of the packet. Let us denote by l_1 the characteristic attenuation length. The physical interpretation of the second term in the brackets in the expression for μ_n is also readily understood if we recall that, when $g_{\alpha\beta} = ig''_{\alpha\beta}$, volume losses become unimportant and there are only losses due to radiation from the boundaries. We shall denote by l_2 the corresponding characteristic length. The above formulas then take the form

$$\mu_0=1/l_0,\quad \mu_v=1/l_0-1/l_1,\quad \mu_n=1/l_0-1/l_1-1/l_2.$$

The two lengths l_1 and l_2 can readily be estimated by introducing one further characteristic length. This length, l will be referred to as the transverse diffusion length and will be defined as the length over which the radius of the beam, given by (9), increases by a factor of $\sqrt{2}$. Assuming that $a_0 \approx 2l$, we obtain

$$l_1/l \approx |g_{01}''/g_{01}'| \gg 1,\quad l_2/l_1 \approx |g_{01}'|l/\sin\theta \gg 1.$$

We note that $l_0 \sim 1$ cm[11] and, for the parameter values used above, $l = 20$ cm.

We may therefore conclude that, for the above numerical values of the parameters of our problem, the plane-wave approximation for the incident waves gives a correct representation of the efficacy of the Borrmann effect as a means of reducing the absorption coefficient provided the width of the beam (crystal) is $a_0 \gg 10^{-2}$ cm, whereas, for $a_0 \leq 10^{-3}$ cm, the aperture effect and the effect connected with radiation from boundaries begin to play an appreciable role.

In general, the plane-wave approximation begins to lose its validity when the characteristic length l approaches l_0.

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Translated by S. Chomet

Polarization effects in a strong field in two-mode lasing

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(Submitted November 12, 1975)
Zh. Eksp. Teor. Fiz. 70, 1720–1728 (May 1976)

The possibility is investigated of two-mode lasing at a large relative excitation of the active medium for an atomic transition with level moments 0 or 1. It is shown that in the case of parallel polarizations of the generated modes a two-mode regime takes place, providing the intermode distance ω_{21} exceeds a certain critical value ω_{21cr}; in the case of orthogonal polarizations the regime sets in at arbitrary intermode distances. The dependence of the size of the two-mode generation region on various parameters is investigated for parallel and orthogonal mode polarizations.

PACS numbers: 42.50.+q

Many recent studies, both experimental and theoretical, have been devoted to problems of nonlinear spectroscopy. A recent book by Letokhov and Chebotaev[1] contains an extensive bibliography on this subject. One of the methods of investigating spectroscopic characteristics is to sound a resonant medium saturated by a strong field with a weak probing wave. We note that the results of the probing depend significantly on the mutual direction of the polarizations of the weak and strong fields, as was first pointed out by Alekseev.[2] The latter considered, for an atomic transition with total angular momentum change $1 \to 0$, the waveform of the weak signal as it passes through a glass laser operating in the single-mode generation regime. The analysis took into account the depolarizing atomic collisions, when the strong field in the laser and the field of the transmitted signal had parallel and orthogonal polarizations.

One more method of spectroscopy of resonant media is to study the two-mode generation regime in them. The principles of theoretical analysis of the two-mode regime were developed in a paper by Lamb.[3] No account was taken of the degeneracy of the resonance levels, and the case was considered when the mode intensities could be regarded as weak (small relative excitation of the medium). The polarization aspect of the two-mode generation method is also of interest. Thus, for example, in[4] they proposed an optical quantum frequency standard based on the two-mode generation regime of a linear gas laser with orthogonal polarizations of the generated modes. The two-mode lasing regime is used for practical purposes also by traveling-wave ring lasers. The theory of two-mode generation, for both linear and ring lasers, has been developed in detail only for the case of weak fields. We note that experimental investigations[4] are usually carried out at values of the relative excitation of the medium η such that the mode intensity cannot be regarded as weak in a position that is symmetrical with respect to the center of the gain line.

In the present article we investigate theoretically the possibility of two-mode degeneration at arbitrary values of the relative excitation of the medium η for an atomic transition with change of total angular momentum $1\to 0$, with account taken of the depolarizing atomic collisions, when the mode polarizations are parallel or orthogonal. The possibility of an analytic solution of this problem is based on the following circumstances. If we denote by ω_1 and ω_2 the frequencies of the generated modes, and by $\omega_{21}=\omega_2-\omega_1$ the intermode distance, then the position of each of the modes relative to the central amplification frequency ω_0 is written in the form

$$\omega_{10}=\omega_1-\omega_0=-{}^1/{}_2\omega_{21}+x,\quad \omega_{20}={}^1/{}_2\omega_{21}+x, \tag{1}$$

where the quantity x characterizes the deviation of the modes from the symmetrical position. The increase of $|x|$ at a fixed value of ω_{21} means that one of these modes approaches the center of the gain line, and the other moves away from it. Therefore, if at $x=0$ the intensities of both modes were large, then, starting with a certain value of x, the intensity of one of the modes, which we shall for concreteness call the second mode, becomes quite weak, and vanishes at $x=x_0$, i.e., the generation regime becomes single-mode. Thus, the quantity x_0 characterizes the size of the region of two-mode generation in the vicinity of the symmetrical mode position: $\omega_{20}=-\omega_{10}=\omega_{21}/2$. To find the value of x_0 it is necessary to calculate the polarization P of the active medium in the presence in it of two standing electromagnetic waves: strong and weak. The calculation procedure is analogous to that developed in[2]. However, in contrast to[2], we have taken into account the influence of the spatial modulation of the excess population, [5,6] which is essential for two-mode generation.

As a result of our investigations, we observed a significant dependence of the region of two-mode generation on the mutual polarization of the modes in the considered atomic transition $1\to 0$. Thus, in the case of orthogonal polarizations of the modes, two-mode generation is possible at any intermode distance ω_{21} in that frequency region in which the gain of the medium exceeds the resonator loss.[1)] In the case of parallel mode polarizations, the two-mode regime takes place if the intermode distance exceeds a certain critical value $\omega_{21\text{cr}}$. The physical cause of the weaker competition of the orthogonal-polarization modes is their interaction with different magnetic sublevels of the resonant transition. The discovered possibility of existence of two-mode generation in the case of orthogonal polarization of the modes in the entire frequency region in which the gain exceeds the losses constitutes a new phenomenon which is not only of theoretical but also of practical interest.

The aforementioned polarization singularities of two-mode generation take place at arbitrary relative excitations of the medium. The appreciable difference between the large relative excitations of the medium η and the small ones manifests itself in the appearance of the dependence of the critical frequency $\omega_{21\text{cr}}$ on η in the case of parallel polarizations of the generated modes.

FUNDAMENTAL EQUATIONS AND RELATIONS

The field existing in the laser in the case of the two-mode lasing regime will be written in the form

$$\mathbf{E}=\mathbf{E}^{(1)}\sin k_1y\cos\omega_1t+\mathbf{E}^{(2)}\sin k_2y\cos\omega_2t, \tag{2}$$

where $\mathbf{E}^{(1)}$ and $\mathbf{E}^{(2)}$ are the amplitudes of the strong and weak fields. We denote by $\rho_{mm'}$ the density matrix of the atoms excited to the level $b(j_b=1)$, and by ρ_a the density matrix of the atoms excited to the level $a(j_a=0)$, while $\rho_{0m}^{(ab)}$ denotes the density matrix describing the transitions between the considered working levels. We expand the matrices $\rho_{mm'}$ and $\rho_{0m}^{(ab)}$ in the irreducible tensor operator[7,8]:

$$\rho_{mm'}=(-1)^{1-m}\frac{1}{\sqrt{3}}\sum_{\varkappa,q}(2\varkappa+1)\begin{pmatrix}1 & 1 & \varkappa\\ m & -m' & q\end{pmatrix}f_q^{(\varkappa)}, \tag{3}$$

$$\rho_{0m}^{(ab)}=\sum_{\varkappa,q}(2\varkappa+1)\begin{pmatrix}0 & 1 & \varkappa\\ 0 & -m & q\end{pmatrix}\psi_q^{(\varkappa)}=(-1)^{1+m}\sqrt{3}\,\psi_m^{(1)}, \tag{4}$$

where the $3j$ symbols are given by

$$\begin{pmatrix}1 & 1 & \varkappa\\ m & m' & q\end{pmatrix}.$$

Taking into account the depolarizing atomic collisions we obtain the following equations from the initial system of equations for the matrices $\rho_{mm'}$, ρ_a, and $\rho_{0m}^{(ab)}$ [9] for the case of a homogeneously broadened spectral line

$$\left(\frac{\partial}{\partial t}+\gamma_b^{(\varkappa)}\right)f_q^{(\varkappa)}=3\gamma_b^{(0)}N_b\delta_{\varkappa 0}\delta_{q0}$$
$$+\frac{i}{\hbar}\sqrt{3}\sum_{q_1q'}E_{-q_1}\begin{pmatrix}1 & 1 & \varkappa\\ q_1 & -q' & q\end{pmatrix}\{(-1)^{q'}d\psi_{q'}^{(1)}+(-1)^{1+\varkappa}d^*\psi_{-q'}^{(1)*}\}, \tag{5}$$

$$\left(\frac{\partial}{\partial t}+\gamma_a\right)\rho_a=\gamma_aN_a+\frac{i}{\hbar}\sum_q E_{-q}\{d\psi_q^{(1)}+(-1)^{1+q}d^*\psi_{-q}^{(1)*}\}, \tag{6}$$

$$\left(\frac{\partial}{\partial t}+\gamma-i\omega_0\right)\psi_q^{(1)}=\frac{i}{\hbar}(-1)^{1+q}\frac{d^*}{3}\left\{\frac{1}{\sqrt{3}}\sum_{\varkappa,q_1,q}(-1)^{1+q_1}E_{-q_1}\right.$$
$$\left.\times(2\varkappa+1)\begin{pmatrix}1 & 1 & \varkappa\\ -q_1 & -q & q'\end{pmatrix}f_{q'}^{(\varkappa)}-E_q\rho_a\right\}. \tag{7}$$

The first terms in the right-hand sides of (5) and (6) describe here the pumping of the exciting atoms to the working levels, E_q is the circular component of the vector E, and d is the reduced dipole-moment matrix element. The quantities $\gamma_b^{(\varkappa)}$ are defined by the expressions[9]

$$\gamma_b^{(\varkappa)}=\gamma_b^{(0)}+\Gamma_b^{(\varkappa)}, \quad \varkappa=0,1,2, \tag{8}$$

where $\gamma_b^{(0)}$ characterizes the decay of the population of the level b, $\Gamma_b^{(\varkappa)}$ characterizes the relaxation of the corresponding components of the density matrix under the influence of the depolarizing collisions, with $\Gamma_b^{(0)}=0$ and $\Gamma_b^{(1)}/\Gamma_b^{(2)}\cong 1.1$. For the level a, which is nondegenerate, we have $\gamma_a=\gamma_a^{(0)}$. The half-width of the spectral line γ with allowance for the atomic collisions is defined by the expression

$$\gamma={}^1/_2(\gamma_a^{(0)}+\gamma_b^{(0)})+\Gamma, \tag{9}$$

where Γ characterizes the broadening of the spectral line due to the atomic collisions. Separating explicitly the dependence on the gas pressure p, we write down $\Gamma_b^{(\varkappa)}$ and Γ in the form

$$\Gamma_b^{(\varkappa)}=A_b^{(\varkappa)}p, \quad \Gamma=Ap, \tag{10}$$

where A and $A_b^{(\varkappa)}$ are constants.

We seek the solution of the system (5)–(7) in the form of a series in powers of the amplitude $E^{(2)}$ of the weak field:

$$\begin{aligned} f_q^{(\varkappa)}(y,t)&=f_{0q}^{(\varkappa)}(y)+L_q^{(\varkappa)}(y)e^{-i\omega_{21}t}+M_q^{(\varkappa)}(y)e^{i\omega_{21}t},\\ \rho_a(y,t)&=\rho_a^{(0)}(y)+\rho_a^{(1)}(y)e^{-i\omega_{21}t}+\rho_a^{(1)*}(y)e^{i\omega_{21}t},\\ \psi_q^{(1)}(y,t)&=\psi_{0q}^{(1)}(y)e^{i\omega_1t}+\psi_{1q}^{(1)}(y)e^{i\omega_2t}+\psi_{kq}^{(1)}(y)e^{2i\omega_1t-i\omega_2t}. \end{aligned} \tag{11}$$

The terms $f_{0q}^{(\varkappa)}$, $\rho_a^{(0)}$, $\psi_{0q}^{(1)}$ constitute the solution of the system of equations at $E^{(2)}=0$, while the remaining terms are linear in the weak-field amplitude $E^{(2)}$. The term containing $\psi_{1q}^{(1)}$ describes the polarization of the medium at the frequency of the weak mode, and that containing $\psi_{kq}^{(1)}$ depolarization at the frequency of the combination tone. Although there is no generation at the tone frequency, the equations for $\psi_{1q}^{(1)}$ and $\psi_{kq}^{(1)}$ are coupled in the presence of a strong field. Substituting expressions (11) in (5)–(7) and solving them, we obtain the following result for the polarization vector of the medium when it is saturated only by the strong field:

$$\mathbf{P}_0=i\mathbf{E}^{(1)}n_-\frac{|d|^2}{6\hbar}(\gamma-i\omega_{10})(\gamma^2I)^{-1}\{1-[1+2\gamma^2I(\gamma^2+\omega_{10}^2)^{-1}]^{-1/2}\}, \tag{12}$$

where

$$I=|d|^2(E^{(1)})^2/3(2\hbar)^2\gamma\gamma_{11} \tag{13}$$

is the dimensionless intensity of the strong field, $n_-=(N_b-N_a)l/L$, l is the length of the tube with the active medium, and L is the resonator length. The quantity γ_{11} is a combination of the widths $\gamma_b^{(\varkappa)}$ and $\gamma_a^{(\varkappa)}$, with coefficients that depend on the angular momenta of the levels. In this case

$$\gamma_{11}^{-1}=\sum_{\varkappa=0}^{2}[a_b^{(\varkappa)}(\gamma_b^{(\varkappa)})^{-1}+a_a^{(\varkappa)}(\gamma_a^{(\varkappa)})^{-1}], \tag{14}$$

where the coefficients $a_b^{(\varkappa)}$ and $a_a^{(\varkappa)}$ are given by

$$a_b^{(0)}={}^1/_2a_b^{(2)}={}^1/_3, \quad a_b^{(1)}=0, \quad a_a^{(0)}=1, \quad a_a^{(1)}=a_a^{(2)}=0.$$

To investigate the possibility of amplifying the weak field in the presence of the strong one it is necessary to have the value of the intensity of the strong field in the stationary single-mode regime. Taking the imaginary part of (12) and equating it to the losses which are taken into account, as usual,[3] in terms of the resonator Q, we obtain the following dependence of the mode intensity on the detuning ω_{10}:

$$I=\frac{1}{4}\left\{4\eta-1-\left(\frac{\omega_{10}}{\gamma}\right)^2-\left[1+\left(\frac{\omega_{10}}{\gamma}\right)^2\right]^{1/2}\left[8\eta+1+\left(\frac{\omega_{10}}{\gamma}\right)^2\right]^{1/2}\right\}, \tag{15}$$

where $\eta=n_-/n_{-\mathrm{thr}}$ is a parameter characterizing the relative excitation of the medium, while $n_{-\mathrm{thr}}$ is the threshold value of the excess population of the active medium.[3]

The polarization of the medium at the weak-field frequency is expressed in terms of the quantity $\psi_{1q}^{(1)}(y)$, which was obtained from the solution of the system (5)–(7) after calculating the quantities $f_{0q}^{(\varkappa)}$, $\rho_a^{(0)}$, $\psi_{0q}^{(1)}$. The vector $\mathbf{P}_1$ of the polarization of the medium at the frequency of the weak field, for both parallel and orthogonal polarizations of the field in question, is given by

$$\mathbf{P}_1=\frac{2}{L}\int_{y_0}^{y_0+l}dy\,\mathbf{D}_1(y)\sin k_2y, \tag{16}$$

where y_0 is the distance from the tube with the active medium to the nearest mirror. For $\mathbf{D}_1(y)$ we obtain the expression

$$\mathbf{D}_1(y)=i\mathbf{E}^{(2)}(N_b-N_a)\frac{|d|^2}{6\hbar}\sin k_2yW(\sin^2k_1y), \tag{17}$$

where

$$W(x)=(T+Bx+Cx^2)(F+Gx)^{-1}(S+Mx+Rx^2)^{-1}. \tag{18}$$

The coefficients T, B, C, F, G, S, M, and R in (18) depend on the frequencies and polarizations of the generated modes, on the spectroscopic characteristics of the working levels, and on the saturation parameter I,

$$\begin{gathered} F=\gamma^2+\omega_{10}^2, \; G=2\gamma^2I, \; T=F(\gamma+i\omega_{20}-2i\omega_{10}),\\ S=\frac{T}{F}(\gamma+i\omega_{20}), \quad B=G\left[\frac{\gamma_{11}}{\gamma_{12}}\frac{T}{F}-if_1(\gamma+i\omega_{10})\frac{\omega_{21}}{2\gamma}\right],\\ C=\gamma^2I^2\left[2\gamma\frac{\gamma_{11}}{\gamma_{12}}f_1-(\gamma-i\omega_{10})(f_1^2-f_2^2)\right],\\ M=2\gamma I(\gamma+i\omega_{21})f_1, \; R=\gamma^2I^2(f_1^2-f_2^2),\\ f_2=\gamma_{11}\sum_{\varkappa=0}^{2}(-1)^\varkappa[c_b^{(\varkappa)}(\gamma_b^{(\varkappa)}+i\omega_{21})^{-1}+c_a^\varkappa(\gamma_a^{(\varkappa)}+i\omega_{21})^{-1}],\\ \gamma_{12}^{-1}=\sum_{\varkappa=0}^{2}[(a_b^{(\varkappa)}-b_b^{(\varkappa)})(\gamma_b^\varkappa)^{-1}+(a_a^\varkappa-b_a^{(\varkappa)})(\gamma_a^{(\varkappa)})^{-1}]. \end{gathered} \tag{19}$$

The expression for f_1 is obtained from f_2 by means of the substitution $(-1)^\varkappa\to 1$. The coefficients $c_b^{(\varkappa)}$, $c_a^{(\varkappa)}$, $b_b^{(\varkappa)}$, $b_a^{(\varkappa)}$ depend on the mutual orientation of the field polarizations. For orthogonal polarizations they are equal to

$$c_b^{(0)}=0, \quad c_b^{(1)}=c_b^{(2)}={}^1/_2, \quad c_a^{(\varkappa)}=0, \quad \varkappa=0,1,2;$$

$$b_b^{(0)}=-b_b^{(2)}={}^1/_3, \quad b_b^{(1)}=0, \quad b_a^{(\varkappa)}=a_a^{(\varkappa)};$$

for parallel polarizations they are given by the equations

$$c_a^{(\times)}=b_a^{(\times)}=a_a^{(\times)}, \quad c_b^{(\times)}=b_b^{(\times)}=a_b^{(\times)}.$$

It is easily seen that in the case of parallel polarizations we have $f_1=f_2$ and $1/\gamma_{12}=0$. This causes the coefficients C and R, which enter in (18) to vanish, greatly simplifying the evaluation of the integral in (16).

Amplification of the weak wave will take place in the presence of the strong wave if the imaginary part of the polarization $\mathbf{P}_1$ exceeds the resonator loss. Substituting in the coefficients (19) the values of ω_{10} and ω_{20} expressed in terms of the intermode distance ω_{21} and the deviation of the modes from the symmetrical position x in accordance with (1), and introducing the function

$$\Phi(\omega_{21}, x)=\frac{8\pi Q}{E^{(2)}}\operatorname{Im}P_1-1, \tag{20}$$

we obtain the condition for the amplification of the second mode in the form

$$\Phi(\omega_{21}, x)>0. \tag{21}$$

The critical intermode distance $\omega_{21\mathrm{cr}}$ is determined by the fact that the condition (21) is violated in an infinitesimally small vicinity of the symmetrical position, i.e., as $x\to 0$:

$$\Phi(\omega_{21\mathrm{cr}}, 0)=0. \tag{22}$$

At a given $\omega_{21}>\omega_{21\mathrm{cr}}$, the boundary of the region of the two-mode regime is determined by the equation

$$\Phi(\omega_{21}, x_0)=0. \tag{23}$$

The strong-mode intensity I entering in the function Φ is determined by expression (15), in which ω_{10} must be expressed in terms of ω_{21} and x. The function Φ (ω_{21}, x) was calculated with a computer after transforming it into

$$\Phi=\frac{2\eta\gamma}{\pi}\int_0^{\pi/2}dz\left(1-\frac{n_2}{n_-}\cos 2z\right)\operatorname{Re}W(\sin^2 z)-1, \tag{24}$$

where

$$n_2=n_-\left(\frac{\omega_{21}}{c}l\right)^{-1}\sin\left(\frac{\omega_{21}}{c}l\right)\cos\left[\frac{\omega_{21}}{c}(2y_0+l)\right] \tag{25}$$

has the meaning of the spatial harmonic of the population in the case of weak fields. [3]

In the case of parallel polarizations of the modes, the integral entering the function Φ can be calculated analytically. The results of this calculation were compared with the results of the numerical calculation of the integral, and this served as a test of the calculation accuracy, inasmuch as the transition from parallel polarizations to orthogonal ones is effected only by interchanging the coefficients $a_{b,a}^{(\times)}$, $b_{b,a}^{(\times)}$ and $c_{b,a}^{(\times)}$. For the numerical calculations we used the following values for the problem parameters: $\gamma_b^{(0)}=8$ MHz, $\gamma_a^{(0)}=3$ MHz, $A_b^{(2)}=2$ MHz/mm Hg, $A=30$ MHz/mm Hg, $l=20$ cm, and $y_0=25$ cm. The calculation was performed for several values of the pressure p. The results presented below pertain to the case $p=2$ mm Hg.

DISCUSSION OF RESULTS

In the case of weak fields, expanding expression (24) in terms of the parameter $I\ll 1$, we obtain for the function Φ the result

$$\Phi\infty(\alpha_2-\theta_{21}I), \tag{26}$$

where α_2 is the linear gain of the second mode with allowance for the losses, θ_{21} is the coefficient of the so-called crossing saturation of the medium, [3] which takes into account the influence of the field with frequency ω_1 on the field with frequency ω_2. Substituting in (26) in place of I the value of the mode intensity in the single-mode stationary lasing regime [3] $I=\alpha_1/\beta_1$, we obtain the following expression for the critical intermode distance $\omega_{21\mathrm{cr}}$ in the case of a symmetrical mode position:

$$\beta_1=\theta_{21}. \tag{27}$$

In the case when the modes deviate from the symmetrical position, at a fixed intermode distance $\omega_{21}>\omega_{21\mathrm{cr}}$, the two-mode regime is realized if the magnitude of this detuning x is smaller than x_0, where x_0 is determined from the equation

$$\alpha_2\beta_1-\theta_{21}\alpha_1=0. \tag{28}$$

For the considered atomic transition $j_b=1\to j_a=0$, the coefficients α_1, β_1, and θ_{21} take the form

$$\alpha_1=[\gamma^2(\eta-1)-\omega_{10}^2](\gamma^2+\omega_{10}^2)^{-1}, \quad \beta_1=6\gamma^2 n_-\gamma_{11}^{-1}(\omega_{10}^2+\gamma^2)^{-2},$$

$$\theta_{21}=n_-\left(2+\frac{n_2}{n}\right)\gamma_{11}^{-1}\Big\{2\gamma^2\left(1-\frac{\gamma_{11}}{\gamma_{12}}\right)(\omega_{10}^2+\gamma^2)^{-1}(\omega_{20}^2+\gamma^2)^{-1}$$
$$+\operatorname{Re}[f_1(\gamma+i\omega_{20})^{-1}[(\gamma+i\omega_{20})^{-1}+(\gamma-i\omega_{10})^{-1}]]\Big\},$$

and the coefficient α_2 is obtained from α_1 by replacing ω_{10} with ω_{20}. Investigations of Eq. (27) for the considered transition have shown that in the case of parallel mode polarizations it has a solution $\omega_{21\mathrm{cr}}$, which lies in the physical region of the values of the intermode distance $0<\omega_{21\mathrm{cr}}\leqslant\omega_{21\max}$. The value of $\omega_{21\max}$ is determined by the size of that frequency interval in which the gain exceeds the loss, i.e., $\alpha_1>0$. For orthogonal mode polarizations, Eq. (27) has no solution in the region $0<\omega_{21\mathrm{cr}}\leqslant\omega_{21\max}$. This means that for orthogonal polarizations two-mode generation is possible at any intermode distance in the frequency region where the gain of the medium exceeds the resonator loss. The conclusions based on the analysis of the equations at $I\ll 1$ are formally valid when the relative excitation of the medium η differs little from unity: $\eta-1\ll 1$.

In experimental investigations, the parameter η can reach a value of two or more. Therefore the equations (22) and (23) derived in the preceding section, with the function Φ defined by (24), enable us to assess the changes in the characteristics of the two-mode regime in the case when I cannot be regarded as a small quantity. An analysis of (22) shows that in the case of parallel mode polarizations the two-mode generation is pos-

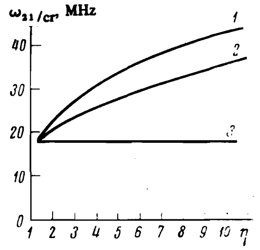

FIG. 1. Dependence of the critical frequency ω_{21cr} on the relative excitation of the medium η for the case of parallel mode polarizations.

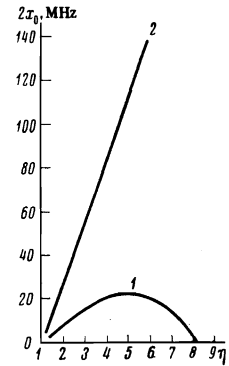

FIG. 3. Dependence of the region $2x_0$ of two-mode generation on η at a fixed intermode distance ω_{21} for parallel and orthogonal mode polarizations.

sible if $\omega_{21} > \omega_{21cr}$, and in the case of orthogonal polarizations it is possible at any intermode distance in that frequency region where the gain exceeds the losses. Thus, the main difference in the competition of the parallel and orthogonal polarization modes is retained in the case of strong fields. The qualitatively new result of a consistent allowance for the strong field is the dependence of ω_{21cr} on η for parallel mode polarizations, since it follows from Eq. (27), which is valid for $I \ll 1$, that ω_{21cr} does not depend on η.

Figure 1 shows a plot of ω_{21cr} against η, obtained by solving Eq. (22) (curve 1) and by solving (27) (line 3). It is seen that ω_{21cr} increases monotonically with increasing η, and the rate of increase decreases with increasing η. Curve 2 of Fig. 1 is obtained from the solution of Eq. (22) in the approximation in which the influence of the combination tone was completely neglected in the calculation of the polarization of the medium, i.e., it was assumed that $\psi_{kq}^{(1)}(y) \equiv 0$. A comparison of curves 1 and 2 shows that allowance for the polarization of the medium at the combination-tone frequency does not change the calculation results qualitatively.

Figure 2 shows the dependence of the region of the two-mode generation $2x_0$ on the intermode distance ω_{21} at a fixed value of η. Curves 1 and 3 were obtained by solving Eq. (23) and correspond to the case of parallel polarizations of the modes at $\eta = 2$ and $\eta = 4$, respectively. Curves 2 and 4 were obtained by solving Eq. (23) and pertain to the case of orthogonal polarizations of the modes at the same values of η. A comparison of these curves shows that the region of two-mode generation is always larger for orthogonal mode polarization

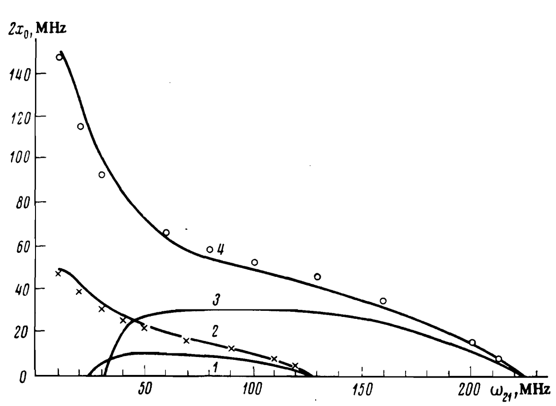

FIG. 2. Dependence of the integral of the two-mode generation $2x_0$ on the intermode distance ω_{21} at fixed η for the case of parallel and orthogonal mode polarizations.

than for parallel polarizations. This fact, as well as the absence of a critical intermode distance, is connected with the fact that in the case of orthogonal polarizations the mutual influence of the modes via the magnetic sublevels of the excited atoms becomes weaker. The crosses and the circles in Fig. 2 show the results of the calculations for $\eta = 2$ and $\eta = 4$, respectively, obtained from the solution of (28) for orthogonal mode polarizations. It is seen that although these results are formally applicable at $\eta - 1 \ll 1$, they can be used also at a much larger value of η. The difference between the behavior of curves 1(3) and 2(4) at small ω_{21} is connected precisely with the presence of ω_{21cr} for parallel polarizations of the modes.

The dependence of ω_{21cr} on η leads to a different behavior of the region of two-mode generation $2x_0$ as a function of η for fixed values of ω_{21} in the case of parallel and orthogonal mode polarizations. Fig. 3 shows this dependence for $\omega_{21} = 40$ MHz. Inasmuch as $\omega_{21cr} = 40$ MHz at $\eta \cong 8$, at this value of η the region of two-mode generation, which corresponds to parallel polarization, vanishes (curve 1), whereas the analogous quantity for orthogonal mode polarizations (curve 2) increases monotonically.

The authors are indebted to E. D. Protsenko and S. A. Gonchukov for a useful discussion of the results.

[1)]As $\omega_{21} \to 0$ it is necessary to take into account the possibility of mode locking, when the lasing is at one frequency rather than at two neighboring frequencies.

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Translated by J. G. Adashko