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Translated by J. G. Adashko

Stimulated Raman scattering and the penetration of an electromagnetic wave into an inhomogeneous plasma

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(Submitted January 20, 1976)
Zh. Eksp. Teor. Fiz. 70, 2161–2177 (June 1976)

We use the equations of non-linear electrodynamics to formulate relations that enable us to determine the noise level and the intensity of the pumping wave in an inhomogeneous medium under conditions of convective parametric instability. Using these relations we study the effect of stimulated Raman scattering on the penetration of an electromagnetic wave into a rarefied inhomogeneous plasma.

PACS numbers: 52.40.Db

INTRODUCTION

Parametric instabilities in an inhomogeneous plasma can be either drift (convective)[1–6] or absolute[7–15] instabilities. In the first case the departure of growing waves from the region where they interact resonantly with the pumping wave leads to the establishing of a stationary state. We can then determine not only the noise amplitude but also the way it varies with the intensity of the pumping wave in an inhomogeneous plasma. In the present paper we consider how a decay-type parametric instability which occurs in a rarefied plasma—stimulated Raman scattering (SRS)—affects the penetration of the pumping wave.

We use in the first section the phenomenological equations of non-linear electrodynamics to formulate the initial relations for a self-consistent determination for the noise level and the intensity of the pumping wave in an inhomogeneous medium. We determine in the second section, from the solution of the dispersion relation, the growth coefficients for SRS in a plasma. We obtain in the third section a non-linear differential equation to determine the pump wave intensity. We give in the fourth section the results of a numerical solution of that equation for a linear variation of the plasma density. In the conclusion we discuss the application of the results to a laser plasma.

We show in the paper that SRS has practically no effect on the propagation of the pumping wave when its intensity is small. When the intensity increases this effect becomes important. The distance over which the pumping wave can travel without practically changing its amplitude is proportional to the wavelength of the incident wave and to the plasma temperature, and inversely proportional to the intensity. At large distances the intensity of the pumping wave decreases with distance according to a hyperbolic law. We show that the nature of the wave penetration does not depend on the steepness of the increase in density when the plasma density varies according to a power law.

1. GENERAL RELATIONS

1. We start our considerations with the equation for the electrical field strength $\mathbf{E}$ in an arbitrary material medium[6]

$$\operatorname{rot}\operatorname{rot}\mathbf{E}+\frac{1}{c^2}\frac{\partial^2\mathbf{D}}{\partial t^2}=0, \tag{1.1}$$

where the induction vector $\mathbf{D}$ is connected with the field $\mathbf{E}$ by a non-linear material equation which in the quadratic approximation has the form[17]

$$D_i(\mathbf{r},t)=\int d\mathbf{r}'\int_{-\infty}^{t}dt'\Big\{\varepsilon_{ij}(\mathbf{r},t;\mathbf{r}',t')E_j(\mathbf{r}',t') + \int d\mathbf{r}''\int_{-\infty}^{t'}dt''\,\varepsilon_{ijl}(\mathbf{r},t;\mathbf{r}',t';\mathbf{r}'',t'')E_j(\mathbf{r}',t')E_l(\mathbf{r}'',t'')\Big\}; \tag{1.2}$$

ε_{ij} and ε_{ijl} are, respectively, the linear and quadratic permittivity tensors of the medium.

We assume that the field in the medium is the sum of the field of a strong pumping wave $\mathbf{E}_0$ and of weaker fluctuation fields $\delta\mathbf{E}$:

$$\mathbf{E}(\mathbf{r},t)=\mathbf{E}_0(\mathbf{r},t)+\delta\mathbf{E}(\mathbf{r},t). \tag{1.3}$$

We substitute Eq. (1.3) into Eq. (1.2) and average over a statistical ensemble. As a result we get for the pumping wave the equation

$$(\operatorname{rot}\operatorname{rot}\mathbf{E}_0)_i+\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\int_{-\infty}^{t}dt'\,d\mathbf{r}'\,\varepsilon_{ij}(\mathbf{r},t;\mathbf{r}',t')E_{0j}(\mathbf{r}',t')=-\frac{1}{c^2}\frac{\partial^2\tilde{D}_i}{\partial t^2}, \tag{1.4}$$

where the vector $\tilde{\mathbf{D}}$ determines the non-linear effect of the fluctuation fields on the pumping wave:

$$D_l(\mathbf{r},t)=\int_{-\infty}^{t}dt'\,d\mathbf{r}'\int_{-\infty}^{t'}dt''\,d\mathbf{r}''\,\varepsilon_{ijl}(\mathbf{r},t;\mathbf{r}',t';\mathbf{r}'',t'')\langle\delta E_j(\mathbf{r}',t')\delta E_l(\mathbf{r}'',t'')\rangle. \tag{1.5}$$

We have dropped in Eq. (1.4) the terms quadratic in the field of the pumping wave which are of no interest for the problems considered by us.

Subtracting Eq. (1.4) from the equation which arises when we substitute Eqs. (1.2), (1.3) into Eq. (1.1) we get an equation for the fluctuation fields. In the approximation linear in the pumping wave field, it has the form

$$(\mathrm{rot\,rot}\,\delta\mathbf{E})_i+\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\int_{-\infty}^{t}dt'\,d\mathbf{r}'\,\varepsilon_{ij}(\mathbf{r},t;\mathbf{r}',t')\delta E_j(\mathbf{r}',t')$$
$$=-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\int_{-\infty}^{t}dt'\,d\mathbf{r}'\int_{-\infty}^{t'}dt''\,d\mathbf{r}''\,\varepsilon_{ijl}(\mathbf{r},t;\mathbf{r}',t';\mathbf{r}''\,t'')$$
$$\times[E_{0j}(\mathbf{r}',t')\delta E_l(\mathbf{r}'',t'')+\delta E_j(\mathbf{r}',t')E_{0l}(\mathbf{r}'',t'')]. \tag{1.6}$$

Equations (1.4), (1.6) are the basis for studying the simultaneous change in the pumping wave and the fluctuation fields in the medium.

2. We consider a medium the properties of which do not change with time, while spatially they change only along the x-axis (one-dimensional inhomogeneity). The permittivity tensors in Eqs. (1.5), (1.6) depend then on the differences of the arguments and on the variable x'. We shall assume that the variation with x' is a slow one and we use for the pumping wave and the fluctuation fields the geometric optics approximation

$$\mathbf{E}_0(\mathbf{r},t)=\frac{1}{2}\Big(\mathbf{E}_0{}^0(x)\exp\Big[-i\omega_0t+i\mathbf{k}_{0\perp}\mathbf{r}_\perp+i\int^x dx_1\,k_{0\parallel}(x_1)\Big]$$
$$+\mathbf{E}_0{}^{0*}(x)\exp\Big[i\omega_0t-i\mathbf{k}_{0\perp}\mathbf{r}_\perp-i\int^x dx_1\,k_{0\parallel}(x_1)\Big]\Big), \tag{1.7}$$

$$\delta\mathbf{E}(\mathbf{r},t)=\sum_\alpha\int_0^\infty d\omega\int d\mathbf{k}_\perp\Big(\delta\mathbf{E}_\alpha(\omega,\mathbf{k}_\perp,x)\exp\Big[-i\omega t+i\mathbf{k}_\perp\mathbf{r}_\perp+i\int^x dx_1\,k_{\parallel,\alpha}(x_1)\Big]$$
$$+\delta\mathbf{E}_\alpha{}^*\exp\Big[i\omega t-i\mathbf{k}_\perp\mathbf{r}_\perp-i\int^x dx_1\,k_{\parallel,\alpha}^*(x_1)\Big]\Big), \tag{1.8}$$

where $\mathbf{k}_{0\perp}$ and $k_{0\parallel}=k_\parallel(\omega_0,\mathbf{k}_{0\perp},x)$ are, respectively, the transverse and longitudinal (with respect to the x-axis) components of the wavevector of the pumping wave, the frequency of which is ω_0; the sum in Eq. (1.8) is over all possible modes of excited waves, and we have to determine the longitudinal components $k_{\parallel,\alpha}(\omega,\mathbf{k}_\perp,x)$ of their wavevectors.

To do this we substitute expressions (1.7), (1.8) into Eq. (1.6) and we shall assume that only two fluctuation waves with frequencies ω and $\omega_0-\omega$ are coupled through the pumping wave. As a result we get

$$M_{ij}(\omega,\mathbf{k}_\alpha,x)\delta E_{\alpha,j}(\omega,\mathbf{k}_\perp,x)$$
$$=-{}^1/_2S_{ijl}(\omega,\mathbf{k}_\perp,k_{0\parallel}-k_{\parallel,\beta}^*(\omega_0-\omega);\omega-\omega_0,\mathbf{k}_\perp-\mathbf{k}_{0\perp},-k_{\parallel,\beta}^*(\omega_0-\omega))$$
$$\times E_{0j}{}^0\delta E_{\beta,l}^*(\omega_0-\omega,\mathbf{k}_{0\perp}-\mathbf{k}_\perp,x)\exp\Big[i\int^x dx_1(k_{0\parallel}-k_{\parallel,\beta}^*(\omega_0-\omega)-k_{\parallel,\alpha}(\omega))\Big], \tag{1.9}$$

where $\mathbf{k}_\alpha=(\mathbf{k}_\perp,k_{\parallel,\alpha}(\omega))$, M_{ij} is the Maxwell tensor:

$$M_{ij}(\omega,\mathbf{k}_\alpha,x)=\varepsilon_{ij}(\omega,\mathbf{k}_\alpha,x)+\frac{c^2k_\alpha^2}{\omega^2}\Big(\frac{k_{\alpha i}k_{\alpha j}}{k_\alpha^2}-\delta_{ij}\Big), \tag{1.10}$$

$$\varepsilon_{ij}(\omega,\mathbf{k}_\alpha,x)=\int d\rho\int_0^\infty d\tau\,\varepsilon_{ij}(\rho,\tau,x)\exp[i\omega\tau-i\mathbf{k}_\perp\rho_\perp-ik_{\parallel,\alpha}(\omega)\rho_\parallel], \tag{1.11}$$

$$S_{ijl}(\omega,\mathbf{k}_\perp,k_{\parallel,\alpha}(\omega);\omega',\mathbf{k}_\perp',k_{\parallel,\beta}(\omega'))$$
$$=\varepsilon_{ijl}(\omega,\mathbf{k}_\perp,k_{\parallel,\alpha}(\omega);\omega',\mathbf{k}_\perp',k_{\parallel,\beta}(\omega'))$$
$$+\varepsilon_{ilj}(\omega,\mathbf{k}_\perp,k_{\parallel,\alpha}(\omega);\omega-\omega',\mathbf{k}_\perp-\mathbf{k}_\perp',k_{\parallel,\alpha}(\omega)-k_{\parallel,\beta}(\omega')), \tag{1.12}$$

$$\varepsilon_{ijl}(\omega,\mathbf{k}_\perp,k_{\parallel,\alpha}(\omega);\omega',\mathbf{k}_\perp',k_{\parallel,\beta}(\omega'))=\int d\rho\int_0^\infty d\tau\int d\rho'\int_0^\infty d\tau'\,\varepsilon_{ijl}(\tau,\rho;\tau',\rho')$$
$$\times\exp[i\omega\tau-i\mathbf{k}_\alpha\rho+i\omega'\tau'-i\mathbf{k}_\beta\rho']. \tag{1.13}$$

In deriving Eq. (1.9) we neglected the x-dependence, due to the inhomogeneity of the medium, of the wave amplitudes, of the permittivities, and of the longitudinal components of the wavevectors. This neglect is justified by the fact that the resonance interaction of a given pair of fluctuation waves with the pumping wave proceeds in a range of x values with a width much smaller than the characteristic dimension of the inhomogeneity of the medium, and the scale along which the pumping wave amplitudes changes.[1,2] We take this interaction into account by means of complex longitudinal components of the wavevectors and assume that the quantity $\delta\mathbf{E}$ determines the amplitudes of the fluctuation waves until they enter the resonance interaction region. In contrast to the work of other authors[1-6] in which the interaction of waves with normal (independent of the pumping wave) dispersion relations (weak parametric coupling) is taken into account through the slow dependence of their amplitudes on the coordinates, in our approach one can consider waves for which the effect of the pumping wave on the dispersion laws is not small (strong parametric coupling[18]).

We perform in Eq. (1.9) the substitution

$$\omega\to\omega_0-\omega,\quad \mathbf{k}_\perp\to\mathbf{k}_{0\perp}-\mathbf{k}_\perp,\quad \alpha\rightleftarrows\beta$$

and we take the complex conjugate of the expression obtained:

$$M_{ij}^*(\omega_0-\omega,\mathbf{k}_{0\perp}-\mathbf{k}_\perp,k_{\parallel,\beta}(\omega_0-\omega))\delta E_{\beta,j}^*(\omega_0-\omega,\mathbf{k}_{0\perp}-\mathbf{k}_\perp,x)$$
$$=-{}^1/_2S_{ijl}^*(\omega_0-\omega,\mathbf{k}_{0\perp}-\mathbf{k}_\perp,k_{0\parallel}-k_{\parallel,\alpha}^*(\omega);-\omega,-\mathbf{k}_\perp,-k_{\parallel,\alpha}^*(\omega))$$
$$\times E_{0j}{}^{0*}\delta E_{\alpha,l}(\omega,\mathbf{k}_\perp,x)\exp\Big[i\int^x dx_1(k_{\parallel,\alpha}(\omega)-k_{0\parallel}+k_{\parallel,\beta}^*(\omega_0-\omega))\Big]. \tag{1.14}$$

Using Eq. (1.14) to get an expression for δE_j^* and substituting it into Eq. (1.9) and using the definitions (1.12), (1.13) we get the relation

$$n_iM_{ij}(\omega,\mathbf{k}_\perp,k_{\parallel,\alpha}(\omega))n_j=-{}^1/_4|E_0{}^0|^2[n_{0j}n_i$$
$$\times S_{ijl}^*(-\omega,-\mathbf{k}_\perp,k_{\parallel,\beta}(\omega_0-\omega)-k_{0\parallel};\omega_0-\omega,\mathbf{k}_{0\perp}-\mathbf{k}_\perp,k_{\parallel,\beta}(\omega_0-\omega))]$$
$$\times M_{lp}^{*(-1)}(\omega_0-\omega,\mathbf{k}_{0\perp}-\mathbf{k}_\perp,k_{\parallel,\beta}(\omega_0-\omega))[n_{0q}n_rS_{pqr}(\omega-\omega_0,$$
$$\mathbf{k}_\perp-\mathbf{k}_{0\perp},k_{\parallel,\alpha}(\omega)-k_{0\parallel};\omega,\mathbf{k}_\perp,k_{\parallel,\alpha}(\omega))], \tag{1.15}$$

where $M_{lp}^{(-1)}$ is the inverse Maxwell tensor; $\mathbf{n}$ and $\mathbf{n}_0$ are unit polarization vectors of the fluctuation wave of frequency ω and of the pumping wave, respectively.

By taking the logarithmic derivative of Eq. (1.9) we can easily check that apart from small quantities (of the order of the inverse of the dimension of the inhomogeneity) in the interaction region the condition $k_{0\parallel} \approx k_{\parallel,\alpha}(\omega)+k^*_{\parallel,\beta}(\omega_0-\omega)$ is satisfied. Using that relation and the symmetry properties of the tensors S[19]

$$S_{ijl}(\omega,\mathbf{k};\omega',\mathbf{k}')=-S_{ilj}(\omega',\mathbf{k}';\omega,\mathbf{k})=-S_{jli}(\omega-\omega',\mathbf{k}-\mathbf{k}';\omega,\mathbf{k}), \tag{1.16}$$

we can reduce Eq. (1.15) to the form

$$n_iM_{ij}(\omega,\mathbf{k})n_j={}^1/_4|E_0{}^0|^2A_l{}^*M_{lp}^{*(-1)}(\omega_0-\omega,\mathbf{k}_0-\mathbf{k})A_p, \tag{1.15'}$$

where

$$\mathbf{k}=(\mathbf{k}_\perp,k_{\parallel,\alpha}(\omega,\mathbf{k}_\perp,x)),\quad A_p=n_{0q}n_rS_{pqr}(\omega-\omega_0,\mathbf{k}-\mathbf{k}_0;\omega,\mathbf{k}).$$

The dispersion relation (1.15′) enables us to determine the quantity $k_{\parallel,\alpha}$.

3. We now consider Eq. (1.14) for the pumping waves.[1)] We substitute Eq. (1.8) into Eq. (1.5) and use the notation (1.12). As a result we get

$$\tilde{D}_i(\mathbf{r},t)=\sum_{\alpha,\gamma}\int_0^\infty d\omega_1\int d\mathbf{k}_{1\perp}\int_0^\infty d\omega_2\int d\mathbf{k}_{2\perp}\Big\{\varepsilon_{ijl}(\omega_1+\omega_2,\mathbf{k}_{1\alpha}+\mathbf{k}_{2\gamma};\omega_2,\mathbf{k}_{2\gamma})$$
$$\times\exp\Big[-it(\omega_1+\omega_2)+i\mathbf{r}_\perp(\mathbf{k}_{1\perp}+\mathbf{k}_{2\perp})+i\int^x dx_1(k_{\parallel,\alpha}(\omega_1)+k_{\parallel,\gamma}(\omega_2))\Big]$$
$$\times\langle\delta E_{\alpha,j}(\omega_1,\mathbf{k}_{1\perp})\delta E_{\gamma,l}(\omega_2,\mathbf{k}_{2\perp})\rangle+\varepsilon_{ijl}(\omega_1-\omega_2,\mathbf{k}_{1\alpha}-\mathbf{k}_{2\gamma}^*;-\omega_2,\mathbf{k}_{2\gamma}^*)$$
$$\times\exp\Big[-it(\omega_1-\omega_2)+i\mathbf{r}_\perp(\mathbf{k}_{1\perp}-\mathbf{k}_{2\perp})+i\int^x dx_1(k_{\parallel,\alpha}(\omega_1)-k^*_{\parallel,\gamma}(\omega_2))\Big]$$
$$\times\langle\delta E_{\alpha,j}(\omega_1,\mathbf{k}_{1\perp})\delta E^*_{\gamma,l}(\omega_2,\mathbf{k}_{2\perp})\rangle+\text{c.c.}\Big\}. \tag{1.17}$$

Owing to the pumping wave, fluctation waves with different frequencies are not independent, but their amplitudes are interrelated by Eq. (1.9). Just such coupled waves interact with the pumping wave, diminishing its energy, and in Eq. (1.17) the average of the product of the amplitudes is given by the expression

$$\langle\delta E_{\alpha,j}(\omega,\mathbf{k}_\perp)\delta E^*_{\gamma,l}(\omega',\mathbf{k}_\perp')\rangle$$
$$=-{}^1/_2M_{jp}^{(-1)}(\omega,\mathbf{k}_\alpha)S_{pqr}(\omega,\mathbf{k}_\perp,k_{0\parallel}-k^*_{\parallel,\beta}(\omega_0-\omega);\omega-\omega_0,\mathbf{k}_\perp-\mathbf{k}_{0\perp},-k^*_{\parallel,\beta}(\omega_0-\omega)$$
$$\times E_{0q}{}^0\exp\Big[i\int^x dx_1(k_{0\parallel}-k^*_{\parallel,\beta}(\omega_0-\omega)-k_{\parallel,\alpha}(\omega))\Big]$$
$$\times\langle\delta E^*_{\beta,r}(\omega_0-\omega,\mathbf{k}_{0\perp}-\mathbf{k}_\perp)\delta E^*_{\gamma,l}(\omega',\mathbf{k}_\perp')\rangle$$
$$-{}^1/_2M_{lp}^{(-1)*}(\omega',\mathbf{k}_\gamma')S^*_{pqr}(\omega',\mathbf{k}_\perp',k_{0\parallel}-k^*_{\parallel,\beta}(\omega_0-\omega');\omega'-\omega_0,\mathbf{k}_\perp'-\mathbf{k}_{0\perp},$$
$$-k^*_{\parallel,\beta}(\omega_0-\omega'))E^{0*}_{0q}\exp\Big[i\int^x dx_1(-k_{0\parallel}+k_{\parallel,\beta}(\omega_0-\omega')+k^*_{\parallel,\gamma}(\omega')\Big]$$
$$\times\langle\delta E_{\alpha,j}(\omega,\mathbf{k}_\perp)\delta E_{\beta,r}(\omega_0-\omega',\mathbf{k}_{0\perp}-\mathbf{k}_\perp')\rangle. \tag{1.18}$$

which is obtained from (1.9). We have already noted that, in the way we have stated the problem, the amplitudes in Eq. (1.18) determine the fluctuation fields until they interact with the pumping wave. We shall assume them to be stationary and uniform random functions and we use the usual averaging procedure:

$$\langle\delta E_{\alpha,j}(\omega,\mathbf{k}_\perp)\delta E_{\beta,i}(\omega',\mathbf{k}_\perp')\rangle=(\delta E_j\delta E_i)^\alpha_{\omega,\mathbf{k}_\perp}\delta(\omega+\omega')\delta(\mathbf{k}_\perp+\mathbf{k}_\perp')\delta_{\alpha\beta}.$$

We substitute this relation in Eq. (1.18) and the resultant expression in Eq. (1.17). We then get for the quantity $\tilde{D}$ the same space-time dependence as for the pumping wave field in Eq. (1.7).

We use the usual rules of the geometric-optics approximation[21] and Eq. (1.7) to transform the left-hand side of Eq. (1.4). Assuming the pumping wave to be transverse, we get in zeroth approximation the dispersion equation

$$(k^2_{0\perp}+k^2_{0\parallel})c^2=\omega_0{}^2\varepsilon^{tr'}(\omega_0,x), \tag{1.19}$$

where $\varepsilon^{tr'}$ is the real part of the transverse permittivity. Equation (1.19) determines how the longitudinal component $k_{0\parallel}$ of the wavevector changes with the x coordinate.

In the first geometric-optics approximation we get an equation for the slowly varying amplitude E_0^0. Taking into account in Eq. (1.17) only terms which describe the decay of the pumping wave into two waves we get

$$\frac{d}{dx}k_{0\parallel}E_{0i}{}^0+k_{0\parallel}\frac{dE_{0i}{}^0}{dx}+\frac{\omega_0{}^2}{c^2}\varepsilon^{tr''}(\omega_0,x)E_{0i}{}^0$$
$$=-i\frac{\omega_0{}^2}{2c^2}\int_0^\infty d\omega\int d\mathbf{k}_\perp S_{ilm}(\omega_0,\mathbf{k}_{0\perp},k_{\parallel,\beta}(\omega_0-\omega)$$
$$+k_{\parallel,\alpha}(\omega);\omega,\mathbf{k}_\perp,k_{\parallel,\alpha}(\omega))M_{ls}^{(-1)}(\omega_0-\omega,\mathbf{k}_{0\perp}-\mathbf{k}_\perp,k_{\parallel,\alpha}(\omega_0-\omega)$$
$$\times S^*_{spj}(\omega-\omega_0,\mathbf{k}_\perp-\mathbf{k}_{0\perp},k_{\parallel,\alpha}(\omega)-k_{0\parallel};\omega,\mathbf{k}_\perp,k_{\parallel,\alpha}(\omega))$$
$$\times E_{0p}{}^0(\delta E_j\delta E_m)^\alpha_{\omega,\mathbf{k}_\perp}\exp\Big[-2\int^x dx_1\,\mathrm{Im}\,k_{\parallel,\alpha}(\omega,x_1)\Big], \tag{1.20}$$

where $\varepsilon^{tr''}$ is the imaginary part of the permittivity.

Multiplying Eq. (1.20) by E^{0*}_{0i} and the complex conjugate equation by E^0_{0i} and taking the difference of these expressions we get an equation for the intensity of the pumping wave. When the condition $k_{0\parallel}\approx k_{\parallel,\alpha}(\omega)+k^*_{\parallel,\beta}(\omega_0-\omega)$ is satisfied we can use Eqs. (1.16) to write this equation in the form

$$\frac{d}{dx}(vW)+2\gamma W=\frac{\omega_0{}^2W}{\partial(\omega_0{}^2\varepsilon')/\partial\omega_0}\int_0^\infty d\omega\int d\mathbf{k}_\perp$$
$$\times\exp\Big[-2\int^x dx_1\,\mathrm{Im}\,k_\parallel(\omega,\mathbf{k}_\perp,x_1)\Big](\delta E^2)_{\omega,\mathbf{k}_\perp}$$
$$\times\mathrm{Im}\{A_l{}^*(\omega,\mathbf{k})M_{lp}^{(-1)}(\omega-\omega_0,\mathbf{k}-\mathbf{k}_0)A_p(\omega,\mathbf{k})\}, \tag{1.21}$$

where we have dropped the superscript tr of the transverse permittivity and introduced the notation

$$\gamma=\frac{\omega_0{}^2\varepsilon''(\omega_0,x)}{\partial(\omega_0{}^2\varepsilon')/\partial\omega_0},\quad W=\frac{|E_0{}^0|^2}{8\pi\omega_0}\frac{\partial(\omega_0{}^2\varepsilon')}{\partial\omega_0},\quad v=\frac{\partial\omega_0}{\partial k_{0\parallel}}.$$

If we neglect non-linear absorption processes, the tensor (1.12) is imaginary and we can use Eq. (1.15) to transform Eq (1.21) to the form

$$\frac{d}{dr}(vW)+2\gamma W=-\frac{\omega_0}{2\pi}\int_0^\infty d\omega\int d\mathbf{k}_\perp(\delta E^2)_{\omega,\mathbf{k}_\perp}$$
$$\times\,\mathrm{Im}\,(n_iM_{ij}(\omega,\mathbf{k})n_j)\exp\Big[-2\int dx_1\,\mathrm{Im}\,k_\parallel(\omega,\mathbf{k}_\perp,x)\Big]. \tag{1.22}$$

Determining the quantity $k_\parallel$ from Eq. (1.15′) and substituting it into Eq. (1.22) we obtain thus a non-linear equation describing the change in the energy of the pumping wave which passes through an inhomogeneous medium.

When $\gamma=0$ and in the weak parametric coupling approximation Eq. (1.22) satisfies the conservation laws for energy fluxes and number of quanta. [21] We can check this using the relation

$$(\delta E^2)_{\omega,\mathbf{k}_\perp}[n_i M_{ij}(\omega,\mathbf{k})n_j]\exp\left[-2\int^x dx_1\,\mathrm{Im}\,k_\parallel(\omega,x_1)\right]$$
$$=(\delta E^2)_{\omega_0-\omega,\mathbf{k}_{0\perp}-\mathbf{k}_\perp}[n_i' M_{ij}(\omega_0-\omega,\mathbf{k}_0-\mathbf{k})n_j']$$
$$\times\exp\left[-2\int^x dx_1\,\mathrm{Im}\,k_\parallel(\omega_0-\omega,x_1)\right]$$

($\mathbf{n}'$ is the polarization vector of the wave with frequency $\omega_0-\omega$) which follows from Eq. (1.14) and the relation

$$\mathrm{Im}[n_i M_{ij}(\omega,\mathbf{k})n_j]=\mathrm{Im}\,k_\parallel\frac{\partial\omega}{\partial k_\parallel}\frac{\partial}{\partial\omega}(n_i M_{ij}n_j),$$

which is valid under the conditions of weak parametric coupling.

An equation for the pumping wave in a plasma was derived in[22, 23] just by using the energy flux conservation law.

2. GROWTH COEFFICIENT FOR SRS IN A PLASMA

1. The tensor (1.12) has been well studied[19] for a rarefied plasma. In the limit when $\omega_0>\omega$ and the phase velocities of the high-frequency waves are large compared to the thermal velocities of the particles, it has a particularly simple form [24] and the vector **A** equals

$$\mathbf{A}=-i\frac{e}{m\omega_0^2}(\mathbf{kn})\,n_0\delta\varepsilon_e^l(\omega,\mathbf{k}),\tag{2.1}$$

where $\delta\varepsilon_e^l$ is the electronic contribution to the longitudinal permittivity. [25]

We shall be interested in the change in the intensity of the pumping wave due to its decay into a longitudinal Langmuir wave (frequency ω) and a transverse wave (frequency $\omega_0-\omega$). This process is often called stimulated Raman scattering. The longitudinal part of the Maxwell tensor on the left-hand side and the transverse part of the inverse Maxwell tensor on the right-hand side of Eq. (1.15′) correspond to this process. Splitting off these parts and using Eq. (2.1) we get from Eq. (1.15′) for the determination of $k_\parallel(\omega)$

$$\varepsilon^l(\omega,\mathbf{k})\left[\varepsilon^{tr}(\omega_0-\omega,\mathbf{k}_0-\mathbf{k})-\frac{c^2(\mathbf{k}_0-\mathbf{k})^2}{(\omega_0-\omega)^2}\right]$$
$$=\frac{k^2|v_E|^2[(\mathbf{k}_0-\mathbf{k})\mathbf{n}_0]^2}{4\omega_0^2(\mathbf{k}_0-\mathbf{k})^2}[\delta\varepsilon_e^l(\omega,\mathbf{k})]^2,\tag{2.2}$$

where $v_E=eE_0^0/m\omega_0$; ε^l and ε^{tr} are, respectively, the longitudinal and transverse permittivities of the plasma. [25] In obtaining Eq. (2.2) we neglected the imaginary parts of the permittivities.

The decay interaction of the waves corresponds to the solution of Eq. (2.2) using perturbation theory with respect to the field of the pumping wave (weak parametric coupling of the waves[18]).[2)] In the zeroth approximation, neglecting the pumping wave, we get the dispersion laws for the Langmuir and the scattered transverse waves. The first of these, $\varepsilon^l(\omega,\mathbf{k})=0$, for a plasma has the form

$$\omega^2=\omega_p^2(x)+3v_{Te}^2[k_\perp^2+k_\parallel^{(l)2}(x)],\tag{2.3}$$

where $\omega_p=(4\pi e^2N(x)/m)^{1/2}$ is the plasma frequency which depends on the coordinate through the electron concentration $N(x)$; $v_{Te}=(T_e/m)^{1/2}$ is the electron thermal velocity which we assume to be constant. Equation (2.3) determines how the longitudinal component $k_\parallel^{(l)}$ of the Langmuir wave wavevector changes with the coordinate.

For normal incidence of the pumping wave onto the plasma ($\mathbf{k}_{0\perp}=0$) the dispersion equation for the scattered wave has the form

$$(\omega_0-\omega)^2=\omega_p^2(x)$$
$$+c^2[k_\perp^2+(k_\parallel^{(0)}(x)-k_0(x))^2].\tag{2.4}$$

Using the dispersion law $\omega_0^2=\omega_p^2+k_0^2(x)c^2$ for the pumping wave to eliminate a number of terms in Eq. (2.4) we get the relation

$$k_\perp^2+k_\parallel^{(0)2}(x)-2k_0(x)k_\parallel^{(0)}(x)+(2\omega\omega_0-\omega^2)/c^2=0,\tag{2.5}$$

which determines essentially the connection between the space and time variation of the high-frequency pressure force produced by the pumping and wave and the scattered wave.

The resonance interaction between the three waves proceeds in the vicinity of the point x_0 where for a given frequency ω and transverse wavevector component $k_\perp$ the quantities $k_\parallel^{(0)}$ and $k_\parallel^{(l)}$, given by Eqs. (2.3) and (2.5), are the same. One checks easily that this point is determined by the relation

$$k_\perp^2=\frac{\omega^2-\omega_p^2(x_0)}{3v_{Te}^2}-\frac{c^2}{4(\omega_0^2-\omega_p^2(x_0))}\left[\frac{\omega^2-2\omega\omega_0}{c^2}+\frac{\omega^2-\omega_p^2(x_0)}{3v_{Te}^2}\right]^2.\tag{2.6}$$

In the limit $\omega_0>\omega$ which is of interest to us it follows from Eq. (2.6) that

$$\omega_p^2(x_0)=\omega^2-6k_0^2v_{Te}^2[1\pm(1-k_\perp^2/k_0^2)^{1/2}],\tag{2.7}$$

where $k_0\approx\omega_0/c$.

Knowing the point x_0 we easily get the quantity $k_\parallel^{(l)}(x_0)$ from Eq. (2.5):

$$k_\parallel^{(l)}(x_0)=k_0(x_0)[1\pm(1-k_\perp^2/k_0^2-2\omega_p(x_0)/\omega_0)^{1/2}].\tag{2.8}$$

Equation (2.8) determines in wavevector space a sphere of radius $(k_0^2-2\,\omega_0\omega_p c^{-2})^{1/2}$ and with a center which lies on the $k_\parallel$ axis in the point k_0. The upper part of the intersection of this sphere with the central plane is shown in Fig. 1.

We note that Langmuir waves with wavevector endpoints which lie on the hemisphere which is furthest away from the origin correspond to transverse waves which are scattered backward, while Langmuir waves with wavevector endpoints which lie on the hemisphere

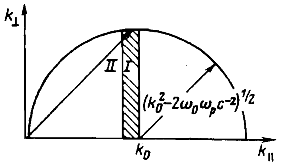

FIG. 1. The upper part of the central section of the sphere in wavevector space on which the decay conditions for Langmuir waves are satisfied.

closest to the origin correspond to waves which are scattered forward.

2. To determine the corrections to the longitudinal wavevector component Δk in the vicinity of the point x_0 we substitute in the left-hand side of Eq. (2.2) $k_\parallel(x) = k_\parallel^{(l)}(x_0) + \Delta k(x - x_0)$ and in the longitudinal and the transverse permittivities we expand the plasma frequency in a series. Using Eqs. (2.3) and (2.4) we get as a result

$$\varepsilon^{l}(\omega, k, x) = -\frac{\Delta x}{L(x_0)} - \frac{6k_\parallel^{(l)} v_{Te}^2 \omega_p^2(x_0)}{\omega^4}\Delta k, \qquad (2.9)$$

$$\varepsilon^{tr}(\omega_0-\omega, k_0-k, x) - \frac{c^2(k_0-k)^2}{(\omega_0-\omega)^2}$$

$$= -\frac{1}{(\omega_0-\omega)^2}\left[\frac{\Delta x k_\parallel^{(l)} \omega_p^2(x_0)}{k_0 L(x_0)} - 2c^2\Delta k(k_0-k_\parallel^{(l)}) + c^2(\Delta k)^2\right], \qquad (2.10)$$

where $\Delta x = x - x_0$; the scale length of the inhomogeneity of the plasma has the form

$$L(x_0) = \omega_p^2(x_0) \Big/ \frac{d\omega_p^2(x_0)}{dx_0}.$$

We substitute Eqs. (2.9), (2.10) into Eq. (2.2), drop the superscript of the quantity $k_\parallel^{(l)}$, and introduce the dimensionless quantities

$$\chi = \Delta k/k_0, \quad \Delta x/L = \Delta\xi.$$

For the determination of χ we get

$$\chi^3 - \left(2\frac{k_0-k_\parallel}{k_0} - \frac{\omega_p^2\Delta\xi}{6k_0k_\parallel v_{Te}^2}\right)\chi^2 - \left(\frac{\omega_p^2(k_0-k_\parallel)}{3k_0^2k_\parallel v_{Te}^2} - \frac{k_\parallel\omega_p^2}{k_0^3c^2}\right)\Delta\xi\chi$$
$$- \frac{\omega_p^4}{6k_0^4c^2v_{Te}^2}\left(\frac{k_0(k_\perp^2+k_\parallel^2)|v_E|^2\sin^2\theta}{4k_\parallel\omega_p^2} - (\Delta\xi)^2\right) = 0, \qquad (2.11)$$

where θ is the angle between the polarization vector $\mathbf{n}_0$ of the pumping wave and the wavevector of the scattered wave $\mathbf{k}_0 - \mathbf{k}$.

3. It is clear from Eq. (1.22) that amplification of the fluctuation waves occurs when $\operatorname{Im} k_\parallel < 0$. We call the quantity $\operatorname{Im} k_\parallel/k_0$ in that case the local growth coefficient and determine it from Eq. (2.11). We consider first the case

$$(k_0-k_\parallel)/k_0 > \chi, \qquad (2.12)$$

which corresponds to the scattering of waves through an angle which is not close to $\pi/2$ (see Fig. 1). In that case Eq. (2.11) becomes quadratic and the growth coefficient which interests us has the form

$$\operatorname{Im}\frac{\Delta k}{k_0} = \operatorname{Im}\chi = -\frac{\sqrt{3}\,\omega_p|v_E|\sin\theta(k_\perp^2+k_\parallel^2)^{1/2}}{cv_{Te}k_0[k_\parallel(k_0-k_\parallel)]^{1/2}}\left(1-\frac{(\Delta\xi)^2}{(\Delta\xi_0)^2}\right)^{1/2}, \qquad (2.13)$$

$$\Delta\xi_0 = \frac{\sqrt{3}\,cv_{Te}|v_E|[(k_\perp^2+k_\parallel^2)k_\parallel(k_0-k_\parallel)]^{1/2}}{\omega_p[3k_\parallel^2v_{Te}^2/k_0+c^2(k_0-k_\parallel)]}\sin\theta. \qquad (2.14)$$

It is clear from Eq. (2.13) that the growth coefficient is non-vanishing only when $|\Delta\xi| < \Delta\xi_0$ and for $k_0 - k_\parallel > 0$, when we have forward scattering. The nature of the instability which leads to backward scattering depends on the boundary conditions.[2,14] We shall discuss below the problem of how to take that instability into account when it is a convective instability.

Using Eq. (2.13) we find the total growth coefficient $\varkappa$:

$$\varkappa = -\int_{-\Delta\xi_0}^{\Delta\xi_0} d(\Delta\xi)\operatorname{Im}\chi(\Delta\xi)k_0L = \frac{\pi(k_\perp^2+k_\parallel^2)|v_E|^2L\sin^2\theta}{8[3k_\parallel^2v_{Te}^2/k_0+c^2(k_0-k_\parallel)]} \qquad (2.15)$$

where the quantity $k_\parallel$ is given by Eq. (2.8). Having available an expression for χ we easily find from inequality (2.12) the conditions for the validity of Eqs. (2.13), (2.15):

$$1 > \frac{k_0-k_\parallel}{k_0} \gg \left(\frac{\omega_p|v_E|}{\omega_0 v_{Te}}\sin\theta\right)^{2/3}. \qquad (2.12')$$

We now turn to the case when the inequality holds which is the opposite of (2.12). Equation (2.11) is cubic and when $k_0 = k_\parallel$ it takes the form

$$\left(\chi + \frac{\omega_p^2}{6k_0^2v_{Te}^2}\Delta\xi\right)\left(\chi^2 + \frac{\omega_p^2}{k_0^2c^2}\Delta\xi\right) - \frac{\omega_p^2|v_E|^2(k_\perp^2+k_\parallel^2)}{24k_0^4c^2v_{Te}^2}\sin^2\theta = 0.$$

This equation has a solution with a negative imaginary part only when $-\Delta\xi' < \Delta\xi < 0$, where

$$\Delta\xi_0' = \frac{3^{1/3}}{4}\left(\frac{k_0c}{\omega_p}\right)^{2/3}\left(\frac{|v_E|\sin\theta(k_\perp^2+k_\parallel^2)^{1/2}}{k_0v_{Te}}\right)^{4/3} \qquad (2.16)$$

and the local growth coefficient is equal to

$$\operatorname{Im}\chi = -\frac{\sqrt{3}}{4}\left(\frac{\omega_p^2(k_\perp^2+k_\parallel^2)|v_E|^2\sin^2\theta}{6k_0^4c^2v_{Te}^2}\right)^{1/3}$$
$$\times\left\{\left[1+\left(1+\left(\frac{\Delta\xi}{\Delta\xi_0'}\right)^3\right)^{1/2}\right]^{1/3} - \left[1-\left(1+\left(\frac{\Delta\xi}{\Delta\xi_0'}\right)^3\right)^{1/2}\right]^{1/3}\right\}.$$

Using that expression we find the total growth coefficient

$$\varkappa = -\int_{-\Delta\xi_0'}^{0} d(\Delta\xi)\operatorname{Im}\chi(\Delta\xi)k_0L = \frac{\pi(k_\perp^2+k_\parallel^2)|v_E|^2L\sin^2\theta}{2^{2/3}\cdot 36k_0v_{Te}^2}. \qquad (2.17)$$

When $k_\parallel = k_\perp = k_0$ (see Fig. 1) we find from Eq. (2.17) an expression which differs by a numerical coefficient ~ 1 from the result obtained in[3] from the solution of the set of equations for parametrically coupled waves.

3. EQUATION FOR THE PUMPING WAVE

1. We have established in the preceding section that a Langmuir wave with a given frequency ω and transverse wavevector component $\mathbf{k}_\perp$ interacts resonantly with a pumping wave only in a narrow neighborhood (see Eqs. (2.14), (2.16)) of the point x_0 determined by Eq. (2.6) or (2.7). This means that in each point the pumping wave interacts with Langmuir waves from a narrow frequency range $\Delta\omega$ (for a given $\mathbf{k}_\perp$), the width

of which is determined by the size of the interaction region. It is therefore convenient to use Eq. (2.7) to change in Eq. (1.22) from an integration over ω to an integration over $\Delta x = x - x_0$:

$$d\omega = \frac{\partial\omega}{\partial x}\bigg|_{x_0} d(\Delta x) \approx \frac{d\omega_p(x_0)}{dx_0} d(\Delta x) = \frac{\omega_p(x_0)}{2L(x_0)} d(\Delta x).$$

Using this relation and also Eq. (2.9) for the longitudinal dielectric permittivity we get from (1.22)

$$\frac{d}{dx}(vW)+2\gamma W = -\frac{\omega_0}{2\pi}\int d(\Delta x)\int d\mathbf{k}_\perp (\delta E^2)_{\omega_p,\mathbf{k}_\perp}$$
$$\times \frac{d\omega_p}{dx}\frac{6k_\parallel v_{Te}^2}{\omega_p^2}\,\mathrm{Im}\,\Delta k(\Delta x)\exp\left[-2\int^{\Delta x} d(\Delta x_1)\,\mathrm{Im}\,\Delta k(\Delta x_1)\right], \quad (3.1)$$

where we have dropped the index of the coordinate x_0 and where the integration over Δx is between the boundaries of the resonance interaction region.

We shall assume that when the fluctuation waves enter the resonance interaction region they have thermal amplitudes. In that case

$$(\delta E^2)_{\omega,\mathbf{k}_\perp} = \frac{(\delta E^2)_\mathbf{k}}{\partial\omega/\partial k_\parallel} = \frac{T}{(2\pi)^2}\frac{\partial k_\parallel}{\partial\omega} = \frac{T\omega_p(x)}{(2\pi)^2 3k_\parallel v_{Te}^2}. \quad (3.2)$$

We integrate over Δx in Eq. (3.2) and using Eq. (3.2) we get

$$\frac{d}{dx}(vW)+2\gamma W = -\frac{\omega_0 T}{2(2\pi)^3 L}\int d\mathbf{k}_\perp (e^{2\varkappa}-1), \quad (3.3)$$

where the quantity $\varkappa$ for different values of $\mathbf{k}_\perp$ is given by Eqs. (2.15), (2.17). We note that although Eq. (2.17) was obtained under the condition $k_0 = k_\parallel$ we shall assume that it remains valid also in a narrow range with width

$$\frac{k_0-k_\parallel}{k_0} \leqslant \left(\frac{\omega_p|v_E|}{\omega_0 v_{Te}}\sin\theta\right)^{2/3} \ll 1$$

(region I in Fig. 1).

2. We introduce a polar system of coordinates in wavevector space with an angle φ which is reckoned from the direction of the vector $\mathbf{n}_0$. Using Eq. (2.8) we then get

$$\sin^2\theta = \sin^2\varphi + \eta^2\cos^2\varphi,$$

where, if we neglect small quantities of order ω_p/ω_0,

$$\eta = (1-k_\perp^2/k_0^2)^{1/2}. \quad (3.4)$$

We substitute Eqs. (2.15), (2.17) in Eq. (3.3) and use (3.4) to change from integration over $k_\perp$ to integration over η. As a result we get, neglecting the linear damping of the pumping wave,

$$\frac{d}{dx}(vW) = -\frac{\omega_0 T}{2(2\pi)^3 L(x)}\int_0^{2\pi} d\varphi\int_0^1 d\eta\,\eta\left[\exp[2\sigma(\sin^2\varphi+\eta^2\cos^2\varphi)]-1\right], \quad (3.5)$$

where

$$\sigma = \beta\frac{1-\eta}{(1-\eta)^2+\gamma_0\eta} \quad \text{when} \quad \sin^2\varphi < \frac{(\eta/a)^3-1}{1-\eta^2}, \quad (3.6)$$

$$\sigma = \frac{2}{3\sqrt[3]{2}}\beta(1-\eta) \quad \text{when} \quad \sin^2\varphi > \frac{(\eta/a)^3-1}{1-\eta^2}. \quad (3.7)$$

We have used the following notation in Eqs. (3.6), (3.7):

$$\beta = \frac{\pi|v_E|^2}{12v_{Te}^2}k_0 L, \quad \gamma_0 = \frac{c^2}{3v_{Te}^2}, \quad a = \left(\frac{\omega_p|v_E|}{\omega_0 v_{Te}}\right)^{2/3}. \quad (3.8)$$

Using Eq. (3.6), (3.7) we can integrate over φ on the right-hand side of Eq. (3.5):

$$\frac{d}{dx}(vW) = -\frac{T\omega_0 k_0^2\pi}{2L(2\pi)^3}(J_1+J_2-1), \quad (3.9)$$

where

$$J_1 = \frac{1}{\pi}\int_0^1 d\eta\,\eta I_0\left[\frac{2\beta}{2^{1/3}\cdot 3}(1-\eta)(1-\eta^2)\right]\exp\left[\frac{2\beta}{2^{1/3}\cdot 3}(1-\eta)(1+\eta^2)\right]$$
$$\times\left\{2\pi - 4\arcsin\left[\min\left(\frac{(\eta/a)^3-1}{1-\eta^2};\,1\right)\right]\right\}, \quad (3.10)$$

$$J_2 = \frac{1}{\pi}\int_0^1 d\eta\,\eta I_0\left[\frac{\beta(1-\eta)(1-\eta^2)}{(1-\eta)^2+\gamma_0\eta}\right]\exp\left[\frac{\beta(1-\eta)(1+\eta^2)}{(1-\eta)^2+\gamma_0\eta}\right]$$
$$\times 4\arcsin\left[\min\left(\frac{(\eta/a)^3-1}{1-\eta^2};\,1\right)\right], \quad (3.11)$$

I_0 is a modified Bessel function of order zero.

In a rarefied plasma ($\omega_p < \omega_0$) and for a relatively low intensity of the pumping wave ($|v_E| < v_{Te}$) the quantity a given by Eq. (3.8) is much less than unity. We shall assume that when

$$z = \frac{(\eta/a)^3-1}{1-\eta^2} < 1$$

we can neglect arc sin$z$ in Eqs. (3.10), (3.11). This leads to the result that the integrand (3.10) is nonvanishing in the interval $0 < \eta < a$ and the integrand (3.11) in the interval $a < \eta < 1$.

3. When $a < 1$ and $\beta > 1$ one can evaluate the integral (3.10) using for the Bessel function the well known asymptotic expression which is valid for large values of its argument. As a result we have

$$J_1 = \frac{(2^{1/3}\cdot 3)^{5/2}}{16\sqrt{\pi}\,\beta^{5/2}}\exp\left(\frac{4\beta}{2^{1/3}\cdot 3}\right)\left[1-\exp\left(-\frac{4\beta a}{2^{1/3}\cdot 3}\right)\left(1+\frac{4a\beta}{2^{1/3}\cdot 3}\right)\right]. \quad (3.10')$$

To estimate the integral (3.11) for $\beta > 1$ we split the integration domain into two parts. In the interval $\beta/(\gamma_0+\beta) > \eta > a$ we use the asymptotic expression for the Bessel function, while we use a series expansion in the interval $1 > \eta > \beta/(\gamma_0+\beta)$. The contribution from the first interval is well defined and for $\beta > 1 + a\gamma_0$ it is equal to

$$J_2 = \frac{(1+a\gamma_0)^{5/2}}{\sqrt{2\pi}\,\gamma_0\beta^{5/2}}\exp\left[\frac{2\beta}{1+a\gamma_0}\right]\left[a+\frac{(1+a\gamma_0)(1+{}^7/_2 a\gamma_0)}{2\beta\gamma_0}\right]. \quad (3.11')$$

Using the approximate expressions for the integrals (3.10′) and (3.11′) we write Eq. (3.9) in the form

$$\frac{dy}{d\xi} = -\frac{k_0 r_0}{48\lambda(\xi)}\left\{\frac{(2^{1/3}\cdot 3)^{5/2}}{16\sqrt{\pi}(\lambda y)^{5/2}}\exp\left(\frac{4\lambda y}{2^{1/3}\cdot 3}\right)\times\right.$$

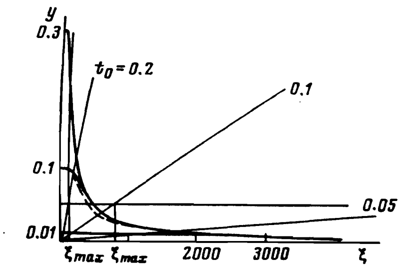

FIG. 2. Change in the intensity of the incident wave $y=\pi v_E^2/12v_{Te}^2$ with coordinate $\xi=k_0L$. The dashed curve is constructed, using the approximate Eqs. (4.5) and (4.7) for $y_0=0.1$.

$$\times\left[1-\exp\left(-\frac{4\mu\lambda y^{4/3}}{2^{1/3}\cdot 3}\right)\left(1+\frac{4\mu\lambda y^{4/3}}{2^{1/3}\cdot 3}\right)\right]$$
$$+\frac{(1+\mu\gamma_0 y^{1/3})^{5/2}}{\sqrt{2\pi}\,\gamma_0(y\lambda)^{3/2}}\exp\left(\frac{2y\lambda}{1+\mu\gamma_0 y^{1/3}}\right)\left[\mu y^{1/3}+\frac{(1+\mu\gamma_0 y^{1/3})(1+{}^7/_2\mu\gamma_0 y^{1/3})}{2y\lambda\gamma_0}\right]\Bigg\},\qquad(3.12)$$

where $\xi=k_0x$,

$$y=\frac{\pi|v_E|^2}{12v_{Te}^2},\quad \lambda=k_0L(\xi)=\frac{\omega_p^2(\xi)}{d\omega_p^2(\xi)/d\xi},\ \mu(\xi)=\left(\frac{12\omega_p^2(\xi)}{\pi\omega_0^2}\right)^{1/3},\ r_0=\frac{e^2}{mc^2}\qquad(3.13)$$

If we specify the manner in which the density changes and solve Eq. (3.12), we can determine how the intensity of the pumping wave decreases due to stimulated Raman scattering when it penetrates into the plasma. We emphasize that Eq. (3.12) is applicable only for a rarefied plasma ($\omega_0>2\omega_p$) when $\mu^3<3/\pi$ and for a sufficiently high intensity of the pumping wave ($y\lambda>1+\mu\gamma_0 y^{1/3}$).

It was shown in[1,2] that when the instability which leads to backward Raman scattering is convective in character the growth coefficients are the same as the corresponding expressions for forward scattering. To take into account backward scattering we must thus double the right-hand side of Eq. (3.12).

4. LINEAR DENSITY PROFILE

1. We consider a linear increase of the plasma density with the coordinate, $N=\alpha x$, where α is a constant coefficient. We get from Eq. (3.13)

$$\lambda=\xi,\quad \mu=t_0\xi^{1/3},\quad t_0=(48\alpha r_0)^{1/3}k_0^{-1}.\qquad(4.1)$$

We substitute these relations into Eq. (3.12) and write it in the form

$$\frac{dy}{d\xi}=-\frac{k_0r_0}{48\xi\sqrt{2\pi}}\left\{\frac{\sqrt{2}(2^{1/3}\cdot 3)^{5/2}}{16(y\xi)^{5/2}}\exp\left(\frac{4y\xi}{2^{1/3}\cdot 3}\right)\right.$$
$$\times\left[1-\exp\left(-\frac{4t_0(y\xi)^{4/3}}{2^{1/3}\cdot 3}\right)\left(1+\frac{4t_0(y\xi)^{4/3}}{2^{1/3}\cdot 3}\right)\right]$$
$$+\frac{[1+t_0\gamma_0(y\xi)^{1/3}]^{5/2}}{\gamma_0(y\xi)^{3/2}}\exp\left(\frac{2y\xi}{1+t_0\gamma_0(y\xi)^{1/3}}\right)$$
$$\left.\times\left[t_0(y\xi)^{1/3}+\frac{(1+t_0\gamma_0(y\xi)^{1/3})(1+{}^7/_2t_0\gamma_0(y\xi)^{1/3})}{2\gamma_0y\xi}\right]\right\}\qquad(4.2)$$

where we have assumed that $y\xi>1+t_0\gamma_0(y\xi)^{1/3}$.

Equation (4.2) was solved with an electronic computer for the following plasma parameters: $T=1$ keV, $v_{Te}=1.3\times10^9$ cm/s, $\gamma_0=200$, $k_0=6\times10^4$ cm^{-1} (this corresponds to the wavelength of a neodymium laser), $\omega_0=1.8\times10^{15}$ s^{-1}, $r_0=2.8\times10^{-13}$ cm. The solution is valid for $\mu<1$ or

$$\xi<\xi_{max}=1/t_0^3,\qquad(4.3)$$

which follows from the condition that the plasma density be small compared to one-fourth the critical density. For the employed parameters, the quantity y and the radiation energy flux q (in W/cm^2) are connected by the relation $q=1.7\times10^{16}y$.

We give in Fig. 2 the behavior of $y(\xi)$ obtained for different intensities of the incident wave ($y(0)=0.3$; 0.1; 0.01; or $q(0)=5\times10^{15}$, 1.7×10^{15}, 1.7×10^{14} W/cm^2) and for different steepnesses of the increase in density ($t_0=0.2$, 0.1, 0.05). It is clear from the curves that the intensity of the pumping wave remains practically unchanged up to a well defined value $\xi=\xi_1$, where the quantity ξ_1 is smaller the larger the intensity of the incident wave. When $\xi>\xi_1$ there occurs a comparatively fast decrease in the intensity. Moreover, it follows from Fig. 2 that the penetration of the wave is independent of the steepness with which the plasma density increases. For a smoother increase the fall in the intensity occurs at lower densities.

2. The parameters for which Eq. (4.2) was solved are typical of many experiments with laser plasmas.[27] In that case one can use an approximate analytical solution of Eq. (4.2).

When $t_0\gamma_0>1$ the first term in the braces on the right-hand side of Eq. (4.2) is the main one. In the range of values $(y\xi)^{4/3}>1/t_0$ this term can be simplified and the equation takes the form

$$\frac{dy}{d\xi}=-\frac{(2^{1/3}\cdot 3)^{5/2}k_0r_0}{48\cdot 16\pi^{1/2}\xi(y\xi)^{5/2}}\exp\left(\frac{4y\xi}{2^{1/3}\cdot 3}\right).\qquad(4.4)$$

The parameter t_0 does not occur in Eq. (4.4) and its solution is therefore independent of the steepness of the density increase.

The right-hand side of Eq. (4.4) contains the small factor k_0r_0. The derivative $dy/d\xi$ is thus small and the function y equals its boundary value y_0 until the other factors occurring on the right-hand side become sufficiently large. For fixed y_0 and increasing ξ the exponent is the fastest growing factor and it cancels the small factor when

$$\xi_1\approx\frac{1}{y_0}\ln\frac{1}{k_0r_0}.\qquad(4.5)$$

Hence, using Eq. (3.13) to reintroduce dimensions we have

$$x_1\approx\frac{12v_{Te}^2}{k_0\pi|v_E^2(0)|}\ln\left(\frac{mc^2}{k_0e^2}\right).\qquad(4.6)$$

When $\xi>\xi_1$ the function y decreases. The increase in the argument is then cancelled by the decrease in y so that the index of the exponential on the right-hand side of Eq. (4.4) remains almost constant ($y\xi=$ const). Hence we get, using Eq. (4.5),

$$y \approx \frac{1}{\xi} \ln \frac{1}{k_0 r_0}, \quad \xi > \xi_1. \tag{4.7}$$

In Fig. 2 we have indicated by dashes the curve constructured using the approximate Eqs. (4.5) and (4.7) for $y_0 = 0.1$. It is clear that it agrees rather well with with the results of the exact solution of Eq. (4.2).[3] Comparing Eqs. (4.5) and (4.3) we easily find the value y_0 for which the pumping wave reaches a quarter of the critical density without change in intensity:

$$y_0 \leqslant \frac{1}{t_0^3} \ln \frac{1}{k_0 r_0}. \tag{4.8}$$

3. We have made a number of assumptions that can be spelled out more concretely and can be verified for a linear density profile.

A. The width of the resonance interaction region was assumed to be small compared to the characteristic scale length of the plasma inhomogeneity. For the parameters considered the change in the intensity of the pumping wave is determined by scattering at an angle close to $\pi/2$ and the width of the interaction region is given by Eq. (2.16). The assumption made by us then reduces to the inequality

$$\xi \geqslant 12 y^2 t_0^{-3}. \tag{4.9}$$

B. When deriving Eq. (1.15′) we neglected small terms of order $(k_0 L)^{-1}$. The correction to the longitudinal component of the wavevector $\Delta k/k_0$ must thus be larger than these dropped terms. Using the appropriate solution of Eq. (2.11) we get the inequality

$$\xi > t_0^{-3/4} y^{-1/4}. \tag{4.10}$$

C. We used the assumption that the amplitude of the pumping wave was constant over the length of the interaction region. Requiring that the width determined by Eq. (2.16) be small compared to the distance over which the intensity of the pumping wave decreases by a factor two, we get by means of Eq. (4.7)

$$y_0 \leqslant 2 t_0 \ln \left(\frac{1}{k_0 r_0} \right)^{1/3}. \tag{4.11}$$

D. When evaluating the integrals (3.10) and (3.11) we assumed that $a < 1$. For a linear law for the change in density this inequality can be changed to the form

$$(y\xi)^{1/3} < 1/t_0. \tag{4.12}$$

Using for the function $y(\xi)$ the approximate expression obtained above, one can check that for the values of the parameters t_0 and y_0 considered by us inequalities (4.9) to (4.12) are satisfied.

4. We elucidate the physical reason for the fact that the decrease in the wave intensity due to the stimulated Raman scattering is independent of the steepness of the increase of the plasma density. The growth coefficient for the Langmuir and the scattered transverse waves characterizes the energy lost per unit length by the pumping wave. This coefficient is larger the larger the distance over which the waves interact, i.e., the less the plasma density changes per unit length. However, the same change in density is less important when it comes to disturbing the resonant interaction in a dense plasma and more important in a rarefied plasma. The growth coefficient and hence also the damping of the pumping wave is thus determined not by the absolute but by the relative change in the plasma density per unit length. For a linear change of density, as well as for any power law, this relative change is constant. The decrease in the intensity of the pumping wave proceeds thus according to the same law regardless of the steepness of the increase in the density.

CONCLUSION

From the considerations given here it follows that:

1. The intensity of the pumping wave in a rarefied inhomogeneous plasma ($\omega_0 > 2\omega_p$) starts to decrease due to stimulated Raman scattering at a well defined depth (Eq. (4.6)) which is inversely proportional to the intensity of the incident wave. Moreover, at distances of the same order of magnitude, the intensity of the pumping wave decreases inversely proportional to the coordinate.[4]

2. A quarter of the critical density, where parametric absorption processes are possible,[10,11,12,23] is reached by a wave with an intensity less than the value given by Eq. (4.8). For parameters which are characteristic for many experiments with a laser plasma ($T = 1$ keV, $k_0 = 6 \times 10^4$ cm^{-1}) this intensity is approximately equal to 3×10^{14} W/cm^2.

3. The law for the fall in the intensity of the pumping wave when the spatial change in the plasma density is slow is the same as for a faster change in the density. Therefore, the steeper the fall in the plasma density, the less important is SRS. And, on the other hand, Raman scattering must manifest itself particularly strongly in a rarefied plasma. It is possible that the results of[27] are connected with this; in that paper a stronger scattering of the second laser pulse was observed when compared to the scattering of the first one.

The model considered by us contains a number of assumptions. One of them is that the plasma is unbounded in directions at right angles to the propagation of the pumping wave. Allowance for the fact that the plasma is bounded leads, apparently, to a diminution of the effect of SRS on the propagation of the pumping wave.

We express our gratitude to L. M. Anosova for performing the numerical calculations and to V. P. Silin for a number of hints.

[1] A similar derivation for a homogeneous bounded plasma can be found in[20].

[2] Andreev[26] has studied SRS in a uniform plasma in the strong wave coupling approximation.

[3] For any power-law change in the density $N \propto \xi^n$ the nature of the penetration of the pumping wave into the plasma remains

the same as for a linear layer. We must in that case in Eqs. (4.5), (4.7) replace ξ by ξ/n.

[4)]The expressions obtained by us for the dimensions that characterize the change in the intensity of the pumping wave differ from the estimates in[3].

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Translated by D. ter Haar

Theory of nonuniform magnetic states in ferromagnets in the vicinity of second-order phase transitions

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(Submitted June 24, 1975; resubmitted January 26, 1976)
Zh. Eksp. Teor. Fiz. **70**, 2178–2188 (June 1976)

A perturbation theory is constructed in a small parameter determined by the degree of closeness to a second-order phase-transition point. This makes it possible to take the demagnetizing field into account and to find all the quantities characterizing the magnetic-moment distribution, with any prescribed degree of accuracy in powers of the small parameter. It is shown that strong correlation effects lead to a nonuniform distribution of the magnetization over the thickness of a ferromagnetic plate. The character of the phase transition to the nonuniform state in finite samples in the vicinity of second-order phase transitions (i.e., near the Curie temperature and the phase-transition point with respect to the magnetic field) is investigated in detail. It is proved that, in ferromagnets of arbitrary shape, the energy of the demagnetizing field does not change the character of the phase transition.

PACS numbers: 75.30.Jy

INTRODUCTION

It is well known that, in samples of finite size, a nonuniform distribution of the magnetization arises simultaneously with the appearance of the spontaneous magnetic moment. Far from the transition point the magnetic-moment distribution consists of an alternation of uniformly magnetized phase domains, separated by narrow intermediate layers inside which the magnetization vector rotates through 180°.[1] In this case the energy of an intermediate layer can be regarded as the surface energy, and the equilibrium configuration determined from the minimum of the energy of the intermediate layers and the magnetic dipole energy. The magnetiza-