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Linear transformation of electromagnetic waves in the region of quasitransverse propagation in a three-dimensionally inhomogeneous magnetoactive plasma

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Transformation of circularly polarized waves in the region of a quasitransverse magnetic field in a three-dimensionally inhomogeneous electron plasma is investigated. Analytic expressions are obtained for the fields in the localized interaction region. The possibilities of determining the local electron concentration at the orthogonality point by measuring the transformation coefficient ("Cotton–Mouton" plasma diagnostics) are discussed.

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1. INTRODUCTION

The interaction of circularly polarized waves in a magnetoactive plasma in the region of quasi-transverse magnetic fields (we shall speak for the sake of brevity of a "quasi-transverse" interaction) has been used to explain the singularities of solar radio emission[1-3] (from among the latest studies we point out also[4]) and certain anomalies of the Faraday effect in the earth's ionosphere.[5,6] All the calculations of this effect were carried out so far for a simplified formulation of the problem: plane waves in a homogeneous plasma placed in an inhomogeneous magnetic field; the only analytic result obtained so far for the transformation coefficients is that of Zheleznyakov and Zlotnik[2] (see also[3]), who used the phase-integral method.

In this paper we report a method of describing the effect of quasi-transverse interaction in a three-dimensional inhomogeneous plasma, based on the quasi-isotropic approximation of geometrical optics (see[7], and also[8,9]), which is applicable to waves in weakly-anisotropic media, particularly to waves in a plasma situated in a weak external magnetic field $\mathbf{H}_0$:

$$u=\omega_H^2/\omega^2=(eH_0/mc\omega)^2\ll 1. \qquad (1)$$

This method takes into account the bending and the torsion of the rays, and also the rotation of the force lines of the magnetic field relative to the ray. In the case when the effect of the quasi-transverse interaction is spatially localized, the equations of the quasi-isotropic approximation can be solved analytically, and the results obtained for the transformation coefficients η are, as expected, those of Zheleznyakov and Zlotnik[2] for a homogeneous plasma. At the same time, it becomes possible to determine the corrections to η necessitated by the extended section of the quasi-longitudinal propagation, i.e., by the "incomplete localization" of the effect.

In the Conclusion we discuss briefly the possibility of using the quasi-transverse interaction for polarization plasma diagnostics, which can be referred to in this case as quasi-transverse or "Cotton-Mouton" diagnostics. In contrast to Faraday diagnostics, Cotton-Mouton diagnostics makes it possible to determine under certain conditions the local parameters of a laboratory or ionosphere plasma at the orthogonality point and, in particular, obtain the local electron density at this point.

2. QUASI-ISOTROPIC APPROXIMATION EQUATIONS FOR INTERACTING CIRCULARLY POLARIZED WAVES

In a weakly-anisotropic medium whose dielectric tensor contains a small anisotropic part $\nu_{ij} \equiv \varepsilon_{ij} - \varepsilon_0\delta_{ij} \ll \varepsilon_0$, the field of the electromagnetic wave in the zeroth approximation of geometrical optics can be assumed to be transverse to the beam, just as in an isotropic medium:

$$\mathbf{E}=(\Phi_1\mathbf{n}+\Phi_2\mathbf{b})e^{i\varphi_0}, \tag{2}$$

where $\mathbf{n}$ and $\mathbf{b}$ are the normal and binormal to the ray with tangent vector $\mathbf{t}$, and

$$\varphi_0=k\int\sqrt{\varepsilon_0}\,d\sigma$$

is the phase advance at $\nu_{ij}=0$. The influence of the weak anisotropy manifests itself in the fact that the amplitudes Φ_1 and Φ_2 depend on ν_{ij} in accordance with the equations of the quasi-anisotropic approximation of geometrical optics.[7]

If we assume $\Phi_{1,2}=\Phi_0\Gamma_{1,2}$, where Φ_0 obeys the equation for the conservation of the energy flux in a ray tube in an isotropic medium: $\operatorname{div}(\sqrt{\varepsilon_0}\,|\Phi_0|^2\mathbf{t})=0$, then we obtain for the quantities Γ_1 and Γ_2, which determine the polarization of the field, the following quasi-isotropic approximation equations ($d\sigma$ is the ray length element):

$$\frac{d\Gamma_1}{d\sigma}=\frac{ik}{2\sqrt{\varepsilon_0}}(\nu_{nn}\Gamma_1+\nu_{nb}\Gamma_2)-\frac{\Gamma_2}{T},$$
$$\frac{d\Gamma_2}{d\sigma}=\frac{ik}{2\sqrt{\varepsilon_0}}(\nu_{bn}\Gamma_1+\nu_{bb}\Gamma_2)+\frac{\Gamma_1}{T}, \tag{3}$$

where T is the torsion radius of the beam ($T^{-1}=\mathbf{n}d\mathbf{b}/d\sigma$). From (3) it follows that in the absence of absorption we have on any ray $|\Gamma_1|^2+|\Gamma_2|^2=1$.

In the case of high-frequency electromagnetic waves

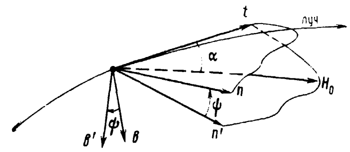

FIG. 1.

in an inhomogeneous magnetoactive plasma, the smallness of ν_{ij} can be ensured by weakness of the magnetic field (1). If we use the known values of the component of the tensor ν_{ij} for this case[10] and write down Eqs. (3) in terms of the unit vectors $\mathbf{n}'$, $\mathbf{b}'$, and $\mathbf{t}$ such that the vector $\mathbf{n}'$ lies in the $(\mathbf{t}, \mathbf{H}_0)$ plane (see Fig. 1), then the quantities

$$\Gamma_1'=\Gamma_1\cos\psi+\Gamma_2\sin\psi,\quad \Gamma_2'=-\Gamma_1\sin\psi+\Gamma_2\cos\psi$$

satisfy at $v\ll 1$ the equations

$$\frac{d\Gamma_1'}{d\sigma}=-iDu^{1/2}\cos^2\alpha\Gamma_1'-\left(-D\cos\alpha+\frac{1}{T_{\text{eff}}}\right)\Gamma_2',$$
$$\frac{d\Gamma_2'}{d\sigma}=\left(-D\cos\alpha+\frac{1}{T_{\text{eff}}}\right)\Gamma_1'-iDu^{1/2}\Gamma_1'. \tag{4}$$

Here $D=kvu^{1/2}/2$, $v=\omega_L^2/\omega^2=4\pi e^2N_e/m\omega^2$, α is the angle between the ray and the magnetic field, ψ is the angle between the normal to the ray $\mathbf{n}$ and the $(\mathbf{t}, \mathbf{H}_0)$ plane, and $T_{\text{eff}}=(1/T-d\psi/d\sigma)^{-1}$ is the effective torsion radius of the ray, which takes into account the rotation of the magnetic-field force lines relative to the ray.

Being interested in transformation of circularly-polarized waves, we change over to the variables[1)]

$$\gamma_{1,2}=\frac{1}{\sqrt{2}}(\Gamma_1'\pm i\Gamma_2')\exp\left\{\frac{i}{2}\int Du^{1/2}(1+\cos^2\alpha)\,d\sigma\right\}, \tag{5}$$

which by virtue of (4) satisfy the equations

$$\frac{d\gamma_1}{d\sigma}=i\left(\frac{1}{T_{\text{eff}}}-D\cos\alpha\right)\gamma_1+\frac{i}{2}Du^{1/2}\gamma_2\sin^2\alpha,$$
$$\frac{d\gamma_2}{d\sigma}=+\frac{i}{2}Du^{1/2}\gamma_1\sin^2\alpha-i\left(\frac{1}{T_{\text{eff}}}-D\cos\alpha\right)\gamma_2 \tag{6}$$

and the normalization condition $|\gamma_1|^2+|\gamma_2|^2=1$. If the wave incident from the side of negative σ is right-circularly polarized, then the system (6) must be solved with the initial conditions

$$\gamma_1(-\infty)=0,\quad |\gamma_2(-\infty)|=1. \tag{7}$$

The system of the quasi-local approximation equations (6) describes the transformation of circularly-polarized waves under rather general conditions: account is taken in (6) of the inhomogeneity of the electron concentration N_e and of the magnetic field H_0, and also the torsion of the rays and the rotation of the force lines.[2)] At the same time, Eq. (6) is much simpler than the second-order system for the transverse components of the field $\mathbf{E}$, which are customarily used for the analysis of wave transformation, and facilitates the qualitative and quantitative analysis of the interaction of the normal waves. We consider below an example of such an analysis—localized interaction of waves in the region of a quasi-transverse field.

3. ESTIMATE OF LENGTH OF THE SECTION OF QUASI-TRANSVERSE INTERACTION

This estimate will be needed later on and can be obtained from the following simple considerations. In Eqs. (6), the Faraday effect corresponds to the terms $D\gamma_{1,2}\cos\alpha$, while the Cotton-Mouton effect corresponds to the terms $\frac{1}{2}Du^{1/2}\gamma_{1,2}\sin^2\alpha$. The "Faraday" terms prevail over the "Cotton-Mouton" terms at $2|\cos\alpha| \gtrsim u^{1/2}\sin^2\alpha$, i.e., in quasi-longitudinal propagation.[10] In this case, the crossing terms in (6) are small, and these equations then describe independent propagation of two circularly-polarized waves. On the other hand, on the quasi-transverse propagation section, the opposite inequality is satisfied

$$2|\cos\alpha| \lesssim u^{1/2}\sin^2\alpha, \tag{8}$$

and in this case the crossing Cotton-Mouton terms responsible for the transformation prevail in (6).

By virtue of the assumed smallness of $u^{1/2}$ it follows from the inequality (8) that the quantity $\cos\alpha$ is also small over the entire quasi-transversality section. On this basis we can assume

$$\cos\alpha \cong \frac{\sigma}{\rho} + O\left(\frac{\sigma^2}{\rho^2}\right), \quad \sin^2\alpha \cong 1 - O\left(\frac{\sigma^2}{\rho^2}\right), \tag{9}$$

where σ is the length of the ray measured from the orthogonality point $\cos\alpha = 0$, and $\rho = (d\alpha/d\sigma)^{-1}$ is the characteristic scale of the variation of the angle α on the ray, and depends both on the curvature of the ray and on the configuration of the magnetic field. In particular, if the ray lies entirely in the plane of the magnetic meridian, then $|\rho_r^{-1} \pm \rho_H^{-1}|^{-1}$, where ρ_r is the curvature radius of the ray and ρ_H is the distance from the orthogonality point to the arbitrary center of the magnetic-field line, where the tangents to the $\mathbf{H}_0$ lines intersect. It follows from (8) and (9) that on the section of the quasi-transverse propagation we have $2|\sigma|/\rho \lesssim u^{1/2}$. As a result, the length of the intersection region

$$l_\perp = 2|\sigma| \cong \rho u^{1/2} \ll \rho \tag{10}$$

is small in comparison with the scale of ρ.

4. TRANSFORMATION COEFFICIENTS FOR THE LOCALIZED REGION OF QUASI-TRANSVERSE INTERACTION

The system of Eqs. (6), of course, cannot be solved analytically at arbitrarily varying parameters. However, if the interaction region is *localized*, i.e., if the length $l_\perp$ of the interaction region is small in comparison not only with ρ but also in comparison with the characteristic plasma inhomogeneity scale L, then the system (6) admits of an approximate solution that is nevertheless sufficiently universal. In fact, at $l_\perp \ll L$ the plasma parameters u and v, as well as the effective radius T_{eff}, can be replaced within the limits of the interaction region by their local values u_0, v_0, and $T_{0\,eff}$ at the orthogonality point. Then, taking (9) into account, we obtain the system of equations

$$\frac{d\gamma_1}{d\sigma} = i\left(-\frac{D_0\sigma}{\rho} + \frac{1}{T_{0\,eff}}\right)\gamma_1 + \frac{1}{2}iD_0u^{1/2}\gamma_2,$$
$$\frac{d\gamma_2}{d\sigma} = +\frac{1}{2}iD_0u^{1/2}\gamma_1 + i\left(\frac{D_0\sigma}{\rho} - \frac{1}{T_{0\,eff}}\right)\gamma_2, \tag{11}$$

where $D_0 = kv_0u_0^{1/2}/2(1-v_0)^{1/2}$.

If we introduce the dimensionless variable $\xi = (D_0/\rho)^{1/2}(\sigma - \rho/D_0T_{0\,eff})$, then we arrive at the system

$$\frac{d\gamma_1}{d\xi} = -i\xi\gamma_1 + \frac{i\sqrt{p}}{2}\gamma_2, \quad \frac{d\gamma_2}{d\xi} = +\frac{i\sqrt{p}}{2}\gamma_1 + i\xi\gamma_2, \tag{12}$$

which contains the single parameter

$$p = D_0u_0\rho = kv_0u_0^{3/2}\rho/2, \tag{13}$$

the physical meaning of which is that it determines the advance of the phase difference of the normal waves on the section of the quasi-transverse propagation. In fact, since the difference of the refractive indices in the region of the quasi-transverse magnetic field is $\Delta n \cong \frac{1}{2}u_0v_0$, the quantity $\Delta\varphi = k\Delta nl_\perp$ is precisely equal to p. The transformation coefficients for localized interaction depends on the value of this parameter (and only on this value).

Elimination of γ_2 from the system (12) leads to the Weber–Hermite equation

$$\frac{d^2\gamma_1}{d\xi^2} + \gamma_1\left(\frac{p}{4} + i + \xi^2\right) = 0, \tag{14}$$

the solution of which is expressed in terms of the parabolic-cylinder functions $D_n(z)$.[11]

Equation (14) and the condition $\gamma_1(-\infty) = 0$ are satisfied by the functions

$$\gamma_1(\xi) = CD_{ip/8-1}(2^{1/2}e^{i3\pi/4}\xi), \tag{15}$$

in which case we have by virtue of (12)

$$\gamma_2(\xi) = \frac{2}{\sqrt{p}}\left(-i\frac{d\gamma_1}{d\xi} + \xi\gamma_1\right) = C\sqrt{\frac{8}{p}}e^{i\pi/4}D_{ip/8}(\sqrt{2}e^{i3\pi/4}\xi), \tag{16}$$

where the constant C is determined from the requirement $|\gamma_2(-\infty)| = 1$ and is equal to (accurate to a phase factor) $(p/8)^{1/2}e^{-\pi p/32}$. Using this value of C and the anisotropic formulas for $D_n(z)$ from[11], we obtain the following value of the intensities of the circularly polarized waves at $\xi \to +\infty$:

$$|\gamma_1(+\infty)|^2 = 1 - e^{-\pi p/4}, \quad |\gamma_2(+\infty)|^2 = e^{-\pi p/4}. \tag{17}$$

Recognizing that the function $\gamma_2(\xi)$ describes a wave with right-hand circular polarization, it is natural to regard the quantity

$$\eta = 1 - e^{-\pi p/4} \tag{18}$$

as the coefficient of transformation of a right-polarized wave into a left-polarized wave. The coefficient for the inverse transformation is obviously the same.

Formula (18) coincides, as is expected, with the values of the transformation coefficients obtained from the localized interaction by the phase-integral method, but in a one-dimensional formulation of the problem. [2] The foregoing analysis leads also to certain additional conclusions (compared with[2]). First, we have obtained not only the value of the transformation coefficient η, but also the value of the field on the ray. As a result, it becomes possible to calculate η under conditions when the source is inside the interaction region, an important factor for ionospheric investigations. Second, it turned out that the torsion of the ray and the rotation of the magnetic force lines do not affect the value of η, since the effective torsion radius T_{eff} has dropped out completely from (12). Third, from the initial equations (6) we can establish the smallness of the transformation of the waves in the extended region of the quasi-longitudinal propagation and obtain the corrections to the transformation coefficient (18) necessitated by the "incomplete localization" of the interaction effect.

The corresponding estimates can be obtained with the aid of Eqs. (6) and (11) by a different method. The simplest method is to solve (6) in the region of quasi-longitudinal propagation by the perturbation method in terms of the parameter $q = u^{1/2}\sin^2\alpha/2\cos\alpha$, which is small precisely in this region. On the other hand, to take the incomplete localization into account in the interaction region $|\sigma| \lesssim l_\perp = u^{1/2}\rho$ we can construct a perturbation-theory series on the basis of Eqs. (11), taking into account in the latter the terms quadratic in σ in the expansions for $\cos\alpha$ and $\sin^2\alpha$, which were discarded in (9), as well as the terms linear in σ of the plasma parameters u, v, and T_{eff}. Matching together the two perturbation-theory series somewhere on the boundary of the interaction region $|\sigma| \sim \rho u^{1/2}$ in such a way that the result depends least on the matching point, we can obtain the correction to the calculated transformation coefficient (18).

The final estimates, which are too cumbersome to present here, indicate that two factors exert a dominant influence: the values of the parameter q_{in} at the point of entrance of the wave into the medium (or at the point where the source is located), and the ratio $\varkappa = l_\perp/L$ of the length $l_\perp$ of the interaction region to the characteristic scale L of the inhomogeneity of the plasma parameters. The correction $\delta\eta$ is in this case of the order of

$$\delta\eta \sim \max(q_{in}\,\varkappa) = \max\left[\left(\frac{u^{1/2}\sin^2\alpha}{2\cos\alpha}\right)_{in}, \frac{\rho u^{1/2}}{L}\right]. \tag{19}$$

5. POSSIBILITIES OF "COTTON-MOUTON" PLASMA DIAGNOSTICS

Unlike the Faraday effect, which yields information on the *integral* characteristics of the plasma, the quasi-transverse interaction due to the Cotton-Mouton effect can serve as a source of information on the *local* characteristics of the plasma at the orthogonality point. In particular, one can speak of determining the local value of the electron concentration N_e if the values of the other parameters are known.

If the effect of the quasi-transverse interaction is spatially localized:

$$l_\perp = \rho u^{1/2} \ll L, \tag{20}$$

then the transformation coefficient is given by expression (18), and from the measured values of η we can then determine the parameter p which characterizes the local properties of the plasma at the orthogonality point:

$$p = D_0 u_0 \rho = \frac{k v_0 u_0^{3/2}\rho}{2} = \frac{4}{\pi}\ln\frac{1}{1-\eta}. \tag{21}$$

A possibility of this kind, which follows from the results of [1,2] has been widely and successfully used to interpret data on the polarization of solar radio emission, particularly to estimate the magnetic field H_0 in the solar corona from reasonably specified values of other plasma parameters. However, as applied to ionosphere and laboratory plasma, the possibilities of Cotton-Mouton diagnostics, insofar as we know, have never been discussed (apparently because there is no theory that takes the bending of the rays into account).

Under the conditions of the earth's ionosphere, the characteristic scale of ρ amounts to (within the framework of the dipole model of the magnetic field) 3000–6000 kilometers, whereas the vertical scale of the plasma inhomogeneity $L_{vert} \sim 100$ km. By virtue of (20), the effect of the quasi-transverse interaction is localized at $u^{1/2} \ll L/\rho \sim \frac{1}{30} - \frac{1}{60}$, i.e., at $\lambda \lesssim 4$–7 meters. Thus, at ultrashort wavelengths one can count on measuring the local electron concentration of the ionosphere plasma at the orthogonality points. In the case of oblique propagation of the radio waves, when the effective scale L of the plasma inhomogeneity along the ray increases by several times (inasmuch as $L_{horiz} \sim 1000$ km), the threshold wavelength increases also by several times (up to 20–30 m).

Under laboratory-plasma conditions, the quantities N_e, H_0, and ρ can vary in a very wide range, and it is therefore quite probable that if the microwave-radiation frequency and the sounding direction are suitably chosen it is possible to "probe" appreciable plasma volumes for the purpose of determining the local electron concentration. This could also be helped by a purposeful control of the magnitude and configuration of the magnetic field, if this can be done under the experimental conditions.

When solving problems in plasma diagnostics it is necessary to strive, besides localization of the effect (the condition (20)), also to a complete "visibility" of the effect, which can be characterized, for example, by the degree of linear polarization $\beta_{lin} = 2\eta(1-\eta)^{1/2}$. This quantity is maximal at $\eta = \frac{1}{2}$, corresponding to the value $p_0 = 4\pi^{-1}\ln 2 = 0.88$ and to a radiation frequency $\omega_0 = (\omega_L^2\omega_H^3\rho/2cp_0)^{1/4}$. [2,3] Generally speaking, at this frequency of the "best visibility" of the effect the inequalities (1) or (20) can be violated. To prevent this, it is necessary, as shown by simple calculations, that the plasma frequency ω_L be in the range

$\omega_H^{3/2}(\rho/c)^{1/2} \gg \omega_L \gg \omega_H^{1/2}(\rho/c)^{-1/2}$,

and this in turn is possible under the condition $\omega_H\rho/c \gg 1$. These inequalities indicate the range of values of ω_H and ω_L in which both spatial localization and good visibility of the effect are possible.

In conclusion, the authors are sincerely grateful to V. V. Zheleznyakov and E. Ya. Zlotnik for attentive discussion of the work and for exceedingly valuable advice.

1)The phase factor is introduced in (5) to simplify the system of equations (6).

2)The region of applicability of these equations is restricted by the condition (1), by the inequality $v \ll 1$, and by the requirement, common to all modifications of the geometrical-optics method, that the parameters of the medium vary slowly in terms of the wavelength (for more details see[9]).

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Relaxation of spin waves with small wave vectors

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It is shown that at small values of the wave vector **k**, the usual kinetic equation for the occupation numbers is not applicable for the calculation of the damping decrement of spin waves $\gamma_{3c}(\mathbf{k})$ due to three-magnon dipole coalescence processes. The effect of four-magnon exchange interaction on spin wave coalescence proceses is taken into account by the diagram technique in all orders of perturbation theory. The expression for $\gamma_{3c}(\mathbf{k})$ for small values of **k** considerably differs from the results derived from the usual kinetic equation. In particular, it is found that three-magnon coalescence processes give a much larger contribution to magnon damping with $\mathbf{k}=0$ than do four-magnon dipole processes.

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At $T \ll T_c$ (T_c is the Curie temperature), many equilibrium and nonequilibrium properties of magnetically ordered crystals are determined by the spin waves.[1,2] In particular, the damping of the spin waves determines the threshold of parametric excitation of the spin system by a variable magnetic field. The relaxation of the magnetization to its equilibrium value, and consequently, the width of the line of magnetic resonance in such crystals are also determined by the damping of the spin waves which, in turn, is due to their interaction with one another, with phonons, impurities, defects of the crystal structure, etc.

In what follows, we shall consider an ideal ferrodielectric and confine ourselves to the case of spin-spin interactions only, to wit, the exchange and relativistic dipole interactions. In this case, the Hamiltonian of the spin waves has the form[1,2]

$$\mathscr{H}=\mathscr{H}_0+V_{3c}^{d}+V_{3sp}^{d}+V_{4s}^{ex}+V_{4s,c}^{d}+W, \tag{1}$$

where $\mathscr{H}_0$ is the Hamiltonian of the free spin waves, V_{3c}^{d} and V_{3sp}^{d} are the contributions from three-magnon processes of coalescence and splitting, due to dipole interaction, V_{4s}^{ex} is the contribution from a four-magnon exchange scattering, $V_{4s,c}^{d}$ is the contribution from four-magnon dipole processes, and W denotes processes of higher order in the creation and annihilation operators of the spin waves, which, at $T \ll T_c$, make a small contribution. We do not include in the Hamiltonian (1) the interactions of spin waves with the vibrations of the crystal lattice, impurities and defects, both from considerations of simplicity and because the experiment allows us to separate the contribution of the "characteristic" mechanisms of magnon damping due only to spin-spin interactions.[2,3]

The damping of spin waves, i.e., the approach to an equilibrium occupation number $n_k=\langle a_k^+ a_k\rangle$ ($\langle\,\rangle$ denotes averaging) is determined in second-order perturbation theory by the usual kinetic equation

$$\frac{dn_k}{dt}=2\pi\left\{\sum_{lm}|V_{lm}|^2\delta(\varepsilon_l-\varepsilon_m)-\sum_{l'm'}|V_{l'm'}|^2\delta(\varepsilon_{l'}-\varepsilon_{m'})\right\}. \tag{2}$$

Here $\delta(x)$ is the delta function, V_{lm} are the matrix elements of the interaction operator in the representation in which the Hamiltonian $\mathscr{H}_0$ is diagonal, l and m denote