The fermion masses in gauge theories with chiral invariance at short distances

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If one imposes chiral invariance at short distances on gauge theories with spontaneous symmetry breaking of the Higgs type one obtains a natural mechanism for the appearance of two classes of fermion masses: light ones and heavy ones. The light fermion masses turn out to be of the order g^2 times the masses of the heavy ones, g being the gauge coupling constant. The possibility of imposing chiral invariance is based on a nontrivial property of the renormalization of the Yukawa coupling constants in gauge theories. As an illustration a simple $SU(3)$ lepton model is considered in which the ratio of the electron and muon masses can be calculated to be of order 1/137.

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1. INTRODUCTION

For many years now particle physics faces the famous electron-muon problem. Although until now no differences in their interactions have been observed, their masses differ by a factor of about 200. Since in order of magnitude this factor coincides with the fine structure constant α the idea has been expressed repeatedly that in a unified theory of weak and electromagnetic interactions the mass of the muon might exist in the zeroth approximation in α whereas the electron mass appears only when the gauge interaction is switched on, and will therefore be of the order αm_μ. The recent experimental news about the existence of heavier quarks and heavy leptons leads to the thought that the e-μ situation is not unique; the masses of the "usual" light quarks could be a fraction of order α of the masses of the heavy quarks and the masses of the light leptons may be a fraction of order α of the masses of the heavy leptons, etc.

In attempts at calculating the electron mass as a "radiative correction" to the muon mass we encounter as a rule divergences. It is true that there are models in which the divergences are absent in the calculation of the electron mass,[1] so that the ratio m_e/m_μ turns out to be finite and of the order 1/137, but these models have an exceptional character and can hardly be considered realistic.

In the present paper we show that there exists a considerably wider class of models in which in spite of the divergences the ratios of the type m_e/m_μ turn out to be finite and of the order g^2 (the square of the renormalized coupling constant of the gauge interaction). This result appears naturally if one requires an additional symmetry of the theory at short distances. In this section we explain the main idea of our approach.

Let us assume that the masses of the fermions appear in some gauge theory as a result of spontaneous symmetry breaking of the Goldstone-Higgs type, i.e., through the appearance of a nonzero vacuum expectation value of some scalar fields. The Yukawa interactions which are responsible for the appearance of these masses will be written in the general form

$$\mathscr{L}=\sum_{air\alpha} h_r\bar\psi_\alpha{}^L\Theta^{(r)}_{ai,\alpha}\varphi_i{}^R\Phi^{(r)}_\alpha + \text{H.c.} \qquad (1)$$

Here ψ_α^L and φ_i^R are left-handed and right-handed fermions transforming according to the representations r_L and r_R of the gauge group G, respectively, with r_R not necessarily the same as r_L; $\Phi_\alpha^{(r)}$ is a scalar field transforming according to the representation r, h_r are the Yukawa coupling constants for the appropriate irreducible representation, $\Theta^{(r)}_{ai,\alpha}$ are the Clebsch–Gordan coefficients of the group G, satisfying the orthonormality conditions

$$\sum_{ai}\Theta^{(r)*}_{ai,\alpha}\Theta^{(r')}_{ai,\alpha'}=\delta_{rr'}\delta_{\alpha\alpha'}. \qquad (2)$$

In the tree approximation the fermion masses appear on account of some nonvanishing vacuum expectation values $\langle\Phi_\alpha^{(r)}\rangle\neq 0$. The corresponding fermion masses are

$$m_{ai}=\sum_{r\alpha}h_r\Theta^{(r)}_{ai,\alpha}\langle\Phi^{(r)}_\alpha\rangle. \qquad (3)$$

It is easy to see that these are the masses of particles described by the four-component spinors

$$\Psi_{ai}=\begin{pmatrix}\varphi_i{}^R\\ \psi_a{}^L\end{pmatrix},$$

since the mass terms which follow from the Lagrangian (1) can be written in the form

$$m_{ai}(\bar\psi_a{}^L\varphi_i{}^R+\bar\varphi_i{}^R\psi_a{}^L)=m_{ai}\bar\Psi_{ai}\Psi_{ai}.$$

If we want some masses to vanish in the tree approximation it follows from Eq. (3) that we have to set equal to zero some definite combinations of the coupling constants and vacuum expectation values of the scalar fields, which does not seem very natural in the general case. We introduce, however, in place of the fields $\Phi_\alpha^{(r)}$ the fields χ_{ai}:

$$\chi_{ai}=\sum_{r\alpha}\Theta^{(r)}_{ai,\alpha}\Phi^{(r)}_\alpha,\quad \Phi^{(r)}_\alpha=\sum_{ai}\Theta^{(r)*}_{ai,\alpha}\chi_{ai}. \qquad (4)$$

(Owing to the orthonormality conditions (2) the kinetic

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energy expressed in terms of the new fields has the canonical form: $\sum|\partial_\mu \chi_{ai}|^2$.) Then, if the Yukawa coupling constants h_r are all equal, $h_r = h$, the Lagrangian (4) can be rewritten in the form

$$\mathscr{L}=h\sum_{ai}\bar{\psi}_a{}^L\varphi_i{}^R\chi_{ai}+\text{H.c.} \tag{5}$$

The Lagrangian (5) exhibits a larger symmetry group than the initial group G, namely, the chiral group $G_L \times G_R$, i.e., it is invariant under independent transformations of the right-handed and left-handed particles. In terms of the chiral group $G_L \times G_R$ the fields transform according to the irreducible representations: $\psi_a^L \sim (r_L, 1)$, $\varphi_i^R \sim (1, r_R)$, $\chi_{ai} \sim (r_L, r_R^*)$. In terms of the gauge group G the fields χ_{ai} transform according to the reducible representation $r_L \otimes r_R$.

Since the mass matrix m_{ai} is now

$$m_{ai}=h\langle\chi_{ai}\rangle, \tag{6}$$

the absence of some masses in the tree approximation means simply the vanishing of the vacuum expectation values of the corresponding field components χ_{ai}, which is, of course, much more natural than the vanishing of combinations of the type (3), since the question of which components $\langle\chi_{ai}\rangle$ are different from zero reduces as a rule to a choice of axes in the "isotopic" space.

In the presence of an interaction with gauge fields the chiral invariance (understood as the quality of the Yukawa coupling constants h_r) can only be approximate. The reason is that the gauge interactions exhibit symmetry with respect to the group G, and not with respect to the chiral group $G_L \times G_R$, and therefore even if one requires equal h_r for some value of the normalization momentum $p^2 = \mu^2$, for any other value of the momentum the quantities h_r will, in general, no longer be equal, since they undergo different renormalizations on account of the gauge interactions.

We now pose the following question: can the chiral symmetry be exact at "small distances"? In other words, is it possible that the ratios of the different Yukawa coupling constants $\bar{h}_r(p)$ should tend to one for $p \to \infty$? Below we solve the Gell-Mann–Low equations for $\bar{h}_r(p)$ and show that

$$\begin{gathered}\bar{h}_r(p)=h_r(\bar{g}(p)/g)^{2\nu}[1+c_r(\bar{g}^2(p)-g^2)+O(g^4)],\\ \bar{g}^2(p)=\frac{g^2}{1+b_1g^2\ln(p/\mu)}.\end{gathered} \tag{7}$$

Here $\bar{g}(p)$ and $\bar{h}_r(p)$ are effective coupling constants, μ is the normalization momentum $\bar{g}(\mu) = g$, $\bar{h}_r(\mu) = h_r$, and ν and c_r are certain numbers. In the derivation of Eq. (7) for $\bar{h}_r$ the second order in g^2 has been taken into account in the Gell-Mann–Low function, but the higher powers of the Yukawa coupling constants have been neglected (on account of the smallness of the fermion masses compared to the intermediate boson masses). We assume that asymptotic freedom holds in all constants of the theory: $\bar{g}(p) \to 0$, $\bar{h}_r(p) \to 0$ for $p \to \infty$, i.e., $b_1 > 0$, $\nu > 0$.

A remarkable property of the expression (7) for $\bar{h}_r$ is the fact that the number ν does not depend on the representation according to which the scalar fields transform, i.e., does not depend on r. This is a consequence of a nontrivial cancellation occurring in perturbation theory. It is for this reason that we can impose the following conditions on the ratios $\bar{h}_r$:

$$\bar{h}_{r_1}(p)/\bar{h}_{r_2}(p)\to 1,\quad p\to\infty. \tag{8}$$

If the power ν were to depend on r, then since $\bar{g}(p) \to 0$ as $p \to \infty$, the functions $h_r(p)$ would tend to zero differently, and the short-distance chiral symmetry would be one hundred percent violated. From (8) and (7) we obtain in the limit $p \to \infty$ $(\bar{g}(p) \to 0)$

$$h_{r_1}(1-c_{r_1}g^2)=h_{r_2}(1-c_{r_2}g^2)=\ldots\equiv h. \tag{9}$$

We see thus that the requirement of short-distance chiral invariance leads to the result that the physical Yukawa coupling constants differ by quantities of the order g^2.

With the help of the relations (9) one can express the fermion masses in the tree approximation (3) in terms of the renormalized constants h, g and the vacuum expectation values of the fields χ_{ai} (making use of the expression of the fields $\Phi_\alpha^{(r)}$ in terms of χ_{ai} given by Eq. (4)). As a result of this we obtain

$$m_{ai}=h\langle\chi_{ai}\rangle+g^2\sum_{a'i'r\alpha}c_r\Theta_{ai,\alpha}^{(r)}\Theta_{a'i',\alpha}^{(r)*}h\langle\chi_{a'i'}\rangle. \tag{10}$$

This implies that the fermion masses can be divided into two classes: light fermions and heavy fermions. The heavy fermions are those for which the masses m_{ai} correspond to nonvanishing vacuum expectation values $\langle\chi_{ai}\rangle$. The masses of the light fermions, corresponding to vanishing $\langle\chi_{ai}\rangle$, are of the order g^2 times the masses of the heavy fermions.

Since the starting equation (3) is strictly valid only in the tree approximation, corrections to it can also yield effects of order g^2. Therefore it is desirable to make use of a formalism in which an equation of the type (3) may be considered as exact. A formalism suitable for this is the effective-action approach[2,3] for a system situated in an external scalar field. Since by definition the effective Lagrangian includes all the radiative corrections, Eq. (3) is indeed exact in this case, albeit with a slightly different definition of the renormalized coupling constants h_r than is usual (this will be explained in more detail below).

There is another essential reason why the use of an effective Lagrangian is convenient. An effective Lagrangian contains effective coupling constants which depend on the external scalar field as a parameter (cf., e.g., [4]). Letting the external field go to infinity we in fact go to short distances[1)] and may set all $\bar{h}_r$ equal to each other. We thus realize exact chiral invariance at small distances directly in the effective Lagrangian, which cannot be achieved by means of the coupling constants occurring in the usual Lagrangian.

Finally, there are purely technical advantages to the use of an effective Lagrangian: the relative simplicity of considering the region of momenta of the order of the particle masses, usually causing difficulties when the renormalization group method is used; the convenient normalization conditions which determine the physical coupling constants as effective coupling constants for values of the scalar fields equal to their vacuum expectation values, etc.

The paper is organized as follows. In Sec. 2 we derive the renormalization group equations for the fermion mass in an external field. At the end we arrive at an exact equation of the type of Eq. (3) in which the constants are functions of the external field. In Sec. 3 we consider the Gell-Mann–Low functions, which determine the behavior of $\bar{h}_r$ for large values of the field within the framework of perturbation theory. We derive Eq. (7) and show that the number ν does not depend on the representation r according to which the scalar fields transform. This proves Eq. (10); however for an explicit determination of the coefficients c_r it is necessary to know the Gell-Mann–Low function for the constants $\bar{h}_r$ in the two-loop approximation, which we will consider in another paper. In Sec. 4 we describe a simple $SU(3)$ model of the leptons, which puts the electron muon and neutrino into one triplet and where one can in principle calculate the ratio $m_e/m_\mu \sim g^2$.

2. THE RENORMALIZATION GROUP EQUATIONS FOR THE FERMION MASS IN AN EXTERNAL FIELD

It is well known[2] that the effective action is the generating functional of the one-particle-irreducible Green's functions of the system. We are interested in that part Γ of the effective action which is related to Green's functions $\Gamma^{(2,n)}$ with two fermions and an arbitrary number of scalar particle lines (for simplicity we first consider the case of one kind of scalar particles only):

$$\Gamma=\sum_n \frac{1}{n!}\int d^4x\, d^4y\, d^4x_1\ldots d^4x_n \times\Gamma^{(2,n)}_{\alpha\beta}(x,y,x_1,\ldots,x_n)\bar{\psi}_\alpha(x)\psi_\beta(y)\Phi(x_1)\ldots\Phi(x_n). \tag{11}$$

Regarding Φ as a constant field and going over to the p-representation, we obtain

$$\Gamma=\sum_n\int\frac{d^4p}{(2\pi)^4}\Gamma^{(2,n)}_{\alpha\beta}(p,p,0,\ldots,0)\bar{\psi}_\alpha(p)\psi_\beta(p)\frac{\Phi^n}{n!}, \tag{12}$$

where

$$\Gamma^{(2,n)}_{\alpha\beta}(p,p',p_1,\ldots,p_n)(2\pi)^4\delta^{(4)}(p-p'-\Sigma p_i) =\int d^4x\, d^4y\, d^4x_1\ldots d^4x_n\Gamma^{(2,n)}_{\alpha\beta}(x,y,x_1,\ldots,x_n)\exp\{-ipx+ip'y-i\Sigma p_ix_i\}. \tag{13}$$

The quantity $\Gamma^{(2,n)}$ which enters into (12) can be split into parts which are even and odd with respect to the fermion momentum:

$$\Gamma^{(2,n)}_{\alpha\beta}(p,p,0,\ldots,0)=\Gamma^{(2,n)}_{\rm odd}(p^2)(\hat{p})_{\alpha\beta}+\Gamma^{(2,n)}_{\rm even}(p^2)\delta_{\alpha\beta}. \tag{14}$$

Hence

$$\Gamma=\int\frac{d^4p}{(2\pi)^4}[Z_\psi(p^2,\Phi)\bar{\psi}(p)\hat{p}\psi(p)-M(p^2,\Phi)\bar{\psi}(p)\psi(p)], \tag{15}$$

$$Z_\psi(p^2,\Phi)=\sum_{n=0}^{\infty}\Gamma^{(2,n)}_{\rm odd}(p^2)\frac{\Phi^n}{n!},\qquad M(p^2,\Phi)=-\sum_{n=1}^{\infty}\Gamma^{(2,n)}_{\rm even}(p^2)\frac{\Phi^n}{n!}. \tag{16}$$

It is clear from (15) that the Green's function of the fermion in the external field Φ is

$$G^{-1}(p,\Phi)=M(p^2,\Phi)-Z_\psi(p^2,\Phi)\hat{p}, \tag{17}$$

where the functions M and Z_ψ are defined by the equations (16).

The one-particle irreducible Green's function $\Gamma^{(2,n)}$ satisfy the Callan–Symanzik equations (these equations are valid separately for $\Gamma_{\rm odd}$ and $\Gamma_{\rm even}$):

$$\left[\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}+2\gamma_\psi(g)+n\gamma_\Phi(g)\right]\Gamma^{(2,n)}(\mu,g,p)=0, \tag{18}$$

where, as usual,

$$\beta(g)=\mu\frac{\partial g}{\partial\mu},\quad \gamma_\psi=-\frac{1}{2}\mu\frac{\partial\ln Z_\psi}{\partial\mu},\quad \gamma_\Phi=-\frac{1}{2}\mu\frac{\partial\ln Z_\Phi}{\partial\mu}, \tag{19}$$

and μ is a normalization parameter.

It follows from (18) and (16) that M and Z_ψ also satisfy an equation of the Callan–Symanzik type with the additional terms $\gamma_\Phi\Phi\,\partial/\partial\Phi$:

$$\left[\mu\frac{\partial}{\partial\mu}+\beta\frac{\partial}{\partial g}+2\gamma_\psi+\gamma_\Phi\Phi\frac{\partial}{\partial\Phi}\right]\left\{\begin{matrix}M\\ Z_\psi\end{matrix}\right\}=0. \tag{20}$$

Similar equations for the effective potential and other quantities of that type have been obtained by Coleman and Weinberg[3] (cf. also[4]).

As can be seen from Eq. (17), the physical mass m in the external field Φ is determined by a solution of the equation

$$M(m^2,\Phi)=Z_\psi(m^2,\Phi)m \tag{21}$$

and obviously depends on μ, g, and Φ. The ratio $M(\mu, g, \Phi)/Z_\psi(\mu, g, \Phi)$ satisfies Eq. (20) without the term $2\gamma_\psi$ (this can be seen simply subtracting the equations for $\ln M$ and $\ln Z_\psi$ from one another). This yields easily that the physical mass $m=m(\mu, g, \Phi)$ also satisfies the equation

$$\left[\mu\frac{\partial}{\partial\mu}+\beta\frac{\partial}{\partial g}+\gamma_\Phi\Phi\frac{\partial}{\partial\Phi}\right]m(\mu,g,\Phi)=0. \tag{22}$$

Until now we have restricted ourselves, for the sake of simplicity, to the case of one scalar field Φ and charge g. In addition we have considered fermions of only one kind. In cases of realistic interest there are, of course, several fields $\Phi^{(r)}$ and charges g_i, fact which will be taken into account in the equations below. The set of constants g_i may include the gauge coupling constant g, the Yukawa couplings h_r and other dimensionless constants which occur in the theory; the fields $\Phi^{(r)}$ refer to different irreducible representations of the group G; the possible indices on the fermion masses will temporarily be omitted for simplicity. In Eq. (22) one must make the substitution:

$$\beta\frac{\partial}{\partial g}\to\sum_i\beta_i\frac{\partial}{\partial g_i},\qquad \gamma_\Phi\Phi\frac{\partial}{\partial\Phi}\to\sum_r\gamma_r\Phi_r\frac{\partial}{\partial\Phi_r}.$$

Dimensional considerations imply that one can write:

$$\left[\mu\frac{\partial}{\partial\mu}+\sum_r\Phi_r\frac{\partial}{\partial\Phi_r}\right]m=m, \tag{23}$$

hence

$$\left[-\sum_r(1-\gamma_r)\Phi_r\frac{\partial}{\partial\Phi_r}+\sum_i\beta_i\frac{\partial}{\partial g_i}+1\right]m(\Phi_r,\mu,g_i)=0. \tag{24}$$

For the gauge theories to be considered below the Gell-Mann–Low functions $\beta_i(g)$ are gauge-invariant, i.e., do not depend on the longitudinal part of the vector boson propagator. The quantities γ_r related to the renormalization of the wave functions depend on the gauge. In general, the gauge-noninvariance of the effective action has been noted repeatedly. We have shown before (cf.[4]) that some physical quantities (e.g., the mass ratios in dynamical spontaneous symmetry breaking) do not depend on the gauge choice. We show here on the example of the quantity $m(\Phi_r, g_i, \mu)$ that the gauge-invariant field-dependence appears if instead of the fields Φ_r one considers the renormalized scalar fields

$$\Phi_r'=\Phi_r\zeta_r^{1/2}(\Phi), \tag{25}$$

where the factors ζ_r characterize the change of the renormalization constant of the wave function of the particle corresponding to the field Φ_r in the presence of external fields (including the field Φ_r itself). More precisely, $\zeta_r(\Phi)$ is that function of the scalar fields which appears in the expression for the effective action as a factor in front of the term $(\partial_\mu\Phi_r)^2$, i.e., (cf.[3])

$$\Gamma=\sum_r\int d^4x\,\zeta_r(\Phi)\,|\partial_\mu\Phi_r|^2. \tag{26}$$

It is easy to see that this term, which is related to the kinetic energy of the scalar field, can be written in the form

$$|\partial_\mu(\Phi_r\zeta_r^{1/2}(\Phi))|^2=|\partial_\mu\Phi_r'|^2. \tag{27}$$

Indeed, the error made by introducing ζ_r under the derivative is small, of the order of g_i^2.

The function $\zeta_r(\Phi)$ itself satisfies an equation of the Callan–Symanzik type[3]:

$$\left[-\sum_s(1-\gamma_s)\Phi_s\frac{\partial}{\partial\Phi_s}+\sum_i\beta_i\frac{\partial}{\partial g_i}+2\gamma_r\right]\zeta_r=0. \tag{28}$$

Making use of this equation one can go over from the variables Φ_r to Φ_r' in the following manner

$$\begin{aligned}\sum_r(1-\gamma_r)\frac{\partial}{\partial\ln\Phi_r}&=\sum_r(1-\gamma_r)\frac{\partial}{\partial\ln\Phi_r'}+\frac{1}{2}\sum_{r,s}(1-\gamma_r)\frac{\partial\ln\zeta_s}{\partial\ln\Phi_r}\frac{\partial}{\partial\ln\Phi_s'}\\&=\sum_r(1-\gamma_r)\frac{\partial}{\partial\ln\Phi_r'}+\sum_s\gamma_s\frac{\partial}{\partial\ln\Phi_s'}+\frac{1}{2}\sum_{s,i}\beta_i\frac{\partial\ln\zeta_s}{\partial g_i}\frac{\partial}{\partial\ln\Phi_s'}.\end{aligned} \tag{29}$$

Cancelling the terms which contain γ and substituting (29) into (24) we obtain

$$\left[-\sum_r\frac{\partial}{\partial\ln\Phi_r'}-\sum_{i,s}\beta_i\left(\frac{\partial\ln\zeta_s^{1/2}}{\partial g_i}\right)_{\Phi=\text{const}}\frac{\partial}{\partial\ln\Phi_s'}+\sum_i\beta_i\left(\frac{\partial}{\partial g_i}\right)_{\Phi=\text{const}}+1\right]m=0. \tag{30}$$

One still needs to replace differentiations with respect to g_i at constant Φ_r by differentiations at constant Φ_r':

$$\left(\frac{\partial}{\partial g_i}\right)_\Phi=\left(\frac{\partial}{\partial g_i}\right)_{\Phi'}+\sum_s\left(\frac{\partial\ln\Phi_s'}{\partial g_i}\right)_\Phi\frac{\partial}{\partial\ln\Phi_s'}. \tag{31}$$

Comparing (31) and (30) we finally obtain:

$$\left[-\sum_r\Phi_r'\frac{\partial}{\partial\Phi_r'}+\sum_i\beta_i\left(\frac{\partial}{\partial g_i}\right)_{\Phi'}+1\right]m(\Phi_r',\mu,g_i)=0. \tag{32}$$

Thus, as a function of the variables (Φ_r', μ, g_i) the quantity m satisfies an equation which does not contain gauge-dependent coefficients.

We now assume that spontaneous symmetry breaking occurs in the system, so that even for switched off external sources some of the fields Φ_r have nonvanishing vacuum expectation values $\langle\Phi_r\rangle=v_r$. We consider a regime in which the fields Φ_r' vary proportionally to their vacuum expectation values v_r (when the sources are switched off), i.e., we assume

$$\Phi_r'=\lambda v_r. \tag{33}$$

This regime is very natural in the framework of the renormalization group: if, as is usual, we would deal with quantities which depend not on fields but on momenta, it would correspond to a proportional variation of all the momenta starting with some fixed set. We understand the transition to short distances in the usual sense: $\lambda\to\infty$.

In the regime (33) the sum of partial derivatives $\sum\Phi_r'\partial/\partial\Phi_r'$ is $\partial/\partial t(t=\ln\lambda)$ and in place of (32) we obtain

$$\left[-\frac{\partial}{\partial t}+\sum_i\beta_i\frac{\partial}{\partial g_i}+1\right]m(t,v_r,\mu,g_i)=0,\qquad t=\ln\lambda. \tag{34}$$

The general solution of the equation (34) is well known:

$$m=F(\bar g_i(t))e^t,\quad d\bar g_i(t)/dt=\beta_i(\bar g),\quad \bar g_i(0)=g_i, \tag{35}$$

where F is an arbitary function of the invariant charges $\bar g$, determined from the boundary conditions, e.g., at $t=0(\Phi_\alpha^{(r)}=v_\alpha^{(r)})$.

We now consider the very essential question of the boundary conditions for the equation (34). This equation was obtained from the usual Callan–Symanzik equations for the usual one-particle irreducible Green's functions $\Gamma^{(2,n)}$, and the boundary conditions for m are, in their derivation, related to the boundary conditions on $\Gamma^{(2,n)}$. These conditions are imposed at $p=\mu$ and determine, in particular, the values of the renormalized coupling constants. Thus, for instance, one can set

$$\Gamma^{(2,1)}(p,p,0)|_{p=\mu}=h, \tag{36}$$

where h is the Yukawa coupling constant of the fermion and the scalar field, etc. The boundary conditions for the other $\Gamma^{(2,n)}$ are determined from (36) and also from the normalization conditions for $\Gamma^{(2,0)}$ and $\Gamma^{(0,2)}$ and therefore depend explicitly on μ. The parameter μ which is an argument in Eq. (34), enters exactly through these normalization conditions. It is clear that such a normalization in the solution of Eq. (34) is inconvenient, since the boundary conditions are imposed not on the quantity $m(t, g, \ldots)$ itself, but on functions which play an auxiliary role. It is considerably more convenient to introduce a new definition of the renormalized coupling constants, directly in term of quantities which depend on the scalar fields and which enter into the expression of the effective action. Such a "field" definition of the charges (coupling constants) (we shall call the usuall definition a "momentum" definition) was proposed in[3] and discussed in detail by us in[4].

We define the Yukawa coupling constants h_r by means of the requirement that Eq. (3) for the Fermion masses m_{ai} which in the usual, "momentum" normalization of the constants is valid only in the tree approximation, should be exact. Then by definition the constants h_r are

$$h_r=\frac{1}{v_\alpha^{(r)}}\sum_{ai} m_{ai}\Theta_{ai,\alpha}^{(r)*}, \tag{37}$$

where m_{ai} are the physical masses of the fermions in external scalar fields $v_\alpha^{(r)}$.[3)] Such a definition of the renormalized Yukawa coupling constants fixes the functions $F(\bar{g}_i(t))$ uniquely in the general solution (35) of the equation (34), written for the masses $m_{ai}(t)$. Indeed, for $t=0$, i.e., for $\Phi_\alpha^{(r)}=v_\alpha^{(r)}$, according to the definition (37), we have

$$m_{ai}(0)=\sum_{r\alpha} h_r\Theta_{ai,\alpha}^{(r)}v_\alpha^{(r)},$$

hence

$$m_{ai}(t)=\sum_{r\alpha}\bar{h}_r(t)\Theta_{ai,\alpha}^{(r)}v_\alpha^{(r)}e^t=\sum_{r\alpha}\bar{h}_r(t)\Theta_{ai,\alpha}^{(r)}\Phi_\alpha^{\prime(r)};$$
$$d\bar{h}_r/dt=\beta_{h_r},\quad \bar{h}_r(0)=h_r. \tag{38}$$

The gauge coupling constant g can also be defined in the "field" manner, namely, in terms of the physical mass of the vector boson via a formula of the type

$$M_W=gv.$$

After defining the "field" normalization of the coupling constants we can calculate the effective coupling constants $\bar{h}_r(t)$ and $\bar{g}(t)$ for large values of the fields. This allows us to formulate the requirement of chiral invariance at "short distances," discussed in the introduction by means of requiring that the constants $h_r(t)$ become equal to $t\to\infty$. (More precisely, $h_{r1}/h_{r2}\to 1$ for $t\to\infty$.)

We explain the procedure of transition from the "momentum" to the "field" definition of the Yukawa coupling constants in the framework of the first approximation of perturbation theory, neglecting details which are related to "isotopic" indices.

From the definitions (14)–(16) for $M(p^2,\Phi)$ and $Z_\psi(p^2,\Phi)$ it is obvious that in the tree approximation

$$M^{tree}(p^2,\Phi)=h\Phi,\quad Z_\psi^{tree}(p^2,\Phi)=1. \tag{39}$$

It is easy calculate $M(p^2,\Phi)$ and $Z_\psi(p^2,\Phi)$ also in the one-loop approximation, similar to[3,4]. If one then expresses the physical mass m, which is a root of Eq. (21), in terms of the renormalized fields $\Phi_r'=\zeta_r^{1/2}(\Phi)\Phi_r$ (the quantity ζ_r can also be obtained by means of perturbation theory), then as a result one can obtain the following expression for the physical mass:

$$m(\Phi,\mu,g_i)=h\Phi'\left[1+ag^2\ln\frac{\mu}{\Phi_r'}f\left(g_i,\frac{v_r}{v_s}\right)\right]. \tag{40}$$

Here a is the coefficient in the Gell-Mann–Low function β_h for the effective coupling constant h, μ is a normalization momentum used for the "momentum" definition of the renormalized coupling constant h, and f is a function of all the coupling constants g_i and of the various ratios v_r/v_s. It is essential that in the regime (33) we have $\Phi_r'/\Phi_s'=v_r/v_s$. The transition to the "field" definition of the renormalized coupling constant h (we temporarily denote it by h^F) consists in setting

$$m(\Phi_r'=v_r,\mu,g_i)=h^F v, \tag{41}$$

i.e.,

$$h\left[1+ag^2\ln\left(\frac{\mu}{v_r}f\right)\right]=h^F. \tag{42}$$

Expressing the function $m(\Phi',\ldots)$ in terms of h^F we obtain

$$m(\Phi',\ldots)=h^F\Phi'(1+ag^2\ln(v_r/\Phi_r'))=h^F\Phi'(1-ag^2\ln\lambda). \tag{43}$$

Thus, after the introduction of the "field" coupling constant h^F, m ceases to depend on μ and becomes a function of $t=\ln\lambda$.

Of course, any definition of the coupling constant leads to the appropriate Gell-Mann–Low function for that constant. In particular, with our "field" definition of h^F (41) the Gell-Mann–Low function is

$$\beta_h=\frac{\partial}{\partial\ln\lambda}\frac{m(\ln\lambda,\ldots)}{\Phi'}\bigg|_{\ln\lambda=0}. \tag{44}$$

To first order of perturbation theory we obtain that $\beta_h=-ah^F g^2$, where a is the same as for any "momentum" definition of h (cf. (40)). In the following section we make use of this fact for the calculation of β_h by means of the simplest Feynman diagrams.

One would like to stress that the use of the "field" definition of the coupling constants is purely a question of convenience. One can, of course, start also from the "momentum" definition, but then the corrections of order g^2 to the "chirally invariant" values of the masses

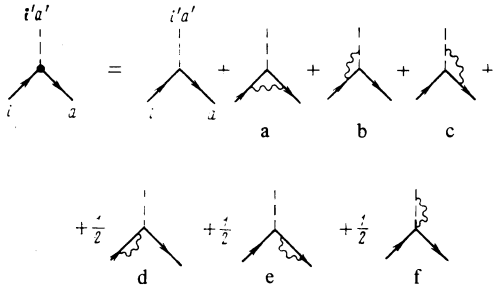

FIG. 1.

would appear from other sources: from the difference between the renormalized constants h_r from each other and from corrections to the tree approximation. The latter can be left out by definition when the "field" approach is used. The definitive expression for such quantities as the mass ratio should not depend on the method of procedure.

3. CALCULATION OF THE FUNCTIONS $\bar{h}_r$ IN THE ONE-LOOP APPROXIMATION

We calculate the coefficient of the term $h_r g^2$ of the function $\beta_{h_r} \equiv \beta_r$ and show that it does not depend on the representation r of the scalar field, but depends only on the representations to which the spinor fields belong. It was already pointed out that it is this circumstance, which does not follow from general group-theoretical considerations, which allows one to impose the requirement of chiral invariance at short distances.

Strictly speaking, as we explained in the preceding section, for our purposes we have to calculate the "field" Gell-Mann–Low function, however, since in the first order of perturbation theory in which we are now interested, the Gell-Mann–Low function does not depend on the method of definition of the coupling constants, and thus we use the "momentum" definition.

A formal calculation of β_r is carried out in the Appendix; here we obtain the same result by means of a more indirect reasoning which may turn out to be useful in the next order of perturbation theory. We start out by setting all the bare coupling constants h_{r0} in the Lagrangian equal to one another: $h_{r0} = h_0$. Then all Yukawa couplings (1) can be written in terms of the fields χ_{ai} in the form (5): $h_0 \bar{\psi}_a^L \varphi_i^R \chi_{ai}$ where the χ_{ai} transform according to the representation $r_L \otimes r_R^*$—a reducible representation of the group G. The expression $h_0 \bar{\psi}_a^L \varphi_i^R \chi_{ai}$ is at the same time an invariant of the chiral group $G_L \times G_R$ in which the left-handed fields (indices a) and the right-handed fields (indices i) transform independently. The bare vertex has now the form $\delta_{aa'} \delta_{ii'}$ where the indices a and i refer to the fermions and the indices a', i' refer to the scalar field. It is obvious that if all the coupling constants h_r are renormalized in the same way to first order in g^2, then to this order the vertex must retain its structure $\delta_{aa'}\delta_{ii'}$. Conversely, if the gauge interaction, which generally does not exhibit the chiral $G_L \times G_R$-invariance, renormalizes the coupling constants h_r differently, then the $\delta_{aa'} \delta_{ii'}$ structure will be destroyed. This must manifest itself in the appearance of a new matrix structure, namely: $(F^n)_{aa'}(F^n)_{ii'}$, which in distinction from the structure $\delta_{aa'} \delta_{ii'}$ does not exhibit chiral $G_L \times G_R$-symmetry, but is an invariant of the group G. We will convince ourselves here that to first order in g^2 the terms proportional to the quantity $(F^n)_{aa'}(F^n)_{ii'}$ cancel in fact, meaning that all coupling constants h_r are renormalized identically, i.e., that the coefficient of $h_r g^2$ in the Gell-Mann–Low function β_r is indeed independent of r (the representation to which the scalar fields belong).

The complete renormalization of the vertex, taking into account the wave function renormalization of the external particles, is determined by the diagrams of Fig. 1.

Let F_{ab}^n denote the generators of the representation r_L and let F_{ij}^n be the generators of the representation r_R; then the generators of the reducible representation $r_L \otimes r_R^*$ are $F_{ab}^n \delta_{ij} - \delta_{ab} F_{ij}^{n*}$, where, on account of the hermiticity of the generators $F_{ij}^{n*} = F_{ji}^n$. The vertex describing the emission of a vector boson by the scalar particle χ_{ai} is proportional to this matrix. Keeping this in mind it is easy to calculate the logarithmically divergent contributions of the individual diagrams in Fig. 1 for an arbitrary choice of gauge α of the vector meson propagator ($D_{\mu\nu} = [g_{\mu\nu} - k_\mu k_\nu (1-\alpha)/k^2]/k^2$). The graph 1, a yields (we omit the common factor $(g_0^2/8\pi^2)\ln\Lambda$)

$$F_{aa'}^n F_{i'i}^n (3+\alpha),$$

the graph b:

$$[F_{aa'}^n F_{i'i}^n - \delta_{aa'} (F^n F^n)_{i'i}](-\alpha),$$

the graph c:

$$[(F^n F^n)_{aa'} \delta_{i'i} - F_{aa'}^n F_{i'i}^n]\alpha,$$

the graph d:

$$\delta_{aa'} (F^n F^n)_{i'i} (-\alpha/2),$$

the graph e:

$$(F^n F^n)_{aa'} \delta_{i'i} (-\alpha/2),$$

and the graph f :

$$[(F^n F^n)_{aa'} \delta_{i'i} + \delta_{aa'} (F^n F^n)_{i'i} - 2F_{aa'}^n F_{i'i}^n](3-\alpha)/2.$$

Summing these contributions we obtain

$$\frac{g_0^2}{8\pi^2} \ln\Lambda \frac{3}{2} [(F^n F^n)_{aa'} \delta_{i'i} + \delta_{aa'} (F^n F^n)_{i'i}] = \frac{3g_0^2}{8\pi^2} \ln\Lambda \frac{C_2(r_L) + C_2(r_R)}{2} \delta_{aa'} \delta_{i'i}, \tag{45}$$

where $C_2(r)$ is the eigenvalue of the Casimir operator for the representation r:

$$(F^n F^n)_{xy} = C_2(r)\delta_{xy}. \tag{46}$$

We note, first, that as expected the result is gauge-invariant, i.e., does not depend on the choice of α.

Secondly, the structure $F^n_{aa'} F^n_{i'i}$ which appeared in the individual graphs cancels in the sum leaving only the $G_L \times G_R$-invariant structure $\delta_{aa'} \delta_{i'i}$, which, as was indicated above means that all the Yukawa coupling constants h_r which couple the irreducible scalar fields are renormalized in the same manner to first order in g^2.

The corresponding term in the Gell-Mann–Low function is

$$\beta_r = -\frac{3h_r g^2}{8\pi^2} \frac{C_2(r_L)+C_2(r_R)}{2} \qquad (47)$$

and does not depend on the representation r of the scalar field.

We emphasize once again that the cancellation of the term $F^n_{aa'} F^n_{i'i}$ is "accidental" (although, as we shall see, it occurs for all groups and all fermion representations). From general group-theoretic considerations one should expect, on the contrary, that in general such a term may appear.[4)]

Thus there is no reason to consider that in order g^4 there will occur the same cancellation; moreover in this order the Gell-Mann–Low function depends in general on the method of definition of the coupling constants. Since we have introduced the "field" definition of the coupling constants (Sec. 2), we have in fact fixed which Gell-Mann–Low function we are dealing with. We assume that for this definition

$$\beta_r = -ah_r g^2 + a_r h_r g^4 + \ldots, \quad a = (3/16\pi^2)[C_2(r_L)+C_2(r_R)],$$
$$\beta_{g^2} = -b_1 g^4 + b_2 g^6 + \ldots \qquad (48)$$

(As was already pointed out, we neglect higher-order terms in the Yukawa coupling constants on account of the fact that the fermion masses are several orders of magnitude smaller than the vector boson masses.) Solving the renormalization group equation (44) together with the equation $d\bar{g}^2/dt = \beta_{g^2}$, we obtain Eq. (7):

$$\bar{g}^2(\lambda) \approx \frac{g^2}{1+b_1 g^2 \ln\lambda}, \quad \bar{h}_r(\lambda) \approx h_r \left(\frac{\bar{g}(\lambda)}{g}\right)^{2\nu} [1+c_r(\bar{g}^2(\lambda)-g^2)], \qquad (49)$$

where

$$\nu = a/b_1, \quad c_r = ab_2/b_1^2 - a_r/b_1.$$

Imposing chiral invariance at short distances, which as explained above means

$$\bar{h}_{r_1}(\lambda)/\bar{h}_{r_2}(\lambda) \to 1, \quad \lambda \to \infty, \qquad (50)$$

(cf. Eq. (9)), we obtain for the physical coupling constants h_r:

$$h_r = (1-c_r g^2) h, \qquad (51)$$

where it suffices to include in the coefficient c_r only the r-dependent part:

$$c_r = -a_r/b_1. \qquad (52)$$

According to Eq. (37) the fermion masses m_{ai} are now determined exactly (i.e., including radiative corrections) by the formula

$$m_{ai} = \sum_{r\alpha} h_r \Theta^{(r)}_{ai,\alpha} v^{(r)}_\alpha, \quad v^{(r)}_\alpha = \langle \Phi^{(r)}_\alpha \rangle.$$

Substituting here the expression (51) for h_r we obtain finally, up to terms of order g^4:

$$m_{ai} = m^{(0)}_{ai} + g^2 \sum_{\substack{a'i' \\ r\alpha}} c_r \Theta^{(r)}_{ai,\alpha} \Theta^{(r)*}_{a'i',\alpha} m^{(0)}_{a'i'},$$
$$m^{(0)}_{ai} = h\langle \chi_{ai} \rangle, \qquad (53)$$

which agrees with Eq. (10) of the Introduction. A simple example of application of Eq. (53) is discussed in the following section.

4. AN EXAMPLE OF APPLICATION OF EQ. (53)

Here we give a simple example of a model in which one can apply the ideas developed above and compute, in principle, the ratio $m_e/m_\mu \sim g^2$.

We consider the $SU(3)$ model of weak and electromagnetic interactions of the leptons ν, e and μ mentioned in[1], but rejected by the authors, since according to their logic the ratio m_e/m_μ in this model remains a free parameter. We show that adding the requirement of small-distance chiral invariance fixes this parameter, and it turns out to be proportional to $\alpha = 1/137$.

We construct two leptonic triplets which exhibit the e-μ universality:

$$\psi_L = \begin{pmatrix} \nu_e \\ e^- \\ \mu^+ \end{pmatrix}_L, \quad \varphi_L = \begin{pmatrix} \nu_\mu \\ \mu^- \\ e^+ \end{pmatrix}_L.$$

It is convienient to replace the triplet φ_L by the antitriplet

$$\varphi_R = CP\varphi_L = \begin{pmatrix} \tilde{\nu}_\mu \\ \mu^+ \\ e^- \end{pmatrix}_R.$$

The quantity φ_L transforms according to the representation $r_L = 3$ of the gauge group $G = SU(3)$, whereas φ_R is in the representation $r_R = 3^*$. Accordingly, the interaction of the leptons with the octet of vector bosons has the form

$$g\bar{\psi}_L \gamma_\mu W^n_\mu F^n \psi_L - g\bar{\varphi}_R \gamma_\mu W^n_\mu F^{n*} \varphi_R,$$

where $F^n = \lambda^n/2$ are the generators of the group $SU(3)$. Since the charge matrix is diag(0, −1, 1) the photon is the combination $A = 1/2\, W_3 - (\sqrt{3}/2) W_8$. The orthogonal combination is the neutral intermediate boson $Z = (\sqrt{3}/2 W_3 + 1/2\, W_8$; it interacts with the leptons proportionally to the matrix $(1/\sqrt{3})$diag(2/3, −1/3, −1/3), *its* interactions with e and μ being purely axial-vector, i.e., parity conserving. Since the group $SU(3)$ contains the Weinberg–Salam group $SU(2) \times U(1)$ as a subgroup, the Weinberg angle is determined to be $\theta_W = -30°$.

This model contains three types of charged vector bosons:

$$W^{\pm}=(W_1\pm iW_2)/\sqrt{2},\quad V^{\pm}=(W_4\pm iW_5)/\sqrt{2},\quad U^{\pm\pm}=(W_6\pm iW_7)/\sqrt{2}.$$

The processes involving the exchange of $W^{\pm}$-bosons lead to the standard theory of weak interactions. Processes with the exchange of doubly charged U-bosons will manifest themselves only via a charge-asymmetry in the reaction $e^+e^-\to\mu^+\mu^-$ at sufficiently high energies. The exchange of a V-boson modifies somewhat the cross section for elastic $\bar{\nu}_\mu e$-scattering and in addition leads to the wrong electron polarization in muon decay. The existing experimental data are not in contradiction with this model if $M_V > 3M_W$.

Assume that the masses of the particles appear as a result of spontaneous symmetry breaking of the gauge invariance, à la Higgs. In particular, let the fermion masses appear as a result of the Yukawa interaction of the form $\bar{\psi}_L\varphi_R\chi$ with scalar fields χ. In this case the scalar fields χ transform according to the reducible representation $3\otimes 3^* = 3^*\otimes 6$. Accordingly the Yukawa interaction must, in general, be written with two independent coupling constants h_3 and h_6:

$$\bar{\psi}_a{}^L\varphi_i{}^R(h_3\varepsilon_{ai\alpha}\Phi_\alpha^{(3)}/\sqrt{2}+h_6(\delta_{a\alpha}\delta_{i\beta}+\delta_{a\beta}\delta_{i\alpha})\Phi_{\alpha\beta}^{(6)}/2)+\text{H.c.},\tag{54}$$

where $\Phi^{(3)}$ and $\Phi^{(6)}$ are scalar fields, transforming according to the irreducible representations 3^* and 6 of $SU(3)$. The coefficients in Eq. (54) have been selected in agreement with the normalization of the Clebsch–Gordan coefficients, Eq. (2). In order that the electron and muon acquire mass it is necessary that the vacuum expectation values of the fields $\Phi_1^{(3)}$ and $\Phi_{23}^{(6)}=\Phi_{32}^{(6)}$ be different from zero. Then

$$m_e=h_3\langle\Phi_1^{(3)}\rangle/\sqrt{2}+h_6\langle\Phi_{32}^{(6)}\rangle,$$
$$m_\mu=-h_3\langle\Phi_1^{(3)}\rangle/\sqrt{2}+h_6\langle\Phi_{32}^{(6)}\rangle,\quad m_\nu=0.$$

It is understood that in general m_e and m_μ are, in general, independent quantities. However, if one imposes the condition of chiral invariance $h_3=h_6=h$, the Yukawa interaction can be written in the form $h\bar{\psi}_a^L\varphi_i^R\chi_{ai}$, where the fields

$$\chi_{ai}=\varepsilon_{ai\alpha}\Phi_\alpha^{(3)}/\sqrt{2}+(\delta_{a\alpha}\delta_{i\beta}+\delta_{a\beta}\delta_{i\alpha})\Phi_{\alpha\beta}^{(6)}/2\tag{55}$$

are irreducible in the framework of the chiral group $SU(3)\times SU(3)$ and the question of which components $\langle\chi_{ai}\rangle$ are nonzero reduces to a choice of axes in the "isotopic" space. If one sets $\langle\chi_{32}\rangle\neq 0$, $\langle\chi_{32}\rangle=0$, then we obtain in zeroth approximation in g^2

$$m_e^{(0)}=0,\quad m_\mu^{(0)}=h\langle\chi_{32}\rangle.\tag{56}$$

Corrections of first order in g^2 are given by the general formula (53). From it we obtain:

$$m_\mu=m_\mu^{(0)}[1+O(g^2)],\quad m_e=g^2m_\mu(c_6-c_3)/2,\quad m_\nu=0,\tag{57}$$

where, according to Eq. (49), $c_6-c_3=(a_3-a_6)/b_1$, $g=2e$, $e^2/4\pi=\alpha=1/137$.

The vector meson masses also appear as a consequence of nonvanishing vacuum expectation values of scalar particles. We note that the presence of $\langle\chi_{32}\rangle\neq 0$ alone does not split the masses of the W and V bosons. In order to make the V and U bosons heavy one can introduce, for example, an octet scalar field ξ (which for group-theoretic reasons does not interact with the leptons) with nonvanishing vacuum expectation value for ξ_8. As a result we obtain the following masses of the vector bosons:

$$M_A^2=0,\quad M_W^2={}^1/_2g^2\langle\chi_{32}\rangle^2,\quad M_Z^2={}^2/_3g^2\langle\chi_{32}\rangle^2,$$
$$M_V^2={}^1/_2g^2\langle\chi_{32}\rangle^2+{}^3/_4g^2\langle\xi_8\rangle^2,\quad M_U^2=g^2\langle\chi_{32}\rangle^2+{}^3/_4g^2\langle\xi_8\rangle^2.$$

subject to the sum rules

$$3M_Z^2=4M_W^2.\quad M_W^2+M_V^2=M_U^2.$$

The example considered here is meant, of course, only as an illustration. For us it is important only that the ratio of the fermion masses is of the order g^2 and can be obtained in a quite natural manner.

We are indebted to V. N. Gribov, Yu. L. Dokshitser and I. T. Dyatlov for useful discussions.

APPENDIX: A DIRECT CALCULATION OF THE GELL-MANN-LOW FUNCTION FOR THE YUKAWA COUPLING CONSTANTS

A Yukawa interaction which is invariant under the gauge group G can be expanded into a sum in terms of the irreducible scalar field representations:

$$\sum_r h_r\bar{\psi}_a{}^L\Theta_{ai,\alpha}^{(r)}\varphi_i{}^R\Phi_\alpha^{(r)},\tag{A.1}$$

where the spinor fields ψ_a^L and φ_i^R and the scalar fields $\Phi_\alpha^{(r)}$ transform respectively according to the representations r_L, r_R, and r of G. The invariance of this interaction Lagrangian implies

$$F_{ab}{}^n\Theta_{bi,\alpha}^{(r)}-\Theta_{aj,\alpha}^{(r)}F_{ji}{}^n=\Theta_{ai,\beta}^{(r)}F_{\beta\alpha}{}^n,\tag{A.2}$$

where F_{ab}^n, F_{ji}^n, $F_{\alpha\beta}^n$ are the generators of the group G respectively in the representations r_L, r_R, and r. According to the general definition of the Gell-Mann–Low function

$$\beta_r=-\frac{\partial}{\partial\ln\Lambda}(h_{r0}Z_{h_r}^{-}Z_{\psi_L}^{1/2}Z_{\varphi_R}^{1/2}Z_{\Phi^{(r)}}^{1/2})_{\ln\Lambda=0,\ g_0^2=g^2,\ h_{r0}=h_r}.$$

In the transverse gauge $Z_\psi=Z_\varphi=1$, and in the expression for Z_h there survives only the diagram represented in Fig. 2a, which together with the graph for $Z_\Phi^{1/2}$ (Fig. 2b) yields

$$\left(F_{ab}{}^n\Theta_{bj,\alpha}^{(r)}F_{ji}{}^n+\frac{1}{2}\Theta_{ai,\beta}^{(r)}F_{\beta\gamma}{}^nF_{\gamma\alpha}{}^n\right)\frac{3g_0^2}{8\pi^2}\ln\Lambda$$
$$\equiv X_{ai,\alpha}^{(r)}\frac{3g_0^2}{8\pi^2}\ln\Lambda.$$

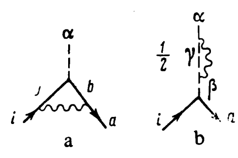

FIG. 2.

It is obvious that the matrix $X^{(r)}_{ai,\alpha}$ must be proportional to $\Theta^{(r)}_{ai,\alpha}$. We determine the proportionality coefficient and show that it does not depend on the representation r of the scalar fields which enter into the Lagrangian (A.1) but is completely determined by the representations r_L and r_R to which the fermions belong.

The matrix $X^{(r)}_{ai,\alpha}$ can be identically rewritten in the form

$$X^{(r)}_{ai,\alpha}={}^1/_2[(F_{ab}{}^n\Theta_{bj,\alpha}-\Theta_{ak,\alpha}F_{kj}{}^n)F_{ji}{}^n$$
$$+\Theta_{ak,\alpha}(F^nF^n)_{ki}]+{}^1/_2[F_{ab}{}^n(\Theta_{bj,\alpha}F_{ji}{}^n-F_{bc}{}^n\Theta_{ci,\alpha})$$
$$+(F^nF^n)_{ac}\Theta_{ci,\alpha}]+{}^1/_2\Theta_{ai,\beta}(F^nF^n)_{\beta\alpha}.$$

We make use of the commutation relation (A.2) and of the fact that $(F^n F^n)_{xy}=C_2(r)\,\delta_{xy'}$, where $C_2(r)$ is the eigenvalue of the Casimir operator in the given representation. We have

$$X^{(r)}_{ai,\alpha}={}^1/_2\Theta_{ai,\alpha}[C_2(r_L)+C_2(r_R)+C_2(r)]$$
$$+{}^1/_2(\Theta_{aj,\gamma}F_{ji}{}^n-F_{ab}{}^n\Theta_{bi,\gamma})F_{\gamma\alpha}{}^n.$$

Using Eq. (A.2) once again, we obtain

$$X^{(r)}_{ai,\alpha}={}^1/_2[C_2(r_L)+C_2(r_R)]\Theta^{(r)}_{ai,\alpha}.$$

We see that the explicit dependence on the representation r of the scalar particles has disappeared. Thus

$$\beta_r=-\frac{3h_rg^2}{8\pi^2}\frac{C_2(r_L)+C_2(r_R)}{2}$$

in agreement with the calculations of Sec. 3.

[1)]The fact that the large-field limit corresponds to a passage to short distances follows from the circumstance that the field-dependence of the effective coupling constants in the effective Lagrangian duplicates the dependence of these constants on the momentum (for some definition of the renormalized charges). Moreover, it can be seen from the Feynman diagrams that large values of the external scalar fields as well as large external momenta cut off the region of integration with respect to small momenta, i.e., the contribution of large distances.

[2)]For switched off sources we shall assume that simultaneously $\Phi_r\to v_r$ and $\Phi'_r\to v_r$, which reduces to the normalization $\zeta_r(\Phi'_s)=1$ condition for ζ_r with all $\Phi'_s=v_s$. This implies some change of the normalization and will be discussed in detail below.

[3)]We note that the constants defined according to (37) and (36) differ by quantities of the order g^2.

[4)]One can verify that this is the situation which arises in the renormalization of the coupling constants in a $\lambda\phi^4$ coupling. If one requires chiral invariance of these interactions by imposing relations on the coupling constants λ, then these relations will be violated already in first order in g^2, so that in this sense the Yukawa couplings are unique.

[1]H. Georgi and S. L. Glashow, Phys. Rev. D7, 2457 (1973).
[2]G. Jona-Lasinio, Nuovo Cimento 34, 1790 (1964).
[3]S. Coleman and E. Weinberg, Phys. Rev. D7, 1888 (1973).
[4]A. A. Ansel'm and D. I. D'yakonov, Zh. Eksp. Teor. Fiz. 68, 1614 (1975) [Sov. Phys. JETP 41, 809 (1975)].

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Nonadiabatic transitions between decaying states

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Processes of the charge exchange type between multiply charged ions and atoms are investigated taking into account the decay of states due to Auger ionization. It is shown that for sufficiently slow collisions with a small resonance defect the decay of states can significantly alter the probability of elastic collisions and the probability of charge exchange. It is also shown that along with the traditional charge exchange scheme a stepwise charge exchange is possible as a result of coherent interaction of states, between which electron transitions occur, via virtual states of the continuous spectrum. The probability of stepwise charge exchange is calculated taking into account the interference between two channels.

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INTRODUCTION

In many problems of the physics of atomic collisions one has to consider transitions between states which decay into the continuous spectrum in the course of the collision. Transitions occurring in collisions of multiply charged ions and atoms,[1,2] in collisions of many-electron atoms with the formation of vacancies in the inner shells[3,4] and in many other cases[5,6] are of such a nature. All the aforementioned processes are characterized by the fact that the lifetime of quasimolecular states between which the transition occurs can be comparable with the time of their interaction in the act of collision.

In investigating the effect of the interactions between states decaying into a common continuum on the probability of a transition it is necessary to distinguish two cases, which practically can be realized in the case of atomic collisions and which in a certain sense are limiting cases. In the first case only the direct interaction between two states and, consequently, only the usual channel for the transition from the initial state to the final state is essential, while the interaction between

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