has been ignored).

The assumption that the product $\nu\tau$ is contant whilst the duration of the muonium stage changes by more than two orders of magnitude means that we are regarding $Mu \to \mu^+$ transitions and reversals of the direction of electron spin in muonium as different consequences of the same process. This process can be regarded, for example, as the transfer of an electron from a local donor level, formed as a result of the presence of a muonium atom between the crystal lattice sites, to the conduction band. The character of the final state of the positive muon depends on the direction of the spin of the delocalized electron relative to the spin of the positive muon.

CONCLUSIONS

The experimental results reported in this paper suggest, in our view, that further studies of the properties of muonium in the crystal lattices of semiconductors should yield useful results. Elucidation of the actual energy level scheme that appears during the interaction between muonium electron shells and the neighboring lattice atoms is an important problem, the solution of which should be helpful in the understanding of the formation of deep donor levels in semiconducting materials.

We emphasize the satisfactory agreement between the calculated barrier heights corresponding to narrow and broad temperature ranges. This may be regarded as evidence for the fact that the hyperfine splitting frequency of muonium in germanium remains constant in the temperature and dopant density ranges we have investigated.

We are indebted to D. G. Andrianov and V. I. Fistul' for supplying specimens with measured electrophysical properties, and for useful discussions.

[1]D. G. Andrianov, G. Myasishcheva, Yu. V. Obukhov, V. S. Roganov, V. G. Firsov, and V. I. Fistul', Zh. Eksp. Teor. Fiz. **56**, 1195 (1969) [Sov. Phys. JETP **29**, 643 (1969)].
[2]D. G. Andrianov, E. V. Minaĭchev, G. G. Myasishcheva, Yu. V. Obukhov, V. S. Roganov, G. I. Savel'ev, V. G. Firsov, and V. I. Fistul', Zh. Eksp. Teor. Fiz. **58**, 1896 (1970) [Sov. Phys. JETP **32**, 1025 (1971)].
[3]I. I. Gurevich, I. G. Ivanter, E. A. Meleshko, B. A. Nikol'skiĭ, V. S. Roganov, V. I. Selivanov, V. P. Smilga, B. V. Sokolov, and V. D. Shestakov, Zh. Eksp. Teor. Fiz. **60**, 471 (1971) [Sov. Phys. JETP **33**, 253 (1971)].
[4]J. H. Brewer, K. M. Crowe, F. N. Gygax, D. G. Fleming, and A. Schenck, Bull. Am. Phys. Soc. **17**, 594 (1972).
[5]V. W. Hughes, Ann. Rev. Nucl. Sci. **16**, 445 (1966).
[6]E. V. Minaĭchev, G. G. Myasishcheva, Yu. V. Obukhov, V. S. Roganov, G. I. Savel'ev, and V. G. Firsov, Zh. Eksp. Teor. Fiz. **58**, 1586 (1970) [Sov. Phys. JETP **31**, 849 (1970)].
[7]I. G. Ivanter, E. V. Minaĭchev, G. G. Myasishcheva, Yu. V. Obukhov, V. S. Roganov, G. I. Savel'ev, V. P. Smilga, and V. G. Firsov, Zh. Eksp. Teor. Fiz. **62**, 14 (1972) [Sov. Phys. JETP **35**, 9 (1972)].
[8]J. Shy-Yih Wang and C. Kittel, Phys. Rev. B **7**, 713 (1973).
[9]J. H. Brewer, K. M. Crowe, F. N. Gygax, R. F. Johnson, B. D. Patterson, D. G. Fleming, and A. Schenck, Phys. Rev. Lett. **31**, 143 (1973).
[10]I. I. Gurevich, B. A. Nikol'skiĭ, V. I. Selivanov, and B. V. Sokolov, Zh. Eksp. Teor. Fiz. **68**, 808 (1975) [Sov. Phys. JETP **41**, 401 (1975)].
[11]V. I. Kudinov, E. V. Minaĭchev, G. G. Myasishcheva, Yu. V. Obukhov, V. S. Roganov, G. I. Savel'ev, V. M. Samoĭlov, and V. G. Firsov, Pis'ma Zh. Eksp. Teor. Fiz. **21**, 49 (1975) [JETP Lett. **21**, 22 (1975)].
[12]I. G. Ivanter and V. P. Smilga, Zh. Eksp. Teor. Fiz. **54**, 559 (1968) [Sov. Phys. JETP **27**, 301 (1968)].
[13]I. G. Ivanter and V. P. Smilga, Zh. Eksp. Teor. Fiz. **55**, 1521 (1968) [Sov. Phys. JETP **28**, 796 (1969)].
[14]G. G. Myasishcheva, Yu. V. Obukhov, V. S. Roganov, and V. G. Firsov, Zh. Eksp. Teor. Fiz. **53**, 451 (1967) [Sov. Phys. JETP **26**, 298 (1968)].
[15]E. V. Minaĭchev, G. G. Myasishcheva, Yu. V. Obukhov, V. S. Roganov, G. I. Savel'ev, and V. G. Firsov, Zh. Eksp. Teor. Fiz. **57**, 421 (1969) [Sov. Phys. JETP **30**, 230 (1970)].

Translated by S. Chomet

A soliton model of particles of the ψ-boson type

I. S. Shapiro

Institute for Theoretical and Experimental Physics
(Submitted January 31, 1976)
Zh. Eksp. Teor. Fiz. **70**, 2050–2059 (June 1976)

Coherent states (condensates) constructed on the basis of classical soliton-like solutions of a one-dimensional field equation are considered. The narrowness of the ψ boson is explained by the smallness of the amplitude for the transition of the condensate into a state with definite number of particles, much smaller than the average number. The ψ' boson is interpreted as a condensate with a mean field which differs little from the mean field of the ψ state, and it is proved that the $\psi' \to \psi$ transition is not suppressed.

PACS numbers: 14.40.—n, 11.10.Qr

INTRODUCTION

The suppression of the number of pion channels characteristic of the ψ particles manifests itself particularly clearly in comparing the decays of the ψ' into 3π and $2\pi\psi$. The three-particle Lorentz invariant phase space for the first of these decays is approximately 260 times larger than for the second mode, whereas the rates are comparable (the $2\pi\psi$ channel is responsible for about 30% of the width of the ψ'). Our fundamental idea (formulated before in[1]) is that ψ and ψ' are states of the coherent type. The norm of such states (Bose condensates) is distributed over an infinite number of bosons and therefore the amplitudes of the components with definite particle number are small. The suppression of the pionic modes of the ψ state depends in such a model, first, on the physical nature of the coherent state, and second, on the mean number of particles in that state. In condensates of the superfluid type (coherent states of the group $SU(1,1)$) the probability of transition from the condensate (a state with indefinite particle number) into an ordinary state (with a finite number of particles) is proportional to $\bar{n}^{-1/3}$, where $\bar{n}$ is the mean particle number in the condensate, and for the suppression of the probability by a factor of 10^{-3} one needs $\bar{n}\simeq 10^9$. More appropriate for the description of the ψ particles are coherent states of the oscillator type (cf. [2]). In this case, there occurs an exponential (in $\bar{n}$) suppression of the transition probability of the coherent state into a normal state, so that a suppression by a factor of 10^3 occurs already for $\bar{n}\simeq 10$. It is our purpose to construct a coherent state which has the properties of a particle. This means that the mass (squared four-momentum) of such a system must not change in interactions with not too strong external fields. As is well known, soliton solutions of classical nonlinear field equations exhibit such a property. Constructing a coherent state on the basis of a classical soliton solution we obtain an object having, in the indicated sense, corpuscular properties and being at the same time a quantum condensate. Its decay into any finite number of components will be suppressed if that number is much smaller than the average particle number in the condensate. Another important property of the soliton-condensate is the fact that it has collective degrees of freedom which interact with the electromagnetic field.

This is the basic content of the proposed model. Following this outline we first describe the formalism of construction of the coherent state (Sec. 1), after that we consider a one-dimensional realization of the model (Sec. 2). In Sec. 3 we discuss electromagnetic transitions.

1. COHERENT STATES OF A NEUTRAL SPINLESS BOSON FIELD

The formalism described in this section is close to the content of the recent publications,[3,4] but does not coincide completely with them.

For definiteness we start from the Schrödinger operators of the canonically conjugate fields $\hat{\varphi}(\mathbf{x})$ and $\hat{\pi}(\mathbf{x})$ satisfying the usual boson commutation relations. The starting point is the splitting of the fields into positive- and negative-frequency components $\hat{\varphi}^{(+)}$, $\hat{\varphi}^{(-)}$ and $\hat{\pi}^{(+)}$, $\hat{\pi}^{(-)}$. This requires defining the vector $|0\rangle$ that describes a state "without particles" and is annihilated by the operators $\hat{\varphi}^{(+)}$ and $\hat{\pi}^{(+)}$; consequently this defines a definite (although in principle arbitrary) realization of the boson creation-annihilation operators, i.e., a specification of their mass.

From the canonical momentum $\hat{\pi}$ and the arbitrary complex-valued function $f(\mathbf{x})$ we construct the unitary operator[1)]

$$U_f=\exp\left\{-i\int(f\hat{\pi}^{(-)}+f^*\hat{\pi}^{(+)})\,d\mathbf{x}\right\}. \tag{1}$$

Acting on the no-particle state $|0\rangle$, this operator gives rise to the normalized coherent state

$$\hat{\pi}^{(+)}|0\rangle=0,\quad |f\rangle=U_f|0\rangle. \tag{2}$$

The scalar product of two coherent states can be expressed through the Fourier transforms $\tilde{f}$ of the functions f:

$$|\langle f_2|f_1\rangle|^2=\exp\left\{-{}^1/_2\int\omega|\tilde{f}_1-\tilde{f}_2|^2(d\mathbf{k})\right\},\quad \omega=\sqrt{k^2+m_0^2}, \tag{3}$$

where m_0 is the mass of the bosons making up the condensate and the volume element $(d\mathbf{k})$ includes the normalization factors. It follows from (3), in particular, that

$$|\langle 0|f\rangle|^2=\exp\left\{-{}^1/_2\int\omega|\tilde{f}|^2(d\mathbf{k})\right\}. \tag{4}$$

The equations (3) and (4) are essential for the ψ-particle model under consideration. They are responsible for the fact that the decays of the type $\psi'\to 2\pi\psi$ are allowed (Eq. (3) at $f_1\simeq f_2$) and the decays $\psi\to 3\pi$ are suppressed (the amplitude of this process is proportional to $\langle 0|f\rangle$).

It follows from (2) that the state $|f\rangle$ is an eigenvector of the positive-frequency part $\hat{\varphi}^{(+)}(\mathbf{x})$ of the field operator:

$$\hat{\varphi}^{(+)}(\mathbf{x})|f\rangle={}^1/_2 f(\mathbf{x})|f\rangle, \tag{5}$$

where, as usual,

$$\hat{\varphi}^{(+)}(\mathbf{x})=\int\frac{\hat{a}(\mathbf{k})}{\sqrt{2\omega}}e^{+i\mathbf{k}\mathbf{x}}(d\mathbf{k})$$

($a(\mathbf{k})$ is the boson annihilation operator). This implies that the integral in the exponent of (4) is simply the mean particle number $\bar{n}$ in the condensate $|f\rangle$. The probability for observing n particles in the condensate is given by a Poisson distribution.

We also note that the operators U_f separate the mean field from $\hat{\varphi}(\mathbf{x})$:

$$\hat{\psi}(\mathbf{x})=U_f\hat{\varphi}(\mathbf{x})U_f^+=\hat{\varphi}(\mathbf{x})-{}^1/_2 f(\mathbf{x}),$$

where, by virtue of (5), the state $|f\rangle$ is the vacuum for $\hat{\psi}(\mathbf{x})$:

$$\hat{\psi}^{(+)}(\mathbf{x})|f\rangle=0.$$

All the above equations are also valid in the Heisenberg picture, since they are based exclusively on the equal-time commutation relations for the field operators.

So far, no restrictions have been imposed on $f(\mathbf{x},t)$.

If one requires that $f(\mathbf{x}, t)$ satisfy the Heisenberg equations for $\hat{\varphi}(\mathbf{x}, t)$ then, as was shown in[5], the state $|f\rangle$ will be the quasiclassical approximation to the exact quantum solution and will determine the classical field in the limit $\bar{n}\to\infty$, $h\to 0$. The second condition is equivalent to smallness of the frequency Ω of the time oscillations of the field, compared to its energy. Since the latter is in any case larger than $\bar{n}m_0$, we are led to the conclusion that the state $|f\rangle$ can be a good approximation in the case when

$$\bar{n}\gg 1,\quad \Omega\ll\bar{n}m_0. \tag{6}$$

In order of magnitude $\Omega\simeq|\dot{f}/f|$ and, as already mentioned, $\bar{n}$ is expressed in terms of f by the integral

$$\bar{n}={}^1/_2\int\omega|\tilde{f}|^2(d\mathbf{k}). \tag{7}$$

As will be seen in the sequel, the conditions (6) are satisfied for solutions of the soliton type in the case of weak coupling.

2. MODELS OF QUANTUM SOLITONS

We consider a one-dimensional equation of the Landau–Ginzburg type for a scalar field

$$\varphi''-\ddot{\varphi}+\mu^2\varphi-\lambda\varphi^3=0 \tag{8}$$

with the condition

$$\mu^2/\lambda\gg 1 \tag{9}$$

("weak coupling"). The equation (8) has a solution corresponding to a constant field $f_0=\pm\mu\lambda^{-1/2}$. The field $\psi_0=\varphi-f_0$ with the condition (9) satisfies approximately the free-field equation with mass $m_0=\mu\sqrt{2}$. The states $|n\rangle$ with definite particle number n and mass m_0 are eigenvectors of the Hamiltonian $\hat{H}$ and of the momentum operator $\hat{P}$

$$\hat{H}={}^1/_2\int(\hat{\pi}^2+\hat{\varphi}'^2-\mu^2\hat{\varphi}^2+\lambda\hat{\varphi}^4/2)\,dx,\quad \hat{P}=-\int\hat{\pi}\hat{\varphi}'\,dx \tag{10}$$

up to corrections of the order $n\lambda/\mu^2$, if $n\ll\mu^2/\lambda$. For values of n comparable to the parameter μ^2/λ the states $|n\rangle$ can no longer be eigenvectors of $\hat{H}$. In this case the system becomes quasiclassical and an appropriate approximation is a coherent state constructed on the basis of the classical field f satisfying the equation (8). Such will in particular be the soliton solution representing the static field translated along the x axis:

$$f_s(u)=\pm\mu\lambda^{-1/2}\,\mathrm{th}\,u, \tag{11}$$

where

$$u=\gamma(x-vt)\mu/\sqrt{2},\quad \gamma=(1-v^2)^{-1/2},$$

and v is the velocity. The energy and momentum of the soliton (11) are given by

$$E_s=-l\mu^4/2\lambda+M_s\gamma,\quad P_s=M_s v\gamma, \tag{12}$$

where

$$M_s=2\sqrt{2}\,\mu^3/3\lambda$$

and l is the normalization "volume." It follows from (12) that the soliton solution corresponds to the motion of a particle of mass M_s, if one renormalizes the energy, removing from it the term which depends on l. The omission of this term is equivalent to counting the energy from that corresponding to the constant field f_0. It is however essential that such a renormalization procedure cannot be extended to the calculation of the divergent integral (7) in the expression (4) for $\langle 0|f_s\rangle$. The only subtraction procedure which is acceptable in this case would consist in the substitution $f_s\to f_s-f_0$. But this does not realize the goal, since $f_s(\pm\infty)=\pm\mu\lambda^{-1/2}$, whereas f_0 is a constant. Any other subtraction is equivalent to a violation of unitarity of the operator U_f or is in contradiction with the equation of motion (8). Thus,

$$\bar{n}_s=\infty,\quad \langle 0|f_s\rangle=\exp\{-\bar{n}_s/2\}=0, \tag{13}$$

with

$$|f_s\rangle=\exp\left\{-i\frac{\mu}{\sqrt{\lambda}}\int_{-\infty}^{+\infty}dx\hat{\pi}(x)\,\mathrm{th}\,u\right\}|0\rangle \tag{14}$$

(in the frame with $v=0$). Equations (13) mean that the "quantum soliton" (14) is stable with respect to transitions into a state with definite particle number n, although in the example under consideration there are no quantum numbers produced by a group symmetry.

The absolute stability of the state (14) stems from the boundary conditions imposed at $x=\pm\infty$. By somewhat modifying Eq. (8) one can construct an example of a soliton-particle for which the decay is strongly suppressed, but not completely forbidden, as happens for the ψ particle. For this purpose we replace (8) by the same equation with a right-hand side $\mu^2\sqrt{2}\delta(u)\lambda^{-1/2}$. Such an equation corresponds to a boson field produced by an external fixed source concentrated at a point. One can imagine that this source is a heavy particle of mass $M_0\gg\mu$ (a fermion or a boson) interacting with φ according to the law

$$V=g\varphi(u)\delta(u),\quad g=\sqrt{2}\,\mu^2/\sqrt{\lambda}. \tag{15}$$

In addition to this interaction one can add to the Hamiltonian also a bare energy M_0 and the appropriate momentum $-v\gamma M_0$. A solution of the equation with the right-hand side will be

$$f_\psi(u)=\varepsilon(u)f_s(u), \tag{16}$$

where $\varepsilon(u)$ is the signature function ($\varepsilon(\pm|u|)=\pm 1$).

Since $f_s(0)=0$ the interaction (15) does not contribute to the energy. For the same reason, and since $\varepsilon'(u)=\delta(u)$

$$(f_\psi')^2=(f_s')^2,\quad f_\psi f_\psi'=f_s f_s'.$$

This implies that the state $|f_\psi\rangle$ (after subtraction of the vacuum energy) will be characterized by a mass, energy, and momentum

$$M_{\psi}=M_0+M_s,\quad E_{\psi}=\gamma M_{\psi},\quad P_{\psi}=v\gamma M_{\psi}.$$

At the same time the subtraction $f_{\psi}^R=f-f_0$, which is now admissible, leads to a regularization of the integral (7). Since $\omega(k)$ varies considerably slower than $|\tilde{f}_{\psi}^R(k)|^2$, we can write:

$$\bar{n}_{\psi}=\frac{\omega(\bar{k})}{2}\int_{-\infty}^{+\infty}|f_{\psi}^R|^2\frac{dk}{2\pi}=\omega(\bar{k})\int_0^{\infty}(f_{\psi}-f_0)^2\,dx.$$

Setting the average $\bar{k}=\mu/\sqrt{2}$ and calculating the integral, we obtain

$$\bar{n}_{\psi}=\sqrt{5}(2\ln 2-1)\mu^2/\lambda\approx 0.9\mu^2/\lambda.$$

This shows that the decay probability of the state $|f_{\psi}\rangle$ into n bosons is proportional to $\exp(-\mu^2/\lambda)$, i.e., will be exponentially small in the weak coupling case.

In addition to (16), Eq. (8) with a right-hand side has oscillating solutions which differ little from f_{ψ}. Such solutions for the homogeneous equation have been considered in the weak-coupling approximation in several papers (cf. [3,6,7]). For the homogeneous equation which interests us the solutions will be the same, but here they are not used for the quantization of the field $\hat{\varphi}-f$, but for the determination of the classical field f_{ψ}, which is close to the field f_{ψ}, and for the construction on the basis of this field of the quantum coherent state $|f_{\psi'}\rangle$. Accordingly we write

$$f_{\psi'}=f_{\psi}(u)+G(u)e^{i(qx-q_0t)},\tag{17}$$

where

$$|G(u)|\ll|f_{\psi}(u)|,\tag{18}$$

and require

$$G(\pm\infty)=0.\tag{19}$$

This boundary condition and the requirement that f_{ψ} satisfy the equation (8) for all $t\geq 0$, determines the function and the frequency q_0 if in addition one gives the normalization integral

$$\int_{-\infty}^{+\infty}G^2(u)\,du=N^2\ll\mu^2/\lambda.\tag{20}$$

Substituting (17) into (8), using (18) and equating to zero the coefficients of the cosine and sine of $qx-q_0t$, we obtain

$$q/q_0=v,$$
$$G''+2(\Omega^2/\mu^2-2+3/\mathrm{ch}^2\,u)G-(\lambda/2\mu^2)G^3=0,\tag{21}$$

where

$$\Omega^2=q_0^2-q^2.$$

Equation (21) has solutions with the boundary conditions (19) only if $\Omega^2<2\mu^2$. In the zeroth approximation in λ (i.e., omitting the nonlinear term in (21)) there exist two eigenvalues of Ω_0^2 satisfying this inequality

$$\Omega_0^2=0;\quad 3\mu^2/2.$$

The value $\Omega_0^2=0$ does not fit, since $v<1$. To first order of the perturbation theory in λ one can obtain

$$\Omega_1^2-3\mu^2/2=\frac{2\lambda}{3N^2}\int_{-\infty}^{+\infty}G_0^4\,du=\frac{9\pi}{2^9}\lambda N^2.$$

In this equation G_0 is a solution of Eq. (21) for $\lambda=0$ and $\Omega_0^2=3\mu^2/2$:

$$G_0=N(^3/_2)^{1/2}\,\mathrm{th}\,u/\mathrm{ch}\,u.$$

The mass difference between the solitons $|f_{\psi'}\rangle$ and $|f_{\psi}\rangle$ to zeroth approximation in λ is given by

$$\Delta M_{\psi}=M_{\psi'}-M_{\psi}=\frac{\sqrt{2}\,\Omega_0^2}{\mu}\int_{-\infty}^{+\infty}G_0^2\,du=\frac{3N^2\mu}{\sqrt{2}}.$$

This shows that in the model under consideration the mass spectrum $M_{\psi'}$ is continuous, and starting from $N^2>\frac{2}{3}(\Delta M_{\psi}>\mu\sqrt{2})$ the decay $|f_{\psi'}\rangle\to|f_{\psi}\rangle$ is energetically allowed. It is also easy to estimate the factor which determines the transition probability. Making use of Eq. (3) and proceeding as before (in the calculation of $\bar{n}_{\psi}$) we obtain

$$|\langle f_{\psi}|f_{\psi'}\rangle|^2=\exp\left\{-{}^1/_2\,\omega(\bar{k})\int_0^{\infty}G_0^2\,dx\right\}=\exp\{-N^2\sqrt{5/4}\}\tag{22}$$

(the factor $\frac{1}{2}$ in front of the integral in the exponential is due to averaging $\cos^2\Omega_0 t$). From Eq. (22) and the condition (20) it follows that

$$|\langle f_{\psi}|f_{\psi'}\rangle|^2\gg|\langle 0|f_{\psi'}\rangle|^2.$$

Since the factor $|\langle 0|f_{\psi'}\rangle|^2$ determines the order of magnitude of the $|f_{\psi'}\rangle\to|n\rangle$ decay probability for $n\ll\bar{n}$, this decay is considerably less likely in the model under consideration than the decay mode $|f_{\psi'}\rangle\to|f_{\psi}\rangle$, similar to what happens for the real bosons ψ' and ψ.

We note that the scalar product $\langle n|f\rangle$ is not exactly equal to the decay amplitude of the state $|f\rangle$ into n observable free particles. It only determines the amplitude to find n particles in the quantum soliton $|f\rangle$. The decay probability will contain in addition a "barrier penetration factor" and a renormalization constant, corresponding to the transition of the particle a of the initial quantization basis into the physical boson b with the same quantum numbers. One can show (cf. Glauber[2]) that a state which is coherent in the basis a is also coherent with respect to the particles b. For this reason the concrete choice of the representation of the Schrödinger operators $\hat{\varphi}(x)$ is of no consequence from the point of view of principle.

We have considered as an example of a solution of a spatially one-dimensional equation of the Landau–Ginzburg type with a right-hand side, solely in order to demonstrate in analytic form the main properties of the proposed model: the exponential smallness (in $1/\lambda$, $\lambda \ll 1$) of the decay probability into light bosons (the analog of the process $\psi \to n\pi$) and the unsuppressed character of the transition from one coherent state into another, which is close to the first as far as the mean field is concerned (the analog of the transition $\psi' \to \psi$). As was recently shown by Kudryavtsev, [8] Eq. (8) has soliton-like bell-shaped solutions similar to (16), but differing from the latter by slow oscillation and decay (the frequency of oscillation is much smaller than the mass M_s of the soliton, and the decay time is much longer than the period of oscillations). The field amplitude of these solutions has the order of magnitude $\mu\lambda^{-1/2}$, i.e., is large in the case of weak coupling (for $\mu\lambda^{-1/2} \ll 1$ such solutions with small field amplitude have been found in the framework of perturbation theory in[9,10]). Apparently, similar solutions exist also in three-dimensional space. [11]

3. ELECTROMAGNETIC PROPERTIES

So far we have considered the condensate of a neutral field. In the case of charged fields in addition to the field $\hat\varphi(x)$ one must introduce the charge-conjugate field:

$$\hat\varphi_c = \hat C \hat\varphi \hat C^-, \quad \hat\pi_c = \hat C \hat\pi \hat C^-,$$

where $\hat C$ is the charge-conjugation operator.

For the charged condensate, the operator U_f will have the form

$$U_f = \exp\left\{ -i \int (f\hat\pi_c^{(-)} + f^*\hat\pi^{(-)})\, dx \right\}. \quad (23)$$

For $\hat\varphi_C = \hat\varphi$ (neutral field) Eq. (23) becomes (1). The operator U_{fC} for the antiparticle condensate is obtained from (23) by means of the substitution f, $\hat\pi$, $\hat\pi_C \to f_C$, $\hat\pi_C$, $\hat\pi$, where $f_C(x)$ is a function which by analogy with $f(x)$ determines the mean field of the antiparticle condensate, with

$$\hat\varphi_c^{(+)} |f_c\rangle = {}^1/_2 f_c |f_c\rangle.$$

Similar to the Cooper-pair condensate, the condensates $|f\rangle$ and $|f_C\rangle$ have undetermined charges. Only the average values of the charge are fixed. Charge and current conservation are valid also only for expectation values. The condensate $|f, f_C\rangle$ containing in the mean $\bar n$ particles and $\bar n_C$ antiparticles and having therefore an average charge $e(\bar n - \bar n_C)$ is obtained in the following manner:

$$|f, f_c\rangle = U_f U_{fc} |0\rangle.$$

We now consider a neutral condensate ($\bar n = \bar n_C$). If $f = f_C$, then the condensate, which is neutral only in the mean, nevertheless transforms into itself under charge conjugation and therefore has a definite charge-conjugation parity. The diagonal matrix elements of the current $\hat j\mu$ will then vanish. The matrix element for the emission of a photon will have the form

$$\langle \gamma f|S|f'\rangle = \frac{ie(1-CC')}{(2\varkappa_0)^{1/2}} \langle f|f'\rangle e_\mu^{(\lambda)} \int \left(f^* \frac{\partial f'}{\partial x_\mu} - \frac{\partial f^*}{\partial x_\mu} f' \right) e^{i\varkappa x}\, d^4x. \quad (24)$$

Here $e_\mu^{(\lambda)}$ is the photon polarization vector, $\varkappa = (\varkappa_0, \boldsymbol{\varkappa})$ is the 4-momentum of the photon, C', C are the charge-conjugation parities of the states $|f'\rangle$, $|f\rangle$ between which the transition occurs (owing to the identity of f and f_C we have used only one function in the notation of the condensate). It is remarkable that Eq. (24) coincides almost exactly with the quantum-mechanical matrix element for the emission of light by a neutral system consisting of two charged particles having the wave functions of the initial and final states equal to f' and f, respectively. The only difference is the presence of the factor $\langle f|f'\rangle$, the nonorthogonality of the functions f' and f, as well as their normalization (whereas the states $|f'\rangle$ and $|f\rangle$ have unit norms, the normalization integrals of the functions f', f yield the average charges of the charged components of the condensate which is neutral in the mean. Thus, it follows from Eq. (24) that each of the charged parts of the condensate interacts coherently with the electromagnetic field, like a point particle with charge $e\bar n$ localized in a region where f is nonzero (the support of f). In this sense the many-particle state $|f\rangle$ differs considerably a usual quantum mechanical system consisting of several particles, like an atom or a nucleus. In the latter case the electromagnetic transition amplitude would contain a summation over the particles, which is absent from (24), where it is replaced by the factor $\bar n$. In other words, whereas in a many-particle quantum-mechanical system at a given time one particle is with a certain probability at the space point 1, another, at the point 2, etc., in the condensate at each instant of time all present particles are situated at one space point (with the mean number of particles at a point being a function of the point). As is well known, the coherent effects lead to the appearance of collective degrees of freedom. In the case of the coherent states considered here such a collective degree of freedom is the mean field of the condensate, determined by the function f. We note that the equation (24) is valid only in the case when the states $|f\rangle$ and $|f'\rangle$ are close to one another. This is due to the fact that the energy and current are conserved only in the mean. For the same reason a consistent calculation of the annihilation transition $|f\rangle \to |e^+e^-\rangle$ runs into difficulties. The amplitude of such a transition is proportional to the integral:

$$(\bar u_1 \gamma_\nu u_2) \int \langle 0|\hat j_\mu(x+y/2)|f\rangle G_{\mu\nu}(y) e^{iQ(x-y/2)}\, d^4x\, d^4y. \quad (25)$$

Here u are the spinor amplitudes of the leptons, Q is their total 4-momentum and $G_{\mu\nu}$ is the photon Green's function. However, in this formula one may not set, as usual,

$$\langle 0|\hat j_\mu(x)|f\rangle = e^{-iPx} \langle 0|\hat j_\mu(0)|f\rangle, \quad (26)$$

where P is the 4-momentum of the state $|f\rangle$, since $|f\rangle$ is not an eigenstate of the translation operator (i.e., of the 4-momentum operator of the system).

This means that, in other words, the Fourier components of the off-diagonal matrix elements $\langle 0|\hat{j}_\mu(\mathbf{x})|f\rangle$ are incorrectly described by the model under consideration, and it is these matrix elements which determine according to Eq. (25) the amplitude of the electromagnetic transition $|f\rangle \to |e^+e^-\rangle$. The matrix element under consideration depends completely on the structure of the two-particle states which enter into $|f\rangle$, since the current $\hat{j}_\mu(\mathbf{x})$ is bilinear in the creation-annihilation operators of the particles. It is clear that for average particle number $\bar{n} \gg 1$ the components of $|f\rangle$ with small numbers of particles may in some respects be very far from the corresponding components of the exact solution. In particular, the momentum distributions must differ strongly. It is easy to show that the mean momentum of the light boson on the two-particle component of the exact solution will be of the order M (M is the mass of the soliton). The average momentum of such a boson in the two-particle component of the approximate solution $|f\rangle$ will be of the order m_0, i.e., the same as in the many-particle components with $n \simeq \bar{n}$, since the whole state $|f\rangle$ is characterized by one function $f(x)$ corresponding to a linear size of the space region of the order $1/m_0$ (here as before m_0 is the mass of the light boson in the condensate). One might think that the exact quantum solution must be described by a countable sequence of functions $f_n(\mathbf{x})$ one for each state $|n\rangle$, so that $f_n(\mathbf{x})$ will be close to $f(\mathbf{x})$ at $n \simeq \bar{n}$. We note that the coherence properties of the system, which is a Bose-condensate, are expressed through the fact that the n-particle states are described not be functions $F_n(\mathbf{x}_1, \ldots, \mathbf{x}_n)$ of n variables, but by functions $f_n(\mathbf{x})$ of one variable (in other words, that the functions F_n are symmetrized sums of products of the $f_n(\mathbf{x}_i)$, $i = 1, 2, \ldots, n$). Although the function $f_n(\mathbf{x})$ for $n \simeq \bar{n}$ must differ strongly from $f(\mathbf{x})$ in its $\mathbf{x}$ dependence, we are entitled to count on closeness of the distributions in the particle numbers in the exact and approximate solutions. In other words, it is reasonable to assume that the normalization integrals of f_n satisfy in order of magnitude the Poisson distribution. Based on this one can estimate the expected theoretical value of the width $\Gamma_{e^+e^-}$ for the electromagnetic decay into the lepton channel, in the following manner. If (26) is valid we can obtain from (25)

$$\Gamma_{e^+e^-} = \frac{\alpha^2}{M^2} \overline{|\langle 0|\hat{\mathbf{j}}(0)|f\rangle|^2} \tag{27}$$

(the bar denotes here averaging over the space components of the current). The square of the matrix element in (27) has the dimension of a spatial density. On the other hand, it is determined by the two-boson component of $|f\rangle$, in which, as shown above, the mean momentum of the particles is of the order of M, and consequently the average radius of the region of spatial localization must be of the order of M^{-1}. Since at the same time we assume the normalization of components with small numbers of bosons to be equal in order of magnitude to the intensity of those components in the approximate model solution, we may set

$$\overline{|\langle 0|\hat{\mathbf{j}}(0)|f\rangle|^2} \approx 2^{-1}\bar{n}^2 e^{-\bar{n}} M^3.$$

$$\Gamma_{e^+e^-} = 2^{-1}\alpha^2\bar{n}^2 e^{-\bar{n}} M, \tag{28}$$

where $\bar{n}$ is the average number of light bosons in the condensate. For $\bar{n} = 10$ and $M \simeq 3$ GeV we obtain from (28) $\Gamma_{e^+e^-} \simeq 1.5$ keV, which is of the same order of magnitude as the widths observed for the ψ particles.

CONCLUSION

It is interesting to compare the probabilities of the decays $\psi' \to 3\pi$ and $\psi' \to 2\pi\psi$ in this model for the same value of $\bar{n}$ for which above we have estimated the value of $\Gamma_{e^+e^-}$. We can write

$$\Gamma_{3\pi} = c^2 \frac{\bar{n}^3}{6} e^{-\bar{n}} W_{3\pi}, \qquad \Gamma_{2\pi\psi} = c'^2 \frac{\bar{n}^2}{2} |\langle f_\psi | f_{\psi'}\rangle|^2 W_{2\pi\psi},$$

where c, c' are effective coupling constants which we will assume to have the same order of magnitude, W are the corresponding invariant phase-space volumes. Taking into account that for the decay of a ψ'

$$W_{3\pi}/W_{2\pi\psi} \approx 260,$$

and setting $|\langle f_\psi | f_{\psi'}\rangle|^2 \simeq 1$, $\bar{n} = 10$, we obtain $\Gamma_{3\pi}/\Gamma_{2\pi} \simeq 8\%$. From these estimates it can be seen that the model under consideration even in its manifestly imperfect shape, is capable of giving the widths of the electromagnetic decays into the leptonic channel, at the same time suppressing the decay mode $\psi' \to 3\pi$ compared to $\psi' \to 2\pi\psi$ by an order of magnitude, although the first of these channels must, by energy considerations, be 260 times more intensive than the second (in other words, the square of the amplitude of the process $\psi' \to 3\pi$ in the given model is by three orders of magnitude smaller than the corresponding quantity for the decay $\psi' \to 2\pi\psi$ for a normal value of $\Gamma_{e^+e^-}$).

We now turn to the problem of the total decay probability of ψ particles. Since the phase-space volume W_n is made dimensionless by the condensate particle mass (this is the mass determining the spatial size of the system), the multiplicity specified by the phase space volume depends on one parameter $\bar{n} = M/m_0$. For $n \simeq \bar{n}$ the volume $W_{n\pi} \sim (1 - n/\bar{n})^{(3n-5)/2}$, and for $n \ll \bar{n}$ the volume is $W_{n\pi} \sim \bar{n}^{2n-4}$. Therefore the average multiplicity n_W allowed by the phase-space volume turns out to be much smaller than $\bar{n}$, with a relatively peaked distribution around n_W. The suppression factor of the decay $\psi \to n\pi$ in the coherent model corresponds to values of the Poisson distribution for an argument $n_W \ll \bar{n}$. It is clear that for $\bar{n} \gg 1$ the suppression may be large (e.g., for $\bar{n} = 20$, $n_W = 5$, the decay will be slowed down by a factor of 10^4). In this connection it does not surprise us that the decay probability of the ψ' into the one channel $2\pi\psi$ (for which there is no exponential suppression) is approximately $\frac{1}{3}$ of the total ψ' width, i.e., makes up about half of all the decays into pionic channels.

Summing up, we arrive at the conclusion that the proposed model is capable of unifying the main qualitative peculiarities of objects of the type of the ψ particles. Two traits of this model seem attractive to us: first, that it gets away without introducing new hypothetical quantum numbers and, second, that the narrowness of the ψ resonances is here related to their large mass (the suppression of the pionic channels requires that $\bar{n} \gg 1$, or, on account of what was said, $M \gg m_0$).

In conclusion we note that one of the simplest types of coherent states was considered here. It is essential that the modern approach to coherent states (cf.[12]) considerably widens the spectrum of theoretically conceivable models of this type.

The author is indebted to V. A. Karmanov for a valuable remark regarding the calculation of the probability of electromagnetic leptonic decays, as well as to A. E. Kudryavtsev and A. M. Perelomov for useful discussions.

[1)]Equation (1) differs from the corresponding expressions in[3,4]. It corresponds to the operator used in[4] in the special case of real functions $f(x)$.

[1]I. S. Shapiro, Pis'ma Zh. Eksp. Teor. Fiz. 21, 624 (1975) [JETP Lett. 21, 293 (1975)].
[2]R. Glauber, Russian transl. in: Kogerentnye sostoyaniya v kvantovoĭ teorii polya (Coherent states in quantum field theory), Mir, 1972, p. 26. A. M. Perelomov, Preprint ITEF-46, 1974.
[3]K. Cahill, Phys. Lett. 53B, 174 (1974).
[4]P. Vinciarelli, Preprint, TH 1993, CERN, 1975.
[5]K. Hepp, Commun. Math. Phys. 35, 265 (1974).
[6]R. Dashen, B. Hasslacher and A. Neveu, Preprint, IAS, COO-2220-30, 1974.
[7]A. M. Polyakov, Pis'ma Zh. Eksp. Teor. Fiz. 20, 430 (1974) [JETP Lett. 20, 194 (1974)].
[8]A. E. Kudryavtsev, Pis'ma Zh. Eksp. Teor. Fiz. 22, 178 (1975) [JETP Lett. 22, 82 (1975)].
[9]A. M. Kosevich and A. S. Kovalev, Zh. Eksp. Teor. Fiz. 67, 1793 (1974) [Sov. Phys. JETP 40, 891 (1975)].
[10]R. Dashen, B. Hasslacher and A. Neveu, Phys. Rev. 11, 3424 (1975).
[11]N. A. Voronov, I. D. Kobzarev and N. B. Konyukhova, Pis'ma Zh. Eksp. Teor. Fiz. 22, 590 (1975) [JETP Lett. 22, 247 (1975)].
[12]A. M. Perelomov, Commun. Math. Phys. 26, 22 (1972).

Translated by Meinhard E. Mayer

Decay of bounded laser beams in nonlinear media

N. N. Rozanov and V. A. Smirnov

(Submitted September 30, 1975)
Zh. Eksp. Teor. Fiz. 70, 2060–2073 (June 1976)

The stability of propagation of an intense laser beam in a medium with quadratic or cubic optical nonlinearities is investigated. The existence of a discrete spectrum and of a set of natural perturbation functions that correspond to the modes of the "waveguide" produced by the laser beam in the nonlinear medium is found. "Branch points" at which, in contrast to the usual reversal point, no reflection of electromagnetic waves take place, are found to play an important role in the formation of the "waveguide." Dispersion equations for the perturbation growth rates are derived for axisymmetric laser beams of arbitrary and smooth intensity profile. Some simple geometric rules for determining the maximal growth rates are formulated and their dependence on the azimuthal number characterizing the perturbation is found. A limiting transition to the case of an unbounded laser beam considered by Bespalov and Talanov {Pis'ma Zh. Eksp. Teor. Fiz. 3, 471 (1966) [JETP Lett. 3, 307 (1966)]} is analyzed and compared with the results by others.

PACS numbers: 42.60.Nj, 42.65.Hw

Intense laser beams propagating in nonlinear media can be unstable to various types of perturbations. Growth of the perturbations leads to decay of the initial beam. For media with cubic nonlinearity, this effect has been most thoroughly investigated as applied to the self-focusing phenomenon. As shown by Bespalov and Talanov, [1] an intense plane wave is unstable to definite perturbations of its profile, and this causes the plane wave to break up into individual filaments.

Although the theory of Bespalov and Talanov explains the main features of the phenomenon and yields for the self-focusing length an estimate that agrees with experiment, it does not take into account so important a factor as the limited dimensions of real laser beams. Attempts to analyze the stability of a bounded laser beam in a cubic medium were undertaken in several places, the best results being obtained by Lyakhov. [2] This procedure, however, is not sufficiently well founded, and some of his results contradict numerical experiments[3] as well as the ideas that have developed by now.

The fundamental difference between the bounded laser beam and an infinite plane wave is the following: In the case of an intense bounded beam, it becomes possible for perturbations that are bounded in the transverse direction to propagate. Such perturbations correspond to the discrete modes of the "waveguide" produced in the nonlinear medium by the laser beam. The waveguide can be regarded as homogeneous, since the distances of interest to us, which are of the order of the characteristic perturbation growth length, are usually much smaller in real laser beams than the self-focusing length of the beam as a whole. For bounded beams it is precisely the discrete spectrum which corresponds to increasing perturbations. An analysis of the stability of intense and broad laser beams without allowance for the waveguide modes (see, e.g., [4]) is therefore only of limited use.

