where the effective cross section of the induced radiation is

$$\sigma_{\text{eff}}^{(k)}=\frac{dI_0}{dt}\left\{\frac{\beta_k k(k-1)}{dI_0/dt}\right\}^{2/(k+1)}. \tag{15}$$

It follows from (15) that the cross section $\sigma_{\text{eff}}^{(k)}$ has an unexpectedly weak dependence on the order $k$ of the radiation process. If, for example, we use the empirical relation $\beta_k\approx 10^{10-30k}$ cm$^{2k}$-sec and assume $dI_0/dt = 10^{35}$ photon/cm$^2$ sec$^2$, then on going from the two-photon process to the three- and four-processes we obtain respectively

$$\sigma_{\text{eff}}^{(3)}/\sigma_{\text{eff}}^{(2)}\approx 0.3,\quad \sigma_{\text{eff}}^{(4)}/\sigma_{\text{eff}}^{(2)}\approx 0.1.$$

This circumstance offers more opportunities for finding systems that are promising for the excitation of coherent radiation on multiphoton transitions.

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[1)] In this case it is possible also to neglect the linear loss, which is characterized by the time $\tau_c$.

[2)] In the case of propagation of very short signals in amplifying media, account must be taken of the coherence of the interaction,[14] but the results of such an analysis are still qualitatively close to those obtained with the aid of the kinetic equation.

[3)] This is equivalent to the condition $2\beta_2\Delta N_2^0\overline{I}_0 L_{\text{ampl}}=1$.

[4)] The durations of a pulse with slope $10^{35}$ photons/cm$^2$ sec$^2$ needed for effective amplification, at the given parameters, are respectively $10^{-8}$ and $10^{-9}$ sec. The amplified-signal powers are then $2\times10^9$ and $2\times10^8$ W/cm$^2$.


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# Exchange spin polarization in a three-particle system


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A theory of spin polarization in a system of three particles, each of which has a spin 1/2, is developed. It is assumed that initially the whole system is in a given spin state. A redistribution of the initial polarization occurs during collisions between the particles as a result of exchange interaction. In particular, a beam of unpolarized electrons becomes polarized to a certain extent upon being scattered by oriented atoms. The obtained results allow a uniform computation of the polarization of the scattered beam and that of the target in elastic and inelastic collisions accompanied by particle exchange in the case of arbitrary initial polarization states. Results are presented of numerical calculations of the polarization of electrons scattered by oriented He($2^3S$) atoms and of the depolarization of electrons as a result of singlet-triplet transitions.


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Interest in spin-dependent processes, which has been displayed in recent years in the physics of atomic collisions, is stimulating the search for efficient sources of polarized electrons and the corresponding theoretical computations. To obtain oriented electron beams at present, the following physical processes are used: Mott scattering, the Fano effect, the photoionization of polarized atoms, photoemission from magnetic materials, and the Penning ionization of oriented atoms. In the first two cases the polarization mechanism is connected with the relativistic spin-orbit interaction, which leads to the appearance of a preferred orientation in the electron beam. In the remaining processes the polarized beams are formed as a result of inelastic transitions leading to the emission of polarized atomic electrons.

Burke and Schey[1] first pointed out the feasibility of producing polarized electrons as a result of the elastic scattering of initially unpolarized electrons by oriented one-electron atoms. In this case the polarization mechanism is, clearly, the exchange electron-atom scattering, which depends on the total spin of the two electrons. Consequently, exchange scattering can be regarded as a special mechanism for producing oriented electron beams which is based essentially on the transfer of a prepared atomic polarization to free electrons. An analogous, although more complex, problem of calculating electron polarization in elastic scattering by oriented targets with spin $S=1$ is considered in[2].

In the present paper we present a theory of polarization in a system of three oriented particles each with spin $S=\frac{1}{2}$. It is assumed that the target is formed by two bound particles (below we shall, for definiteness, speak of electrons) in the singlet or triplet states. These states are described by a set of vector and tensor characteristics defining the initial spin orientation of the target. If a beam of unpolarized electrons is scattered from such a target, then as a result of the exchange interaction the spin polarization of the atomic electrons is partially transferred to the electrons being scattered. If, on the other hand, a beam of polarized electrons is scattered by an unpolarized target, then there occurs a depolarization of the electron beam, and the target turns out to be polarized to a certain extent. In the general case, when all the three particles are initially oriented, a redistribution of the polarization occurs. A specific character of the problem under consideration is the presence of an inelastic channel (singlet–triplet transitions) together with the elastic-scattering channel. The object of the undertaken investigation is the uniform computation, for arbitrary initial polarization states, of electron-beam and target polarization in elastic and inelastic collisions accompanied by particle exchange.

The obtained formulas are used to compute the exchange electron polarization in scattering by oriented He($2^3S$) atoms and to determine electron depolarization as a result of a singlet–triplet transition. In the latter case the results are compared with the data of[3], in which electron depolarization occurring in the inelastic exchange transition was measured for the first time.

## 1. FORMULATION OF THE PROBLEM

To describe the spin state of the system of three particles, each with spin $S=\frac{1}{2}$, let us use the density-matrix apparatus. Let us introduce the basis matrices according to the definition:

$$\begin{aligned} &I_{(8)}=I\times I\times I, \quad \hat{Q}^{12}_{\alpha\alpha'}=\sigma_{1\alpha}\times\sigma_{2\alpha'}\times I,\\ &\hat{\sigma}_{1\alpha}=\sigma_{1\alpha}\times I\times I, \quad \hat{Q}^{13}_{\alpha\alpha''}=\sigma_{1\alpha}\times I\times\sigma_{3\alpha''},\\ &\hat{\sigma}_{2\alpha'}=I\times\sigma_{2\alpha'}\times I, \quad \hat{Q}^{23}_{\alpha'\alpha''}=I\times\sigma_{2\alpha'}\times\sigma_{3\alpha''},\\ &\hat{\sigma}_{3\alpha''}=I\times I\times\sigma_{3\alpha''}, \quad \hat{R}_{\alpha\alpha'\alpha''}=\sigma_{1\alpha}\times\sigma_{2\alpha'}\times\sigma_{3\alpha''},\\ &\alpha,\ \alpha',\ \alpha''=1,\ 2,\ 3. \end{aligned} \tag{1}$$

Here $I$ is the unit $2\times 2$ matrix, $\sigma_{1\alpha}$, $\sigma_{2\alpha'}$, and $\sigma_{3\alpha''}$ are the two-dimensional Pauli matrices for the three particles, and the sign $\times$ denotes the direct product of the matrices. As follows from (1), the complete basis is formed by $(2S+1)^6=64$, $8\times 8$ Hermitian matrices. The mean operator values $\langle\hat{\sigma}_{i\alpha}\rangle=p_{i\alpha}$ in the state defined by the density matrix $\rho$ characterize the vector polarization of the $i$-th particle, while the mean values $\langle\hat{Q}^{ik}_{\alpha\alpha'}\rangle=Q^{ik}_{\alpha\alpha'}$ and $\langle\hat{R}_{\alpha\alpha'\alpha''}\rangle=R_{\alpha\alpha'\alpha''}$ are respectively the pair and triple spin correlations of the particles.

The initial density matrix in its general form is given by the expansion

$$\rho=\frac{1}{8}\Big[I_{(8)}+\sum_{i=1}^{3}\sum_{\alpha}p_{i\alpha}\hat{\sigma}_{i\alpha}+\sum_{\substack{i=1\\k>i}}^{3}\sum_{\alpha\alpha'}Q^{ik}_{\alpha\alpha'}\hat{Q}^{ik}_{\alpha\alpha'}+\sum_{\alpha\alpha'\alpha''}R_{\alpha\alpha'\alpha''}\hat{R}_{\alpha\alpha'\alpha''}\Big]. \tag{2}$$

We shall assume that, initially, the target is formed by the particles 1 and 2. If the spin states of the target and the third particle are independent, i.e., if $\rho$ can be represented in the form $\rho=\rho(1,\ 2)\times\rho(3)$, then we have for the components of the correlation operators the following expressions:

$$Q^{13}_{\alpha\alpha'}=p_{1\alpha}p_{3\alpha'},\quad Q^{23}_{\alpha\alpha'}=p_{2\alpha}p_{3\alpha'},\quad R_{\alpha\alpha'\alpha''}=Q^{12}_{\alpha\alpha'}p_{3\alpha''}. \tag{3}$$

For the sum

$$q_{12}=\sum_{\alpha}Q^{12}_{\alpha\alpha}$$

we obtain the condition

$$q_{12}=\begin{cases}-3 & S=0\ (\text{singlet})\\ \ \ 1 & S=1\ (\text{triplet}),\end{cases} \tag{4}$$

which follows directly from the equality

$$\langle\hat{\sigma}_1\hat{\sigma}_2\rangle=\mathrm{Sp}(\hat{\sigma}_1\hat{\sigma}_2\rho)=\sum_{\alpha}Q^{12}_{\alpha\alpha}.$$

Below the values of all the quantities after scattering will be indicated by primes. In elastic scattering, in which the target spin is conserved, we should have $q'_{12}=q_{12}$.

Of particular interest is inelastic excitation accompanied by a change in the target spin. In this case, according to Wigner's rule, the total spin $j$ of the system is conserved. It is easy to verify that when the target is initially in the singlet state, so that $j=\frac{1}{2}$, we have the following conditions:

$$q_{12}=-3,\quad q_{13}+q_{23}=\sum_{\alpha}(Q^{13}_{\alpha\alpha}+Q^{23}_{\alpha\alpha})=0. \tag{5}$$

In the triplet state $j=\frac{1}{2},\ \frac{3}{2}$. Assuming that the transition to the singlet state is allowed only for the value $j=\frac{1}{2}$, we obtain as the initial conditions the following conditions:

$$q_{12}=1,\quad q_{13}+q_{23}=-4. \tag{6}$$

Transitions accompanied by changes in the target spin are purely exchange transitions, the third particle is captured by the target and one of the atomic electrons appears in the continuous spectrum. Therefore, after the transition $S=0\rightarrow S=1$ (the singlet–triplet excitation), we should have

$$q_{13}'=1,\quad q_{12}'+q_{23}'=-4, \tag{7a}$$

or

$$q_{23}'=1, \quad q_{12}'+q_{13}'=-4$$

and, similarly, after the transition $1\to 0$:

$$q_{13}'=-3, \quad q_{12}'+q_{23}'=0, \tag{7b}$$

or

$$q_{23}'=-3, \quad q_{12}'+q_{13}'=0.$$

Let us draw attention to the fact the quantity

$$\sum_{k>i} q_{ik}=\sum_{k>i} q_{ik}'=-3 \tag{8}$$

is conserved in inelastic collisions in which the spin changes by unity.

The final values of the polarization characteristics of the system depend on the amplitude transition matrix $M$. The algebraic structure of the matrix $M$ is determined by the invariants that can be formed from the axial vectors $\hat{\sigma}_1$, $\hat{\sigma}_2$, $\hat{\sigma}_3$ and the vectors depending on the geometry of the collisions. In the region of comparatively low energies, when the exchange scattering is still comparable to the potential scattering and when, consequently, the spin-orbit interaction can be neglected, the structure of the matrix $M$ can be uniquely re-established. This matrix has the form

$$M=aI_{(8)}+b\sigma_1\times\sigma_2\times I+c\sigma_1\times I\times\sigma_3+dI\times\sigma_2\times\sigma_3, \tag{9}$$

where the amplitudes $a$, $b$, $c$, and $d$ are complex functions depending on the energy and the scattering angle. There exist between these amplitudes relations whose form is determined by the type of process.

Below we give a complete formal solution to the problem of polarization redistribution as a result of the exchange interaction in collisions in a three-particle system. The values of all the polarization characteristics of the electrons after scattering, namely, $p_{i\alpha}'$, $Q_{\alpha\alpha'}^{(ik)\prime}$ and $R_{\alpha\alpha'\alpha''}'$, are computed with the final-state density matrix $\rho'=M\rho M^{+}$ by conventional methods. All the details of the computations are omitted, in view of the exceptional unwieldiness, and the results are given in the Appendix. Let us only note that the total volume of the work includes the computation of $2^{13}$ terms containing the traces (Sp) of the products of various numbers of the matrices $\hat{\sigma}_1$, $\hat{\sigma}_2$, and $\hat{\sigma}_3$ in different combinations.

Below we consider two of the most interesting processes: elastic scattering and the singlet–triplet transition.

## 2. ELASTIC SCATTERING

Let us first of all consider this problem for electron scattering by a target in the triplet state. Introducing the amplitudes of the quadruplet $F(j=\frac{3}{2})$ and doublet $G(j=\frac{1}{2})$ scattering, and setting $c=d$ on the basis of symmetry arguments, we have

$$M_{3/2}=F=a+b+2c, \quad M_{1/2}=G=a+b-4c,$$

whence $a+b=\frac{1}{3}(2F+G)$ and $c=\frac{1}{6}(F-G)$. Moreover, requiring that the operator $a+b\sigma_1\times\sigma_2\times I$ coincide with the operator of projection $\Pi_1=\frac{1}{4}(3I_{(8)}+\sigma_1\times\sigma_2\times I)$ onto the $S=1$ spin state, we obtain for $M$ the expression

$$M={}^1/_4(3I_{(8)}+\sigma_1\times\sigma_2\times I)\,{}^1/_3(2F+G)+{}^1/_6(\sigma_1\times I+I\times\sigma_2)\times\sigma_3(F-G). \tag{10}$$

Let us note for comparison that in the theory of electron scattering by a structureless target with spin $S=1$ the analog of the matrix $M$ has the form[2]

$$M={}^1/_3(2F+G)+{}^1/_3\mathbf{S}\sigma(F-G).$$

Using the formulas (A.1)–(A.4), and taking into account the fact that $a=\frac{1}{4}(2F+G)$, $b=\frac{1}{3}a$, and $c=\frac{1}{6}(F-G)$, we obtain the following results.

1. The elastic-scattering cross section

$$\sigma={}^1/_3(2F\bar{F}+G\bar{G})+{}^1/_6(F\bar{F}-G\bar{G})(q_{13}+q_{23}) \tag{11}$$

or, assuming that the initial particle polarization states (2) and (3) are independent,

$$\sigma={}^1/_3[(2F\bar{F}+G\bar{G})+(F\bar{F}-G\bar{G})\mathbf{p}_3\mathbf{P}], \tag{12}$$

where we have introduced the notation $P=\frac{1}{2}(p_1+p_2)$ for the polarization vector of the atom.

2. The polarization of an electron after scattering is

$$\sigma p_{3\alpha}'={}^1/_9(5F\bar{F}-2F\bar{G}-2\bar{F}G-G\bar{G})P_\alpha+{}^1/_{27}(10F\bar{F}+8F\bar{G}+8\bar{F}G+G\bar{G})p_{3\alpha}+{}^1/_3 i(\bar{F}G-F\bar{G})[\mathbf{p}_3\mathbf{P}]_\alpha. \tag{13}$$

Let us note two particular cases in this result. First, after the scattering of unpolarized electrons by an oriented atom, there arises in the scattered beam the polarization

$$\mathbf{p}_3'=\frac{1}{3}\frac{5F\bar{F}-2\bar{F}G-2F\bar{G}-G\bar{G}}{2F\bar{F}+G\bar{G}}\mathbf{P}. \tag{14}$$

Second, for $F=G$ we have $\mathbf{p}_3'=\mathbf{p}_3$, i.e., the initial polarization state of the electron does not change. Below the formula (14) will be used for numerical calculations of polarization in the $e-\mathrm{He}(2^3S)$ system.

3. The polarization of the atom after the scattering is

$$\sigma P_\alpha'={}^1/_2\sigma(p_{1\alpha}'+p_{2\alpha}')={}^1/_9(5F\bar{F}+\bar{F}G+F\bar{G}+2G\bar{G})P_\alpha+{}^1/_{54}(19F\bar{F}-7\bar{F}G-7F\bar{G}-5G\bar{G})p_{3\alpha}+{}^1/_6 i(\bar{F}G-F\bar{G})[\mathbf{P}\mathbf{p}_3]_\alpha. \tag{15}$$

For $F=G$ we have $P_\alpha'=P_\alpha$. The function

$$D_\alpha=\frac{P_\alpha'}{P_\alpha}=\frac{1}{3}\frac{5F\bar{F}+2G\bar{G}+F\bar{G}+\bar{F}G}{2F\bar{F}+G\bar{G}} \tag{16}$$

characterizes the depolarization of the atom because of the exchange scattering in the case when the initial value $p_3=0$.

It follows from the formula (A.5) that $q_{12}'=1$ for any values of $q_{13}+q_{23}$ and any values of the amplitudes $a$ and $c$. This implies the conservation of the target spin in elastic scattering, and confirms the self-consistency of all the results.

The correlations of the third particle with the target after the scattering change, since the initial values of the vectors $\mathbf{p}_3$ and $\mathbf{P}$ are not conserved in the scattering:

$$\sigma(q_{13}'+q_{23}')={}^4/_9(F\bar{F}-G\bar{G})+{}^2/_3(\mathbf{p}_3\mathbf{P})(2F\bar{F}+G\bar{G}). \tag{17}$$

The elimination, i.e., the preservation, of the pair correlations should be expected under conditions when $\mathbf{p}_3'=\mathbf{p}$ and $\mathbf{P}_3'=\mathbf{P}$. Indeed, these equalities are attained under the condition that $F=G$, under which it follows from (17) that

$$q_{13}'+q_{23}'=2\mathbf{p}_3\mathbf{P}=q_{13}+q_{23}.$$

Thus, under the condition that $F=G$ after the scattering, all the initial polarization characteristics are preserved.

For the triple correlations, let us give the result for one particular case, when, initially, $\mathbf{P}\neq 0$, $\mathbf{p}_3=0$, and $R_{\alpha\alpha'\alpha''}=0$. After the scattering, $R'_{\alpha\alpha'\alpha''}=0$ for different values of the indices, and

$$\sigma\sum_\alpha R'_{\alpha\alpha\alpha''}=\frac{1}{12}(2F\bar{F}-\bar{F}G-F\bar{G})P_{\alpha''}. \tag{18}$$

In electron scattering by a target in the singlet state, all the initial polarization characteristics should, as is *a priori* clear, remain unchanged. Let us give the results.

Since

$$M={}^1/_4[(I_{(8)}-\sigma_1\times\sigma_2\times I)a+(\sigma_1\times I+I\times\sigma_2)\times\sigma_3 c], \tag{19}$$

it follows from (A.1)–(A.7) that $(a=-b,\ c=d)$:

$$\sigma=a\bar{a},\quad p_3'=p_3,\quad \mathbf{P}'=0,\quad q_{12}'=-3,\quad R'_{\alpha\alpha'\alpha''}=R_{\alpha\alpha'\alpha''}. \tag{20}$$

The amplitude $c$ remains arbitrary, as it should be, since $\langle(\sigma_1+\sigma_2)\rangle=0$. It is convenient to set

$$M={}^1/_4[(I_{(8)}-\sigma_1\times\sigma_2\times I)+(\sigma_1\times I+I\times\sigma_2)\times\sigma_3]a,$$

where $a$ is, by implication, $f_{\rm dir}-g_{\rm exc}$, the difference between the direct and exchange amplitudes.

## 3. THE SINGLET-TRIPLET TRANSITION

The singlet–triplet excitation cross section is determined by one exchange amplitude, $G$. Therefore, there exists between all the functions $a$, $b$, $c$, and $d$ in (9) a proportionality relation, for the determination of which we use the invariant, (8), of the transition.

Substituting into the sum $\sum q'_{ik}$ for $k>i$ the explicit expressions for $q'_{ik}$ given by (A.5)–(A.7), we find that the equation

$$\sum_{k>i} q_{ik}'=-3$$

is satisfied identically provided

$$a\bar{a}+b\bar{b}+c\bar{c}+d\bar{d}+[\bar{a}b+a\bar{b}+\bar{a}c+a\bar{c}+\bar{a}d+a\bar{d}$$
$$+\bar{b}c+b\bar{c}+\bar{b}d+b\bar{d}+\bar{c}d+c\bar{d}]=0. \tag{21}$$

Let us, on the basis of the symmetry of the problem, set $b=c=d=\lambda$. Then (21) goes over into the equation

$$a\bar{a}+3\bar{a}\lambda+3a\bar{\lambda}+9\lambda\bar{\lambda}=0, \tag{22}$$

whence $a=-3\lambda$. This solution allows us to represent the transition amplitude, which is a matrix in spin space, as follows:

$$M={}^1/_2[I_{(8)}-{}^1/_3(\sigma_1\times\sigma_2\times I+\sigma_1\times I\times\sigma_3+I\times\sigma_2\times\sigma_3)]G. \tag{23}$$

It follows from (A.1) with allowance for the initial conditions (5) that $\sigma=G\bar{G}$, as it should be. Further, using the obtained—for the inelastic transition—relation, $-\frac{1}{3}a=c=b=d$, between the amplitudes in the formulas (A.5)–(A.7) for the $0\rightarrow 1$ transition in the case when, initially, $q_{12}=-3$ and $q_{13}+q_{23}=0$, we find

$$4q_{13}'={}^{10}/_3q_{13}-{}^2/_3q_{23},$$
$$4q_{23}'=-{}^2/_3q_{13}+{}^{10}/_3q_{13},\quad q_{12}'=-3. \tag{24}$$

Setting $q_{13}=-q_{23}=1$ (the Eqs. (3) for $q_{13}$ and $q_{23}$ are valid only for elastic scattering), we have

$$q_{13}'=1,\quad q_{23}'=-1,$$

and, consequently, the conditions (7a): $q'_{13}=1$ and $q'_{23}+q'_{12}=-4$ for the singlet–triplet excitation are fulfilled. For the case when $-q_{13}=q_{23}=1$ we obtain the second of the conditions (7a), which is equivalent to the first because of the equivalence of the atomic electrons. Thus, the relation obtained for the amplitudes and the initial conditions $q_{12}=-3$, $\pm q_{13}=\mp q_{23}=1$ correspond to the $0\rightarrow 1$ transition.

Let us give the results for the vector polarizations of the atom in the triplet state and the scattered electron and the depolarization, $D_e$, of the electron of the continuous spectrum:

$$P_\alpha={}^1/_2(p_{1\alpha}'+p_{3\alpha}')={}^1/_2(p_{2\alpha}'+p_{3\alpha}')={}^1/_{36}(13-Q_{\alpha\alpha}{}^{12})p_{3\alpha},$$
$$p_{2\alpha}'=p_{1\alpha}'=-{}^2/_9(1+Q_{\alpha\alpha}{}^{12})p_{3\alpha},$$
$$D_3=p_{2\alpha}'/p_{3\alpha}=-{}^2/_9(1+Q_{\alpha\alpha}{}^{12}) \tag{25}$$

under the condition that

$$\sum_\alpha Q_{\alpha\alpha}^{12}=-3.$$

The depolarization of an electron in a purely exchange transition is a constant that does not depend on the energy and the scattering angle. Below the result obtained for $D_e$ is compared with the data obtained in the experimental investigation.[3]

## 4. NUMERICAL CALCULATIONS

### 1. The $e-\mathrm{He}(2^3S)$ elastic scattering

Let us find the polarization $p_3'$ arising in a beam of electrons elastically scattered by oriented metastable helium atoms $\mathrm{He}(2^3S)$.

With that end in view we have computed $p_3'$ from the formula (14) with the amplitudes $F$ and $G$ found by the method of partial phases. We used in the computation the quadruplet, $\delta_l^{(4)}$, and doublet, $\delta_l^{(2)}$, phases computed in[4] for the orbital angular momenta $l=0$, 1, and 2 and for energies $0.003\leqslant E\leqslant 10$ eV in the Hartree–Fock approximation with allowance for the distortion of the electron density of the atom by the electron of the continuous spectrum. As a sample, we present in the table (for $P=1$) the results obtained for the polarization and the differential cross section for one of the energy values ($E=0.5$ eV).

As the energy increases, the maximum value of $p_3'$ decreases (for example, for $E=10$ eV we have $p'_{3\,\max}=0.36$ at $x=-0.8$). This result is general and is connected with the predominance of the direct scattering over the exchange scattering when the collision energy is increased.

| $x=\cos\theta$ | $p_3'$ | $\sigma$ | $x=\cos\theta$ | $p_3'$ | $\sigma$ | $x=\cos\theta$ | $p_3'$ | $\sigma$ |
|---|---|---|---|---|---|---|---|---|
| 0.8 | −0.144 | 19.85 | 0.2 | 0.688 | 9.01 | −0.4 | −0.005 | 6.16 |
| 0.6 | 0.431 | 6.86 | 0.0 | 0.401 | 10.78 | −0.6 | 0.346 | 5.48 |
| 0.4 | 0.993 | 6.01 | −0.2 | 0.172 | 9.41 | −0.8 | 0.701 | 15.03 |

## 2. Depolarization in the singlet-triplet transition

In[3] the depolarization of an electron beam inelastically scattered as a result of the excitation of the $6^1S_0$–$6^3P_1$ line in the Hg atom was measured. Under the conditions of the experiment this electron beam was not energetically resolved from the electrons which excited the adjacent level $6^1P_1$ and which did not undergo depolarization in this transition. Consequently, the experimentally found depolarization is determined by the relation between the amplitudes of the direct and exchange scattering in the transitions ${}^1S_0-{}^1P_1$ and ${}^1S_0-{}^3P_1$ respectively. Therefore, with the theoretical value $D_e = -\frac{2}{9}(1+Q^{12}_{\alpha\alpha})$ can be compared only those $D_{exp}$ values that were determined near the $6^3P_1$-excitation threshold, where the exchange scattering predominates.

The experimental value $D_{exp} = -(0.1-0.2)$ for $E = 5-6$ eV. The theoretical value lies in the interval $-0.22 \leqslant D_e \leqslant 0$, since $-1 \leqslant Q^{12}_{\alpha\alpha} \leqslant 0$, and, consequently, we can speak of a qualitative agreement of the results. To the maximum depolarization $D_e = 0$ corresponds the maximum polarization ($P=\frac{1}{2}$) of the product atom, as it should be.

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## APPENDIX

A. The scattering cross section is

$$\sigma = \mathrm{Sp}\,\rho' = \Big[a\bar{a}+3(b\bar{b}+c\bar{c}+d\bar{d})+q_{12}(\bar{a}b+a\bar{b}+\bar{c}d+c\bar{d}-2b\bar{b}) +q_{13}(\bar{a}c+a\bar{c}+\bar{b}d+b\bar{d}-2c\bar{c})+q_{23}(\bar{a}d+a\bar{d}+\bar{b}c+b\bar{c}-2d\bar{d}) +i(b\bar{c}-\bar{b}c+b\bar{d}-\bar{b}d+\bar{c}d-c\bar{d})\sum R_{ijk}\varepsilon_{ijk}\Big]. \qquad (\mathrm{A.1})$$

Here $\varepsilon_{ijk}$, the unit third-rank antisymmetric tensor ($\mathrm{Sp}\,\sigma_i\sigma_j\sigma_k = 2i\varepsilon_{ijk}$), is summed over repeated indices. The bar denotes complex conjugation.

B. The vector-polarization components are

$$\sigma p_{1\alpha}' = \Big\{p_{1\alpha}(a\bar{a}-b\bar{b}-c\bar{c}+3d\bar{d})+p_{2\alpha}(\bar{a}b+a\bar{b}+\bar{c}d+c\bar{d}+2b\bar{b}) +p_{3\alpha}(\bar{a}c+a\bar{c}+\bar{b}d+b\bar{d}+2c\bar{c})+i\Big[(a\bar{b}-\bar{a}b+\bar{c}d-c\bar{d})\sum Q^{12}_{ss'}\varepsilon_{ss'\alpha} +(a\bar{c}-\bar{a}c+\bar{b}d-b\bar{d})\sum Q^{13}_{ss'}\varepsilon_{ss'\alpha}+(b\bar{c}-\bar{b}c-b\bar{d}-\bar{b}d+\bar{c}d-c\bar{d})\sum Q^{23}_{ss'}\varepsilon_{ss'\alpha}\Big] +(\bar{a}d+a\bar{d})\sum R_{ijk}\delta_{jk}\delta_{i\alpha}+(\bar{c}d+c\bar{d}-\bar{b}d-b\bar{d})\sum R_{ijk}\varepsilon_{sjk}\varepsilon_{si\alpha} +(\bar{b}c+b\bar{c})\sum R_{ijk}(\delta_{ij}\delta_{k\alpha}+\delta_{j\alpha}\delta_{ik}-\delta_{i\alpha}\delta_{jk})-d\bar{d}\sum R_{ijk}\varepsilon_{rjs}\varepsilon_{rks}\delta_{i\alpha}\Big\}, \qquad (\mathrm{A.2})$$

$$\sigma p_{2\alpha}' = \Big\{p_{1\alpha}(\bar{a}b+a\bar{b}+\bar{c}d+c\bar{d}+2b\bar{b})+p_{2\alpha}(\bar{a}a-b\bar{b}+3c\bar{c}-d\bar{d}) +p_{3\alpha}(\bar{a}d+a\bar{d}+\bar{b}c+b\bar{c}+2d\bar{d})+i\Big[(\bar{a}b-a\bar{b}+\bar{c}d-c\bar{d})\sum Q^{12}_{ss'}\varepsilon_{ss'\alpha} +(b\bar{c}-\bar{b}c+b\bar{d}-\bar{b}d+c\bar{d}-\bar{c}d)\sum Q^{13}_{ss'}\varepsilon_{ss'\alpha}+(c\bar{b}-\bar{c}b+a\bar{d}-\bar{a}d)\sum Q^{23}_{ss'}\varepsilon_{ss'\alpha}\Big] +(\bar{a}c+a\bar{c})\sum R_{ijk}\delta_{j\alpha}\delta_{ki}+(\bar{c}d+c\bar{d}-\bar{b}c-b\bar{c})\sum R_{ijk}\varepsilon_{rj\alpha}\varepsilon_{rik} +(\bar{b}d+b\bar{d})\sum R_{ijk}(\delta_{ij}\delta_{k\alpha}+\delta_{i\alpha}\delta_{jk}-\delta_{j\alpha}\delta_{ik})-c\bar{c}\sum R_{ijk}\varepsilon_{rjs}\varepsilon_{rks}\delta_{j\alpha}\Big\}, \qquad (\mathrm{A.3})$$

$$\sigma p_{3\alpha}' = \Big\{p_{1\alpha}(\bar{a}c+a\bar{c}+\bar{b}d+b\bar{d}+2c\bar{c})+p_{2\alpha}(\bar{a}d+a\bar{d}+\bar{b}c+b\bar{c}+2d\bar{d}) +p_{3\alpha}(a\bar{a}+3b\bar{b}-c\bar{c}-d\bar{d})+i\Big[(c\bar{b}-\bar{c}b+b\bar{d}-\bar{b}d+c\bar{d}-\bar{c}d)\sum Q^{12}_{ss'}\varepsilon_{ss'\alpha} +(\bar{a}c-a\bar{c}+\bar{b}d-b\bar{d})\sum Q^{13}_{ss'}\varepsilon_{ss'\alpha}+(c\bar{b}-\bar{c}b+\bar{a}d-a\bar{d})\sum Q^{23}_{ss'}\varepsilon_{ss'\alpha}\Big] +(\bar{a}b+a\bar{b})\sum R_{ijk}\delta_{k\alpha}\delta_{ij}+(\bar{b}c+b\bar{c}-\bar{b}d-b\bar{d})\sum R_{ijk}\varepsilon_{rk\alpha}\varepsilon_{rji} +(\bar{c}d+c\bar{d})\sum R_{ijk}(\delta_{ik}\delta_{j\alpha}+\delta_{kj}\delta_{i\alpha}-\delta_{k\alpha}\delta_{ji})-b\bar{b}\sum R_{ijk}\varepsilon_{rjs}\varepsilon_{ris}\delta_{k\alpha}\Big\}. \qquad (\mathrm{A.4})$$

C. The pair-correlation components. Since the components $Q^{(ik)\prime}_{\alpha\alpha'}$ enter into the final results only in the form of the sums

$$\sum Q^{(ik)\prime}_{\alpha\alpha} = q_{ik}',$$

let us give the formulas for these quantities:

1) $$\sigma q_{12}' = 3(\bar{a}b+a\bar{b}+\bar{c}d+c\bar{d})-6b\bar{b}+q_{12}[a\bar{a}+7b\bar{b}-c\bar{c}-d\bar{d} -2(\bar{a}b+a\bar{b}-\bar{c}d-c\bar{d})]+q_{13}[\bar{a}d+a\bar{d}+3\bar{b}c+3b\bar{c}-2(\bar{b}d+b\bar{d}+\bar{c}d+c\bar{d}-d\bar{d})] +q_{23}[\bar{a}c+a\bar{c}+3\bar{b}d+3b\bar{d}-2(\bar{b}c+b\bar{c}+\bar{c}d+c\bar{d}-c\bar{c})] +i\Big[(\bar{c}d-c\bar{d})\sum R_{ijk}\varepsilon_{ri\alpha}\varepsilon_{js\alpha}\varepsilon_{rks}+(\bar{a}c-a\bar{c}+\bar{a}d-a\bar{d}+2b\bar{c}-2\bar{b}c +2\bar{b}d-2b\bar{d})\sum R_{ijk}\varepsilon_{ijk}\Big]. \qquad (\mathrm{A.5})$$

2) $$\sigma q_{13}' = 3(\bar{a}c+a\bar{c}+\bar{b}d+b\bar{d})-6c\bar{c}+q_{12}[\bar{a}d+a\bar{d}+3\bar{b}c+3b\bar{c}-2(\bar{b}d+b\bar{d} +\bar{c}d+c\bar{d}-d\bar{d})]+q_{13}[a\bar{a}+7c\bar{c}-b\bar{b}-d\bar{d}-2(\bar{a}c+a\bar{c}-\bar{b}d-b\bar{d})]+q_{23}[\bar{a}b+a\bar{b} +3\bar{c}d+3c\bar{d}-2(\bar{b}c+b\bar{c}+\bar{b}d+b\bar{d}-b\bar{b})]+i\Big[(b\bar{d}-\bar{b}d)\sum R_{ijk}\varepsilon_{ri\alpha}\varepsilon_{js\alpha}\varepsilon_{rks} +(a\bar{b}-\bar{a}b+\bar{a}d-a\bar{d}+2b\bar{c}-2\bar{b}c+2c\bar{d}-2\bar{c}d)\sum R_{ijk}\varepsilon_{ijk}\Big]. \qquad (\mathrm{A.6})$$

3) $$\sigma q_{23}' = 3(\bar{a}d+a\bar{d}+\bar{c}b+c\bar{b})-6d\bar{d}+q_{12}[\bar{a}c+a\bar{c}+3\bar{b}d+3b\bar{d}-2(\bar{b}c+b\bar{c} +\bar{c}d+c\bar{d}-c\bar{c})]+q_{13}[\bar{a}b+a\bar{b}+3\bar{c}d+3c\bar{d}-2(\bar{b}c+b\bar{c}+\bar{b}d+b\bar{d}-b\bar{b})] +q_{23}[a\bar{a}+7d\bar{d}-b\bar{b}-c\bar{c}-2(a\bar{d}+\bar{a}d-\bar{b}c-b\bar{c})] +i\Big[(\bar{b}c-b\bar{c})\sum R_{ijk}\varepsilon_{ri\alpha}\varepsilon_{js\alpha}\varepsilon_{rks}+(a\bar{c}-\bar{a}c+a\bar{b}-\bar{a}b+2c\bar{d}-2\bar{c}d +2\bar{b}d-2b\bar{d})\sum R_{ijk}\varepsilon_{ijk}\Big]. \qquad (\mathrm{A.7})$$

The values of $R'_{\alpha\alpha'\alpha''}$ are not given here because of the unwieldiness of the result in the general form. The values of $R'_{\alpha\alpha'\alpha''}$ in one particular case of elastic scattering are given in the main text (the formulas (18) and (20)).

*Note added in proof.* (July 6, 1976). The operator $M$ (formula (23)) describes singlet-triplet transition in a three-particle system, resulting from particle exchange of the substitution type. In the case of three identical particles the exchange excitation with change of target spin does not reduce fully to replacement of one particle by another. The comparison of the numerical data (formula (25)) with the experimental results on electron depolarization due to excitation of the triplet level of the mercury atom must therefore be admitted to be inconsistent. A complete analysis of the situation involving transitions with change of the spin of the atom will be given for the case of identical particles in a separate paper. The authors thank G. F. Drukarev for calling our attention to this circumstance.

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