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Pion interaction in nuclear matter and π condensation

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A number of topics is considered related to the problem of π condensation in nuclear matter ($N = Z$) and in neutron stars ($N \gg Z$). A consistent multiparticle approach is developed to describe exact excitations of a medium with quantum numbers characterizing π mesons—pion quasiparticles. A method is given for calculating the effective Lagrangian of a pion field, the nonlinear terms of which are interpreted as an interaction between pion quasiparticles. An exactly soluble model for the $\pi^+\pi^-$ condensation in a neutron medium is studied, which enables us to calculate by numerical methods the energy of the system in the presence of a π condensate of arbitrary amplitude. In order to illustrate the computation methods the high frequency approximation $\omega \gg k v_F$ is considered within the framework of which we succeed in calculating analytically the critical parameters and the energy of the $\pi^+\pi^-$ condensate. It is shown that the instability discovered by Sawyer and Scalapino [R. F. Sawyer, Phys. Rev. Lett. 29, 382 (1972); D. I. Scalapino, Phys. Rev. Lett. 29, 386 (1972)] is of the same nature as the $\pi^+\pi^-$-instability. The problem of the spatial and isotopic structure of the π condensate in the system with $N = Z$ is investigated. A broad class of solutions is investigated by the Thomas-Fermi method and it turns out that the one-dimensional isotopically asymmetric configurations of the condensate field have the lowest energy. The amplitude of the modulations of the particle density and of the spin density of nucleons in the condensate field is calculated.

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I. FORMULATION OF THE PROBLEM

1. The physical picture

The possibility of a phase transition with the formation of a π-meson condensate was investigated for the first time in[1]. The Klein–Gordon–Fock (KGF) equation was solved in external fields of different type: scalar, electric, in a field produced by nucleons (nuclear matter). It was found that in sufficiently strong external fields two types of instability of the pion field arise which correspond in a scalar field to $\omega^2_{\pi^+,\pi^-,\pi^0} < 0$ and in an electrical field to $(\omega_{\pi^+} + \omega_{\pi^-})^2 < 0$, where ω_π is the energy of the pions in the field. In these cases the single particle treatment becomes inapplicable and in order to obtain the condensate field and the energy of the system it is necessary to solve the nonlinear field problem. In[1] this problem was solved for the pion interaction of the form $\lambda\varphi^4$. The appearance of a Bose-condensate makes the system stable (the energy of all possible excitations becomes greater than zero). In that paper the possibility of a phase transition in nuclei and in neutron stars was demonstrated. But the influence of the nucleon medium was taken into account in the gaseous approximation in terms of the external field. Possible excitations of the nucleon medium were taken into account independently in references[2,3] by essentially different methods. In accordance with this the further development of the theory proceeded along two paths.

Sawyer and Scalapino[2] have put forward the idea of the instability of the matter of a neutron star with respect to the reaction $n \to p + \pi^-$. In order to verify this assertion a model was considered in which the nucleons interact with π^- mesons which are present in the only state with the propagation vector **k**. Since only one type of pions was taken into consideration the approach of[2] based on the method of the average field corresponded to the description of the pion field by a Schrödinger equation and not by the KGF equation. An instability was discovered in this model the meaning of which has become entirely clear only recently. This instability does not correspond to the initial idea of[2]. The instability with respect to the reaction $n \to p + \pi^-$ could have arisen only if the obvious condition $\mu^{(n)} \geq \mu^{(p)} + (\omega_{\pi^-})_{\min}$ is satisfied, which in the absence of an interaction between pions and nucleons goes over into the condition $\mu^{(n)} \geq \mu^{(p)} + m_\pi c^2$ (m_π is the pion mass). At the same time the instability observed in[2] disappears in the case of a weak interaction. Below we shall return to the question of the nature of the instability observed in[2]. We shall show that it represents a manifestation of the instability observed in a realistic model.[3,4]

The stability of the model under consideration with respect to the reaction $n \to p + \pi^-$ was demonstrated in[5] where an exact formulation of the problem was given concerning the interaction of nucleons with the traveling wave of a pion field of arbitrary amplitude. An effective Lagrange function was constructed for a system corresponding to the description of the pion field by the KGF equation, i.e., introducing the π^- and π^+ mesons automatically. Analogous results were obtained in[6] with the aid of the Hamiltonian formalism, and also in[7] by the method of the average field extended to two types of charged pions.

The other path along which the development of the theory proceeded representing an extension of[1] enabled one to discuss the problem of the π condensation corresponding to a considerably more realistic formulation of the problem (taking into account N^* resonances and s scattering, taking nucleon correlations into account, nuclear matter with an arbitrary ratio Z/N). This path consisted of a consistent application of the methods of the many-body problem. In[3] a method was developed for determining the polarization operator for the pions $\Pi(\mathbf{k}, \omega)$ in a nucleon medium, based on picking out those graphs which are essentially altered in the case of four-momenta of the order of $m_\pi c$. Self-energy graphs for pions which in the intermediate states have a nucleon and a nucleon hole or the isobar N^*_{33} (1232) and a nucleon hole have turned out to be of such a nature. The other less sensitive graphs were replaced by constants which must be determined experimentally, such as the constant f of the πN interaction, the constant g^{nn} and g^{np} for the spin–spin NN interaction in a medium. The constants characterizing the nucleon–nucleon interaction in a medium ($g^{nn}, g^{np}, f^{nn}, f^{np}$), are known for a nucleus ($Z \approx N$), [8] and are not known for a neutron star ($Z \ll N$). But they can be sufficiently well estimated with the aid of sensible model calculations, and in the case of high pion frequencies $\omega > \varepsilon_F$, which is realized in the case of condensation of charged mesons in neutron stars, they must not differ appreciably from the corresponding constants in vacuo. Knowing $\Pi(\mathbf{k}, \omega)$ one can from the poles of the pion propagator $D(\mathbf{k}, \omega)$ determine the excitation spectrum with the quantum numbers of π mesons (pion quasiparticles):

$$D^{-1}(\mathbf{k}, \omega) \equiv \omega^2 - \omega_\mathbf{k} - \Pi(\mathbf{k}, \omega) = 0, \quad \omega_\mathbf{k} = 1 + \mathbf{k}^2. \tag{1}$$

We utilize the units $\hbar = m_\pi = c = 1$.

In[3] this method was used for a detailed investigation of the case $N = Z$ (atomic nucleus), and in[4] of the case $N \gg Z$ (neutron star). It was found that there exist several branches of the spectrum of pion quasiparticles. Since the interaction between pions and nucleons is not small, strong mixing occurs of pion states with states of the type $(N, \overline{N})$, $(N^*, \overline{N})$. Therefore the nature of a particular excitation can be determined only from the behavior of the corresponding branch when the interaction is switched off. Thus, in addition to the "pion" branch there exist the "N^* resonance" and the "spin-isospin-acoustic" branches of the excitations of pion quasiparticles. Classification of the branches is carried out according to the type of excitation into which the corresponding branch goes over when the pion-nucleon interaction is switched off.

In[3] it was shown that in a medium with $N = Z$ with a sensible choice of constants for the NN interaction, for a density $n_c < n_0$ (n_0 is the nuclear density) for a definite wave number k_0 ($k_0 \approx p_F$) an instability appears in the spin-isospin-acoustic branch corresponding to $\omega^2_{\pi^+,\pi^-,\pi^0} < 0$. In virtue of the isotopic symmetry for $N = Z$ this instability develops simultaneously for all three types of pions and leads to the formation of an electroneutral condensate of π^+, π^-, π^0 mesons.

A still more distinctive situation arises in the case $N \gg Z$.[4,9] The spectrum of π^0 mesons remains the same as in the case $N = Z$, while the spectrum of charged mesons is significantly altered. First of all at a density of $n_c^+ \approx 0.4 n_0$ yet another branch arises in the π^+-meson spectrum of energy $\omega_{\pi^+} < -\varepsilon_F^{(n)}$. These excitations correspond to a bound state $(p, \overline{n})$ (in the same sense in which zero sound is a bound state of particle-hole) and possess the symmetry $\sigma\tau_+$. Of course these excitations represent a superposition of all competing states, and only when the pion-nucleon interaction is switched off do they go over into zero spin-isospin sound. We denote this branch of excitations by π_s^+. Since the proton density is small ($Z \ll N$), no analogous branch exists for π^- mesons. The appearance of the π_s^+ branch leads to an important consequence: all the free protons present in the medium go over for $n > n_c^+$ into neutrons and into excitations of the type π_s^+ ($p \to n + \pi_s^+$). The charge of π_s^+ mesons is compensated by electrons, and their equilibrium concentration is determined by the equation $\omega_{\pi_s^+} + \varepsilon_F^{(el)} = 0$. Analogous conclusions are reached by Anderson *et al.*, [10] who give the name of isospin waves to excitations of the π_s^+ type.

As the neutron density increases, the quantity $\omega_{\pi_s^+} + \omega_{\pi^-}$ diminishes and at the density of $n_c^\pm \approx 0.9 n_0$ vanishes for a wave number $k = k_c^\pm \approx 1.6$. An instability arises which leads to the formation of an electroneutral condensate of $\pi_s^+\pi^-$ pairs. The field of π^0 mesons becomes unstable at approximately the same density ($n_c^0 \approx 0.8 n_0$).

All these questions are investigated in detail in[9].

In this paper we study the interaction of pion excitations in nuclear matter ($Z \approx N$) and in the matter of neutron stars ($Z \ll N$) as it relates to the problem of π condensation, the energy of the π condensate is calculated using simple models and the isotopic and spatial structure of the condensate field is investigated.

In the next section of this part of the paper a method is given for calculating the effective Lagrangian of the system which is obtained from the usual Lagrangian by summing over all the degrees of freedom except for the condensate field. The nonlinear terms of this Lagrangian are interpreted as the effective interaction of pion quasiparticles replacing the model expression $\lambda\varphi^4$ ($0 < \lambda \ll 1$), utilized in[1,3] for a qualitative investigation of the π condensation. In Sec. 3 of part I the nature of the possible solutions for the condensate field is discussed.

In part II the exactly soluble model of the π condensation proposed in[2] is studied in which the condensate field of charged mesons has the form of a traveling wave $\varphi = a\exp(-i\omega t + i\mathbf{k}\cdot\mathbf{r})$. The energy of such a condensate can be obtained without assuming nearness to the critical point. A simple analytic expression is obtained for the energy of the π condensate in the high frequency approximation ($\omega \gg kv_F^{(n)}$). It is shown that the instability found by Sawyer and Scalapino[2,11] is of the same nature as the $\pi_s^+\pi^-$ instability found in[1].

Part III is devoted to the investigation of the static π condensate in the system with $N=Z$. The Thomas–Fermi method is used to investigate the spatial and the isotopic structure of the condensate field $\boldsymbol{\varphi} = \{\varphi_1, \varphi_2, \varphi_3\}$. It is shown that the one-dimensional configurations satisfying the condition $\mathrm{Sp}_\sigma \mathrm{Sp}_\tau (\sigma_\alpha \tau_\beta \partial_\alpha \varphi_\beta)^2 = \mathrm{const}$, i.e., the isotopically asymmetric solutions, have the lowest energy. The problem is investigated concerning the modulation of particle density and spin density of nucleon in the condensate field. It is noted, in particular, that solutions of the traveling wave type do not lead to such modulations.

Possible consequences of the π condensation for nuclear physics and for astrophysics are briefly discussed in the Conclusion.

2. Evaluation of the effective Lagrangian

In order to obtain the structure of the π condensate (the magnitude, the coordinate dependence and the isotopic character of the condensate field) it is necessary to take into account the effect of interaction between the pions. In[1] the $\lambda\varphi^4$ model of the interaction with $0<\lambda\ll 1$ was utilized for a qualitative solution of this problem. Our problem is to obtain the effective interaction of the pions and the structure of the π condensate in real nuclear matter. The effective $\pi\pi$ interaction is obtained as the sum of the interaction in vacuo which is described by the Weinberg Lagrangian and the interaction via the nucleon excitations. In order to obtain the effect of the nucleon medium on the $\pi\pi$ interaction and in order to obtain the structure of the π condensate it is convenient to describe the condensate field with the aid of an effective Lagrangian averaged over the motions of the nucleons. The present section is devoted to finding this Lagrangian.

The total Lagrangian density describing the system of interacting nucleons and π mesons can be represented in the form

$$L = L_\pi + L_N + L_{\pi N} + L_{NN} + L_{\pi\pi}, \tag{2}$$

where L_π and L_N are the free Lagrangians of the pion and the nucleon fields, while $L_{\pi N}$, L_{NN} and $L_{\pi\pi}$ are the Lagrangians for the πN, NN, and $\pi\pi$ interactions. In describing the ground state of a system with a condensate one can in expression (2) average over all the degrees of freedom of the nucleon and the meson fields leaving only the classical part of φ, which describes the condensate field. Moreover, it is convenient to deal with exact excitations of the medium (pion quasiparticles), and not with "bare" particles. For this purpose it is necessary to go over from L to the effective Lagrangian $\tilde{L}$, which is obtained from (2) by averaging over the exact states of the nucleons in the condensate field. In carrying this out the role of the nucleon medium reduces to a change in the pion spectrum and in their interaction compared with the same quantities in vacuo. Since the interaction of pion quasiparticles with each other and with the average nucleon field is related to exchange of low frequency particle-hole excitations of the medium, then it is essentially a retarded one, i.e., the effective Lagrangian contains high order derivatives of φ with respect to time. But in the case when φ describes a stationary state, i.e., depends on time as $e^{-i\omega t}$, this does not lead to complications. Instead of dependence on $\dot{\varphi}$, $\ddot{\varphi}$, etc. one can introduce into the effective Lagrangian of the pion field as independent variables the frequency ω and the amplitude $\varphi(\mathbf{r})$ of the condensate field.

For clarification we consider a Lagrangian which contains only a πN interaction:

$$L = \sum_{\mathbf{p}} \Psi_{\mathbf{p}}^+ (w-H)\Psi_{\mathbf{p}} + \frac{1}{2}\sum_{\mathbf{k}} \Big[(\omega^2-\omega_{\mathbf{k}}^2)\varphi_{\mathbf{k}}\varphi_{-\mathbf{k}} + i\sum_{\mathbf{k},\mathbf{p}} f\Psi_{\mathbf{p}}^+(\boldsymbol{\sigma}\mathbf{k})\boldsymbol{\tau}\Psi_{\mathbf{p}-\mathbf{k}}\varphi_{\mathbf{k}} \Big].$$

Here w and ω are the frequencies of the nucleon and the meson fields; we have omitted the isotopic indices; H is the Hamiltonian of a single nucleon. Variation with respect to $\Psi_{\mathbf{p}}^+$ and $\varphi_{\mathbf{k}}$ gives a system of equations for the field $\boldsymbol{\varphi}$ and the operator Ψ:

$$(w-H)\Psi_{\mathbf{p}} = i\sum_{\mathbf{k}} f(\boldsymbol{\sigma}\mathbf{k})\boldsymbol{\tau}\Psi_{\mathbf{p}-\mathbf{k}}\varphi_{\mathbf{k}},$$

$$(\omega^2-\omega_{\mathbf{k}}^2)\varphi_{\mathbf{k}} = i\sum_{\mathbf{p}} f\Psi_{\mathbf{p}}^+(\boldsymbol{\sigma}\mathbf{k})\boldsymbol{\tau}\Psi_{\mathbf{p}-\mathbf{k}}.$$

In order to obtain the effective Lagrangian for the pions we obtain the energy of a system of nucleons in the field φ. The part of the nucleon energy depending on φ plays the role of "potential energy" for mesons. Then the effective Lagrangian $\tilde{L}$ in the momentum representation can be written in the form[5]

$$\tilde{L} = \sum_{\mathbf{p}} (w^{(n)} - \tilde{\varepsilon}^{(n)}(\mathbf{p}))\tilde{n}_{\mathbf{p}}^+\tilde{n}_{\mathbf{p}} + \sum_{\mathbf{p}} (w^{(p)} - \tilde{\varepsilon}^{(p)}(\mathbf{p}))\tilde{p}_{\mathbf{p}}^+\tilde{p}_{\mathbf{p}} + \frac{1}{2}\sum_{\mathbf{k}} (\omega^2-\omega_{\mathbf{k}}^2)\varphi_{\mathbf{k}}\varphi_{-\mathbf{k}}. \tag{3}$$

Here $\tilde{\varepsilon}^{(n,p)}(\mathbf{p})$ are the exact single particle energies of the neutron and the proton in the condensate field; $\tilde{n}_{\mathbf{p}}^+$ and $\tilde{p}_{(\mathbf{p})}^+$ are the creation operators for the "new" neutron and the "new" proton defined in such a way that

$$\tilde{n}_{\mathbf{p}}^+\tilde{n}_{\mathbf{p}} = \begin{cases} 1, & \tilde{\varepsilon}^{(n)}(\mathbf{p}) < \tilde{\varepsilon}_F^{(n)} \\ 0, & \tilde{\varepsilon}^{(n)}(\mathbf{p}) > \tilde{\varepsilon}_F^{(n)} \end{cases}, \quad \tilde{p}_{\mathbf{p}}^+\tilde{p}_{\mathbf{p}} = \begin{cases} 1, & \tilde{\varepsilon}^{(p)}(\mathbf{p}) < \tilde{\varepsilon}_F^{(p)} \\ 0, & \tilde{\varepsilon}^{(p)}(\mathbf{p}) > \tilde{\varepsilon}_F^{(p)} \end{cases}$$

where $\tilde{\varepsilon}_F^{(n)}$ and $\tilde{\varepsilon}_F^{(p)}$ are the limiting energies for the Fermi-filling of "new" particles, which are obtained from the condition of the conservation of the total number of nucleons in the condensate field. In terms of such a formulation the problem consists of finding the change in the nucleon energy in the external field which is provided by the pion condensate field.

Expression (3) gives the correct equations of motion: the Schrödinger equation for the nucleon field and the KGF equation for the meson field. In order to verify this one can independently derive the equation for the amplitude $\varphi_{\mathbf{k}}$ considering, for example, the poles of the correlation function along the particle-hole channel in the condensate field.

The total energy of the system (the average Hamiltonian density) is related to the effective Lagrangian (3) by the equation[9,5]

$$E(\omega, \varphi_{\mathbf{k}}) = \sum w^{(n)} \frac{\partial \tilde{L}}{\partial w^{(n)}} + \sum w^{(p)} \frac{\partial \tilde{L}}{\partial w^{(p)}} + \omega \frac{\partial \tilde{L}}{\partial \omega} - \tilde{L}. \qquad (4)$$

In future we shall also need the formula relating the four-vector for the current of charged quasiparticles with the effective Lagrangian (3)[9]:

$$j_\mu = e\partial \tilde{L}/\partial k_\mu, \qquad (5)$$

where $k_\mu = (\mathbf{k}, \omega)$ is the four-momentum of the particle under consideration.

It should be noted that when Eq. (5) for the zero component of the four-vector current is taken into account relation (4) takes on a form analogous to the relation between the free energy of the system and the thermodynamic potential Ω. Thus, the effective Lagrangian $\tilde{L}$ is equivalent to the potential $-\Omega$, the role of chemical potentials in which is played by the frequencies of the meson and the nucleon fields.

Below we shall add to the expression (3) the interaction terms $L_{\pi\pi}$ and L_{NN}.

3. The nature of possible solutions

As has been noted already, in sufficiently dense neutron matter ($n^{(n)} > n_c^{+}$) production of $\pi^- \pi_s^+$ pairs of pion quasiparticles becomes possible. The particles being produced macroscopically populate one energetically most favorable state forming a Bose condensate. In the future we shall regard such a condensate as a coherent classical field $\varphi(\mathbf{r}, t)$ which represents the average value of the operator of the pion field with respect to the new ground state with broken symmetry. The quantity $\varphi(\mathbf{r}, t)$ plays the role of a complex order parameter characterizing the new phase.

From the condition of thermodynamic equilibrium with respect to the processes of creation and annihilation of $\pi^- \pi_s^+$ pairs it follows that the frequencies (chemical potentials) of the π^- and π_s^+ mesons in the condensate are related by the equation

$$\omega_{\pi^-} + \omega_{\pi_s^+} = 0,$$

not only at the critical point, but also for $n^{(n)} > n_c^{\pm}$.

Thus, the wave function for the condensate field $\varphi(\mathbf{r}, t)$ which simultaneously describes the π^- and π_s^+ components of the meson field must have the following form

$$\varphi(\mathbf{r}, t) = {}^1/_2 (\varphi_{\pi^-} + \varphi^*_{\pi_s^+}) = \varphi(\mathbf{r}) e^{-i\omega t}, \qquad (6)$$

where the notation $\omega \equiv \omega_{\pi^-} = -\omega_{\pi_s^+}$ has been introduced. The same character of the time dependence of the condensate field is obtained in the Hamiltonian formalism.[6] As regards the coordinate dependence, the exact form of $\varphi(\mathbf{r})$ must be determined from the equation of motion which in the present case is a nonlinear integro-differential equation. Setting aside attempts to find an exact solution we utilize the variational method: we specify different trial functions $\varphi(\mathbf{r})$ and choose that one which corresponds to the lowest energy. Since the instability arises for a nonvanishing momentum $k \sim 1$, the trial functions must be periodic functions of $\mathbf{r}$. The simplest functions are

$$\varphi(\mathbf{r}) = a e^{i\mathbf{kr}} \quad \text{(traveling wave)}, \qquad (7a)$$

$$\varphi(\mathbf{r}) = a\sqrt{2} \sin \mathbf{kr} \quad \text{(standing wave)}. \qquad (7b)$$

In this case the effective Lagrangian $\tilde{L}$ is a function of three independent variables ω, k and a. The equation of motion reduces to the algebraic equation for the determination of the optimum amplitude of the condensate field

$$\partial \tilde{L}/\partial a = 0. \qquad (8)$$

In a sufficiently large system we must specify the condition of electroneutrality $j_0 = e\partial \tilde{L}/\partial \omega = 0$. Then from the condition $d\tilde{E}/d\mathbf{k} = 0$ that the energy should be a minimum it follows that $\partial \tilde{L}/d\mathbf{k} = 0$, i.e., the four-current is absent in the ground state. Thus,

$$\partial \tilde{L}/\partial \omega = 0, \quad \partial \tilde{L}/\partial \mathbf{k} = 0. \qquad (9)$$

From relations (8) and (9) we determine a, ω, and k.

In the limit as $a \to 0$, when we can restrict ourselves to terms of order a^2 in the Lagrangian $\tilde{L}$, the three equations (8) and (9) go over to a system determining the critical parameters of the $\pi_s^+ \pi^-$ condensation: $n_c^{\pm}$, ω_c, and k_c. At the critical point the coefficient in front of a^2 changes sign and for the determination of the parameters of the condensate it is necessary to include in $\tilde{L}$ terms nonlinear with respect to a^2. The evaluation of such terms represents quite a difficult problem in a realistic formulation. It turns out that this problem can be successfully solved exactly for a condensate field of the form (7a) to the investigation of which the following section is devoted.

II. $\pi_s^+ \pi^-$ CONDENSATION IN NEUTRON STARS. THE EXACTLY SOLUBLE MODEL.

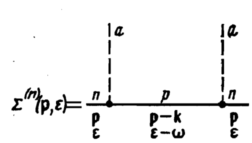
1. Nucleon energy in the condensate field

We consider the field of the $\pi_s^+ \pi^-$ condensate which has the form of a traveling wave

$$\varphi(\mathbf{r}, t) = a e^{-i\omega t + i\mathbf{kr}}, \qquad (10)$$

where a is real. Such a field corresponds to the fact that all the π^- mesons "occupy" a single state–of frequency ω and with momentum $\mathbf{k}$, while all the π_s^+ mesons "occupy" the state of frequency–ω and of momentum–$\mathbf{k}$.

The irreducible self-energy part of the neutron in the momentum representation $\Sigma^{(n)}(\mathbf{p}, \varepsilon)$ in the field (10) is represented by the one graph



$$= |\mathcal{T}_0^{(\pm)}|^2 a^2 G_0^{(p)}(\mathbf{p} - \mathbf{k}, \varepsilon - \omega). \qquad (11)$$

Here the dotted lines represent the amplitude of the condensate field, the points correspond to a π^+np vertex: $|\mathcal{F}_0^{(\pm)}|^2 = 2f^2k^2$ (nonrelativistic coupling, $f=1.0$). For the sake of simplicity we do not for the present include in our consideration the resonance and the S-wave πN interaction, and also the nucleon correlations. It can be seen that the intermediate proton line need no longer be made more precise by including the condensate field, since every such increase in precision leads to the appearance in the intermediate state of a neutron with the initial $\mathbf{p}$ and ε, and such graphs by definition do not appear in $\Sigma^{(n)}(\mathbf{p}, \varepsilon)$.

The exact Green's function for the neutron $\tilde{G}^{(n)}(\mathbf{p}, \varepsilon)$ is obtained from the Dyson equation

$$\tilde{G}^{(n)} = G_0^{(n)} + G_0^{(n)}\Sigma^{(n)}\tilde{G}^{(n)}. \tag{12}$$

Here and in (11) $G_0^{(n)}$ and $G_0^{(p)}$ are the Green's functions for the particles without taking into account the effect of the condensate field:

$$G_0^{(n,p)}(\mathbf{p}, \varepsilon) = [\varepsilon - \varepsilon_p + i\gamma \operatorname{sign}(\varepsilon - \varepsilon_F^{(n,p)})]^{-1}, \quad \gamma \to +0 \tag{13}$$

where $\varepsilon_p = \mathbf{p}^2/2m$ ($m = 6.7$ is the nucleon mass). From (12) we obtain

$$\tilde{G}^{(n)}(\mathbf{p}, \varepsilon) = \left[\varepsilon - \varepsilon_p - \frac{|\mathcal{F}_0^{(\pm)}|^2 a^2}{\varepsilon - \varepsilon_{p-k} - \omega}\right]^{-1}. \tag{14}$$

The corresponding expression for the proton Green's function $\tilde{G}^{(p)}(\mathbf{p}, \varepsilon)$ differs by the sign of ω and $\mathbf{k}$. The single particle energies of the "new" neutrons $\tilde{\varepsilon}^{(n)}(\mathbf{p})$ and of the "new" protons $\tilde{\varepsilon}^{(p)}(\mathbf{p})$ are obtained from the poles of the exact Green's functions, i.e., they are determined by the equations $[\tilde{G}^{(n,p)}(\mathbf{p}, \varepsilon)]^{-1} = 0$. From this we obtain[2]

$$\tilde{\varepsilon}^{(n)}(\mathbf{p}) = \frac{\varepsilon_p + \varepsilon_{p-k} + \omega}{2} + \frac{\varepsilon_p - \varepsilon_{p-k} - \omega}{2}\left[1 + \frac{4|\mathcal{F}_0^{(\pm)}|^2 a^2}{(\varepsilon_p - \varepsilon_{p-k} - \omega)^2}\right]^{1/2}, \tag{15a}$$

$$\tilde{\varepsilon}^{(p)}(\mathbf{p}) = \frac{\varepsilon_p + \varepsilon_{p+k} - \omega}{2} + \frac{\varepsilon_p - \varepsilon_{p+k} + \omega}{2}\left[1 + \frac{4|\mathcal{F}_0^{(\pm)}|^2 a^2}{(\varepsilon_p - \varepsilon_{p+k} + \omega)^2}\right]^{1/2}. \tag{15b}$$

The signs in front of the square root are here chosen in such a manner that as $a \to 0$ the energies $\tilde{\varepsilon}^{(n,p)}(\mathbf{p})$ should go over into the energy of the free particles $\varepsilon_p^{(n,p)} = \mathbf{p}^2/2m$. Since for $n > n_c^{\pm}$ in the given model the inequality $\omega > \varepsilon_F^{(n)}$ holds, then the filling of the "new" proton states is energetically unfavorable. Indeed, as was shown in[5], the conversion of a small number of neutrons into protons and exact excitations with the quantum numbers of the π^- mesons ($n \to p + \pi^-$) leads to a change in the energy of the system:

$$\delta E = (\omega_{\pi^-} - \tilde{\varepsilon}_F^{(n)})\tilde{\nu}_p,$$

where $\tilde{\nu}_p$ is the density of the "new" protons equal to the density of the π^- mesons. Thus, the filling of the "new" proton states which must be accompanied by the formation of a π^- condensate would have been possible only under the condition $\omega_{\pi^-} - \tilde{\varepsilon}_F^{(n)} < 0$, which, as has been shown in[9], is not satisfied right up to very great densities of neutron matter.

In the case of a $\pi_s^+\pi^-$ condensation only the "new" neutron states turn out to be occupied. At the same time the Fermi surface $\tilde{S}_F^{(n)}$ is no longer a sphere, as in the normal phase. Its equation is found from the condition

$$\tilde{\varepsilon}^{(n)}(\mathbf{p})|_{\tilde{S}_F^{(n)}} = \tilde{\varepsilon}_F^{(n)}. \tag{16}$$

We choose the z axis along the $\mathbf{k}$ vector and introduce cylindrical coordinates $(p_\perp, p_\parallel, \theta)$, where $p_\parallel = p_z - k/2$ and $p_\perp$ is the component of the vector $\mathbf{p}$ in the plane perpendicular to $\mathbf{k}$. In terms of these variables, the equation for the Fermi limit $p_\perp^2(p_\parallel)$ assumes the form

$$\frac{p_\perp^2}{2m} + \frac{1}{2}\left[\frac{p_\parallel^2 + k^2/4}{m} + \omega + \left(\frac{p_\parallel k}{m} - \omega\right)\left(1 + \frac{4|\mathcal{F}_0^{(\pm)}|^2 a^2}{(p_\parallel k/m - \omega)^2}\right)^{1/2}\right] = \tilde{\varepsilon}_F^{(n)}. \tag{17}$$

The boundary points $p_{\parallel 1,2}$ of the Fermi-filling along the z axis are determined from the condition $p_\perp^2(p_{\parallel 1,2}) = 0$, which, as follows from (17), leads to an algebraic equation of the fourth degree.

The value of the exact Fermi energy $\tilde{\varepsilon}_F^{(n)}$ is fixed by the requirement that after a redistribution in the condensate field the total number of nucleons should not be changed, i.e.,

$$n^{(n)} = \sum_{\tilde{\varepsilon}^{(n)}(\mathbf{p}) < \tilde{\varepsilon}_F^{(n)}} 1 = \frac{1}{4\pi^2}\int_{p_{\parallel 1}}^{p_{\parallel 2}} p_\perp^2(p_\parallel)\,dp_\parallel = \frac{(p_F^{(n)})^3}{3\pi^2}, \tag{18}$$

where $p_F^{(n)}$ is the Fermi momentum for $a = 0$. The kinetic energy of the nucleons in the condensate field appearing in the effective Lagrangian (3) is obtained from the formula

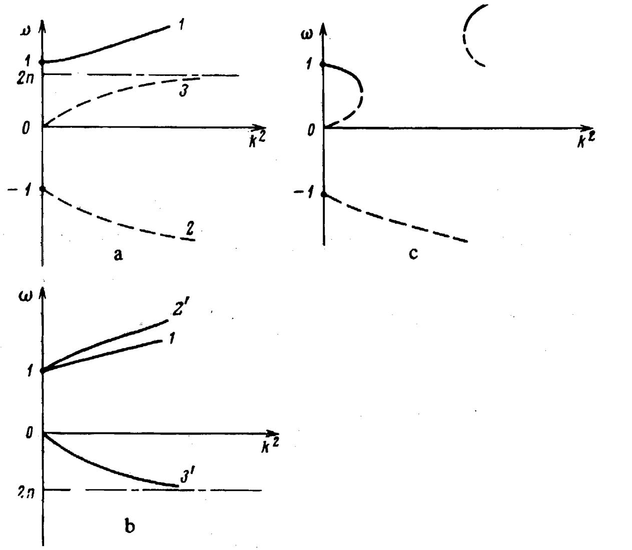
$$\tilde{E}^{(N)} = \sum_{\tilde{\varepsilon}^{(n)}(\mathbf{p}) < \tilde{\varepsilon}_F^{(n)}} \tilde{\varepsilon}^{(n)}(\mathbf{p}) = \tilde{\varepsilon}_F^{(n)} n - \frac{1}{16m\pi^2}\int_{p_{\parallel 1}}^{p_{\parallel 2}} p_\perp^4(p_\parallel)\,dp_\parallel. \tag{19}$$

The integrals (18) and (19) can be expressed in terms of elementary functions[12] and, thus, the problem of determining the energy of the system in the presence of a $\pi_s^+\pi^-$ condensate of the form (10) can be solved without assuming smallness of the amplitude of the condensate field. The calculations are carried out in the following manner. From equation (18) the exact Fermi energy for the neutrons $\tilde{\varepsilon}_F^{(n)}$ is obtained as a function of the parameters n, ω, k, and a^2, which is then substituted into expression (19) for $\tilde{E}^{(N)}$. After this, the effective Lagrangian for the condensate field $\tilde{L}(k, \omega, a^2)$ is constructed in accordance with formula (3) and from it the system of equations (8), (9) is obtained for the determination of a^2, ω, and k. Substitution of these parameters into expression (4) gives the total energy of the system $\tilde{E}(n)$ as a function of the nucleon density n. The gain in energy due to the formation of the $\pi_s^+\pi^-$ condensate is by definition equal to

$$E^{(\pi)}(n) = \tilde{E}(a(n)) - \tilde{E}(a=0). \tag{20}$$

In actual fact the evaluation of $E^{(\pi)}(n)$ is associated with computational difficulties and use of numerical methods is required even in that simplified variant of the theory which was considered above.[6,13]

We succeed in obtaining a simple analytic expression for $E^{(\pi)}(n)$ in the high frequency approximation: $\omega \gg kv_F$.[14] In this approximation the Fermi surface remains spherically symmetric and the integration in



The spectrum of charged mesons in a neutron medium in the high frequency approximation $\omega \gg kv_F^{(n)}$: a) solutions of the dispersion equation for $n < \frac{1}{2}f^2 = n_c^{\pm}$, b) the spectrum of charged mesons for $n < n_c^{\pm}$, where 1 and 2 are the meson branches, and 3 is the spin-isospin acoustic branch (π_s^{+}); c) the spectrum of charged pions for $n > n_c^{\pm}$.

(18) and (19) can be carried out in an elementary manner. The effective Lagrangian of the condensate field (3) in this case takes on the form[5]

$$\mathcal{L}_n = \mathcal{L}(a) - \mathcal{L}(0) = (\omega^2 - \omega_k^2)a^2 + \frac{n\omega}{2}\left[1 - \left(1 + \frac{8f^2k^2a^2}{\omega^2}\right)^{1/2}\right]. \quad (21)$$

The critical parameters of the $\pi_s^+\pi^-$ condensate in this approximation have the values $n_c = (2f^2)^{-1}$, $k_c = \sqrt{2}$, $\omega_c = 1$. The energy of the condensate (without assuming that we are close to a critical point) is given by the expression

$$E^{(n)}(n) = -(n - n_c)^2/4n_c, \quad n \geqslant n_c. \quad (22)$$

An analogous approximation was utilized in[15,16] to calculate the energy of a developed condensate in a more realistic model.

An analytic solution of the problem is also possible near a critical point, when the amplitude of the condensate field is small. In[17] the effective Lagrangian of the condensate field which takes into account the P- and S-wave πN interaction, the vacuum $\pi\pi$ interaction, the nucleon correlations and the N_{33}^* (1232) resonance is calculated with an accuracy up to terms of order a^4. In the same paper a comparison is made between the energies of configurations (10) and (7b) of the condensate field, and also the possibility of a phase transition of the first kind is discussed.

It should be noted that in[15,16] the results of the interesting paper by Campbell, Dashen and Manassah, [18] in which the pion condensation is investigated from the point of view of chiral symmetry, have been utilized in an essential manner.

2. Remarks on the Sawyer-Scalapino model.

In this section we show that the instability found in[2] is of the same nature as the $\pi_s^+\pi$- instability described above.

On the example of a simple model ($\omega \gg kv_F^{(n)}$) we show how the nature of the solutions $\omega(k)$ of the dispersion equation (1) changes as a result of passage through the critical density $n_c^{\pm}$. In this model the dispersion equation for the determination of the exact energies of the pion quasiparticles in the normal phase of pure neutron matter ($Z = 0$) is given by the expression $(\partial\bar{L}/\partial a^2)_{a=0} = 0$, or

$$D^{-1}(\mathbf{k}, \omega) = \omega^2 - \omega_k^2 + 2n^{(n)}f^2k^2/\omega = 0. \quad (23)$$

The last term in this equation is the polarization operator for the π^- meson in the approximation $\omega \gg kv_F^{(n)}$. Exactly the same equation is obtained from formula (8) of Ref. 5 under the condition $\varphi^*\varphi = 0$ and $\tilde{\nu}_p = 0$ where $\tilde{\nu}_p$ is the density of the "new" proton states forming a set of occupied Fermi states. (We note that the instability with respect to the reaction $n \to p + \pi^-$ corresponds to $\tilde{\nu}_p > 0$.) The solution of Eq. (23) can be written in the form

$$k^2(\omega) = \frac{\omega^2 - 1}{1 - \alpha/\omega},$$

where $\alpha = 2n^{(n)}f^2$. For $\alpha < 1$ there exist three branches of the spectrum (cf., case in the diagram).

As has been shown in[9] classification of the branches is determined by the sign of the residue of $D(\mathbf{k}, \omega)$, i.e., by the sign of the quantity

$$\frac{\partial D^{-1}}{\partial\omega} = 2\omega - \frac{\partial\Pi^{(-)}}{\partial\omega} = 2\omega - \frac{\alpha k^2}{\omega^2}.$$

The branches for which $2\omega - \partial\Pi^{(-)}/\partial\omega > 0$ correspond to π^- mesons, while the branches for which $2\omega - \partial\Pi^{(-)}/\partial\omega < 0$ yield, after replacement of ω by $-\omega$, the dispersion law for π^+ mesons.[1)] In the diagram, case a, branch 1 corresponds to π^- mesons, since for it $2\omega - \partial\Pi^{(-)}/\partial\omega > 0$. For branches 2 and 3 we have $2\omega - \partial\Pi^{(-)}/\partial\omega < 0$, and therefore after the replacement of ω by $-\omega$ these branches give the dispersion law for quasiparticles with quantum numbers characteristic of π^+ mesons. Thus, in the medium there exist two types of π^+-meson excitations: π^+, and π_s^+. The spectrum of quasiparticles has the form shown in the diagram as case b. If the interaction between pions and nucleons is switched off branches 1 and 2′ go over into the vacuum spectra for π^- and π^+ mesons ($\omega(\mathbf{k}) \to \omega_k$), while branch 3′ goes over into the dispersion law for spin-isospin sound ($\omega \sim kv_F^{(n)}$). The results quoted here are valid only for $\omega \gg kv_F^{(n)}$. It follows from exact calculations (cf.,[9]), that the segment of the branch 3′ with $|\omega| \lesssim kv_F^{(n)}$ vanishes, while for $\alpha < kv_F^{(n)}$ the π_s^+ branch is absent entirely. The appearance of this branch is what corresponds to the instability indicated above with respect to the reaction $p \to n + \pi_s^+$.

As the density increases further the branches π^- and π_s^+ become lower, the quantity $(\omega_{\pi^-} + \omega_{\pi_s^+})_{\min}$ diminishes and for $\alpha = 1$ there appears for the first time in the spectrum the point ($k^2 = 2$) at which $\omega_{\pi^-} + \omega_{\pi_s^+} = 0$. Thus, the value $\alpha = 1$ corresponds to the critical density of the $\pi_s^+\pi^-$ condensation: $n_c^{\pm} = 1/2f^2$. It is evident that at the critical point $2\omega - \partial\Pi^{(-)}/\partial\omega = 0$, and this corresponds to the merging of two roots of (23), i.e., to a double pole of the meson propagator $D(\mathbf{k}, \omega)$. For $\alpha = 1$ the spectrum turns out to be degenerate, it breaks up into two curves: $\omega = 1$ and $k^2 = \omega(\omega + 1)$. The coordinates of the point of intersection of these curves are the critical parameters of the $\pi_s^+\pi^-$ condensate in this model: $k_c^{\pm} = \sqrt{2}$ and $\omega_c = 1$ (cf., (8), (9), and (21)).

For $\alpha > 1$ the spectrum, as can be easily shown, has the form shown in diagram c. It is distinguished by a characteristic feature: the spectrum has a discontinuity at the edges of which $d\omega/dk = \infty$. Within the region of discontinuity $(\omega_{\pi^-} + \omega_{\pi_s^+})^2 < 0$. This is an indication of the instability of the system with respect to the formation of the $\pi_s^+\pi^-$ condensate. Thus, this simplified model correctly reproduces the principal results of the exact calculations.[4,9]

We now turn to the model of Sawyer and Scalapino.[2,11] In this model a system is considered consisting of neutrons, protons and "bare" π^- mesons, "occupying" the single state with propagation vector $\mathbf{k}$. In this discussion the degrees of freedom of the meson field are artificially restricted: the "bare" π^+ mesons are not taken into consideration, and this corresponds to describing the π^--meson field by the Schrödinger equation instead of the KGF equation. Therefore the equation for determining the energy (chemical potential) of the π^- meson in contrast to (23) has the form (cf.,[5], formula (7), $\tilde{\nu} = 0$, $\varphi \to 0$)

$$\omega = \omega_{\mathbf{k}} - n^{(n)} f^2 k^2 / \omega_{\mathbf{k}} \omega. \tag{24}$$

The second term on the right hand side represents the self-energy part $\Sigma^{(-)}$ for the π^- meson.

Equation (24) has two roots:

$$\omega = \frac{\omega_{\mathbf{k}}}{2}\left(1 \pm \left[1 - \frac{4n^{(n)} f^2 k^2}{\omega_{\mathbf{k}}^3}\right]^{1/2}\right).$$

The upper sign corresponds to a π^- meson. For this solution the residue of the Green's function for the π^- meson $(1 - \partial\Sigma^{(-)}/\partial\omega)$ is positive, as it should be. Moreover, when the interaction is switched off $\omega \to \omega_{\mathbf{k}}$. It is this particular branch that was utilized in[5] in order to prove the stability of the system with respect to the reaction $n \to p + \pi^-$.

Within the framework of the model considered in[2,11] it is difficult to give a sensible physical interpretation for the second root of Eq. (24) for which $1 - \partial\Sigma^{(-)}/\partial\omega < 0$. As follows from the analysis of the solutions of (23) given above, the second root should be interpreted (after replacing ω by $-\omega$) as the π_s^+ meson branch, i.e., as the bound state of a proton and a neutron hole $(p, \bar{n})$. When the condition $1 - 4n^{(n)} f^2 k^2 / \omega_{\mathbf{k}}^3 = 0$, which coincides with the critical condition obtained in[2,11], is satisfied, the sum of the energies $\omega_{\pi^-} + \omega_{\pi_s^+}$ vanishes (the two roots merge), and the system becomes unstable with respect to the appearance of the electroneutral $\pi_s^+\pi^-$ condensate. It is not difficult to obtain the values of the critical parameters of the $\pi_s^+\pi^-$ condensate in this model:

$$n_c^{\pm} = \frac{3\sqrt{3}}{8f^2} \approx 1.3 n_0, \quad k_c^{\pm} = \sqrt{2}, \quad \omega_c = \frac{\omega_{k_c}}{2} = \frac{\sqrt{3}}{2}.$$

Due to the more complicated dependence of the energy of the system on k the expression for the gain in energy from the new phase $E^{(\pi^\pm)}(n)$ has the form analogous to (22) only near the critical point.

It is clear that the results obtained in[2,11] describe only the qualitative aspects of the phenomenon since the pion is described by a Schrödinger equation. Subsequently this model was improved for the case of two types of pions, π^+ and π^-.[17] The simplicity of this model also made it possible to include in the discussion nucleon correlations, the $\pi\pi$ interaction[12] and the N_{33}^* resonance,[15] and this made it possible to calculate the energy of the developed condensate in a sufficiently realistic manner.

We now make a few remarks concerning the paper by Sawyer[20] which contains a critique of the method of investigating the π condensation developed in[3,4]. This critique is based on an incorrect classification of the roots of (23). Sawyer is surprised that when the πN interaction is switched off there exists a solution with $\omega \to 0$, which he erroneously ascribes to the π^- meson. Indeed, such a root is possessed both by Eq. (23), and by Eq. (24) of the simplified model,[2] but it corresponds to a π_s^+ meson. As has been indicated above, such a solution is valid only for $\omega \gg kv_F^{(n)}$. There is nothing strange in the fact that the energy of the bound state $(p, \bar{n})$ tends to zero as $m_N \to \infty$. Thus, this critique is based on a misconception.

III. π CONDENSATION IN A SYSTEM WITH $N=Z$

This section is devoted to an investigation of the π-condensation in isotopically symmetric nuclear matter $(N=Z)$. As has been already noted above, due to the isotopic symmetry the instability in such a system occurs simultaneously for all three types of pions π^+, π^-, π^0, and $n_c \approx 0.6 n_0$.[3,9] The pion condensate arising for $n > n_c$ differs in two respects from the condensate of charged mesons in a neutron medium considered until now. First, it is static, i.e., the frequencies of the pions in the condensate are equal to zero, and second, the isotopic vector of the condensate field $\varphi = \{\varphi_1, \varphi_2, \varphi_3\}$ $(\varphi_\pi^{\pm} = (\varphi_1 + i\varphi_2)/\sqrt{2}, \varphi_{\pi 0} = \varphi_3)$ has, generally speaking, all three components different from zero. Therefore to the difficulties of determining the spatial structure of the condensate field which we have already encountered above, another one is added associated with the choice of the optimal isotopic composition of the condensate. A problem arises which is quite difficult in a realistic formulation concerning the finding of the energy of the system in the presence of a condensate which has a complicated spatial and isotopic structure.

For a qualitative solution of this problem the Thomas–Fermi approximation is a convenient method. The application of this method to the problem of the π condensation is given in Sec. 1 of this chapter. We note at once that the condition for the applicability of the Thomas–Fermi method in the case of periodic potentials is the condition $k^2/4p_F^2 \ll 1$. In actual fact the π condensation occurs for $k \sim p_F$, and therefore, utilizing this approximation, we cannot expect good quantitative accuracy.

Below in Sec. 2 the question is considered of the spatial distribution of the particle density and the spin density of the nucleons in the condensate field. It is shown that in a condensate field of a standing wave type modulations arise of the particle and spin density, while in a field of the form of a traveling wave these quantities do not change.

1. The spatial and isotopic structure of the π condensate. The Thomas-Fermi method.

We consider a system of nucleons interacting with the classical field of a static pion condensate $\varphi(\mathbf{r})$. If we do not take into account transitions of nucleons into N^*_{33} states, then such an interaction can be described by a time-independent potential

$$U(\mathbf{r})=f\sigma_\alpha\tau_\beta\partial_\alpha\varphi_\beta,\quad \alpha,\beta=1,2,3, \tag{25}$$

which is an operator in the space of spin and isospin variables.

In future we shall normalize the condensate field $\varphi(\mathbf{r})$ in such a manner that

$$\frac{1}{V}\int\varphi^2(\mathbf{r})\,d^3r=a^2. \tag{26}$$

$U(\mathbf{r})$ has a particularly simple form when we are concerned with π^0 condensation in a pure neutron medium.[14] In this case, $\varphi_1=\varphi_2=0$ and the field

$$U(\mathbf{r})=-f\sigma\nabla\varphi_3. \tag{27}$$

acts on the neutrons. In view of the fact that this case is of independent interest and is at the same time sufficiently simple, we shall use it to derive the basic formulas which we then generalize to the case of a π condensation in a system with $N=Z$. Independently of the nature of the coordinate dependence of the field $\varphi_3(\mathbf{r})$ (it is important only that it should vary sufficiently slowly) the axis of quantization of the spin at each spatial point can be chosen along the direction of $\nabla\varphi_3(\mathbf{r})$. Then the total energy of the neutron with its spin parallel and antiparallel to $\nabla\varphi_3$ ($\varepsilon_\pm(\mathbf{p},\mathbf{r})$, respectively) will be determined by the expression

$$\varepsilon_\pm(\mathbf{p},\mathbf{r})=\mathbf{p}^2/2m\mp f|\nabla\varphi_3|.$$

The local values of the limiting momenta of the Fermi distribution of particles with the two components of spin $p_+(\mathbf{r})$ and $p_-(\mathbf{r})$ are determined from the condition of constancy in space of the maximum energy of the occupied states:

$$\varepsilon_+(p_+,\mathbf{r})=\varepsilon_-(p_-,\mathbf{r})=\tilde{\varepsilon}_F.$$

The particle number density $n(\mathbf{r})$ and the spin density $s(\mathbf{r})$ at the point $\mathbf{r}$ are given by the obvious relations

$$n(\mathbf{r})=n_+(\mathbf{r})+n_-(\mathbf{r}), \tag{28}$$

$$\mathbf{s}(\mathbf{r})=(n_+(\mathbf{r})-n_-(\mathbf{r}))\frac{\nabla\varphi_3}{2|\nabla\varphi_3|}, \tag{29}$$

where

$$n_\pm(\mathbf{r})=\frac{p_\pm^3(\mathbf{r})}{3\pi^2}=\frac{n}{2}\left(\frac{\tilde{\varepsilon}_F}{\varepsilon_F}\right)^{3/2}\left[1\pm\frac{f|\nabla\varphi_3|}{\tilde{\varepsilon}_F}\right]^{3/2}. \tag{30}$$

In formula (30) n and $\varepsilon_F=p_F^2/2m$ are the particle density and the Fermi energy in the absence of the condensate field. The value of $\tilde{\varepsilon}_F$ is obtained from the condition of conservation of the total number of particles after a redistribution in the field i.e.,

$$\langle n(\mathbf{r})\rangle=n=p_F^3/3\pi^2 \tag{31}$$

(the angle brackets denote averaging over the coordinates). The total energy of the new Fermi distribution of neutrons is calculated in accordance with the formula

$$E^{(N)}(\mathbf{r})=\sum_{p<p_+(\mathbf{r})}\varepsilon_+(\mathbf{p},\mathbf{r})+\sum_{p<p_-(\mathbf{r})}\varepsilon_-(\mathbf{p},\mathbf{r})$$
$$=\frac{1}{20m\pi^2}[p_+^5(\mathbf{r})+p_-^5(\mathbf{r})]-f|\nabla\varphi_3|[n_+(\mathbf{r})-n_-(\mathbf{r})]. \tag{32}$$

The formulas obtained above enable us to find the energy of the system without assuming that the meson field is small. But for simplicity we restrict ourselves to a consideration of weak fields. By restricting ourselves to terms of the fourth degree in the condensate field and utilizing relations (30) and (31), we obtain the following expression for the average value of the total energy of the system:

$$\tilde{E}=\langle E^{(N)}(\mathbf{r})\rangle+\frac{1}{2}[\langle\varphi_3^2(\mathbf{r})\rangle+\langle(\nabla\varphi_3(\mathbf{r}))^2\rangle]$$
$$=E_0+\frac{1}{2}\langle\varphi_3^2+(\nabla\varphi_3)^2\rangle-f^2\frac{mp_F}{\pi^2}\langle(\nabla\varphi_3)^2\rangle$$
$$+\frac{f^4}{4\pi^2v_F^3}[\langle(\nabla\varphi_3)^2\rangle^2+\frac{1}{3}\langle(\nabla\varphi_3)^4\rangle]. \tag{33}$$

In the table are given the values of the energy of the different configurations of the field $\varphi_3(\mathbf{r})$, among which there are one-dimensional, two-dimensional and three-dimensional ones. We can see that the lowest energy belongs to the meson field of the form of a three-dimensional lattice:

$$\varphi_3(\mathbf{r})=\sqrt{2/3}\,a(\sin kx+\sin ky+\sin kz), \tag{34}$$

for which

$$a^2=-\frac{27}{25}\frac{\tilde{\omega}_0^2}{\lambda_0},\quad \tilde{E}=E_0-\frac{27}{100}\frac{\tilde{\omega}_0^4}{\lambda_0}\quad(\tilde{\omega}_0^2<0). \tag{35}$$

Here

$$\tilde{\omega}_0^2=1+k^2-f^2k^2\frac{mp_F}{\pi^2}(1+g^{nn})^{-1},$$
$$\lambda_0=\frac{3f^4k^4}{4\pi^2v_F^3}(1+g^{nn})^{-4}, \tag{35'}$$

The factor $(1+g^{nn})^{-1}$ takes into account the principal contribution of nucleon correlations.

Thus, one can conclude that the most probable structure of the field of the π^0 condensate in a neutron medium is the three-dimensional lattice (34). It should be emphasized that this conclusion is based on the approximate method of calculation ($k^2\ll 4p_F^2$) and that for a final solution of the problem concerning the structure of the π^0 condensate in a neutron medium it is necessary to carry out the calculation for $k\approx p_F$. Moreover, it is necessary to take into account the interaction of the π^0 condensate with the condensate of charged mesons which also influences the structure of the con-

The energies* of certain configurations of the field of a π^0 condensate in a neutron medium in the Thomas-Fermi approximation.

№	Meson field	ζ
1	$\varphi_{\pi^0}=a(\cos kx+\cos ky+\cos kz)$	1
2	$\varphi_{\pi^0}=a(\cos kx+\cos ky)$	$^{50}/_{51}$
3	$\varphi_{\pi^0}=a\cos kx$	$^{25}/_{27}$
4	$\varphi_{\pi^0}=a\cos kx\ \cos ky\ \cos kz$	$^{20}/_{21}$

*The quantity $\zeta\equiv{}^{100}/_{27}\,(\lambda_0/\tilde{\omega}^4)\,E^{(\pi)}$.

densates in an essential manner.[14,21]

Utilizing formulas (28) and (29), we easily obtain the spatial distribution of the particle density and the spin density for neutrons in the field (34). Restricting ourselves to only the lowest terms in φ_3, we obtain:

$$n(\mathbf{r})=n[1+\xi^2(\cos 2kx+\cos 2ky+\cos 2kz)],$$
$$s(\mathbf{r})=\frac{mp_F}{\pi^2}\nabla\varphi_3,\quad \xi^2=\frac{f^2k^2a^2}{8\varepsilon_F^2}. \qquad (36)$$

We now go over to the case of symmetric nuclear matter ($N=Z$). As has been noted already, instability arises simultaneously for all three types of pions. As a result a static electroneutral condensate is formed in which all three components of the meson field $\boldsymbol{\varphi}$ can differ from zero. The diagonalization of the operator $U(\mathbf{r})$ (25) in this case is accomplished in a somewhat more complex manner, since in addition to a rotation in coordinate space it is also necessary to carry out a rotation in isotopic space. In the space of the states $(p\uparrow, p\downarrow, n\uparrow, n\downarrow)$ the operator $U(\mathbf{r})$ is represented by a 4×4 matrix, the eigenvalues of which λ_i are determined by a secular equation of the fourth degree: $|U-\lambda\cdot 1|=0$. It is not difficult to obtain the explicit form of this equation:

$$(\lambda^2-(\nabla\varphi_1)^2-(\nabla\varphi_2)^2-(\nabla\varphi_3)^2)^2-4[\nabla\varphi_1\nabla\varphi_2]^2$$
$$-4[\nabla\varphi_1\nabla\varphi_3]^2-4[\nabla\varphi_2\nabla\varphi_3]^2-8\lambda[\nabla\varphi_1\nabla\varphi_2]\nabla\varphi_3=0. \qquad (37)$$

The single-particle energies corresponding to two spin and two isospin states of the nucleon at each spatial point are given by the relation

$$\varepsilon_i(\mathbf{p},\mathbf{r})=\mathbf{p}^2/2m+\lambda_i(\mathbf{r}),\quad i=p\uparrow,\ p\downarrow,\ n\uparrow,\ n\downarrow.$$

It is clear that the eigenfunctions of the operator $U(\mathbf{r})$ are linear combinations of the neutron and the proton functions.

Utilizing the scheme of calculations described above it is not difficult to obtain the expression for the total energy of the system in the presence of the condensate in the general case. Obvious generalizations consist of the fact that in the system with $N=Z$ not two, as before, but four types of nucleon states will undergo filling, while the value of the new Fermi energy of nucleons $\tilde{\varepsilon}_F$ must be obtained from the condition of conservation in the condensate field of the total number of nucleons. Restricting ourselves to terms of the fourth degree in the condensate field and expressing the combinations

$$\sum_i\langle\lambda_i^2\rangle,\quad \sum_i\langle\lambda_i^4\rangle$$

appearing in $\tilde{E}$ in terms of the coefficients of the secular equation (37), we obtain

$$E=E_0+\frac{1}{2}\langle\varphi_1^2+\varphi_2^2+\varphi_3^2\rangle+\frac{1}{2}\left(1-f^2\frac{2mp_F}{\pi^2}\right)\langle(\nabla\varphi_1)^2+(\nabla\varphi_2)^2+(\nabla\varphi_3)^2\rangle$$
$$+\frac{1}{2}\frac{f^4}{\pi^2v_F^3}\Big\{\langle(\nabla\varphi_1)^2+(\nabla\varphi_2)^2+(\nabla\varphi_3)^2\rangle^2+\frac{1}{3}\langle[(\nabla\varphi_1)^2+(\nabla\varphi_2)^2+(\nabla\varphi_3)^2]^2\rangle$$
$$+\frac{1}{3}\langle[\nabla\varphi_1\nabla\varphi_2]^2+[\nabla\varphi_1\nabla\varphi_3]^2+[\nabla\varphi_2\nabla\varphi_3]^2\rangle\Big\}. \qquad (38)$$

The fact that the coefficients in front of f^2 and f^4 differ by a factor of two compared with (33) is associated with the fact that in an isosymmetric medium the nucleon density is defined as $n=2p_F^3/3\pi^2$. From expression (38) it can be seen that for given values of $|\nabla\varphi_1|$, $|\nabla\varphi_2|$, $|\nabla\varphi_3|$ the lowest energy corresponds to the case when they are all parallel. Moreover, for a given value of

$$\langle(\nabla\varphi_1)^2+(\nabla\varphi_2)^2+(\nabla\varphi_3)^2\rangle$$

it is energywise advantageous to have the lowest value of

$$\langle[(\nabla\varphi_1)^2+(\nabla\varphi_2)^2+(\nabla\varphi_3)^2]^2\rangle.$$

It is evident that the optimal relationship between these two averages is attained under the condition[21]

$$(\nabla\varphi_1)^2+(\nabla\varphi_2)^2+(\nabla\varphi_3)^2=\text{const}. \qquad (39)$$

It can be seen that this condition can be satisfied only by isotopically asymmetric solutions. Examples of one-dimensional configurations of this type are

$$\boldsymbol{\varphi}(\mathbf{r})=a\{\cos kx,\ -\sin kx,\ 0\}, \qquad (40a)$$
$$\boldsymbol{\varphi}(\mathbf{r})=a\left\{\frac{1}{\sqrt{2}}\cos kx,\ \frac{1}{\sqrt{2}}\cos kx,\ \sin kx\right\}. \qquad (40b)$$

All one-dimensional configurations satisfying condition (39) have the same energy

$$E^{(\pi)}=\tilde{E}-E_0=-\tilde{\omega}^4/4\lambda_0, \qquad (41)$$

where

$$\tilde{\omega}^2=1+k^2-f^2k^2\frac{2mp_F}{\pi^2}\frac{1}{1+g^-},$$
$$\lambda_0=f^4k^4\frac{4}{3\pi^2v_F^3}\frac{1}{(1+g^-)^4}.$$

At the same time $a^2=-\tilde{\omega}^2/\lambda_0$. By the introduction of the constant g^- we take into account the contribution of nucleon correlations in a system with $N=Z$.[3,8] In heavy nuclei $g^-\approx1.6$.[22]

For comparison we exhibit the energy of an isotopically symmetric solution of the form

$$\boldsymbol{\varphi}(\mathbf{r})=\sqrt{2/3}\,a\{\sin kx,\ \sin kx,\ \sin kx\},$$
$$E^{(\pi)}=-{}^2/_9\,\tilde{\omega}^4/\lambda_0,\quad a^2=-{}^8/_9\,\tilde{\omega}^2/\lambda_0. \qquad (42)$$

In addition to the one-dimensional solutions (40) condition (39) is also satisfied by spherically symmetric configurations, for example by

$\varphi(\mathbf{r})=a\{\cos kr, -\sin kr, 0\}$.

It should be noted that meson fields satisfying condition (39) do not lead to modulations of density either of the neutrons or of the protons.[21]

Thus, the Thomas–Fermi method has enabled us to calculate the energy of sufficiently complicated spatial and isotopic configurations of the condensate field. An essential defect of this method is the condition that the meson field should be of long wavelength: $k^2 \ll 4p_F^2$. In[17] the energy of the two simplest configurations (40a) and (42) is evaluated utilizing perturbation theory in terms of the amplitude of the condensate field for a realistic case $k=p_F$. The results of the exact calculation confirm the qualitative conclusions reached in this section.

2. Influence of the π condensate on the distribution of density and spin of nucleons.

We obtain the spatial distribution of the particle density $n(\mathbf{r})$ and the spin density $\mathbf{s}(\mathbf{r})$ of nucleons in the condensate field. For this purpose it is convenient to introduce the density matrix for the nucleons ρ. By definition, the variation of the density matrix in an external field is given by the expression[8]

$$\delta\rho_{\lambda\lambda'}=\int\frac{d\varepsilon}{2\pi i}[\tilde{G}_{\lambda\lambda'}(\varepsilon)-G_\lambda(\varepsilon)\delta_{\lambda\lambda'}]. \tag{43}$$

Here, $\tilde{G}_{\lambda\lambda'}(\varepsilon)$ is the exact Green's function for a nucleon in an external field, in the present case the field of the condensate; G_λ is the Green's function for $\varphi=0$. The subscript λ denotes the set of variables uniquely determining the state of the nucleon. In homogeneous nuclear matter $\lambda=(\mathbf{p},\sigma)$.

A change in the density of particles in the field of the condensate is associated with the change in the density matrix by the relation[8]

$$\delta n(\mathbf{r})=\sum_{\lambda\lambda'}(\delta\rho)_{\lambda\lambda'}\Psi_{\lambda'}^{*}\Psi_\lambda, \tag{44}$$

where $\Psi_\lambda(\mathbf{r})$ is the complete system of wave functions for the nucleons in the unperturbed system. In our case these are plane waves:

$$\Psi_\lambda(\mathbf{r})=\chi_\sigma e^{i\mathbf{pr}}$$

(the normalization volume is taken to be equal to unity); χ_σ is a two-component spinor.

In the periodic field of the condensate the average density of the neutrons and of the protons does not vary, and therefore the diagonal elements do not make any contribution to (44). It is also clear that the contribution to the scalar quantity $\delta n(\mathbf{r})$ can be made only by the nondiagonal matrix elements corresponding to transitions of a nucleon with emission or absorption of an even number of condensate mesons. In an isotopically symmetric field (42) such transitions in second order in terms of the amplitude are described by the Green's function

$$\tilde{G}(\mathbf{p},\varepsilon,\mathbf{p}-2\mathbf{k},\varepsilon)=f^2k^2\frac{a^2}{2}G(\mathbf{p})G(\mathbf{p}-\mathbf{k})G(\mathbf{p}-2\mathbf{k})$$

$$-2\pi F_R^{(+)}(k,\omega=0)G(\mathbf{p})G(\mathbf{p}-2\mathbf{k})\frac{a^2}{2}. \tag{45}$$

The first term corresponds to the nucleon, and the second to the N^* (1232) resonance in the intermediate state. $F_R^{(+)}(k,\omega=0)$ is the amplitude of the resonance πN scattering[9]:

$$4\pi F_R^{(+)}(k,\omega=0)=\frac{4k^2}{1+0.23k^2}\frac{1}{\omega_R},\quad \omega_R=2.36.$$

From (44) and (45) we obtain

$$\delta n^{(n)}(\mathbf{r})=\delta n^{(p)}(\mathbf{r})=\xi^2 n\cos 2\mathbf{kr},\quad n=\bar{n}^{(n)}+\bar{n}^{(p)}. \tag{46}$$

The quantity ξ^2 taking nucleon correlations into account is given by the formula

$$\xi^2=3\frac{a^2}{v_F^2}\left\{f^2\frac{\Phi(k/2p_F)-\Phi(k/p_F)}{[1+g^-(n/n_0)^{1/3}\Phi(k/2p_F)]^2}+\pi F_R^{(+)}\frac{1}{m}\Phi\left(\frac{k}{p_F}\right)\right\}. \tag{46'}$$

For the critical values of n and k and $g^-=1.6$ we have

$$\xi^2\approx 0.5\frac{a^2}{v_F^2}\infty\frac{n-n_c}{n_c}.$$

In an analogous manner, one obtains the distribution of the spin density of the nucleons in the condensate field:

$$\mathbf{s}(\mathbf{r})=\sum_{\lambda\lambda'}(\delta\rho)_{\lambda\lambda'}\Psi_{\lambda'}^{*}\boldsymbol{\sigma}\Psi_\lambda. \tag{47}$$

A contribution to $s(\mathbf{r})$ is given only by matrix elements corresponding to the absorption or emission of an odd number of condensate mesons. For the field (42) in the lowest order in a we obtain

$$s_z^{(n)}(\mathbf{r})=-s_z^{(p)}(\mathbf{r})=e_q fk\Phi\left(\frac{k}{2p_F}\right)\left[1+g^-\left(\frac{n}{n_0}\right)^{1/3}\Phi\left(\frac{k}{2p_F}\right)\right]^{-1}\frac{2m^*p_F}{\pi^2}a\sin kz.$$

We note that in the field of the $\pi_s^+\pi^-$ condensate of the form of a standing wave (7b) there also occurs a modulation of the nucleon density with the propagation vector $2\mathbf{k}$ and an amplitude which near the critical point is proportional to the square of the amplitude of the condensate field.

At the same time in the field of the $\pi_s^+\pi^-$ condensate of the form of a traveling wave both for $Z=0$, and also for $N=Z$ the density of particles of each kind remains homogeneous while the local spin density is equal to zero. This is associated with the fact that in such a field nondiagonal transitions $\mathbf{p}\to\mathbf{p}\pm 2\mathbf{k}$ do not occur, while transitions which have spin symmetry, $\mathbf{p}\to\mathbf{p}\pm\mathbf{k}$ are accompanied by a change in the isotopic index of the nucleon. It can be easily verified that in such a field only the matrix elements of the operator $\sigma_z\tau_+$ describing the local spin-isospin distribution are different from zero.

IV. CONCLUSION

In conclusion we give a brief formulation of the principal consequences of π condensation for nuclear physics and astrophysics.

In[3,4,9] and in the present paper a theory of π condensation has been developed for infinite nuclear matter. But the calculations of the critical density of a π condensation in a system with $N=Z$ undertaken in Refs.

3, 9 have shown that it is less than the nuclear density ($n_c \approx 0.6 n_0$). This has enabled us to draw a conclusion concerning the possibility of existence of a pion condensate in atomic nuclei. In order to apply this theory to atomic nuclei in article[23] a study has been made of π condensation in a finite system. It turned out that in the interior regions of a heavy nucleus a periodic meson field arises whose amplitude vanishes at the boundary in a layer $\delta \approx 1 \ll R$. The surface energy arising as a result of this is proportional not to the whole surface of the nucleus but to the surface of the equatorial section and this favors an elongated shape of nuclei and can lead to isomerism of shape. The existence of a periodic structure must lead to the appearance of rotation bands in the spectra of spherical nuclei.

As analysis has shown, the available experimental data do not contradict an assumption of the existence of a pion condensate in atomic nuclei. Moreover, this assumption enables one to make a number of phenomena agree with experimental data. Thus, the probabilities of M1 transitions calculated taking into account a one-meson graph and π condensation[24] have turned out to be in agreement with experiment and by a factor of severalfold greater than those calculated using the shell model or according to the theory of finite Fermi systems but without taking condensation into account. The same also applies to the probabilities of β transitions and to the position of $0^-, T=1$ levels.[25] All these phenomena show that the nuclei in any case are very close to the point of condensation. In[26] it is shown that the presence of a π condensate in nuclei leads to an additional repulsive contribution to the P-wave terms of the optical potential of the π meson in a π mesic atom, which is not contradicted by experiment.

If a pion condensate does in fact actually exist in nuclei, then one should seek its manifestation in experiments involving scattering. In Ref. 27 it is shown that the anomalies in the differential cross-section for the scattering of an electron by a nucleus in the range of transmitted momenta $q \approx 600$ MeV/c can be explained by the presence in the nucleus of modulations of charge density of the form $\cos 2k_0 z$ (cf., (46)) which result from a condensate field (42) of amplitude $a \approx 0.1$. Inelastic scattering of polarized electrons and protons by oriented nuclei must depend critically on the presence of a π condensate. Corresponding anomalies in this case must become apparent at values of transferred momentum lower by a factor of two $q \approx 300$ MeV/c.

Another, and possibly the most interesting, consequence of the theory which was formulated already in 1971[1] is the possibility of existence of new objects of a type similar to nuclei: superdense neutron and supercharged nuclei.[28] The stability of such nuclei is determined by the negative energy of the π condensate. So far it is difficult to say whether such anomalous nuclei will be stable or metastable. In order to obtain an answer to this question detailed calculations are required of the energy of a developed π condensate, which are now only beginning to be undertaken. But there is no doubt that attempts to discover anomalous nuclei experimentally in collisions of heavy ions, in cosmic rays etc. are both useful and timely.

One of the most important effects of π condensation in neutron stars is the "softening" of the equation of state of neutron matter compared to the one calculated without taking the phase transition into account. This must lead to a noticeable decrease in the maximum mass of stable neutron stars. The presence of a π condensate in neutron stars can lead to interesting electromagnetic effects. In particular, a system with such a condensate must apparently possess the property of superconductivity. So far it is not clear what kind of a macrostructure will be characteristic of a π condensate in a large system. Apparently, domains will be formed with different directions of the propagation vector in neighboring domains. Of interest is the question of the temperature properties of a π condensate: the critical temperature of transition into the normal state, the heat capacity of the system etc. The resolution of these questions will enable one to determine the role played by π condensation in the cooling of neutron stars.

All these questions are under investigation at the present time.

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1)From time to time these results are derived anew (cf., for example,[19]).

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Description of deep inelastic processes in the compound quark model

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Deep-inelastic lepton–hadron scattering processes are considered within the compound quasi-nuclear quark model. Both incoherent processes of scattering by individual quarks and coherent processes of scattering by bound systems of quarks (diquarks and triquarks) are taken into account. Scattering by diquarks is dominant for $x\approx 2/3$, and by triquarks for $x\approx 1$. The total contribution of diquarks to the structure functions is estimated to be approximately 10%, and that of triquarks to be about 1%.

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1. INTRODUCTION

The study of deep-inelastic lepton-hadron scattering processes makes it possible to obtain valuable information on the structure of hadrons. The most striking experimental fact has been the Bjorken scale invariance,[1] which can be explained in the framework of Feynman's parton picture.[2] According to the parton ideas a hadron consists of structureless particles—partons, each of which carries a definite fraction x of the total hadron momentum. We may consider quarks as the partons, and such a quark-parton model gives a fairly good description of deep-inelastic processes.[3–6]

There are, however, fairly serious indications that the quarks in a hadron exist not only as carriers of quantum numbers but also as real spatially separated objects. Such a quasi-nuclear nonrelativistic model of hadrons enables us to give a good description of their static properties,[7,8] makes the successes of $SU(6)$ symmetry comprehensible, and leads to a number of relations, in agreement with experiment,[9] between the total hadron cross sections. The spatial separation of the quarks in a hadron is indicated by the presence of dips in the differential cross sections for pp-scattering at high energies[10] and by the symmetry of the inclusive spectra in the center-of-mass frame of the colliding quarks in meson-nucleon interactions.[11] In a model with spatially separated quarks each quark carries, on average, $\frac{1}{3}$ of the momentum of the nucleon. Therefore, deep-inelastic processes in the region $x>\frac{1}{3}$ can arise as a result of coherent scattering of the lepton by several quarks.[12,13]

It is obvious that, in the given case, in the scattering of a lepton with a large momentum-transfer squared q^2, we can disregard the coherent interaction with quark-partons only when they are positioned at the same impact-parameter distance: the contribution of such processes falls rapidly for $-q^2R^2\gg 1$, where R is the hadron radius ($R^2\approx 2$ GeV^{-2}). For $-q^2>1$ GeV^2 the lepton interacts coherently with quarks separated by relatively small distances of the order of their radius r_0. The lepton can begin to "feel" the structure of such formations of quarks (two quarks form a diquark and three quarks a triquark) only when $-q^2r_0^2\gg 1$. (The quark-parton picture for strong interactions makes it possible to estimate the square of the quark radius: $r_0^2\sim 0.25$ GeV^{-2}; see below, and also in[10,14].) For small transfers $-q^2\lesssim r_0^{-2}\approx 4$ GeV^2, the lepton interacts with diquarks and triquarks as with a single whole. The problem of the interaction of a lepton with diquarks and triquarks for $-q^2>4$ GeV^2 cannot be solved uniquely *a priori* at the present time. Here it is possible that there is a rapid decrease of the form factors of the diquarks and triquarks and equally possible that there is a rather long delay before the onset of the fall-off regime (if the intrinsic radii of the diquarks and triquarks are less than the quark radius). The interactions between the quarks can also be such that the form factors of the diquarks or triquarks will not fall with increasing q^2. In this case, the corresponding diquark or triquark can be regarded as a new type of parton.

In the present paper we consider the coherent interaction of leptons with quarks, when they are at com-