

Spin states of $d\mu$ atoms in gaseous hydrogen and measurement of the fusion rate in $pd\mu$ molecule

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(Submitted May 6, 1976)

Zh. Eksp. Teor. Fiz. **71**, 1680-1688 (November 1976)

The fusion reaction rate λ_F in the $pd\mu$ mesic molecule is measured in experiments performed with a target $H_2+7\%D_2$ gas mixture at a pressure 42 atm and exposed to the muon beam from the 680-MeV JINR synchrocyclotron. The upper limit of the transition rate λ_d between the hyperfine structure levels of the $d\mu$ atom in $d\mu(3/2)+d\rightarrow d\mu(1/2)+d$ collisions is also determined. The values $\lambda_F = (.287 \pm 0.022) \cdot 10^6 \text{ sec}^{-1}$ and $\lambda_d < 15 \cdot 10^6 \text{ sec}^{-1}$ are obtained. From the estimate of λ_d it follows that at low deuterium concentrations (several percent) the population of the $d\mu$ -atom spin states in H_2+D_2 mixtures is close to the statistical value at the instant of nuclear capture by the deuteron.

PACS numbers: 36.10.Dr

An experimental study of the fundamental reaction of nuclear μ capture by a deuteron



is best carried out in a gaseous mixture of hydrogen with a small admixture of deuterium, sufficient to ensure practically complete transfer of the muons from the protons to the deuterons. The choice of these conditions leads to an appreciable suppression of the intense neutron background from the fusion reaction in the $dd\mu$ molecule $dd\mu \rightarrow He^3 + n + \mu^-$. Even in this case, however, difficulties remain, due to the need for taking into account some other mesic-atom and mesic-molecule processes.

One of the problems is connected with the need for knowing the spin state of the μd system immediately at the instant of the nuclear capture of the muon by the deuteron, since the probability of this process depends essentially on the mutual orientation of the muon and deuteron spins.^[1] Although in the case of production of the $d\mu$ atoms both hyperfine-structure states with spin $E = \frac{3}{2}$ and $F = \frac{1}{2}$ are statistically populated, but by the instant of capture the character of the population can change, nonetheless, owing to the possible $\frac{3}{2} \rightarrow \frac{1}{2}$ transitions in the collisions



and



Another problem is to account correctly for the main background processes connected with the production of $pd\mu$ molecules.

Such processes are the following: a) capture of a muon by a proton or deuteron in this mesic molecule at a rate that differs from the capture rate in the isolated $d\mu$ atom; b) μ capture in the $He^3\mu$ system produced in one of the channels of the fusion reaction



Thus, to choose the optimal experimental conditions for the measurements of the rate of μ capture in deuterium and to interpret their results it is necessary to know the following quantities: 1) the populations of the spin states of the $d\mu$ atom at the instant of capture; 2) the rates of formation of the $pd\mu$ molecule ($\lambda_{pd\mu}$); 3) the rates of the nuclear fusion reactions (3a) and (3b).

The rates of formation of the $pd\mu$ molecule in the gaseous and liquid hydrogen have by now been measured with good accuracy,^[2,3] the rate of the nuclear fusion reaction (3) in the $pd\mu$ mesic molecule was measured only in one experiment,^[2] and the question of the character of the population of the spin states of the $d\mu$ atom at the instant of capture remains open. The results of the experiment^[2,4] point to a statistical character of the population of the spin states of the $d\mu$ system at the instant of μ capture, whereas the capture rate (1) obtained in^[5] agrees with the theoretical calculations of this quantity^[1] only under the assumption that all $d\mu$ atoms go over into a state with $F = \frac{1}{2}$ at the instant of capture.

The mechanism considered by Zel'dovich and Gershstein^[6] for the transitions $d\mu(\frac{3}{2}) \rightarrow d\mu(\frac{1}{2})$ implies that they take place only in the collisions (2a), and it follows furthermore from the calculated rate of the process (2a) that its influence should be negligibly small both under the conditions of the experiments of^[2,4] and under the conditions of^[5]. To explain the discrepancy with^[6], the authors of^[5] have proposed that the intense transition to the $d\mu(\frac{1}{2})$ state is due to the collisions (2b). This, however, does not eliminate the discrepancy between the results of^[5] and of^[2,4].

The purpose of the present study was to measure the rates of the transition $d\mu(\frac{3}{2}) \rightarrow d\mu(\frac{1}{2})$ in hydrogen gas mixed with deuterium, and the rate of the fusion reaction (3).

MEASUREMENT METHOD

The measurement method is based on the use of the connection of the yield of γ quanta from the reaction (3a)

and of the form of their distribution in time with the character of the population of the spin states of the $d\mu$ atoms when $pd\mu$ molecules are produced. The sequence of the processes occurring in the $H_2 + D_2$ mixture after the stopping of the μ^- meson is shown in Fig. 1. The mesic atom $d\mu$ produces when colliding with a proton a $pd\mu$ molecule, the ground state of which is split into four sublevels with total angular momentum values $J=0$, $J=1$ (two levels) and $J=2$. The populations of the four sublevels depend on the spin distribution of the $d\mu$ atoms at the instant of molecule production. The weights of the sublevels of the $pd\mu$ system (a_i, b_i, c_i) for the different states of the $d\mu$ atoms are listed in Table I. The rate of the fusion reaction in the $pd\mu$ molecule depends on its spin state: the reaction is forbidden at $J=2$ and proceeds at relative rates d_i in the remaining states (Table I). If we designate by λ_1 and λ_2 the rates of the processes (3a) and (3b) from the state $pd\mu(J=0)$, and by λ_F their sum, then the corresponding rates of the reactions from the i -th sublevel are given by

$$d_i \lambda_i, d_i \lambda_2, d_i \lambda_F.$$

As seen from Table I, the γ yield from the reaction (3a) depends essentially on the distribution of the $d\mu$ atoms over the spin states: in the case when all the $d\mu$ atoms are in a state with $F=\frac{1}{2}$, the yield is approximately double than in the case of statistical population of the states $d\mu(\frac{3}{2})$ and $d\mu(\frac{1}{2})$.

The temporal distribution of the γ quanta from the reaction (3a) can be represented as follows:

$$\frac{dN_\gamma}{dt} = A e^{-\lambda_0 t};$$

$$P = \lambda_{pd\mu} e^{-\lambda_0 t} \left(\frac{1}{3} \sum_{i=1}^3 \frac{d_i \lambda_i b_i (e^{-r_i t} - e^{-r_i t})}{r_i - q} + \frac{2}{3} \left\{ \sum_{i=1}^3 \frac{d_i \lambda_i a_i (e^{-(q+s)t} - e^{-r_i t})}{r_i - q - s} + s \sum_{i=1}^3 \frac{d_i \lambda_i b_i (e^{-r_i t} - e^{-r_i t})}{r_i - q - s} \right\} \right), \quad (4)$$

$$q = \varphi(1 - C_D) \lambda_{pd\mu}, \quad r_i = d_i \lambda_i, \quad s = q C_D \lambda_{pd\mu}$$

where $\lambda_0 = 0.455 \cdot 10^6 \text{ sec}^{-1}$ is the rate of decay of the free muon; φ is the ratio of the density of the $H_2 + D_2$ gas mixture to the density of liquid hydrogen ($\rho_0 = 4.22 \cdot 10^{22} \text{ cm}^{-3}$); C_D is the atomic concentration of the deuterium; ε_γ is the efficiency of registration of the γ

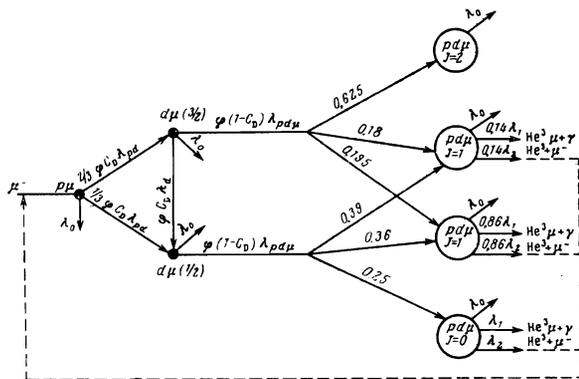


FIG. 1. Diagram of the processes occurring in the $H_2 + D_2$ mixture after the stopping of a μ^- meson.

TABLE I. Weights of $pd\mu$ -molecule states with different angular momenta J and relative values of the fusion reaction rates for these states.

| Angular momentum J | Spin states of $d\mu$ atom forming the $pd\mu$ molecule | | | Relative rate of fusion reaction |
|----------------------|---|------------|--|----------------------------------|
| | $F=3/2$ | $F=1/2$ | Statistical mixture of $F=3/2$ and $F=1/2$ | |
| 0 | $a_1=0$ | $b_1=0.25$ | $c_1=0.083$ | $d_1=1$ |
| 1 | $a_2=0.195$ | $b_2=0.36$ | $c_2=0.25$ | $d_2=0.86$ |
| 1 | $a_3=0.180$ | $b_3=0.39$ | $c_3=0.25$ | $d_3=0.14$ |
| 2 | $a_4=0.625$ | $b_4=0$ | $c_4=0.417$ | $d_4=0$ |

quanta from the reaction (3a); A is a coefficient proportional to the number of stopped muons in the target. In the derivation of formula (4) it was assumed in accordance with^[6] that the transitions $d\mu(\frac{3}{2}) \rightarrow d\mu(\frac{1}{2})$ occur only in the collisions (2a) with a rate λ_d . Using the previously obtained^[3] value $\lambda_{pd\mu} = (5.53 \pm 0.16) \cdot 10^6 \text{ sec}^{-1}$ and the value of ε_γ measured in the present study, and also the relation $\lambda_F = (1.182 \pm 0.015) \lambda_1$ obtained from the data of^[7, 8], we were able to determine the sought parameters λ_d and λ_F by approximating the experimental temporal distributions of the γ quanta by means of expression (4).

EXPERIMENTAL SETUP AND PROCEDURE

The arrangement of the experimental setup and a block diagram of the electric apparatus were published by us earlier.^[1] The setup, which includes a hydrogen-gas target and scintillation detectors, was placed in a low-background laboratory and exposed to the beam of negative muons with momentum 130 MeV/c from the meson channel of the 680-MeV synchrocyclotron of our Institute.

After passing through counters 1-3, the muons were slowed down in absorber 6 and were stopped in the target. The construction of the gas target and the methods of registering the stopped muons with the aid of CsI(Tl) scintillators are described in^[9, 10].

The γ quanta from the reaction (3a) was registered by two detectors (γ_1 and γ_2) with NaI(Tl) crystals, of 150 mm diameter and 100 mm thickness. The discrimination thresholds in the spectrometric channels of these detectors corresponded to an energy 1.5 MeV. To determine the number of stopped muons in the target, we registered the electrons from the μ decay with the aid of four detectors (e_1-e_4) with stilbene crystals (70 mm diameter and 30 mm thickness) placed around the target.

The muon-stopping signal 2345 triggered time gates of duration 10 μsec , during which pulses from the γ and e detectors were registered. The time of appearance of these pulses was measured with the aid of time-amplitude converters TAC γ and TAC e . The gates G γ and G e generated logical characters with the aid of which the type and number of the detector producing the count were identified.

To suppress the "instantaneous" background due to the stopping of the muons in the γ and e detectors and in the housing of the target, we introduced anticoincidences $23\bar{7}$ and $23\bar{e}$ with resolution time 0.15 μsec .

TABLE II. Experimental data.

| Experiment | Number of stopped muons in gas | Content of target and mixture pressure | Number of γ quanta registered by the detectors at an energy threshold $E_{thr} = 4.1$ MeV | | Efficiency of registration of γ quanta, $E_{thr} = 4.1$ MeV | |
|------------|--------------------------------|--|--|---------------------|--|---------------------|
| | | | Detector γ_1 | Detector γ_2 | Detector γ_1 | Detector γ_2 |
| | | | | | | |
| A | $2.9 \cdot 10^6$ | $H_2 + 7\% D_2$; 42 atm | 1550 | 2020 | 0.0229 ± 0.0025 | 0.0297 ± 0.0036 |
| B | $1.57 \cdot 10^6$ | He; 47 atm | 280* | 350* | | |

*These values were normalized for the number of stopped muons in run A.

The information concerning the events consisted of the following: 1) the time of appearance of the signals from the γ and e detectors relative to the instant of the stopping of the muon, 2) the amplitude of the signal from the γ detector; 3) logical characters indicating the type and number of the operating detector; 4) a logical character indicating the presence of a second muon registered by detector 1 in the time 5 μ sec prior to stopping and 10 μ sec after the stopping; 5) a monitor count (23), a stopped-muon count (2435), and a count of the events registered by the γ and e detector; 6) auxiliary information (number of the measurement, time, etc.).

The recording system operated in line with an HP-2116 computer that accumulated the information and reduced it, making it possible to monitor the course of the experiment.

Two experimental runs were made: A—with $H_2 + D_2$ mixture, and B—with helium (background). The experimental conditions and the principal data that characterizes the experiments are summarized in Table II. Impurity atoms with $Z > 1$ in hydrogen (run A) did not exceed a volume fraction 10^{-8} .^[10,11]

During the course of the experiment we calibrated periodically the time and energy scales with the aid of a standard-frequency generator and γ -quantum sources (Co^{60} , PO-Be).

On the basis of the obtained information we plotted the temporal spectra of the γ quanta (γ_5) and of the elec-

trons (e_5) registered by the γ and e detectors in anti-coincidence and coincidence with the signal from counter 5. The use of detector 5 in the anticoincidence regime has made possible very effective separation of the γ quanta of the reaction (4a) from the electrons due to the muon decay and entering the γ detector (the efficiency of registration of the electrons by counter 5 was 99.5%).

The characteristic $\gamma \bar{5}$ temporal distributions obtained in runs A and B are shown in Fig. 2. Figure 3 shows the amplitude distribution of the γ quanta from reaction (3a), measured in run A.

DETERMINATION OF EFFICIENCY OF γ -QUANTUM REGISTRATION

The efficiency of registration of the γ quanta produced in the reaction (3a) was determined in an experiment with a tantalum target. The choice of tantalum was dictated by the fact that the energy of the $2p \rightarrow 1s$ transition in the Ta μ atom ($E(K_\alpha) = 5.35$ MeV) is close in value to the energy of the γ quanta (5.5 MeV) from the reaction (3a). A tantalum plate 1 mm thick in a foamed-plastic mount was placed at different points inside the detector 5, which was constructed in the form of a hollow vessel of CsI(Tl) (experiment I).

The background was determined in runs with the mount placed at the same points of the vessel (experiment II).

Figure 4 shows the amplitude spectrum of the mesic

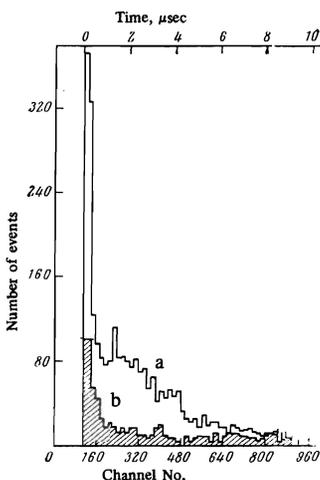


FIG. 2. Temporal spectra of events registered by γ detectors in anticoincidences with the counter 5 (γ_5): a) run A $H_2 + D_2$ mixture). b) Data of run B(He) normalized to the number of stopped muons in run A. The abscissas represent the time of the γ quantum measured from the time of stopping of the muon, while the ordinates represent the number of events in a time intertergal 0.189 μ sec.

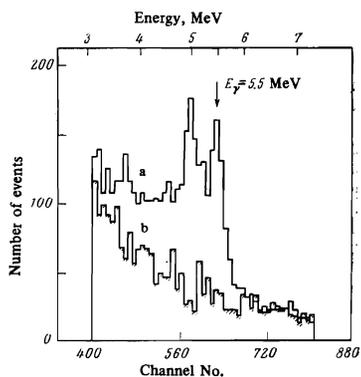


FIG. 3. Amplitude distributions of events registered by γ detectors in anticoincidence with counter 5: a) run A, b) normalized data of run B. The ordinates represent the number of events in a 0.092-MeV interval.

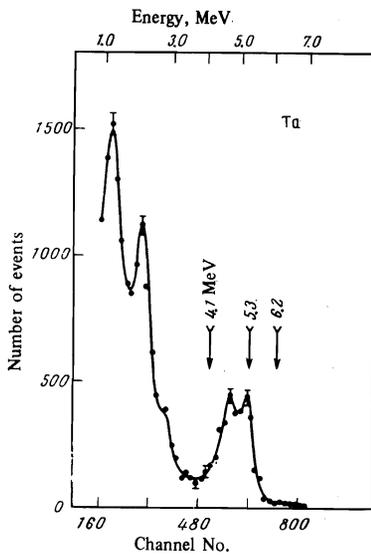


FIG. 4. Amplitude distribution of events registered by γ detectors in experiment I after subtracting the normalized of experiment II. The arrows indicate the position of the photopeak of the K_α line of the $Ta\mu$ atom and the limits of the range in which the events were reduced.

x-rays of $Ta\mu$ atoms registered with the γ detector (experiment I). The reduction of the amplitude spectra obtained in experiments I and II was carried out in a region corresponding to γ -quantum energies from 4.6 to 6.1 MeV. The choice of these limits has made it possible to exclude completely the contributions of the L , M , and other series of γ quanta. The contribution of the harder lines of the K series was $\leq 2\%$ and was taken into account in the reduction. The efficiency of registration of the γ quanta by the γ detector was determined with the aid of the relation

$$\epsilon_\gamma = \frac{N_\gamma^I - N_\gamma^{II}}{(N_\mu^I - N_\mu^{II}) a_{Ta} (1 - \beta)}, \quad (5)$$

where N_γ^I and N_γ^{II} are the numbers of γ quanta registered by the γ detector in experiments I and II (N_γ^{II} was normalized to run I); N_μ^I and N_μ^{II} are the numbers of stopped muons in runs I and II (N_μ^{II} was normalized to run I); β is a correction that takes into account the absorption of the γ quanta in the tantalum target; the value of β for each position of the target was calculated by the Monte Carlo method and turned out to be ~ 0.05 ; $a_{Ta} = 0.590 \pm 0.077$ is the intensity of the $2p - 1s$ transition in the $Ta\mu$ atom and was obtained from the relation $a_{Ta} = a_{Pb} k$ using the value $a_{Pb} = (0.59 \pm 0.06)$ obtained in [12], and $k = a_{Ta} / a_{Pb} = (1.0 \pm 0.08)$. [13]

Using the measured distribution of the stopped muons

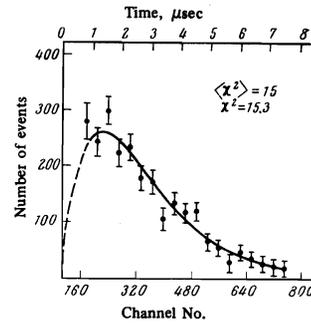


FIG. 5. Temporal distribution of γ quanta from the fusion reaction in the mesic molecule $pd\mu$ (the normalized background was subtracted). Ordinates—number of events in $0.378 \mu\text{sec}$ interval. Solid line—calculated curve.

over the volume of the target and the obtained values of the γ -quantum registration efficiencies for the different positions of the Ta target, we calculated the average values of the γ -quantum registration efficiencies for both γ detectors (γ_1, γ_2). The results of the calculations are given in Table II.

REDUCTION OF RESULTS

The temporal distributions of the γ quanta ($\gamma\bar{5}$) obtained in run B (background experiment) and normalized to the conditions of the run A were subtracted channel by channel from the corresponding temporal distributions measured in run A. The integrated contribution of the background at a recording-apparatus energy threshold 4.1 MeV did not exceed 18%. To exclude the contribution of the background due to the stopped muons in the scintillators of the detectors 4 and 5, followed by emission of the products of the μ^- capture ($\tau_\mu(\text{Cs, I}) = 0.08 \mu\text{sec}$), the spectra were reduced starting with a time $t \geq 0.5 \mu\text{sec}$.

The temporal spectrum of the γ quanta from the reaction (3a) is shown in Fig. 5.

The temporal distributions $\gamma\bar{5}$ obtained with the aid of both γ detectors were reduced by a least-squares fit to expression (4) for the purpose of determining the transition rate λ_d and the rate of the fusion in the mesic molecule $pd\mu$. The quantity A in (4) was obtained from the relation

$$A = N_\mu (1 - F_t) \varphi, \quad (6)$$

where $N_\mu = N_e / e_e f_t$ is the number of stopped muons in the $H_2 + D_2$ gas mixture; N_e is the number of muon-decay electrons registered by the e detectors in run A (in the time interval 1–7 μsec from the instant of stopping of the muon) after subtracting the normalized background; e_e is the efficiency of registration of the electrons by the e detector, which was calculated earlier [14]; F_t is

TABLE III.

| Value in units of 10^{-6} sec^{-1} | Experiment | | Theory | | |
|--|---|---------------------------------------|---------------|--------------|-----------------------------|
| | Columbia University, [21] liquid H_2 | Present study, H_2 gas (42 atm). | Gershtein [6] | Gallone [15] | Matveenko, Pnomarev [16] |
| λ_P | 0.305 ± 0.010 | 0.287 ± 0.022 | — | 0.263^* | — |
| $\lambda_d(\frac{3}{2} \rightarrow \frac{1}{2})$ | — | < 15 | 7.0 | — | 47.0 |

* This value corresponds to λ_1 .

the probability of muon decay in the given time interval; f_t is the probability of registration of the muon-decay electrons by detector 5 during the resolution time 2.2 μsec of the anticoincidences ($\gamma\bar{5}$).

The data obtained as a result of the analysis are given in Table III. For the transition rate λ_d the upper bound is indicated with a confidence level 90%. The obtained value of λ_p is in good agreement with the results of the experiments at Columbia University by Bleser *et al.*^[2] and with the calculations of Gallone *et al.*^[15]

The obtained upperbound of the rate of the transition (2a)

$$\lambda_d < 15 \cdot 10^6 \text{ sec}^{-1} \quad (7)$$

agrees with the experimental data of Gershtein.^[6] At the same time it is much less than the value calculated by Matveenko and Ponomarev^[16] under the same approximations, but with more accurate potentials. This lack of agreement, however, should not be regarded as decisive, since merely refinement of the potential is apparently insufficient in this case: it is necessary at the same time to take accurate account of the interaction of the spins of the muons and nuclei, since the hyperfine splittings in the $d\mu$ atoms $\Delta E_{\text{hfs}} = 0.049 \text{ eV}$ are comparable with the energy $E \sim 0.04 \text{ eV}$ of the thermal motion of the atoms.

From the estimate (7) obtained by us it follows that at small deuterium concentrations (on the order of several percent) in $\text{H}_2 + \text{D}$ mixtures the character of the population of the spin states of the $d\mu$ atoms at the instant of nuclear capture of the muon by the deuteron is close to statistical. This conclusion disagrees with the deductions of^[7], and the disparity remains in force regardless of the assumption made concerning the process (2b) in the $d\mu(\frac{3}{2}) \rightarrow d\mu(\frac{1}{2})$ transitions. Indeed, if we do not neglect the contribution of this process, then our result can be represented in the form

$$\lambda_d + \frac{1-C_D}{C_D} \lambda_p < 15 \cdot 10^6 \text{ sec}^{-1} \quad (8)$$

and it follows that the rate of the process (2b) is

$$\lambda_p < 1.1 \cdot 10^6 \text{ sec}^{-1} \quad (9)$$

The estimates (8) and (9) for the conditions of the ex-

periment^[5] ($C'_D = 5\%$, $\varphi' = 9.7 \cdot 10^{-3}$) then lead to the limitation

$$\lambda(\frac{3}{2} \rightarrow \frac{1}{2}) = \varphi' C'_D \lambda_D + \varphi' (1 - C'_D) \lambda_p < 0.2 \cdot 10^6 \text{ sec}^{-1},$$

which differs greatly from the estimate^[5]

$$\lambda(\frac{3}{2} \rightarrow \frac{1}{2}) > 5 \cdot 10^5 \text{ sec}^{-1}.$$

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Translated by J. G. Adashko