tor $\exp\{-2t/\alpha - 1/2t^2\}$ so that we use the saddle point method which yields

$$f(\alpha,\beta)=\frac{\pi^{3/2}}{3^{1/2}\cdot 2^{7/6}}\alpha^{-4/3}\left|\varepsilon\left[\beta,\left(\frac{\alpha}{2}\right)^{1/3}\right]\right|^{-2}\exp\left(-\frac{3}{(\alpha\sqrt{2})^{2/3}}\right). \qquad (A.6)$$

Using (11) and (12), we have

$$\left|\varepsilon\left[\beta,\left(\frac{\alpha}{2}\right)^{1/3}\right]\right|^2_{\alpha\to 0}=\begin{cases}(1-\beta)^2, & \beta\neq 1,\\ 9(\alpha/2)^{4/3}, & \beta=1,\end{cases} \qquad (A.7)$$

so that the final expression for $\alpha \ll 1$ becomes

$$f(\alpha,\beta)=\begin{cases}\dfrac{\pi^{3/2}}{3^{1/2}\cdot 2^{7/6}}\alpha^{-4/3}(1-\beta)^{-2}\exp\left(-\dfrac{3}{(\alpha\sqrt{2})^{2/3}}\right), & \beta\neq 1\\ \dfrac{2^{1/6}}{3}\left(\dfrac{\pi}{3}\right)^{3/2}\alpha^{-8/3}\exp\left(-\dfrac{3}{(\alpha\sqrt{2})^{2/3}}\right), & \beta=1.\end{cases} \qquad (A.8)$$

1)We are using the atomic system of units.

2)This result could be foreseen because the plasma field $V(\mathbf{r},t)$ in (2) is assumed to be classical, which, in collision language, corresponds to the description of the relative motion of colliding particles in terms of classical trajectories. Strictly speaking, this approach is valid for cross sections only when $\hbar\omega_0 < E$, where E is the energy of the relative motion of the colliding particles. However, a slight modification enables us to extend this approach to the region where $\hbar\omega_0 \sim E$, at least for optically allowed transitions.[7]

3)The presence of the factor $\lambda\omega_0/v^2$ on the left of (24) is specific precisely for transitions with small resonance defect ω_0.[9]

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Collisionless emission of radiation by an inhomogeneous plasma

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Collisionless radiation by an inhomogeneous plasma, due to the finite motion of charges in the field of external forces and collective interaction forces, is studied. The radiation intensity is inversely proportional to the square of the transverse dimensions of the plasma. It apparently makes the main contribution to the radiation from a vacuum spark and other relativistic beams compressed to a small size by collective interaction forces. The intensity of the collisionless radiation is calculated with account taken of the Fermi statistics of the electrons. The spectral radiance in the low frequency range increases with frequency, reaches a maximum at the frequency of the finite motion of the emitters, and then decreases. Measurement of the collisionless radiation by a plasma compressed to a small size by the pinch effect is a natural way of diagnosing the plasma.

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1. INTRODUCTION

A widely used method of plasma diagnostics is the determination of its parameters from the experimentally measured radiation. The electrons and ions of the plasma are under the influence of external forces and of the collective-interaction forces. These forces accelerate the charges and cause them to radiate.

If the plasma is a gas, i.e., if the time between collisions of the particles is large in comparison with the duration of the collision, then the charges are subjected for the greater part of the time to smooth acceleration under the influence of the forces of the external field and of the average collective interaction. In the case of short-range collisions, the charges are appreciably accelerated within short time intervals (on the order of the duration of the collision). The radiation produced by a gaseous fully ionized plasma is of two types, the radiation accompanying the short-range pair collisions, and the radiation due to the external forces and the average collective-interaction forces. The latter type has no bearing on the particle collisions and can be naturally called collisionless.

If the plasma is uniform and is not situated in an external field, then there is no collisionless radiation. The radiation connected with the Coulomb collisions of

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the charges plays the principal role in a low-temperature plasma, when the average collective-interaction forces, and also the external fields, alter the charge velocity little during the time between the collisions.

With increasing plasma temperature, the efficiency of the Coulomb collisions decreases sharply. To compare the collisionless radiation with the bremsstrahlung in the case of collisions with ions, it suffices to estimate the accelerations experienced by the charges in the two cases. The difference between the radiation intensities lies precisely in the expressions for the electron acceleration. In a hot plasma, when the mean free path l is comparable with the characteristic scale of the inhomogeneity r, the charge accelerations due to the inhomogeneity of the plasma exceed the accelerations due to the pair collisions. Collisionless radiation becomes predominant already at $l \gtrsim r$, inasmuch as in Coulomb collisions the relative change of the velocity is small.

The need for taking into account collisionless radiation at sufficiently large currents, in connection with the growth of the forces of the collision interaction, was indicated by Budker.[1] However, collisionless radiation of an inhomogeneous plasma has not received its due attention in the literature. The only exception is synchrotron radiation or the radiation of the charges in an external magnetic field.

At a fixed energy, the acceleration of a charge is inversely proportional to the dimension of the region of the finite motion. The intensity of the collisionless radiation, which is proportional to the square of the acceleration, increases rapidly with decreasing inhomogeneity scale. Thus, the intensity of collisionless radiation per unit length of a beam of relativistic electrons with a nonrelativistic energy spread is proportional to $e^2W^2N_e/c^3$. Here $W \sim v_T/r$ is the acceleration of the electron, v_T is its thermal velocity, N_e is the number of electrons per unit length of beam, and r is the radius of the beam. Since $ecN_e/r^2 \sim j$ is the current density, we obtain an estimate for the radiation intensity: $dJ/dz \sim j(v_T/c)^4$ erg/cm-sec (j is in A/cm^2).

Large values of the current density $\sim 10^{11}$ A/cm^2 are reached in vacuum sparks (micropinches) observed in strong-current diodes.[2-5] A micropinch is accompanied by a flash of radiation both in the x-ray and in the submillimeter region of the spectrum. The mechanism of this radiation has not yet been established. It appears that the main contribution to the microwave flash is made by the collisionless radiation connected with the finite character of the transverse motion of the charges in the spark. Submillimeter radiation was observed only at fixed frequencies,[3,4] and no absolute values of the intensity are cited at all. However, the dependences of the total and spectral intensities of the radiation on the time can be reconstructed with the aid of the evolution of the compression and the decay of the spark. Photographs in the x-ray region yielded only the micropinch radius averaged over the flash time ($\sim 8\ \mu$).[5]

Recently Sukhorukov and the present author[6] have shown that among the predominant equilibrium stationary configurations of a relativistic beam in vacuum are those of a beam compressed to very small dimensions by the collective-interaction forces. Measurement of collisionless radiation of relativistic beams compressed as a result of the pinch effect contains information on the collective interaction and serves as a natural method of their diagnostics. The present paper is devoted to the calculation of the intensity of collisionless radiation out of an inhomogeneous plasma, and also to an analysis of the spectral composition of the irradiation. The general formulas for the total (Sec. 2) and spectral (Sec. 4) intensities are used to calculate the radiation of a stationary relativistic electron beam.

2. INTENSITY OF COLLISIONLESS RADIATION

We assume that the plasma is in a stationary state, and that the average force acting on the charges and leading to inhomogeneity of their concentration are potential. In this case we disregard a plasma situated in an external magnetic field, the synchronous radiation of which has been well investigated. Let furthermore the electron density n_e in the plasma and the characteristic dimension r of the region occupied by them be small, so that the radiation of each charge takes place within the confines of the plasma without noticable absorption by other charges. The corresponding condition takes the form

$$\delta \gg r, \tag{2.1}$$

where δ is the depth of penetration of the field into the plasma. Let the characteristic frequency ω of the radiation be small in comparison with the temperature:

$$\hbar\omega \ll T. \tag{2.2}$$

Conditions (2.1) and (2.2) make it possible to calculate the radiation intensity by using the classical expressions for the field produced by the moving charges. To be able to employ the formulas for relativistic beams, it is expedient to use the expressions for the radiation intensity of rapidly moving beams ([7], Sec. 73). The intensity of the irradiation of an individual charge of sort α (α takes on the values e and i for the electrons and ions, respectively), moving along a trajectory $\mathbf{r}=\mathbf{r}(t)$, i.e., the energy irradiated per unit time into a solid-angle element dO in a given direction $\mathbf{n}$ is equal to

$$dI_\alpha = \frac{e_\alpha^2}{4\pi c^3}\left\{\frac{2(\mathbf{nW})(\boldsymbol{\beta}\mathbf{W})}{(1-\boldsymbol{\beta}\mathbf{n})^5} + \frac{W^2}{(1-\boldsymbol{\beta}\mathbf{n})^4} - \frac{(1-\beta^2)(\mathbf{nW})^2}{(1-\boldsymbol{\beta}\mathbf{n})^6}\right\}dO. \tag{2.3}$$

Here $\beta = \mathbf{v}/c$. $\mathbf{v}$ is the velocity, $\mathbf{W}$ is the acceleration of the charge moving along a trajectory at the retarded instant of time

$$t' = t - R/c, \tag{2.4}$$

t is the observation time, and R is the radius vector drawn from the point on the trajectory in which the radiation took place to the observation point. Knowing the radiation intensity (2.3) of the individual charge, we can obtain the total intensity by summing over all the charges.

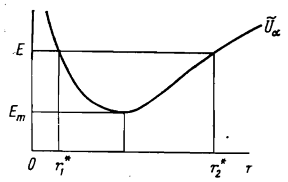

FIG. 1. Region of finite motion in the radial direction. r_1^* and r_2^* are the classical turning points of a particle of energy E and angular momentum M, $E_m = E_m(M)$ is the minimum value of the energy at a fixed value of M.

It is convenient to classify the moving charges with the aid of quantities that are constant along the trajectory, i.e., the integrals of motion, the number of which for one particle is 6. Let $C_1, \ldots, C_6$ be such integrals of the motion, the transition from which to the variables **r** and **p** (coordinates and momenta) constituting a canonical transformation. According to the Liouville theorem the phase volume is not altered by the canonical transformations ([8], Sec. 46). Therefore $d\Gamma = d^3r d^3p = dC_1 \ldots dC_6$. By definition, the distribution functions f_α of the number of particles of sort α in a volume element $d\Gamma$ of phase space is equal to $f_\alpha \cdot 2d\Gamma/(2\pi\hbar)^3$. Taking into consideration the possibility of electron degeneracy, it must be recognized that the probability of the radiation is proportional also to the number of free places in the final state, which is equal to $1 - f_\alpha$ under the condition (2.2). For the intensity of the radiation by charges of sort α from the element $d\Gamma$ of the phase volume of the plasma into the element dO of solid angle in a given direction **n** we obtain ultimately

$$dJ_\alpha = dI_\alpha f_\alpha (1-f_\alpha) \frac{2\,d\Gamma}{(2\pi\hbar)^3} = dI_\alpha f_\alpha (1-f_\alpha) \frac{2}{(2\pi\hbar)^3} dC_1 \ldots dC_6, \qquad (2.5)$$

where dI_α is the intensity of the radiation of the individual charge of sort α and is given by (2.3).

3. RADIATION INTENSITY OF A CYLINDRICALLY SYMMETRICAL PLASMA

We apply the general formula (2.5) for the intensity of collisionless radiation to a cylindrically symmetrical plasma of a relativistic beam. The average electron velocity is directed along the beam axis (the z axis). Let the energy scatter of the particles be nonrelativistic

$$T_\alpha \ll m_\alpha c^2, \quad \alpha = i, e. \qquad (3.1)$$

Here T_α is the temperature of the charges of sort α in their co-moving reference frame. We consider the behavior of the beam during times that are large in comparison with the times of t_e and t_i of establishment of electron–electron and ion–ion equilibrium, but small in comparison with the time of relaxation t_{ei} of the electrons with the ions. The state of the beam during these times can be regarded as stationary.

Since z, φ, and the time are all cyclic variables, the generalized momenta conjugate to them (p_z—the projection of the momentum, $M = rp_\varphi$—the angular momentum about the z axis, and the total energy) are integrals of the motion. The motion of a charge in a cylindrically symmetrical field with a relativistic average velocity v_0 along the z axis with a small velocity scatter in the transverse direction is described in the laboratory frame by the formulas

$$\begin{gathered} t = t_0 + \int_{r_1^*}^{r} \frac{dr}{v_r(r)}, \quad \varphi = \varphi_0 + \frac{M}{m_\alpha^*} \int_{r_1^*}^{r} \frac{dr}{r^2 v_r(r)}, \\ z = z_0 + \frac{p_z}{m_\alpha^*}(t - t_0), \\ v_r(r) = \left\{ \frac{2}{m_\alpha^*} [E - \tilde{U}_\alpha(r)] \right\}^{1/2}, \\ \tilde{U}_\alpha(r) = U_\alpha(r) + \frac{M^2}{2 m_\alpha^* r^2}, \end{gathered} \qquad (3.2)$$

where $m_\alpha^* = m_\alpha(1 - \beta_0^2)^{-1/2}$ is the effective mass, $v_r(r)$ is the radial velocity, $\tilde{U}_\alpha(r)$ is the effective potential energy of the simultaneous motion in the radial direction, $U_\alpha(r)$ is the potential of the force acting on the charges of sort α, and E is the energy of motion in a frame perpendicular to the beam axis. The values of the coordinates $r = r_{1,2}^*$ at which the radial velocity vanishes determine the boundaries of the region of finite motion of the charges (with specified E, M, and β_0) in the radial direction. At a fixed value of the angular momentum M the region of the finite motion exists for energies $E > E_m$, where $E_m = E_m(M)$ is the minimum value of the effective potential $\tilde{U}_\alpha(r)$ (Fig. 1).

Since the forces acting on the charges in a relativistic beam are perpendicular to its axis, the first term in the curly brackets in (2.3) can be omitted. We choose as three (out of the six) integrals of motion $C_1 = p_z$, $C_2 = M$, $C_3 = E$. The remaining three integrals of motion are conveniently chosen in the form of additive constants z_0, φ_0, and t_0, which are connected with the leeway in the choice of the origin of the cyclic variables $dC_4 = dz_0 = dz$, $dC_5 = d\varphi_0$, $dC_6 = dt_0$. We denote by θ the angle between the emission direction and the beam axis. Taking (3.1) into account, we obtain for the emission (in per unit length of beam and per unit solid angle) the following expression:

$$\begin{gathered} \frac{dJ_\alpha}{dz\,dO} = \frac{e^2}{4\pi c^3} \frac{2}{(2\pi\hbar)^3} \int_{-\infty}^{+\infty} dp_z \int_{-\infty}^{+\infty} dM \int_{E_m}^{+\infty} dE\, f_\alpha(1-f_\alpha) \\ \times \int_0^{2\pi} d\varphi_0 \int_0^T dt_0 \left\{ \frac{W^2}{(1-\beta_0\cos\theta)^4} - \frac{(1-\beta_0^2)(\mathbf{nW})^2}{(1-\beta_0\cos\theta)^6} \right\}. \end{gathered} \qquad (3.3)$$

The acceleration **W** in (3.3) depends on φ_0 and t_0 in the combinations $\varphi - \varphi_0$ and $t' - t_0$, where t' is the retarded instant of time (2.4), $T = \oint dr/v_r(r)$ is the period of the finite motion. The dependence on $\varphi - \varphi_0$ is contained only in the term $(\mathbf{n} \cdot \mathbf{W})^2 = W^2 \sin^2\theta \cos^2(\varphi - \varphi_0)$. Therefore the integration with respect to φ_0 is trivial. The summation of charges having different values of t_0 can be carried out by integrating with respect to the coordinates r' at which they are located at the instant t'. In other words, we can use the first formula of (3.2) to change from integration with respect to t_0 to integration with respect to r':

$$\int_0^T dt_0\, W^2(t' - t_0) = \oint W^2(r') \frac{dr'}{v_r(r')}.$$

The integration with respect to r' is carried out over the entire region of the finite motion (both "forward" and "backward"), so that the prime can be omitted.

Thus, the irradiation intensity does not depend on the time and is given by

$$\frac{dJ_\alpha}{dz\,dO}=\frac{e^2}{2c^3}F(\beta_0,\theta)\frac{2}{(2\pi\hbar)^3}\int_{-\infty}^{+\infty}dp_z\int_{-\infty}^{+\infty}dM\int_{E_m}^{\infty}dE\,f_\alpha(1-f_\alpha)\oint\frac{dr}{v_r(r)}W^2(r). \tag{3.4}$$

The angular distribution of the radiation is given by the factor

$$F(\beta_0,\theta)=\frac{1}{(1-\beta_0\cos\theta)^4}-\frac{(1-\beta_0^2)\sin^2\theta}{2(1-\beta_0\cos\theta)^6} \tag{3.5}$$

and as $\beta_0\to1$ it has a strongly pronounced directivity along the beam, which is typical of the emission of fast particles. The total radiation of the length of the beam is obtained by integrating over the solid angle. The corresponding interval is equal to

$$2\pi\int_0^\pi F(\beta_0,\theta)\sin\theta\,d\theta=\frac{8\pi}{3}\gamma^6\left(1+\frac{2}{5}\beta_0^2\right)=\begin{cases}8\pi/3, & \beta_0\ll1,\\ 56\pi\gamma^6/15, & \gamma\gg1,\end{cases}$$

where $\gamma=(1-\beta_0^2)^{-1/2}$ is a relativistic factor.

In the state of equilibrium the distribution function f_α does not depend on the angular momentum M if the beam as a whole does not rotate. For nonrelativistic temperatures (3.1) and for relativistic velocities, f_α takes the form[6]

$$f_\alpha=[1+\exp(x+\psi_\alpha(r))]^{-1},$$
$$\psi_\alpha(r)=\psi_\alpha(0)+\frac{U_\alpha(r)-U_\alpha(0)}{T_\perp},\quad x=\frac{(p_z-p_0)^2}{2m_\alpha^*T_\parallel}+\frac{E-U_\alpha(r)}{T_\perp}, \tag{3.6}$$

where $T_\perp=T_\alpha/\gamma$, $T_\parallel=T_\alpha\gamma$. The constant $\psi_0=\psi_\alpha(0)$ determines the degree of degeneracy on the beam axis.

Noting that $W^2=(T_\perp/m_\alpha^*)^2(d\psi_\alpha/dr)^2$, after interchanging the order of integration and calculating the integrals with respect to M and p_z we have

$$\frac{dJ_\alpha}{dz\,dO}=\frac{e^2}{2c^3}F(\beta_0,\theta)\frac{2}{(2\pi\hbar)^3}\frac{4\pi T_\perp^3(2m_\alpha^*T_\parallel)^{1/2}}{m_\alpha^*}\int_0^\infty\psi_\alpha'^2(r)\,r\,dr\int_0^\infty x^{1/2}f_\alpha(1-f_\alpha)\,dx.$$

After integrating with respect to r by parts ($f_\alpha(1-f_\alpha)=-\partial f_\alpha/\partial\psi_\alpha$) we can easily see that the intensity of the radiation contains the same integral as the volume density of the charges[6]:

$$n_\alpha(r)=\frac{8\pi m_\alpha^*T_\perp(2m_\alpha^*T_\parallel)^{1/2}}{(2\pi\hbar)^3}\int_0^\infty x^{1/2}f_\alpha\,dx.$$

The intensity of the collisionless radiation of particles of sort from a unit length of the beam is expressed thus in the form of a single integral

$$\frac{dJ_\alpha}{dz\,dO}=\frac{e^2}{2c^3}\left(\frac{T_\alpha}{m_\alpha}\right)^2\frac{1}{\gamma^4}F(\beta_0,\theta)\int_0^\infty(r\psi_\alpha')'n_\alpha(r)\,dr. \tag{3.7}$$

Formula (3.7), which was derived with allowance for the Fermi statistics of the radiators, is at the same time convenient also in the case of a Boltzmann distribution. If the deviations from classical statistics are insignificant, then we must put in (3.7) $n_\alpha(r)=n_{\alpha0}\exp\{-\psi_\alpha(r)\}$. Further calculations call for knowledge of the actual form of $\psi_\alpha(r)$.

We consider by way of example the radiation of a beam in which the electrons and ions have a Bennett distribution[9] (see also[6]):

$$n_\alpha^B(r)=\frac{N_\alpha}{\pi r_0^2}\exp\{-\psi_\alpha^B(r)\},\quad \psi_\alpha^B(r)=2\ln\left[1+\left(\frac{r}{r_0}\right)^2\right], \tag{3.8}$$

where N_α is the number of charges of sort α per unit length of the beam and r_0 is the radius of the beam and can be arbitrary because of the scale invariance of the theory. After substitution in (3.7) we have

$$\frac{dJ_\alpha^B}{dz\,dO}=\frac{2e^2}{3\pi c^3}\left(\frac{T_\alpha}{m_\alpha}\right)^2\frac{N_\alpha}{r_0^2\gamma^4}F(\beta_0,\theta).$$

The current of the ultrarelativistic beam is equal to $I_0=eN_ec$, and we arrive at the conclusion that at the given values of the parameters T_α, γ, and r_0 the intensity of collisionless radiation of electrons in the case of Boltzmann statistics is proportional to the beam current. At a given current and electron temperature the intensity of the collisionless radiation is inversely proportional to the square of the beam radius.

It was shown earlier[6] that if the energy of the collective compression of the beam is not offset by the energy of the thermal scatter, then there exist only stationary states of beams that are compressed to such small dimensions, that the equations of classical statistics of ideal gases no longer holds. The Bennett distribution (3.8) is the particular example in which the beam parameters are specially chosen in such a way that a balance exists between the collective compression and the thermal spreading. In the absence of balance, the stationary structure of the compressed beam is determined by non-ideality effects if $T_e<E_a=27.2$ eV, or by electron degeneracy at $T_e>E_a$. In the former case the stationary states of the relativistic beam have not been calculated. Not knowing the structure of the band, at $T_e<E_a$ it is impossible to draw any exact quantitative conclusions with respect to its collisionless radiation. On the other hand, if the electron temperature is not low, $T_e\gg E_a$, then the structure of the beam is determined by the energy balance of the collective compression and the energy of the Fermi exchange interaction of the electrons. In the limiting case of strong degeneracy, under the condition $|\psi_0|\gg1$, $\psi_0<0$, $T_i\gg T_\perp$, the structure of the band was obtained analytically:

$$\psi_i=\frac{|\psi_0|^{3/2}}{6}\left(\frac{r}{r_0}\right)^2,\quad \psi_e=\psi_0+2K_i\int_0^{\psi_i^{1/2}}\frac{dz}{z}(1-e^{-z^2}), \tag{3.9}$$
$$K_i=\frac{e^2N_i}{T_\perp},\quad r_0^2=\frac{\pi}{2^{1/2}}a^2\left(\frac{E_a}{T_e}\right)^{1/2}\frac{T_i}{T_e\gamma},$$

where a is the Bohr radius. The distribution of the electron density is given by the integral

$$n_e(r)=\frac{T_i}{4\pi e^2r_0^2}\int_0^\infty\frac{x^{1/2}dx}{1+e^{x+\psi_e(r)}} \tag{3.10}$$

and takes in the region of strong degeneracy the form

$$n_e(r)=\frac{T_i}{4\pi e^2 r_0^2}\frac{2}{3}|\psi_e(r)|^{3/2},\quad |\psi_e(r)|\gg 1,\quad \psi_e<0.$$

The degree of the degeneracy of the electrons is characterized by the value of the parameter $\psi_0=\psi_e(0)$, which is connected with the particle numbers and with the temperatures by the relation

$$K_2\equiv\frac{e^2N_e}{T_i}=\frac{3}{4}\frac{\pi/K_1}{\sin(\pi/K_1)}\left(\frac{\pi K_1^3}{|\psi_0|^3}\right)^{1/2}\exp\left\{\frac{|\psi_0|}{K_1}-C\right\},\qquad(3.11)$$

where $C=0.577\ldots$ is the Euler constant. The relation (3.10) makes it possible to find the upper limit of the current density of the relativistic beam, which in principle cannot be exceeded because of degeneracy, and also its dependence on the total current and temperature of the electrons. The value of the current density on the beam axis in the case of strong degeneracy is equal to

$$j_0=en_e(0)c=\frac{2^{3/2}}{3\pi^2}\gamma|\psi_0|^{3/2}\left(\frac{T_e}{E_a}\right)^{3/2}\frac{cE_a}{a^2e}=3.1\cdot10^{15}\gamma|\psi_0|^{3/2}\left(\frac{T_e}{E_a}\right)^{3/2}\quad\left[\frac{\text{A}}{\text{cm}^2}\right].$$

According to (3.11), $|\psi_0|$ is proportional to the logarithm of N_e, and consequently also to the logarithm of the total beam current. The maximum current density is proportional to the logarithm of the total current raised to the power $\frac{3}{2}$.

Substituting (3.9) and (3.10) in (3.7) and integrating over the solid angle, we obtain the intensity of the collisionless emission of the electrons:

$$\frac{dJ_e}{dz}=\frac{112}{45\pi}\frac{e^2v_TT^3K_1|\psi_0|^{3/2}\gamma^3}{(\hbar c)^3}=1.8\cdot10^{-24}K_1v_TT_e^3|\psi_0|^{3/2}\gamma^3\quad[\text{W/cm}].\quad(3.12)$$

Here T_e is in degrees, $v_T=(2T_e/m_e)^{1/2}$ is in cm/sec, and the remaining parameters are dimensionless. If we assume $T_e=10^6$ K and $v_T=6\cdot10^8$ cm/sec, then $dJ_e/dz=1.6\cdot K_1|\psi_0|^{3/2}\gamma^3$ [kW/cm]. The high radiation intensity in the stationary state denotes that the radiation can manifest itself during the final stage of the beam compression by the collective-interaction forces. Just like the current density, the intensity of collisionless radiation is proportional to the logarithm of the total current raised to the $\frac{3}{2}$ power. Thus, from the deviation from linearity of the radiation intensity in the beam current (at fixed T_e and γ) we can ascertain whether the electrons are degenerate in a relativistic beam compressed by collective-interaction forces.

Substituting ψ_i from (3.9) in (3.7) and putting $\beta_0=0$, we obtain the intensity of the emission of the beam ions:

$$\frac{dJ_i}{dz\,dO}=\frac{e^2N_i}{\pi c^3r_i^2}\left(\frac{T_i}{m_i}\right)^2\left(1-\frac{1}{2}\sin^2\theta\right),\quad r_i^2=r_0^2\frac{6}{|\psi_0|^{3/2}}.\qquad(3.13)$$

Owing to the large mass of the ions, the intensity of their collisionless radiation is small. For a motion in a quadratic potential, however, the period of the finite oscillations does not depend on the energy or the momentum. As a result, the energy radiated by the ions lies in a narrow spectral interval about the frequency of their collisions. The width of this interval is determined by the collisions and by the Doppler effect (see Sec. 5 below).

4. SPECTRAL RESOLUTION OF COLLISIONLESS RADIATION

The experimental measurements of the radiation make it possible to determine not only the total intensity but also its spectral composition, which contains additional information concerning the plasma. The spectral composition of the radiation of an individual charge can be easily determined if the motion of the charge (which is assumed to be finite with respect to part of the degrees of freedom) admit of separation of the variables in the Hamilton–Jacobi method ([8], Sec. 50). To solve the problem it is convenient to carry out a canonical transformation to the action variables and the angle variables.

For charges moving in a cylindrically symmetrical field, the motion along the coordinates r and φ is finite, while the motion along the coordinate z is infinite. The action variables are the quantities

$$I_r=\frac{1}{2\pi}\oint p_r\,dr,\quad I_\varphi=\frac{1}{2\pi}\int_0^{2\pi}M\,d\varphi=M,$$

and the angle variables are

$$w_r=w_{r0}+\omega_r\int_{r_1^*}^{r}\frac{dr}{v_r(r)},$$
$$w_\varphi=w_{\varphi0}+\varphi+\omega_\varphi\int_{r_1^*}^{r}\frac{dr}{v_r(r)}-\frac{I_\varphi}{m_\alpha^*}\int_{r_1^*}^{r}\frac{dr}{r^2v_r(r)}.\qquad(4.1)$$

The fundamental frequencies ω_r and ω_φ of the radial and azimuthal motions are

$$\omega_r=2\pi\Big/\oint\frac{dr}{v_r(r)},\quad \omega_\varphi=\frac{\omega_r\Delta\varphi}{2\pi},$$
$$\Delta\varphi=\frac{I_\varphi}{m_\alpha^*}\oint\frac{dr}{r^2v_r(r)},\qquad(4.2)$$

where $\Delta\varphi$ is the change of the coordinate φ during the period of the radial motion.

The acceleration $\mathbf{W}$ of the charge, being a unique function of its state, can be expanded with respect to the angle variables (4.1) in a double Fourier series

$$\mathbf{W}(w_r,w_\varphi)=\sum_{l,m=-\infty}^{+\infty}\mathbf{W}_{lm}\exp[i(lw_r+mw_\varphi)],$$
$$\mathbf{W}_{lm}=\frac{1}{(2\pi)^2}\int_{-\pi}^{\pi}dw_r\exp(-ilw_r)\int_{-\pi}^{\pi}dw_\varphi\exp(-imw_\varphi)\mathbf{W}(w_r,w_\varphi).\qquad(4.3)$$

By virtue of the azimuthal symmetry we have

$$W_x=-\frac{1}{m_\alpha^*}U_\alpha'\cos\varphi,\quad W_y=-\frac{1}{m_\alpha^*}U_\alpha'\sin\varphi.$$

Expressing in (4.3) w_φ in terms of φ with the aid of (4.1), we see that the integral with respect to φ is different from zero only for $m=\pm1$. Introducing the notation

$$\varphi(w_r)=\frac{I_\varphi}{m_\alpha^*}\int^{r(w_r)}\frac{dr}{r^2v_r(r)},$$
$$W_{lm}=-\frac{1}{4\pi m_\alpha^*}\int_{-\pi}^{\pi}dw_r\,U_\alpha'(w_r)\exp\left\{-i\left(l+m\frac{\omega_\varphi}{\omega_r}\right)w_r+im\varphi(w_r)\right\},\qquad(4.4)$$

$l=0,\pm 1,\ldots, m=\pm$, we obtain the following expressions for the spectral components of the acceleration:

$$(W_x)_{l,\pm 1}=W_{l,\pm};\quad (W_y)_{l,\pm 1}=\pm\frac{1}{i}W_{l,\pm}.$$

Comparing (4.1) and (3.2), we verify that the angle variables are linear functions of the time:

$$w_r=w_{r0}+\omega_r(t-t_0),\quad w_\varphi=w_{\varphi 0}+\varphi_0+\omega_\varphi(t-t_0). \tag{4.5}$$

The spectral resolution of the acceleration

$$\mathbf{W}(t)=\sum_{\substack{l=-\infty\\ m=\pm 1}}^{+\infty}\mathbf{W}_{lm}\exp\{ilw_{r0}+im(w_{\varphi 0}+\varphi_0)\}\exp\{i(l\omega_r+m\omega_\varphi)(t-t_0)\} \tag{4.6}$$

takes the form of a sum of harmonic oscillations with frequencies $l\omega_r\pm\omega_\varphi$, while $\mathbf{W}$ is an almost periodic function if the frequencies are not commensurate.

To determine the spectral resolution of the collisionless radiation, we substitute (4.6) in (3.3). It is convenient to change from integration with respect to φ_0 and t_0 to integration with respect to w_{r0} and $w_{\varphi 0}$ with the aid of relations (4.5). In the double sums over l, l' and m, m' there remain only terms with $l=-l'$ and $m=-m'$, and we obtain

$$\frac{dJ_\alpha}{dz\,dO}=\frac{2\pi e^2}{c^3}F(\beta_0,\theta)\frac{2}{(2\pi\hbar)^3}\int_{-\infty}^{+\infty}dp_z\int_{-\infty}^{+\infty}dM\int_{E_m}^{+\infty}dE\,f_\alpha(1-f_\alpha)\sum_{\substack{l=-\infty\\ m=\pm}}^{+\infty}\frac{|W_{lm}|^2}{\omega_r}. \tag{4.7}$$

We call attention to the fact that formula (3.3) contains W at the retarded instant of time (2.4). In the wave zone we have

$$R(t')=R_0-\mathbf{r}(t')\mathbf{n}\approx R_0-v_0\cos\theta(t'-t_1).$$

Therefore

$$t'=t_1+(t-R_0/c)/(1-\beta\cos\theta).$$

The time dependence of the waves corresponding to the individual terms of the sum (4.7) is of the form $\exp\{i(l\omega_r+m\omega_\varphi)t'\}$, corresponding at the instant of observation to $\exp\{it(l\omega_r+m\omega_\varphi)/(1-\beta\cos\theta)\}$. Thus, each term of the sum (4.7) yields a monochromatic-oscillation radiation intensity with frequency

$$\omega_{lm}=(l\omega_r+m\omega_\varphi)/(1-\beta\cos\theta). \tag{4.8}$$

The radiation intensity in a unit frequency interval

$$\frac{dJ_\alpha(\omega)}{dz\,dO\,d\omega}=\frac{2\pi e^2}{c^3}F(\beta_0,\theta)\sum_{\substack{l=-\infty\\ m=\pm}}^{+\infty}\frac{2}{(2\pi\hbar)^3}\times\int_{-\infty}^{+\infty}dp_z\int_{-\infty}^{+\infty}dM\int_{E_m}^{+\infty}dE\,f_\alpha(1-f_\alpha)\frac{|W_{lm}|^2}{\omega_r}\delta(\omega-\omega_{lm}) \tag{4.9}$$

takes the form of a sum of intensities of monochromatic oscillations emitted by individual charges. The frequency of each oscillation depends on the energy, angular momentum, and number of the harmonic. Since the energy and the angular momentum are continuously distributed parameters, the collisionless radiation of an inhomogeneous plasma has a continuous spectrum.

5. SPATIAL OSCILLATOR. ION EMISSION SPECTRUM

If the origin $r=0$ for the potential is not a singular point, then the first terms of the expansion $U_\alpha(r)$ near zero are given by

$$U_\alpha(r)=U_0+{}^1/{}_2m_\alpha{}^*\omega_0{}^2r^2+\ldots. \tag{5.1}$$

The fundamental frequencies of the oscillations in the field (5.1) do not depend on the energy E and the angular momentum M of the particles:

$$\omega_r=2\omega_0,\quad \omega_\varphi=\omega_0,\quad \omega_{0\pm}=\pm\omega_0/(1-\beta\cos\theta).$$

The Fourier component (4.4) differs from zero only at $l=0$:

$$W_{0\pm}=-\frac{\omega_0}{2}\left(\frac{E-U_0}{m_\alpha{}^*}\right)^{1/2}\exp\left\{\mp\frac{i}{2}\arccos\frac{M\omega_0}{E-U_0}\right\}.$$

For the potential (5.1) we have $E_m=U_0+\omega_0|M|$, and formula (4.9) becomes

$$\frac{dJ_\alpha(\omega)}{dz\,dO\,d\omega}=\frac{\pi e^2}{2m_\alpha{}^*c^3}F(\beta_0,\theta)\frac{2}{(2\pi\hbar)^3}\int_{-\infty}^{+\infty}dp_z\int_{U_0}^{\infty}dE(E-U_0)^2f_\alpha(1-f_\alpha)\delta(\omega-\omega_{0\pm}).$$

The dependence of $\omega_{0\pm}$ on the velocity of the motion of the radiator (the Doppler effect) leads to a finite width of the spectral emission lines. To calculate the Doppler form of the line it is necessary to take into account exactly the energy scatter. On the other hand, the total radiation intensity contained in the line can be obtained by taking the δ function outside the integral signs. For the spectral intensity of the collisionless emission of the ions of a relativistic beam we obtain (cf. (3.13))

$$\frac{dJ_i(\omega)}{dz\,dO\,d\omega}=\frac{e^2N_i}{2\pi c^3r_i^2}\left(\frac{T_i}{m_i}\right)^2\left(1-\frac{1}{2}\sin^2\theta\right)\delta(\omega-\omega_{0\pm}').$$

If the temperature scatter is small, so that the Doppler broadening is smaller than the collision frequency, then the line width is determined by the collisions. Allowance for the collision leads to replacement of the δ function by the quantity $\nu/\pi[(\omega-\omega_{0\pm})^2+\nu^2]$, which determines the broadening.

6. COLLISIONLESS-RADIATION SPECTRUM OF A BEAM ELECTRON

Let r_e be the characteristic radius of the region occupied by the electrons. The maximum of the intensity of the collisionless radiation takes place, naturally, at the frequencies

$$\omega\sim\omega_{max}=\omega_e(1-\beta_0{}^2)^{1/2}/(1-\beta_0\cos\theta),$$

where $\omega_e=v_T/r_e$ is the characteristic frequency of the finite motion and the co-moving reference frame, and $v_T=(2T_e/m_e)^{1/2}$ is the thermal velocity of the electrons.

The structure of a beam compressed by the collective interaction in the case of strong degeneracy (3.9) is such that in practically the entire region occupied by the electrons, $r \sim r_e \gg r_i$, the potential of the forces acting on the electrons depends logarithmically on r:

$$U_e(r)=U_0+2K_1T_\perp \ln(\lambda r),\quad \lambda^2=e^C/r_i^2,\quad r\gg r_i. \tag{6.1}$$

For the potential (6.1), the limiting value of the momentum $M^*(E)$—a function inverse to $E_m(M)$—is

$$M^*=M^*(E)=\frac{1}{\lambda}(2K_1T_\perp m_\alpha^*)^{1/2}\exp\left(\frac{E-U_0}{2K_1T_\perp}-\frac{1}{2}\right).$$

In place of E, M, and r it is convenient to introduce the dimensionless variables

$$\varepsilon=\frac{E-U_0}{K_1T_\perp},\quad \mu=\frac{M}{M^*},\quad s=\frac{\lambda r}{\mu}\exp\left(-\frac{\varepsilon-1}{2}\right). \tag{6.2}$$

Changing over in (4.2) to the variables (6.2), we find that in the case of motion in a logarithmic potential, the fundamental frequencies decrease exponentially with increasing energy:

$$\omega_r=\frac{\lambda v_T K_1^{1/2}}{\gamma|\mu|}\frac{\pi}{\sigma_1(\mu)}\exp\left(-\frac{\varepsilon-1}{2}\right),\quad \omega_\varphi=\omega_r\frac{\mu}{|\mu|}\frac{\sigma_2(\mu)}{\pi},$$
$$\sigma_1(\mu)=\int_{s_1}^{s_2}ds\left(\ln\frac{e}{\mu^2s^2}-\frac{1}{s^2}\right)^{-1/2},\quad \sigma_2(\mu)=\int_{s_1}^{s_2}\frac{ds}{s^2}\left(\ln\frac{e}{\mu^2s^2}-\frac{1}{s^2}\right)^{-1/2}. \tag{6.3}$$

Here s_1 and s_2 are the turning points in the dimensionless variables and are determined from the equation $\ln(e/\mu^2s^2)-s^{-2}=0$.

The Fourier component of the acceleration (4.4) takes the form

$$W_{lm}=-\frac{K_1^{1/2}\omega_r v_T}{2\pi\gamma}S_{lm}(\mu),$$
$$S_{lm}(\mu)=\int_{s_1}^{s_2}\frac{ds}{s}\left(\ln\frac{e}{\mu^2s^2}-\frac{1}{s^2}\right)^{-1/2}\cos\left[\left(l+m\frac{\omega_\varphi}{\omega_r}\right)w_r-m\varphi(w_r)\right],$$

and the angle variable w_r and the phase φ as functions of s do not depend on the energy

$$w_r=\frac{\pi}{\sigma_1(\mu)}\int_{s_1}^{s}ds'\left(\ln\frac{e}{\mu^2s'^2}-\frac{1}{s'^2}\right)^{-1/2},\quad \varphi(w_r)=\frac{\mu}{|\mu|}\int_{s_1}^{s}\frac{ds'}{s'^2}\left(\ln\frac{e}{\mu^2s'^2}-\frac{1}{s'^2}\right)^{-1/2}.$$

Assuming $p_z-p_0=(2m_\alpha^*T_\parallel)^{1/2}\tau$ and changing over to integration with respect to μ in the region $\mu>0$, we reduce the spectral intensity of the radiation to the form

$$\frac{dJ_e(\omega)}{dz\,dO\,d\omega}=\frac{4e^2}{c^3}\frac{K_1^2T_e^3v_T}{\gamma^3}F(\beta_0,\theta)\frac{2}{(2\pi\hbar)^3}$$
$$\times\sum_{\substack{l=-\infty\\m=\pm}}^{+\infty}\int_{-\infty}^{+\infty}d\tau\int_{\varepsilon_0}^{\infty}d\varepsilon\int_0^1\frac{d\mu}{\mu}\frac{|S_{lm}(\mu)|^2}{\sigma_1(\mu)}f_e(1-f_e)\delta(\omega-\omega_{lm}). \tag{6.4}$$

The frequency ω_{lm} in terms of the variables (6.2) is equal to

$$\omega_{lm}=\frac{(1-\beta_0^2)^{1/2}}{1-\beta_0\cos\theta}\frac{\lambda v_T K_1^{1/2}\pi}{\mu\sigma_1(\mu)}\left[l+m\frac{\sigma_2(\mu)}{\pi}\right]\exp\left(-\frac{\varepsilon-1}{2}\right). \tag{6.5}$$

The first factor in (6.5) gives the change of the frequency due to the Doppler effect ([6], Sec. 48). The distribution function enters in (6.4) in the form of the factor

$$f_e(1-f_e)=\exp(\psi_0+K_1\varepsilon+\tau^2)[1+\exp(\psi_0+K_1\varepsilon+\tau^2)]^{-2}, \tag{6.6}$$

which under the conditions $\psi_0<0$ and $|\psi_0|\gg 1$ decreases rapidly in the region $\varepsilon>|\psi_0|/K_1$. Therefore $\varepsilon\sim|\psi_0|/K_1$ is the characteristic energy scale of the electrons, and $r_e=\exp(|\psi_0|/2K_1-\frac{1}{2})/\lambda K_1^{1/2}$ is the characteristic radius of the region occupied by them.

Low-frequency branch of the spectrum

At low frequencies $|\omega|\ll\omega_{max}$ the main contribution is made by electrons with high energy $\varepsilon\gg|\psi_0|/K_1$, for which (6.6) corresponds to a classical Boltzmann statistics: $f_e(1-f_e)=\exp(|\psi_0|-K_1\varepsilon-\tau^2)$. Calculating in (6.4) the integrals with respect to τ and ε, we obtain

$$\frac{dJ_e(\omega)}{dz\,dO\,d\omega}=\frac{8\sqrt{\pi}Be^2K_1^3T_e^3v_T}{c^3\gamma^3\omega_{max}}\frac{2}{(2\pi\hbar)^3}F(\beta_0,\theta)\left(\frac{|\omega|}{\omega_{max}}\right)^{2K_1-1},$$
$$|\omega|\ll\omega_{max},$$
$$B=\sum_{l,m}\int_0^1\frac{d\mu}{\mu}\frac{|S_{lm}(\mu)|^2}{\sigma_1(\mu)}\left\{\frac{\mu\sigma_1(\mu)}{l\pi+m\sigma_2(\mu)}\right\}^{2K_1}.$$

Here B is a constant on the order of unity and $K_1=e^2N_i/T_\perp$.

Thus, the spectral intensity of the collisionless radiation in the low-frequency region depends on the frequency in power-law fashion. This result is connected only with the logarithmic behavior of the potential $U_e(r)$ at large distances and is therefore valid for arbitrary relativistic beams (and not only those compressed to degeneracy), the electrons of which are prevented from spreading by the forces of the collective interaction with the ions.

Region of high frequencies

The main contribution to the high-frequency part of the spectrum is made by electrons with low energy, the overwhelming part of which are in the degenerate state. Therefore $f_e(1-f_e)$ reduces to a δ function. The integration with respect to τ and ε is carried out in (6.4) with the aid of δ functions. As a result we get

$$\frac{dJ_e(\omega)}{dz\,dO\,d\omega}=\frac{4e^2K_1^3v_T}{c^3\gamma^3|\omega|}\frac{2}{(2\pi\hbar)^3}F(\beta_0,\theta)\sum_{\substack{|l|<L\\m=\pm}}\int_0^1\frac{d\mu}{\mu}\frac{|S_{lm}(\mu)|^2}{\sigma_1(\mu)(K_1\zeta_{lm}(\mu,\omega))^{1/2}}$$
$$\zeta_{lm}(\mu,\omega)=2\ln\left(\frac{\omega\mu\sigma_1(\mu)}{\omega_{max}(l\pi+m\sigma_2(\mu))}\right). \tag{6.7}$$

The main contribution to the sum over l is made by the higher harmonics $|l|\gg 1$; the maximum number of harmonics L is determined by the positiveness of $\zeta_{lm}(\mu,\omega)$. To calculate the integral with respect to μ we can use the asymptotic expression of $S_{lm}(\mu)$ at large l.

In the calculation of the asymptotic form (4.4) at $|l|\gg 1$ it is convenient (at $l<0$) to integrate with respect to w_r along the contour shown in Fig. 2 in the complex w_r plane. In view of the periodicity of $U_e(w_r)$, the integrals along the vertical lines cancel each other and do not

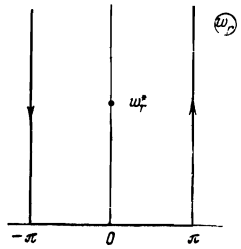

FIG. 2. Integration contour in complex w_r plane at $l < 0$.

change the value of W_{lm}. At $l > 0$ the integration contour should be symmetrical in the lower half-plane. At $|l| \gg 1$, the main contribution to the integral is made by the integrand singularity closest to the real axis. For a logarithmic potential this is a simple pole at the point $w_r = w_r^*$ on the imaginary axis corresponding to the origin $r = 0$. By shifting the integration contour into the region $|\mathrm{Im}\, w_r| > |w_r^*|$, we separate the first term of the asymptotic expression of W_{lm} at $|l| \gg 1$ in the form of a residue at the point $w_r = w_r^*$. Neglecting the dependence of $\zeta_{lm}(\mu, \omega)$ on m at $|l| \gg 1$, we obtain

$$\sum_{m=\pm}|S_{lm}|^2 = \frac{\pi^2 s_1^2}{4}\exp\left\{-\frac{2\pi|l|}{\sigma_1(\mu)}\int_0^{s_1}\left(\frac{1}{s^2}-\ln\frac{e}{\mu^2 s^2}\right)^{-1/2}ds + 2\int_0^{s_1}\frac{ds}{s}\left[\left(1-s^2\ln\frac{e}{\mu^2 s^2}\right)^{-1/2}-1\right]\right\}, \quad |l|\gg 1. \tag{6.8}$$

The main contribution to the integral with respect to μ in (6.7) is made at $|l| \gg 1$ by small μ. Calculating the integrals in (6.8) at $\mu \ll 1$, we have

$$\sum_{m=\pm}|S_{lm}|^2 = \frac{\pi^2}{\ln(e/\mu^2)}\exp\left\{-\left(\frac{8\pi}{e}\right)^{1/2}\frac{\mu}{\ln(e/\mu^2)}|l|\right\}, \quad |l|\gg 1 \quad \mu\ll 1.$$

Integrating with respect to μ and replacing the summation over l by integration, we obtain with logarithmic accuracy the following expression for the spectral intensity of the collisionless radiation in the high-frequency region:

$$\frac{dJ_e(\omega)}{dz\,dO\,d\omega} = \frac{\sqrt{2}}{\pi}\frac{e^2 K_1^{1/2} T_e^3 v_T}{(\hbar c\gamma)^3}F(\beta_0,\theta)\frac{[\ln(|\omega|/\omega_{max})]^{1/2}}{|\omega|}. \tag{6.9}$$

This formula is valid for frequencies $\omega_{max} \ll \omega \ll \omega_{max}(r_e/r_i)$. The intensity decreases with increasing frequency so slowly that this region accounts for the bulk of the total energy of the radiation. After integrating over the frequencies we obtain

$$\frac{dJ_e}{dz\,dO} = \frac{2}{3\pi^2}\frac{e^2 K_1 T_e^3 v_T}{(c\hbar\gamma)^3}|\psi_0|^{3/2}F(\beta_0,\theta),$$

$$\frac{dJ_e}{dz} = \frac{112}{45\pi}\frac{e^2 v_T T_e^3 K_1 |\psi_0|^{3/2}}{(\hbar c)^3}\gamma^3 \quad (\gamma\gg 1).$$

The last formula coincides with the expression for the total intensity of the collisionless radiation (3.12).

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