ships observed may be valid to some degree in other multilayer systems also, if the DS in a magnetically hard layer (or layers) is sufficiently stable and does not change the change of the external conditions.

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Electron density distribution for localized states in a one-dimensional disordered system

A. A. Gogolin

L. D. Landau Institute of Theoretical Physics, USSR Academy of Sciences
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The explicit form of the electron density distribution $p_\infty(x)$ is calculated for a localized state in a one-dimensional disordered system. A general formula is obtained for the moments of $p_\infty(x)$.

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The question of the character of the electronic states in a one-dimensional disordered system was investigated by a number of workers (see the review by Mott[1]). Mott and Twose[2] have shown that all states in such a system are localized. The asymptotic form of the electron density for a localized state as $|x|\to\infty$ is essentially exponential. The argument of the exponential for certain models was determined by a number of workers.[3–6] A more correct asymptotic expansion, which includes the pre-exponential factor, was obtained by Mel'nikov, Rashba, and the author[7] with the aid of a method developed by Berezinskiĭ.[3] In the present paper the same method is used to obtain the explicit form of the distribution of the electron density of the localized state $p_\infty(x)$ for arbitrary x.

We consider a system of noninteracting electrons with a dispersion law $\varepsilon(p)$, situated in the field of randomly disposed centers $V(x)$. The random potential $V(x)$ is characterized by a correlator $U(x-x')$

$$U(x-x')=\langle V(x)V(x')\rangle. \tag{1}$$

The angle brackets denote here averaging over the realizations of the random potential. The electron scattering is considered in the Born approximation.

It was shown in the preceding paper[7] that for this model the distribution of the electron density of the localized state $p_\infty(x)$, obtained from the expression for the long-time density correlator, is given by

$$p_\infty(x)=\frac{2}{\pi^2 l_i^-}\int_0^\infty \eta\, d\eta \operatorname{sh}\pi\eta \exp\left(-\frac{\eta^2+1}{4l_i^-}|x|\right) \times\int_0^\infty z\,dz K_1(z)K_{i\eta}(z)\int_z^\infty \xi\, d\xi\, K_1(\xi)K_{i\eta}(\xi), \tag{2}$$

where K_1 and $K_{i\eta}$ are Bessel functions, and l_i^- is the mean free path calculated from the Born amplitude of the impurity backscattering:

$$\frac{1}{l_i^-}=\frac{1}{v^2(\varepsilon)}\int_{-\infty}^{\infty} U(x)e^{2ip(\varepsilon)x}\,dx, \tag{3}$$

here $v(\varepsilon)$ is the velocity of an electron with energy ε, and $p(\varepsilon)$ is its momentum.

The integral with respect to z and ξ in (2) can be calculated exactly (see[8], formula (6.576)):

$$\int_0^\infty z\,dz\,K_1(z)K_{i\eta}(z)\int_z^\infty \xi\,d\xi\,K_1(\xi)K_{i\eta}(\xi)=\frac{1}{2}\left[\int_0^\infty z\,dz\,K_1(z)K_{i\eta}(z)\right]^2 =\frac{1}{8}\left[\Gamma\left(\frac{3+i\eta}{2}\right)\Gamma\left(\frac{1+i\eta}{2}\right)\Gamma\left(\frac{3-i\eta}{2}\right)\Gamma\left(\frac{1-i\eta}{2}\right)\right]^2. \tag{4}$$

Using also the known identity for the Γ function (formula (8.332) from[8]), we obtain

$$\Gamma\left(\frac{3+i\eta}{2}\right)\Gamma\left(\frac{1+i\eta}{2}\right)\Gamma\left(\frac{3-i\eta}{2}\right)\Gamma\left(\frac{1-i\eta}{2}\right)=\frac{\pi^2}{2}\frac{1+\eta^2}{1+\operatorname{ch}\pi\eta},$$

$$p_\infty(x)=\frac{\pi^2}{16l_i^-}\int_0^\infty \eta\,d\eta\operatorname{sh}\pi\eta\left(\frac{1+\eta^2}{1+\operatorname{ch}\pi\eta}\right)^2\exp\left(-\frac{1+\eta^2}{4l_i^-}|x|\right). \tag{5}$$

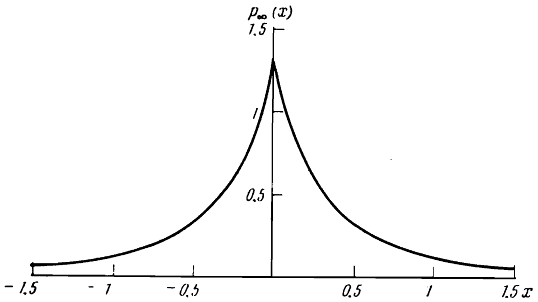

FIG. 1. Distribution of electron density for a localized state in a one-dimensional disordered system $p_\infty(x)$ in dimensionless units with $4l_i^- = 1$.

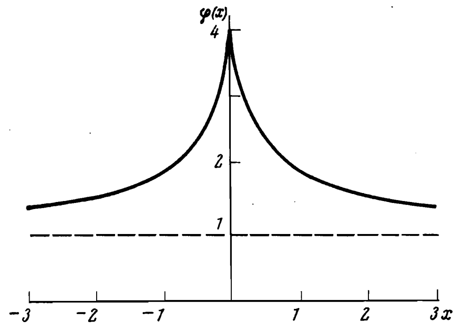

FIG. 2. Plot of the absolute values of the logarithmic derivative of $p_\infty(x)$: $\varphi(x) = |d \ln p_\infty(x)/dx|$.

We shall henceforth use dimensionless units in which $4l_i^- = 1$. The asymptotic form of (5) at $|x| \gg 1$ is

$$p_\infty(x) = \frac{1}{\pi^{1/2}}\left(\frac{\pi^2}{8}\right)^2 \frac{1}{|x|^{3/2}} e^{-|x|} \tag{6}$$

which agrees with our results.[7] It is also easy to calculate the values of the function $p_\infty(x)$ and its derivatives at $x=0$, in particular,

$$p_\infty(0) = \frac{4}{3}, \qquad \left|\frac{dp_\infty(x)}{dx}\right|\Bigg|_{x=0} = \frac{16}{3}. \tag{7}$$

The first two moments of $p_\infty(x)$ can be easily obtained after integrating twice by parts

$$p_0 \equiv \int_{-\infty}^{\infty} p_\infty(x)\,dx = 1, \qquad p_1 \equiv \int_{-\infty}^{\infty} |x| p_\infty(x)\,dx = {}^1/_2. \tag{8}$$

From (8) it follows, in particular, that the average dimension of the localized state in dimensional units is $2l_i^-$. This is half the value obtained from the asymptotic form (6).

It is possible also to calculate all the succeeding moments. Indeed, after integrating twice by parts we obtain

$$p_n \equiv \int_{-\infty}^{\infty} |x|^n p_\infty(x)\,dx = \frac{n!}{2}\Bigg[1 + 4(n-1) \times \int_0^\infty \frac{\eta\,d\eta}{e^{\pi\eta}+1}\left\{\frac{2n-3}{(1+\eta^2)^n} - \frac{2n}{(1+\eta^2)^{n+1}}\right\}\Bigg]. \tag{9}$$

Next, using the formula (see[8], (3.415))

$$\int_0^\infty \frac{\eta\,d\eta}{(\eta^2+\beta^2)(e^{\mu\eta}-1)} = \frac{1}{2}\left[\ln\left(\frac{\beta\mu}{2\pi}\right) - \frac{\pi}{\beta\mu} - \psi\left(\frac{\beta\mu}{2\pi}\right)\right] \tag{10}$$

and the identity

$$1/(e^{\mu\eta}+1) = 1/(e^{\mu\eta}-1) - 2/(e^{2\mu\eta}-1),$$

we obtain ultimately ($n \geq 2$)

$$p_n = \frac{n!}{2}\Bigg\{1 + \frac{(-1)^{n-1}}{2^{n-3}(n-2)!}\left(2n-3+\frac{1}{\beta}\frac{d}{d\beta}\right)\left(\frac{1}{\beta}\frac{d}{d\beta}\right)^{n-1} \times \left(\psi(\beta) - \frac{1}{2}\psi\left(\frac{\beta}{2}\right) - \frac{1}{2}\ln 2\beta\right)\Bigg\}\Bigg|_{\beta=1}, \tag{11}$$

$$\psi^{(n)}(1) = (-1)^{n+1} n!\,\zeta(n+1), \quad \psi^{(n)}({}^1/_2) = (-1)^{n+1} n!\,(2^{n+1}-1)\cdot\zeta(n+1).$$

Here ψ is the logarithmic derivative of the Γ function and ζ is the Riemann zeta function. From (11), in particular, we obtain at $n=2$

$$p_2 = \zeta(3)/4 \tag{12}$$

in accord with[3,7]. This expression determines the electronic polarizability.

A comparison of p_0, p_1, and p_2 shows that the function $p_\infty(x)$ is concentrated mainly in the region $|x| < \frac{1}{2}$. This circumstance can be clearly seen from the plot of $p_\infty(x)$ (see Fig. 1). The decrease in the region $|x| < \frac{1}{4}$ is mainly proportional to $e^{-4|x|}$, going over gradually to the asymptotic form $|x|^{-3/2}e^{-|x|}$ at $|x| \gg 1$. The change of the rate of decrease of the electron density is particularly clearly demonstrated in Fig. 2, which shows a plot of the absolute value of the logarithmic derivative $|d \ln p_\infty(x)/dx|$. This curve characterizes the deviation of the behavior of $p_\infty(x)$ from a pure exponential function, which would correspond to $|d \ln p_\infty(x)/dx| = \text{const}$. We note that the general form of the plot in Fig. 1 is similar to the results of the computer calculation within the framework of the model of Frisch and Lloyd.[9]

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