μ_r are the Fermi degeneracy energies of the corresponding semiconductors.

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1. Much attention has been paid recently to an investigation of high-density nonequilibrium carriers in semiconductors.[1] If the lifetimes of the electrons and holes are long enough, these systems lend themselves to a quasi-equilibrium description, wherein thermodynamic equilibrium is established in each band separately within a time much shorter than the carrier lifetime. Allowance for the Coulomb interaction of the electrons and holes from different bands leads to the possibility of the formation of electron–hole pairs in the system below a certain temperature T_c, at which the system undergoes a second-order phase transition. We note, however, that the phase produced in the transition differs from the excitonic dielectric which is produced by Coulomb interaction of the electrons and holes in a semimetal,[2,3] and in which the character of the Bose condensation of the electron–hole pairs in the quasi-equilibrium semiconductors is determined only by a Coulomb interaction of the density–density type.[1] As a result, in a quasi-equilibrium semiconductor, just as in the case of a superconductor, the phase of the order parameter is indeterminate below the point T_c, and this leads to the existence of specific quantum effects in such systems.

From the macroscopic point of view, the new coherent state into which the quasi-equilibrium system of electrons and holes goes over below T_c is characterized on the whole by the existence of a certain generally speaking complex function $\psi=fe^{i\varphi}$, which has the meaning of the electron–hole pair. The fact that there exists for the entire sample a single function ψ that characterizes the entire ensemble of particles makes it possible for phase differences φ, which are fixed at a given instant of time, to occur between two arbitrary points of the crystal. It is precisely this phase coherence which produces in a quasi-equilibrium semiconductor–insulator–semiconductor (SIS) tunnel structure electric-current oscillations[4] analogous to Josephson oscillations in superconducting tunnel junctions. This result is not trivial because, in contrast to a Cooper pair, the charge of which is $2e$, the total charge of the electron–hole pair is zero.

Coherent tunneling of electron-hole pairs and the associated energy transport in a non-equilibrium system was considered also in[4]. We note, however, that the tunneling of the particles can be accompanied by the transport of not only charge and energy, but also spin, since each tunneling particle has spin 1/2. Furthermore, the presence of a Bose condensate of electron-hole pairs in quasi-equilibrium semiconductors gives grounds for hoping that the spin tunneling will be accompanied by a number of specific effects, the investigation of which is in fact the subject of the present communication.

It will be shown below that when a potential difference is applied to an S-I-S tunnel structure in which an external source produces optical pumping, oscillations of the magnetic moment are produced in the quasi-stationary state if the conditions necessary for the realization of electron–hole pairing are satisfied. The frequency of these oscillations is

$$\omega=2V,$$
$$V=V_r-V_l,\quad V_r={}^1/_2D_r+\mu_r,\quad V_l={}^1/_2D_l+\mu_l, \tag{1}$$

D_r and D_l are the widths of the forbidden bands of the right and left semiconductors, respectively, while μ_r and μ_l are the Fermi degeneracy energies of the corresponding semiconductors. There is no ferromagnetic ordering in the two semiconductors.

2. We consider an S-I-S tunnel structure in which an external field has produced a stationary carrier density. We confine ourselves for the time being to high carrier density in the bands, when a substantial restructuring of the spectrum takes place in a narrow energy layer at the Fermi surface. This is the simplest case from the mathematical point of view, in view of the formal analogy with superconductivity theory. Just as before,[4] to investigate the phenomena that occur in the tunnel structure we use an approach based on the tunnel-Hamiltonian method.[5] As applied to theory of the Josephson effect, this approach was developed by many authors and is described in detail in the book by Kulik and Yanson.[6]

In the tunnel-Hamiltonian method, the tunnel junction of two semiconductors is regarded as a weakly-coupled system described by a Hamiltonian that consists in the zeroth approximation of two parts corresponding to isolated semiconductors—one on the right and the other on the left:

$$H_0=H_{r0}+H_{l0}, \tag{2}$$

and containing in the next approximation a term

$$H_W=\iint W_{lr}(\mathbf{rr}')(\psi^+_{1r\alpha}(\mathbf{r})\psi_{1l\alpha}(\mathbf{r}')+\psi^+_{2r\alpha}(\mathbf{r})\psi_{2l\alpha}(\mathbf{r}'))\,d\mathbf{r}\,d\mathbf{r}'+\text{h.c.}, \tag{3}$$

that describes the tunneling of the electrons from one semiconductor to the other (we consider tunneling without spin flip). Here $\psi^+_{1r\alpha}$ and $\psi_{2r\alpha}$ are the electron creation and annihilation operators in the electron and hole bands of the right-hand semiconductor, and $\psi^+_{1l\alpha}$ and $\psi_{2l\alpha}$ are the corresponding operators of the left-hand semiconductor.

We assume for the semiconductors making up the tunnel structure a simple model with isotropic dispersion laws in the electron and hole bands. We assume for simplicity that the extrema E_1 and E_2 of the electron and hole bands are located at the point $\mathbf{p}=0$ of momentum space and, in addition, the electron and hole masses are equal. Then we can write, for example, for the right-hand semiconductor

$$E_{1r}(\mathbf{p})=\frac{\mathbf{p}^2}{2m}+\frac{1}{2}D_r=\frac{\mathbf{p}^2}{2m}-\mu_r+\left(\frac{1}{2}D_r+\mu_r\right)=\varepsilon(\mathbf{p})+V_r, \tag{4}$$
$$E_{2r}(\mathbf{p})=-\frac{\mathbf{p}^2}{2m}-\frac{1}{2}D_r=-\frac{\mathbf{p}^2}{2m}+\mu_r-\left(\frac{1}{2}D_r+\mu_r\right)=-\varepsilon(\mathbf{p})-V_r,$$

where D_r is width of the forbidden band, μ_r is the Fermi degeneracy energy of the electrons and holes of the right-hand semiconductor, and $\varepsilon(\mathbf{p})=\mathbf{p}^2/2m-\mu_r$.

Just as before,[4] in the investigation of the case of high density $na_0^3\gg 1$ (n is the carrier density and a_0 is the exciton Bohr radius) we use a model similar to the BCS model in superconductivity theory, inasmuch as in this case the Coulomb interaction between the electrons and the holes is strongly screened, as a result of which only carriers situated in a narrow layer of momentum space near the Fermi surface take part in the production of the electron-hole pairs. In this model the interaction between the electrons and the holes is described by a certain effective interaction Hamiltonian which, for example in the right-hand semiconductor, can be written in the form

$$H_g=g_1\int\psi^+_{2r\alpha}(\mathbf{r})\psi_{2r\alpha}(\mathbf{r})\psi^+_{1r\beta}(\mathbf{r})\psi_{1r\beta}(\mathbf{r})\,d\mathbf{r}$$
$$+g_2\int\psi^+_{2r\alpha}(\mathbf{r})\sigma_{\alpha\gamma}\psi_{2r\gamma}(\mathbf{r})\psi^+_{1r\beta}(\mathbf{r})\sigma_{\beta\delta}\psi_{1r\delta}(\mathbf{r})\,d\mathbf{r}, \tag{5}$$

where g_1 and g_2 are the constants of the Coulomb and exchange interactions, respectively, with energy cutoff at the frequency $\omega_0\ll\mu_r$; $\psi_{1r\alpha}$ and $\psi_{2r\alpha}$ are the electron operators in the electron and hole bands, while $\sigma_{\alpha\gamma}$ is a vector whose commutators are Pauli matrices.

Just as in[2], we do not include in the Hamiltonian the interaction of carriers within the same band. In addition, we have left out of (5) the interaction terms corresponding to the transformation of electrons of one band into electrons of another,[1] inasmuch as the states in the different bands are separated in energy by an amount $D_r\gg me^4/\varkappa^2\hbar^2$. As a result, the Hamiltonian of the right-hand semiconductor takes the form

$$H_{r0}=\int(\varepsilon(\hat{\mathbf{p}})+V_r+U)\psi^+_{1r\alpha}(\mathbf{r})\psi_{1r\alpha}(\mathbf{r})\,d\mathbf{r}-\int(\varepsilon(\mathbf{p})+V_r-U)\psi^+_{2r\alpha}(\mathbf{r})\psi_{2r\alpha}(\mathbf{r})\,d\mathbf{r}$$
$$+(g_1\delta_{\alpha\gamma}\delta_{\beta\delta}+g_2\sigma_{\alpha\gamma}\sigma_{\beta\delta})\int\psi^+_{2r\alpha}(\mathbf{r})\psi_{2r\gamma}(\mathbf{r})\psi^+_{1r\beta}(\mathbf{r})\psi_{1r\delta}(\mathbf{r})\,d\mathbf{r}. \tag{6}$$

The letter U denotes here the applied potential difference (the contact potential difference between the semiconductors is assumed to be zero). The expression for H_{l0} is similar.

The statement that a bound state of an electron and a hole exists in a system with Hamiltonian (6) was made in[2,7]. There, however, the reference was to a semimetal with overlapping bands, and it is therefore natural to raise the question whether the conclusions of [2,7] apply to the system considered by us. The answer is in the affirmative, since the lifetime of the nonequilibrium carriers in a semiconductor is usually long enough in comparison with the time required to establish thermodynamic equilibrium in each band separately,[1] and also with time of the electron-hole pairing accompanied by the restructuring of the spectrum. In this case, our system becomes fully equivalent to that considered earlier,[2,7] and the appearance of terms with V_r and U in the Hamiltonian leads to no physical effects whatever (so long as tunnel transitions are disregarded), and their influence reduces to the appearance of phase factors in the particle creation and annihilation operators and in the Green's functions.[8]

3. We derive an expression for the current due to the electron magnetic moment from semiconductor to semiconductor. In the tunnel-Hamiltonian method, this quantity is determined from the rate of change of the magnetic moment of the electrons in one of the semiconductors, say on the right,

$$\mathbf{J}_M(t)=\langle\dot{\mathbf{M}}_r(t)\rangle, \tag{7}$$

where $\dot{\mathbf{M}}_r(t)$ is the operator of the rate of change of the magnetic moment in the right-hand semiconductor, written in the Heisenberg representation. The averaging in (7) is over an equilibrium Gibbs canonical ensemble with Hamiltonian $H_0=H_{l0}+H_{r0}$, which conserves the number of the electrons, and consequently also the electronic magnetic moment in each semiconductor. The expression for $\dot{\mathbf{M}}_r$ obviously takes the form

$$\mathbf{M}_r=\mu_0\,\mathrm{Sp}\left\{\sigma_{\alpha\beta}\left[\int\psi^+_{1r\beta}(\mathbf{r})\psi_{1r\alpha}(\mathbf{r})\,d\mathbf{r}+\int\psi^+_{2r\beta}(\mathbf{r})\psi_{2r\alpha}(\mathbf{r})\,d\mathbf{r}\right]\right\}, \tag{8}$$

where $\mu_0=e\hbar/2mc$. The procedure for deriving the formula for the tunnel current is well known (see, e.g.,[6,8]) albeit laborious, since it is necessary to operate with cumbersome expressions. We shall therefore not present here all the details of the calculations, and write out only the final results.

The expression for the magnetic-moment current is

$$\mathbf{J}_M(t)=\mathbf{I}_1\sin 2Vt+\mathbf{I}_2\cos 2Vt+\mathbf{I}_3, \tag{9}$$

where $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$ can be represented as follows,

$$\mathbf{I}_1=\frac{\mu_0}{e}R^{-1}\operatorname{Re}\iint_{-\infty}^{+\infty} i\left[\frac{\operatorname{Im}(G_{21l\alpha\gamma}(\omega_1)\sigma_{\alpha\beta}G_{12r\gamma\beta}(\omega_2))}{\omega_2-\omega_1+V+U+i\delta}-\frac{\operatorname{Im}(G_{12l\alpha\gamma}(\omega_1)\sigma_{\alpha\beta}G_{21r\nu\beta}(\omega_2))}{\omega_2-\omega_1-V+U+i\delta}\right]\frac{d\omega_1}{2\pi}\frac{d\omega_2}{2\pi}, \quad (10)$$

$$\mathbf{I}_2=-\frac{\mu_0}{e}R^{-1}\operatorname{Re}\iint_{-\infty}^{+\infty}\left[\frac{\operatorname{Im}(G_{21l\alpha\gamma}(\omega_1)\sigma_{\alpha\beta}G_{12r\gamma\beta}(\omega_2))}{\omega_2-\omega_1+V+U+i\delta}+\frac{\operatorname{Im}(G_{12l\alpha\nu}(\omega_1)\sigma_{\alpha\beta}G_{21r\gamma\beta}(\omega_2))}{\omega_2-\omega_1-V+U+i\delta}\right]\frac{d\omega_1}{2\pi}\frac{d\omega_2}{2\pi} \quad (11)$$

$$\mathbf{I}_3=\frac{\mu_0}{e}R^{-1}\operatorname{Re}\iint_{-\infty}^{+\infty}\left[\frac{\operatorname{Im}(G_{1l\alpha\gamma}(\omega_1)\sigma_{\alpha\beta}G_{1r\gamma\beta}(\omega_2))}{\omega_2-\omega_1+V+U+i\delta}+\frac{\operatorname{Im}(G_{2l\alpha\gamma}(\omega_1)\sigma_{\alpha\beta}G_{2r\gamma\beta}(\omega_2))}{\omega_2-\omega_1-V+U+i\delta}\right]\frac{d\omega_1}{2\pi}\frac{d\omega_2}{2\pi}. \quad (12)$$

Here

$$G_{\alpha\beta}(\omega)=\int G_{\alpha\beta}(\omega\xi)\,d\xi$$

are the system Green's functions integrated with respect to energy, with $G_{1\alpha\beta}$ and $G_{2\alpha\beta}$ the usual Green's functions of the electrons in the electron and hole bands, respectively, while $G_{12\alpha\beta}$ and $G_{21\alpha\beta}$, which are analogous to the anomalous Gor'kov functions in the theory of superconductivity, correspond to the electron–hole pairing that exists in the system,[2,7] $R=[4e|W_{rl}|^2(mp_F^2/2\pi^2)^2]^{-1}$ is the resistance of the insulating layer between the normal semiconductors.

As follows from (9)–(12), the value of $\mathbf{J}_M(t)$ depends significantly on the spin structure of the Green's functions of the system, which in turn is determined by the character of the restructuring of the initial spectrum in the semiconductors that enter in the tunnel structure. Volkov, Kopaev, and Rusinov have shown[8] that, depending on the ratio of the constants g_1 and g_2, the Hamiltonian (6) admits, below a certain critical temperature T_c, of restructuring of the initial spectrum followed by formation of either singlet or triplet pairs, or else by coexistence of both types of electron-hole pairs. Accordingly, we shall consider in succession all the possible variants that can occur in our problem. The direction of the magnetic moment $\mathbf{M}_r$ is taken to be the z axis of the coordinate system, i.e., the integrals $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$ will take the form $\mathbf{I}_i=I_i\mathbf{n}_z$ ($i=1,2,3$, $\mathbf{n}_z$ is a unit vector along the z axis).

a) Singlet pairing is realized in both semiconductors. In this case $G_{\alpha\beta r(l)}=\delta_{\alpha\beta}G_{r(l)}$ and, as is easily seen from (10)–(12), all three integrals $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$ are equal to zero, i.e., $\mathbf{J}_M(t)\equiv 0$.

b) When triplet pairing takes place in both semiconductors the result is similar. Now $G_{\alpha\beta 1r(l)}=\delta_{\alpha\beta}G_{1r(l)}$, $G_{\alpha\beta 2r(l)}=\delta_{\alpha\beta}G_{2r(l)}$, $G_{\alpha\beta 12r(l)}=\sigma_{\alpha\beta z}G_{12r(l)}$, $G_{\alpha\beta 21r(l)}=\sigma_{\alpha\beta z}G_{21r(l)}$, but a simple calculation yields in this case $I_1=I_2=I_3=0$ and $\mathbf{J}_M(t)\equiv 0$. We note, however, that in both indicated cases an oscillating tunnel current arises in the structure, and its dependence on the structure parameters and on their applied voltage U is described by formulas (8)–(18) of [4].

c) We consider now the case when triplet pairs are produced in, say, the right-hand semiconductor and singlet pairs in the left one. Direct calculation yields the following result: the integral I_3 vanishes indentically, while the integrals I_1 and I_2 differ from zero and the expressions for them are

$$I_1=\frac{2\mu_0}{e}R^{-1}\Big\{\int_{-\infty}^{+\infty}\operatorname{th}\frac{\omega}{2T}(\operatorname{Im}G_{12l}^R(\omega)\operatorname{Re}G_{21r}^R(\omega_{1+})+\operatorname{Im}G_{21r}^R(\omega)\operatorname{Re}G_{12l}^R(\omega_{2-}))\,d\omega-\int_{-\infty}^{+\infty}\operatorname{th}\frac{\omega}{2T}(\operatorname{Im}G_{21l}^R(\omega)\operatorname{Re}G_{12r}^R(\omega_{1-})+\operatorname{Im}G_{12r}^R(\omega)\operatorname{Re}G_{21l}^R(\omega_{2+}))\,d\omega\Big\}, \quad (13)$$

$$I_2=\frac{2\mu_0}{e}R^{-1}\Big\{\int_{-\infty}^{+\infty}\Big(\operatorname{th}\frac{\omega}{2T}-\operatorname{th}\frac{\omega_{2+}}{2T}\Big)\operatorname{Im}G_{12r}^R(\omega)\operatorname{Im}G_{21l}^R(\omega_{2+})\,d\omega+\int_{-\infty}^{+\infty}\Big(\operatorname{th}\frac{\omega}{2T}-\operatorname{th}\frac{\omega_{2-}}{2T}\Big)\operatorname{Im}G_{21r}^R(\omega)\operatorname{Im}G_{12l}^R(\omega_{2-})\,d\omega\Big\}, \quad (14)$$

where the function G^R is an analytic continuation of the corresponding temperature Green's function on the real axis of the variable $i\omega_n$,[9] and the notation $\omega_{1\pm}=\omega\pm V-U$ and $\omega_{2\pm}=\omega\pm V+U$ has been introduced.

The formulas obtained are valid for both pure and doped semiconductors. We consider first semiconductors without impurities. In this case

$$G_1^R(\omega)=G_2^R(\omega)=-\frac{\omega}{(\Delta^2-\omega^2)^{1/2}},\quad G_{21}^R(\omega)=-G_{12}^R(\omega)=\frac{\Delta}{(\Delta^2-\omega^2)^{1/2}}. \quad (15)$$

Substituting these expressions in (13) and (14), we can obtain the explicit forms of the integrals I_1 and I_2. However, we shall not write them out because of the cumbersome resultant expressions. We shall dwell only on the final results.

Consider the case $T=0$. Then at $|U+V|$, $|U-V|<\Delta_l+\Delta_r$ the integral I_2 vanishes and the integral I_1 is equal to

$$I_1=(2\mu_0/eR)2\Delta_l\Delta_r\{K(x_{1-})[(\Delta_l+\Delta_r)^2-(U-V)^2]^{-1/2}-K(x_{1+})[(\Delta_l+\Delta_r)^2-(U+V)^2]^{-1/2}\} \quad (16a)$$

at $|U-V|,|U+V|<|\Delta_l-\Delta_r|$;

$$I_1=(2\mu_0/eR)\{2\Delta_l\Delta_r K(x_{1-})[(\Delta_l+\Delta_r)^2-(U-V)^2]^{-1/2}-(\Delta_l\Delta_r)^{1/2}K(x_{2+})\} \quad (16b)$$

at $|U-V|<|\Delta_l-\Delta_r|$, $|U+V|>|\Delta_l-\Delta_r|$;

$$I_1=(2\mu_0/eR)(\Delta_l\Delta_r)^{1/2}\{K(x_{2-})-K(x_{2+})\} \quad (16c)$$

at $|U+V|$, $|U-V|>|\Delta_l-\Delta_r|$, where $K(x)$ is a complete elliptic integral of the first kind

$$x_{1\pm}=\Big[\frac{(\Delta_l-\Delta_r)^2-(U\pm V)^2}{(\Delta_l+\Delta_r)^2+(U\pm V)^2}\Big]^{1/2},\quad x_{2\pm}=\Big[\frac{(U\pm V)^2-(\Delta_l-\Delta_r)^2}{4\Delta_l\Delta_r}\Big]^{1/2}.$$

At

$$\max(|U+V|,|U-V|)=\Delta_l+\Delta_r$$

a magnetic-moment current component appears jumpwise and is described by the integral I_2, while the integral I_1 has a singularity. The singular part of the integral I_1 is

$$I_1=\frac{2\mu_0}{eR}\frac{(\Delta_l\Delta_r)^{1/2}}{4}\ln\Big(\frac{\Delta_l+\Delta_r}{|\beta-(\Delta_l+\Delta_r)|}\Big), \quad (17)$$

where $\beta=\max(|U-V|, |U+V|)$, and the jump of the integral I_2 is equal to

$$I_2(\Delta_l+\Delta_r+0)-I_2(\Delta_l+\Delta_r-0)=-\mu_0 e^{-1}R^{-1}\pi(\Delta_l\Delta_r)^{1/2}. \tag{18}$$

At $U+V \gg \Delta_l+\Delta_r$ and $U=V$ we have

$$I_1=\frac{2\mu_0}{e}R^{-1}\left[\frac{2\Delta_l\Delta_r}{\Delta_l+\Delta_r}K\left(\frac{|\Delta_l-\Delta_r|}{\Delta_l+\Delta_r}\right)-\frac{\pi\Delta_l\Delta_r}{2U}\right],$$
$$I_2=-\frac{2\mu_0}{e}R^{-1}\frac{\Delta_l\Delta_r}{U}\ln\frac{U}{(\Delta_l\Delta_r)^{1/2}}. \tag{19}$$

Thus, in the case when different types of pairing are realized in the semiconductors making up the tunnel structure, singlets on the left and triplets on the right, application of a potential difference on the structure gives rise to magnetic-moment oscillations described by formulas (9)–(19). It is easy to show that in this case there are no electric-current oscillations corresponding to those considered earlier.[4]

At low voltages, in formula (9), only the integral I_1 differs from zero. With increasing applied voltage, the expression for $\mathbf{J}_M(t)$ acquires a singularity at the point $\max(|U-V|, |U+V|)=\Delta_l+\Delta_r$, in view of the fact that breaking of the electron–hole pair is possible and one electron or one hole can pass through the barrier. Since there is no magnetic ordering in either of the semiconductors making up the tunnel structure, there is no injection of magnetic moment in this case, i.e., $I_3=0$. It is easy to show, however, that a normal electric current appears jumpwise at this point and is described by formula (14) of [4]. In strong fields $U \gg V$ and $\Delta_l+\Delta_r$, the integrals I_1 and I_3 decrease and tend to zero, while the normal electric current, in accordance with formula (18) of [4], is proportional to U, in accord with the usual Ohm's law. At $U=V$ and $U \gg \Delta_l+\Delta_r$, however, the integral I_1 does not tend to zero, i.e., oscillations of the magnetic moment can exist also when strong fields are applied to the structure.

d) We consider the case when singlet pairing is realized in the left-hand semiconductor, while singlet and triplet pairs can coexist in the right semiconductor. The spin structure of the Green's functions can now be chosen in the following manner: $G_{\alpha\beta l}=\delta_{\alpha\beta}G_l$, $G_{\alpha\beta r}=G_{r0}\delta_{\alpha\beta}+G_{rz}\sigma_{\alpha\beta z}$. By simple transformations of formulas (10)–(12) we find that in this case all three integrals I_1, I_2, and I_3 differ from zero. The expressions for I_1 and I_2 are similar to formulas (13) and (14), in which the right-hand semiconductor is represented by the z components of the Green's functions and the integral I_3 can be written as

$$I_3=\frac{2\mu_0}{e}R^{-1}\left\{\int_{-\infty}^{+\infty}\left(\operatorname{th}\frac{\omega}{2T}-\operatorname{th}\frac{\omega_{1-}}{2T}\right)\operatorname{Im}G_{1l}{}^R(\omega)\operatorname{Im}G_{1rz}(\omega_{1-})d\omega\right.$$
$$\left.+\int_{-\infty}^{+\infty}\left(\operatorname{th}\frac{\omega}{2T}-\operatorname{th}\frac{\omega_{1+}}{2T}\right)\operatorname{Im}G_{2l}{}^R(\omega)\operatorname{Im}G_{2rz}^R(\omega_{1+})d\omega\right\}. \tag{20}$$

It was shown in[7] that the coexistence of singlet and triplet states of electron-hole pairs is impossible in a crystal without impurities, in which the number of electrons is equal to the number of holes. The inequality of the number of carriers of opposite sign as a result of doping makes possible this coexistence, which furthermore is accompanied by ferromagnetic ordering of the carrier spins. Formulas describing $\mathbf{J}_M(t)$ can be easily obtained in this case by substituting in (13), (14), and (20) the corresponding expressions for the Green's functions. It is easy to show, however, that the integrals I_1 and I_2 can be obtained from (16)–(19) by making the formal substitution

$$U\pm V \to U\pm V-(\delta\mu_r-\delta\mu_l),$$

where $\delta\mu_r$ and $\delta\mu_l$ are the doping-induced changes in the Fermi degeneracy energies of the right and left semiconductors, respectively (we assume the carrier density n to be given for each semiconductor).

Since, however, the dependence of $\delta\mu_r$, Δ_{sr}, and Δ_{tr} on the concentration of the doping impurities is rather complicated,[7] the explicit forms of the integrals I_1, I_2, and I_3 will not be written out here for simplicity. We shall dwell only on one result of physical importance. The right substitution of the expressions for the Green's functions in the integral I_3 shows that this integral, which describes the injection of ordered spins from the ferromagnetic[7] right-hand semiconductor into the nonferromagnetic left semiconductor differs from zero at arbitrary small U and V. The reason is the following. It was shown in[7] that the nature of the ferromagnetic ordering in a system in which singlet and triplet pairs coexist is connected with the inequality of the number of carriers with up and down spins above the energy gap in the restructured spectrum. It becomes clear therefore that these very carriers will contribute to the normal injection of the spins from the one semiconductor to the other.

Thus, in our case, in contrast to the case (c), all three integrals in (9) differ from zero, with I_3 contributing to the expression for $\mathbf{J}_M(t)$ at arbitrarily small U and V.

The cases (e) (triplet pairing in the left semiconductor and coexistence of singlet and triplet pairs in the right one) and (f) (coexistence of singlet and triplet pairings in both semiconductors) lead to results that are qualitatively similar to the case (d). We shall therefore not dwell on them in detail.

The results can be easily generalized to include the case when the fact that the mean free path of the carriers in the crystal is finite becomes significant.

4. We have thus shown that quantum phenomena connected with the coherence of the phase of the wave function in quasi-equilibrium semiconductors with electron-hole pairing can lead to the appearance of oscillations of the magnetic moment in a tunnel structure consisting of two such semiconductors separated by an insulating layer. In analogy with[4], the character of these oscillations is determined by two parameters: V, which depends on the rate of carrier generation and on the difference between their lifetimes in the semiconductors making up the tunnel structure, and the external voltage U applied to the structure. The value of U determines the magnitude of the oscillations, the frequency of which is equal to $\omega=2V$.

From the physical point of view, we regard as the most interesting the results obtained in case (c), that magnetic-moment oscillations can appear when an external potential difference is applied to a tunnel structure in which both semiconductors have no magnetic ordering. This case is interesting also because the magnetic-moment oscillations are more pronounced here than elsewhere. The reason is that the case (c) is the only one in which the magnetic-moment oscillations are not accompanied by oscillations of the tunnel electric current. It is easy to show that in cases (d), (c), and (f), in addition to oscillations of the magnetic moment there will exist in the tunnel structure an oscillating electric tunnel current described by the formulas of [4], which can greatly hinder the observation of the magnetic-moment oscillations. Therefore the case (c) is the most favorable when attempts are made to investigate experimentally the oscillations of the magnetic moment in a tunnel structure.

The result obtained in the present study can be qualitatively interpreted in the following manner. As shown by Kozlov and Maksimov,[10] the restructuring of the initial spectrum and the formation of electron–hole pairs in the crystal are accompanied by the appearance of standing waves of charge-density (in the case of singlet pairing) or of spin density (in the case of triplet pairing). When the extrema of the electron and hole bands are at one point of the Brillouin zone, the period of such a wave coincides with the period of the lattice. The coexistence of two such waves in a homogeneous crystal, as shown by Volkov, Kopaev, and Rusinov,[7] leads to appearance of ferromagnetic ordering.

In our case of a quasi-equilibrium semiconductor, when the Fermi quasilevels of the electrons and holes are separated in energy $2V_{r(l)}$, the situation becomes somewhat more complicated because of the distinctive time dependence of the equations on the problem. This was already discussed earlier, when we noted that the appearance in the Hamiltonian (6) of terms with $V_{r(l)}$ leads to the appearance of factors of the type $\exp(2iV_{r(l)}t)$ in the Green's functions of the problem. It is easy to show that in this case the density of the electron–hole-pair Bose condensate in each of the quasi-equilibrium semiconductors making up in the tunnel structure oscillates in time with frequency $\omega = 2V_{r(l)}$. When an external potential difference U is applied to the structure between the semiconductors, intense carrier exchange takes place, with superposition of time-oscillating charge- and spin-density waves on the different semiconductors. It is this superposition which causes oscillations of the magnetic moment. The concrete arguments that explain each of the cases (a), (b), (c), (d), and (e) are obvious and will not be presented here.

We note that, as follows from (9) and is also easily understood from the qualitative reasoning presented above, no magnetic-moment oscillations are produced at $V=0$. In other words, there are no oscillations in the equilibrium case when the Fermi levels of the electrons and holes coincide. In this case, however, the tunneling from semiconductor to semiconductor (or from semimetal to semimetal) leads to a mutual penetration and superposition of standing charge- and spin-density waves from the different semiconductors (semimetals), and consequently gives rise to magnetic ordering near the tunnel junction. The result is particularly interesting when singlet pairing is produced in one equilibrium semiconductor (semimetal), and triplet pairing in the other. In this case each of the semiconductors (semimetals) is nonferromagnetic, and ferromagnetic ordering of the carrier spins is produced near the interface, owing to the superposition of the charge and spin waves from the different crystals. A detailed examination of the proximity effect in the equilibrium case will be the subject of a separate study.

We note in conclusion that although the final expressions for the magnetic-moment current were obtained for the case $na_0^3 \gg 1$, these results remain qualitatively in force also for an arbitrary carrier density, if the conditions for the Bose condensation of the electron–hole pairs are satisfied.

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