tioned in Sec. 3 that the requirement that the absorption be small in comparison with the scattering can in all cases be satisfied for gases.

5. CONCLUSION

Thus, the cooling of matter by a high-frequency field, which seems paradoxical at first glance, does indeed follow from rather elementary considerations. The effect lends itself to experimental observation and one might think also to applications.

In this paper the cooling problem was considered under conditions of a relatively weak external alternating field. With increasing field, various unaccounted-for nonlinear processes in the system will come into play (in particular, the nonlinearity of Eqs. (1), (3), and (11) with respect to q). It is important that higher-order nonlinearities can either weaken or enhance the cooling effect (depending on the sign of the anharmonicity). We note also (see[4] on this subject) that unidirectional energy fluxes are produced not only at the fundamental resonance $\omega \sim \omega_k$, but also in nonlinear resonances of higher orders (e.g., at the parametric resonance $\omega \sim 2\omega_k$).

1)The quenching of slow motions in high-frequency resonances is always accompanied by changes in the effective rigidity of these motions. For the examples discussed in the text, the changes in the susceptibilities of the slow motions are connected by the Kramers-Kronig relations.

2)At $f_k \equiv 0$ in the thermodynamic state, the average work above the thermal radiation background at fluctuations of ϵ_- is always equal to zero. It can be disregarded also in the case of small deviations from this regime. On the other hand if $f_k(t)$ causes accumulation of a finite difference between the temperatures of the medium and of the radiation (i.e., the temperature of the resonator walls), then work is performed on the average, i.e., a correlation appears between ϵ_- and $f^{(c)}$. The resultant heat flux ultimately balances the flux (13); it is this which determines the establishment of the stationary temperature regime of the system.

3)Excitations with large free paths usually fluctuate slowly in time, i.e., they contribute to $B_k(\Omega)$ at $\Omega \approx 0$. However, as $\Omega \to 0 (\Omega \ll \delta_k)$ the factor preceding B_k in (13) is small, so that the contribution to P from these fluctuations is negligible.

4)We recall that H^{-1} is the thickness of the medium over which the light intensity is attenuated by the scattering by a factor 2.7.

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Photoelectric and acousto-electric fields in superconductors

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If a superconductor is exposed to microwave radiation or if sound propagates through a superconductor, stationary electric fields arise in such a superconductor and decrease with the distance from the boundary. We obtain equations which describe the distribution of these fields and the boundary conditions for them. We discuss methods of observation and find the correction to the frequency of the Josephson radiation if one of the superconductors is irradiated by uhf radiation or sound.

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1. INTRODUCTION

We study in the present paper how electric fields arise in a superconductor under the action of microwave radiation or longitudinal sound. Rieger, Scalapino, and Mercereau[1] and Tinkham and Clarke[2] were the first to indicate the possibility that there might exist stationary electric fields in superconductors. They showed that if a current passes through a S–I–N contact, electric fields arise in the superconductor which decrease far from the boundary at a diffusion distance $L^2 = D\tau_Q$ in superconductors with a gap[2] and at a coherence length in gapless superconductors.[1] The time τ_Q of the relaxation of the excitations between two branches of the spectrum $\xi_p > 0$ and $\xi_p < 0$ ($\xi_p = p^2/2m - \mu$, $\mathbf{p}$ is the quasi-momentum of the electrons, μ the chemical potential of the normal metal, and D the diffusion coefficient of the normal electrons) caused, for in-

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stance, by phonons, increases when the transition point T_c is approached like $(T_c - T)^{-1/2}$.[2]

Artemenko and Volkov have in a recent paper[3] analyzed the appearance of a thermo-electric field in a superconductor. The existence of these fields is connected with the fact that under the action of an external perturbation there occurs a redistribution of quasi-particles over the two branches of the spectrum $\xi_{\mathbf{p}} > 0$ and $\xi_{\mathbf{p}} < 0$. Such an asymmetric distribution means that the quasi-particles produce a space charge. Because of the neutrality condition this charge is compensated by a redistribution of the superfluid component and this leads to the occurrence of a gradient in the chemical potential Φ. However, since the force acting on the condensate and leading to its acceleration is equal to

$$\partial \mathbf{p}_s/\partial t = e\mathbf{E} + \nabla\Phi = e\mathbf{F}, \tag{1}$$

there appears in the stationary state an electric field

$$\mathbf{E} = -\frac{1}{e}\nabla\Phi. \tag{2}$$

We emphasize that this field is the gradient of a scalar potential φ and only in the conditions given is it proportional to the gradient of the chemical potential.

We use for the description of the photo- and acousto-electric fields the kinetic equation[4]

$$\frac{\partial n_{\mathbf{p}}}{\partial t} + \frac{\partial \tilde{\varepsilon}_{\mathbf{p}}}{\partial \mathbf{p}}\frac{\partial n_{\mathbf{p}}}{\partial \mathbf{r}} - \frac{\partial \tilde{\varepsilon}_{\mathbf{p}}}{\partial \mathbf{r}}\frac{\partial n_{\mathbf{p}}}{\partial \mathbf{p}} + J\{n_{\mathbf{p}}\} = 0 \tag{3}$$

and the neutrality condition

$$\delta N = \delta \int d\tau_{\mathbf{p}} [u_{\mathbf{p}}^2 n_{\mathbf{p}} + v_{\mathbf{p}}^2 (1 - n_{\mathbf{p}})]. \tag{4}$$

When a sound wave or uhf radiation is present

$$\tilde{\varepsilon}_{\mathbf{p}} = \varepsilon_{\mathbf{p}} + \mathbf{p}_s\mathbf{v}, \quad \varepsilon_{\mathbf{p}} = (\tilde{\xi}_{\mathbf{p}}^2 + \Delta^2)^{1/2},$$
$$\tilde{\xi}_{\mathbf{p}} = \xi_{\mathbf{p}} + \Phi + \lambda_{ik}(\mathbf{p}) u_{ik} + \gamma_{iklm}(\mathbf{p}) u_{ik} u_{lm} + p_s^2/2m; \tag{5}$$

$$\Phi = \frac{1}{2}\left(\frac{\partial \chi}{\partial t} + e\varphi\right), \quad \mathbf{p}_s = \frac{1}{2m}\left(\nabla\chi - \frac{2e}{c}\mathbf{A}\right), \tag{6}$$

$$u_{\mathbf{p}}^2 = {}^1/_2 (1 + \tilde{\xi}_{\mathbf{p}}/\varepsilon_{\mathbf{p}}), \quad v_{\mathbf{p}}^2 = {}^1/_2 (1 - \tilde{\xi}_{\mathbf{p}}/\varepsilon_{\mathbf{p}}). \tag{7}$$

Here $d\tau_{\mathbf{p}} = 2d^3p/(2\pi)^3$, χ is the phase of the condensate wavefunction, $\lambda_{ik}(\mathbf{p})$ and $\gamma_{iklm}(\mathbf{p})$ are the constants of the linear and non-linear interaction between the electrons and sound in a normal metal.

It follows from (1) that $\frac{1}{2}\partial\chi/\partial t$ is the electrochemical potential. If we write the distribution function in the form $n_{\mathbf{p}} = n_0(\tilde{\varepsilon}_{\mathbf{p}}) + f_{\mathbf{p}}$, where $n_0(\tilde{\varepsilon}_{\mathbf{p}})$ is the equilibrium distribution function of the excitations, we have from (4)

$$\Phi + \overline{\lambda_{ik}(\mathbf{p})}\, u_{ik} + \overline{\gamma_{iklm}(\mathbf{p})}\, u_{ik} u_{lm} + \frac{p_s^2}{2m} = \frac{\partial \mu}{\partial N} \int d\tau_{\mathbf{p}} \frac{\tilde{\xi}_{\mathbf{p}}}{\varepsilon_{\mathbf{p}}} f_{\mathbf{p}}. \tag{8}$$

Here $\overline{\lambda_{ik}(\mathbf{p})}$ and $\overline{\gamma_{iklm}(\mathbf{p})}$ are the quantities $\lambda_{ik}(\mathbf{p})$ and $\gamma_{iklm}(\mathbf{p})$ averaged over all directions of $\mathbf{p}$. Equation (8) is the equation for determining Φ which through Eq. (2) is connected with the electric field $\mathbf{E}$.

2. THE ACOUSTO-ELECTRIC FIELDS IN SUPERCONDUCTORS

Let there be a longitudinal sound wave incident on the surface of a superconductor. It then follows in zeroth approximation from the neutrality condition that (see, e.g. [5])

$$\Phi = -\overline{\lambda_{ik}(\mathbf{p})}\, u_{ik},$$

while the effective linear potential for the interaction with the wave is

$$\Phi_{\mathbf{p}} = [\lambda_{ik}(\mathbf{p}) - \overline{\lambda_{ik}(\mathbf{p})}]\, u_{ik}. \tag{9}$$

Linearizing the kinetic equation and changing to Fourier components we have for the linear correction to the distribution function

$$f_{\mathbf{p}}^{(1)}(\mathbf{q}, \omega) = \frac{i\omega}{-i\omega + i\mathbf{q}\mathbf{v}\xi_{\mathbf{p}}/\varepsilon_{\mathbf{p}} + I_{\mathbf{p}}} \Phi_{\mathbf{p}} \frac{\xi_{\mathbf{p}}}{\varepsilon_{\mathbf{p}}} \frac{\partial n_0}{\partial \varepsilon_{\mathbf{p}}}, \tag{10}$$

where $I_{\mathbf{p}}$ is the linearized operator for the collisions of the excitations with impurities and phonons, while we have chosen $n_0(\tilde{\varepsilon}_{\mathbf{p}})$ as the zeroth approximation.

To obtain an equation for the stationary chemical potential we average Eq. (8) over time and take the Fourier transform with respect to the coordinates. We then have

$$\langle\Phi\rangle_{\mathbf{k}} + \overline{\gamma_{iklm}(\mathbf{p})}\, \langle u_{ik} u_{lm}\rangle_{\mathbf{k}} = \frac{\partial \mu}{\partial N} \int d\tau_{\mathbf{p}} \frac{\xi_{\mathbf{p}}}{\varepsilon_{\mathbf{p}}} \langle f_{\mathbf{p}}^{(2)}\rangle_{\mathbf{k}}. \tag{11}$$

The angle brackets indicate time-averages. We have from the kinetic equation for the function $\langle f_{\mathbf{p}}^{(2)}\rangle_{\mathbf{k}}$

$$\left(i\mathbf{k}\mathbf{v}\frac{\xi_{\mathbf{p}}}{\varepsilon_{\mathbf{p}}} + \hat{I}_{\mathbf{p}}\right)\langle f_{\mathbf{p}}^{(2)}\rangle_{\mathbf{k}} = \frac{\xi_{\mathbf{p}}}{\varepsilon_{\mathbf{p}}}\left\langle \frac{\partial \Phi_{\mathbf{p}}}{\partial \mathbf{r}} \frac{\partial f_{\mathbf{p}}^{(1)}}{\partial \mathbf{p}} \right\rangle_{\mathbf{k}}$$
$$-\mathbf{v}\frac{\partial}{\partial \xi_{\mathbf{p}}}\frac{\xi_{\mathbf{p}}}{\varepsilon_{\mathbf{p}}}\left\langle \Phi_{\mathbf{p}} \frac{\partial f_{\mathbf{p}}^{(1)}}{\partial \mathbf{r}} \right\rangle_{\mathbf{k}} - \langle \hat{I}^{(1)}\{f_{\mathbf{p}}^{(1)}\}\rangle_{\mathbf{k}} - \langle \mathbf{p}\dot{\mathbf{v}}_s\rangle_{\mathbf{k}} \frac{\partial n}{\partial \varepsilon_{\mathbf{p}}}. \tag{12}$$

Here $\hat{I}^{(1)}\{f_{\mathbf{p}}^{(1)}\}$ is the term in the expansion of the collision operator which is linear in the wave amplitude and which in the case of impurity scattering can be written in the form (see Appendix)

$$\langle \hat{I}_{imp}^{(1)}\{f_{\mathbf{p}}^{(1)}\}\rangle = \left\langle \Phi_{\mathbf{p}} \frac{\partial}{\partial \xi_{\mathbf{p}}} \hat{I}_{imp}\{f_{\mathbf{p}}^{(1)}\} \right\rangle_{\mathbf{k}} - \hat{I}_{imp}\left\{\left\langle \Phi_{\mathbf{p}} \frac{\partial f_{\mathbf{p}}^{(1)}}{\partial \xi_{\mathbf{p}}} \right\rangle_{\mathbf{k}}\right\}. \tag{13}$$

Introducing a new function

$$\psi_{\mathbf{p}} = f_{\mathbf{p}}^{(2)} - \Phi_{\mathbf{p}} \partial f_{\mathbf{p}}^{(1)}/\partial \xi_{\mathbf{p}}, \tag{14}$$

we can use the equation for $f_{\mathbf{p}}^{(1)}$ to write Eq. (12), up to terms of order $\omega\tau_{ph} \ll 1$, where ω is the frequency of the wave, in the form

$$\left\{i\mathbf{k}\mathbf{v}\frac{\xi_{\mathbf{p}}}{\varepsilon_{\mathbf{p}}} + \hat{I}_{ph} + \hat{I}_{imp}\right\}\langle\psi_{\mathbf{p}}\rangle_{\mathbf{k}} = \frac{\partial}{\partial \xi_{\mathbf{p}}}\left\langle \Phi_{\mathbf{p}} \frac{\partial f_{\mathbf{p}}^{(1)}}{\partial t} \right\rangle_{\mathbf{k}}$$
$$-(\mathbf{p}\dot{\mathbf{v}}_s)\frac{\partial n_0}{\partial \varepsilon_{\mathbf{p}}} + \left\langle \frac{\partial \Phi_{\mathbf{p}}}{\partial \mathbf{r}} \frac{\xi_{\mathbf{p}}}{\varepsilon_{\mathbf{p}}} \left[\frac{\partial f_{\mathbf{p}}^{(1)}}{\partial \mathbf{p}} - \mathbf{v}\frac{\partial f_{\mathbf{p}}^{(1)}}{\partial \xi_{\mathbf{p}}}\right] \right\rangle_{\mathbf{k}} = S_{\mathbf{p}}. \tag{15}$$

We have already said that the scale length for changes in Φ and hence in $\langle\psi_{\mathbf{p}}\rangle$ is much longer than the mean free path so that we can use the diffusion approximation for $\langle\psi_{\mathbf{p}}\rangle$. To do this we multiply the equation by $\delta(\xi-\xi_{\mathbf{p}})$ and integrate over all $\mathbf{p}$. We write

$$\langle\bar{\psi}_{\mathbf{p}}\rangle=\int d\tau_{\mathbf{p}}\delta(\xi-\xi_{\mathbf{p}})\langle\psi_{\mathbf{p}}\rangle.$$

Since $\overline{I_{imp}\langle\psi_{\mathbf{p}}\rangle}=0$ we get then, subtracting from (15) the equation summed over $\mathbf{p}$ and expanding in the small parameter $kl\ll 1$,

$$\langle\psi_{\mathbf{p}}\rangle_{\mathbf{k}}=\langle\bar{\psi}_{\mathbf{p}}\rangle_{\mathbf{k}}-\hat{I}_{imp}^{-1}i\mathbf{kv}\frac{\xi}{\varepsilon}\langle\bar{\psi}_{\mathbf{p}}\rangle_{\mathbf{k}}+\hat{I}_{imp}^{-1}(S_{\mathbf{p}}-\bar{S}_{\mathbf{p}}). \tag{16}$$

Substituting (16) into (15) we get

$$\left\{Dk^2\frac{|\xi|}{\varepsilon}+\hat{I}_{ph}\right\}\langle\bar{\psi}_{\mathbf{p}}\rangle=\bar{S}_{\mathbf{p}}-\overline{i\mathbf{kv}\frac{\xi}{\varepsilon}\hat{I}_{imp}^{-1}(S_{\mathbf{p}}-\bar{S}_{\mathbf{p}})}. \tag{17}$$

Here $D=v_F^2\tau_{tr}/3$, where τ_{tr} is the transport relaxation time in the normal metal.

Multiplying (17) by $\operatorname{sign}\xi$ and integrating over all ξ we get after simple transformations

$$Dk^2\int d\tau_{\mathbf{p}}\frac{\xi_{\mathbf{p}}}{\varepsilon_{\mathbf{p}}}\langle\psi_{\mathbf{p}}\rangle_{\mathbf{k}}+\int d\tau_{\mathbf{p}}\operatorname{sign}\xi_{\mathbf{p}}\hat{I}_{ph}\langle\psi_{\mathbf{p}}\rangle_{\mathbf{k}}$$
$$=\frac{i\mathbf{k}}{m}\int d\tau_{\mathbf{p}}\tau_{tr}\frac{\xi_{\mathbf{p}}}{\varepsilon_{\mathbf{p}}}\left\langle f_{\mathbf{p}}^{(1)}\frac{\partial\Phi_{\mathbf{p}}}{\partial\mathbf{r}}\right\rangle_{\mathbf{k}}+i\mathbf{k}\dot{\mathbf{p}}_s D\int d\tau_{\mathbf{p}}\frac{|\xi_{\mathbf{p}}|}{\varepsilon_{\mathbf{p}}}\frac{\partial n_0}{\partial\varepsilon_{\mathbf{p}}}. \tag{18}$$

We have used here the fact that $f_{\mathbf{p}}^{(1)}$ changes sign if we make simultaneously the substitutions $\mathbf{p}\to-\mathbf{p}$ and $\xi_{\mathbf{p}}\to-\xi_{\mathbf{p}}$, while the collision operator preserves the symmetry properties of the function on which it acts with respect to $\xi_{\mathbf{p}}$ and $\mathbf{p}$, and we have introduced the transport relaxation time, defined as

$$\int d\tau_{\mathbf{p}}\mathbf{v}\hat{I}\{f_{\mathbf{p}}^{(1)}\}\delta(\xi_{\mathbf{p}}-\xi)=\frac{1}{\tau_{tr}}\frac{|\xi|}{\varepsilon}\int d\tau_{\mathbf{p}}\mathbf{v}f_{\mathbf{p}}^{(1)}\delta(\xi-\xi_{\mathbf{p}}). \tag{19}$$

Using the property of the collision-operator matrix elements[4]

$$I_{\mathbf{pp}_1}\frac{\partial n_0}{\partial\varepsilon_{\mathbf{p}_1}}=I_{-\mathbf{p}_1,-\mathbf{p}}\frac{\partial n_0}{\partial\varepsilon_{\mathbf{p}}}, \tag{20}$$

we can write the second term on the left-hand side of Eq. (18) in the form

$$\int d\tau_{\mathbf{p}}\langle\psi_{\mathbf{p}}\rangle_{\mathbf{k}}\left(\frac{\partial n_0}{\partial\varepsilon_{\mathbf{p}}}\right)^{-1}\hat{I}_{ph}\operatorname{sign}\xi_{\mathbf{p}}\frac{\partial n_0}{\partial\varepsilon_{\mathbf{p}}}. \tag{21}$$

Artemenko and Volkov[3] have shown that close to the transition point we can with good accuracy replace the expression

$$\left(\frac{\partial n_0}{\partial\varepsilon_{\mathbf{p}}}\right)^{-1}\hat{I}_{ph}\operatorname{sign}\xi_{\mathbf{p}}\frac{\partial n_0}{\partial\varepsilon_{\mathbf{p}}}$$

by the relaxation time so that (21) takes the form

$$\tau_Q^{-1}\int d\tau_{\mathbf{p}}\frac{\xi}{\varepsilon}\langle\psi_{\mathbf{p}}\rangle_{\mathbf{k}},\quad \tau_Q^{-1}\approx a\frac{T^3}{\Theta_D^2}\frac{\Delta}{T}\quad (a\sim 1).$$

According to Gal'perin *et al.*[6] the spatial change in the sound energy density S is

$$\frac{\partial S}{\partial\mathbf{r}}=-\Gamma_s S(\mathbf{r})=-\int d\tau_{\mathbf{p}}\left\langle\frac{\xi_{\mathbf{p}}}{\varepsilon_{\mathbf{p}}}f_{\mathbf{p}}^{(1)}\frac{\partial\Phi_{\mathbf{p}}}{\partial\mathbf{r}}\right\rangle, \tag{22}$$

where Γ_s is the absorption coefficient for sound in a superconductor. Bearing in mind that in (18) the electrons near the Fermi surface are the important ones we can take the transport relaxation time from under the integration sign. Since the term $\partial f_{\mathbf{p}}^{(1)}/\partial\xi_{\mathbf{p}}$ in $f_{\mathbf{p}}^{(2)}$ does not contribute to the expression for the potential Φ, because $f_{\mathbf{p}}^{(1)}$ is odd under the simultaneous substitutions $\mathbf{p}\to-\mathbf{p}$ and $\xi\to-\xi$, we can use (22) to get from (11) and (18)

$$\left(k^2+\frac{1}{L^2}\right)\left\{\langle\Phi\rangle_{\mathbf{k}}+\frac{\gamma}{\rho w^2}S_{\mathbf{k}}\right\}=\frac{i\mathbf{k}\Gamma_s}{N}S_{\mathbf{k}}-i\mathbf{k}\dot{\mathbf{p}}_s\frac{N_n}{N}. \tag{23}$$

Here w is the sound speed, N the total electron density, $S_{\mathbf{k}}$ the $\mathbf{k}$-th Fourier component of $S(\mathbf{r})$, $N_n=N-N_s$ the normal component density, and N_s the superfluid component density. Equation (23) describes in the $\mathbf{r}$-representation the diffusive spatial distribution of the chemical potential $\langle\Phi\rangle$ when a source is present.

Before discussing the boundary conditions to Eq. (23) we find an expression for the current in the superconductor when $\nabla\Phi$ and the second wave are present:

$$\mathbf{J}=e\int d\tau_{\mathbf{p}}\mathbf{v}f_{\mathbf{p}}+eN_s\mathbf{v}_s. \tag{24}$$

Using (15), (16), and (14) we get the following expression for the current:

$$\langle J_i\rangle=eN_s v_{si}+\frac{1}{e}\sigma_{ik}\dot{p}_{sk}+J_i^{ac}-eD_{ik}\frac{\partial\nu_s}{\partial x_k}, \tag{25}$$

where

$$J_i^{ac}=-\frac{e}{m}\int d\tau_{\mathbf{p}}\tau_{tr}(\varepsilon)\operatorname{sign}\xi_{\mathbf{p}}\left\langle f_{\mathbf{p}}^{(1)}\frac{\partial\Phi_{\mathbf{p}}}{\partial\mathbf{r}}\right\rangle$$

is the acousto-electric current, evaluated and analyzed in[6]

$$\nu_s=\int d\tau_{\mathbf{p}}\operatorname{sign}\xi_{\mathbf{p}}\langle\psi_{\mathbf{p}}\rangle,$$

$$\sigma_{ik}=-e^2\int d\tau_{\mathbf{p}}v_i I_{\mathbf{p}}^{-1}v_k\partial n_0/\partial\varepsilon_{\mathbf{p}}.$$

We see from (25) that there is a contribution to the total current from the diffusion current which is connected with the gradient of ν_s, which turns out to be proportional to $\nabla\Phi$ only when $T\to T_c$, differing from it by a quantity of the order of $\Delta/T\ll 1$. To find the complete set of equations it is therefore necessary to find an equation for ν_s. This can be done in a way similar to the one used to obtain the equation for Φ. To do this we multiply Eq. (17) by ε/ξ and integrate over all ξ. We then have

$$Dk^2\nu_s(\mathbf{k})+\int d\tau_{\mathbf{p}}\frac{\varepsilon_{\mathbf{p}}}{\xi_{\mathbf{p}}}\hat{I}_{ph}\langle\psi_{\mathbf{p}}\rangle$$
$$=i\mathbf{k}\int d\tau_{\mathbf{p}}\frac{\tau_{tr}}{m}\operatorname{sign}\xi_{\mathbf{p}}\left\langle f_{\mathbf{p}}^{(1)}\frac{\partial\Phi_{\mathbf{p}}}{\partial\mathbf{r}}\right\rangle_{\mathbf{k}}-i\mathbf{k}\dot{\mathbf{p}}_s\frac{1}{e^2}\sigma. \tag{26}$$

Again, using the property (20) of the operator I_{ph} and introducing the relaxation time, which by definition equals

$$\left(\frac{\partial n_0}{\partial \varepsilon_p}\right)^{-1} \hat{I}_{ph} \frac{\varepsilon_p}{\xi_p} \frac{\partial n_0}{\partial \varepsilon_p} = \frac{1}{\tau_s} \operatorname{sign} \xi_p,$$

we get an equation for $\nu_s(\mathbf{k})$ using the expression for the acousto-electric current:

$$\left(Dk^2 + \frac{1}{\tau_s}\right) \nu_s(\mathbf{k}) = -\frac{i\mathbf{k}}{e} \left\{ \mathbf{J}^{ac} + \frac{1}{e} \sigma \dot{\mathbf{p}}_s \right\}. \quad (27)$$

Since $\operatorname{div} \mathbf{J} = 0$, we have from (25) and (27)

$$\operatorname{div} N_s \mathbf{v}_s - \nu_s/\tau_s = 0. \quad (28)$$

Near the transition point τ_s^{-1} is the same as τ_Q^{-1}, differing from it by small terms of the order Δ/T. The equation for ν_s has the form of a continuity equation for the superfluid current, so that $-\nu_s$ plays the role of the deviation of the superfluid density from its equilibrium value while τ_s is the speed of coming into equilibrium. Tinkham and Clarke[2] call ν_s the imbalance. At the superconductor-vacuum boundary the component of the current density normal to the surface must vanish, $\mathbf{J}_n = 0$. On the other hand, since quasi-particles are elastically reflected from the boundary, their distribution function satisfies the condition that the flux of quasi-particles with a given energy through the boundary must vanish, i.e.,

$$\int d\Omega_p \mathbf{p}_n n_p = 0, \quad (29)$$

where $\mathbf{p}_n$ is the quasi-momentum component normal to the surface. The condition (29) is valid for both specular and for diffuse reflection from the boundary. Since the total current is described by Eq. (24), it follows from (24) and (29) that $\mathbf{v}_{sn} = 0$, i.e., the normal component of the superfluid velocity also vanishes at the boundary. These boundary conditions were used in[3].

We now consider the S–S' boundary to two superconductors and restrict ourselves to situations when the magnitude of the gap in superconductor 1 is much larger than in superconductor 2 which is close to its transition point, i.e., $T \sim T_{c2} \ll \Delta_1$. The number of quasi-particles with energies $\varepsilon > \Delta_1$ is then exponentially small. On the other hand, quasi-particles with energies $\varepsilon < \Delta_1$ undergo Andreev reflection at the boundary, i.e., their distribution function satisfies the condition

$$n(\mathbf{p}, \xi_p) = n(\mathbf{p}, -\xi_p). \quad (30)$$

From (30) and the definition of ν_s it follows that ν_s vanishes together with its derivatives up to terms of order T/μ.

We now turn to the boundary conditions for the determination of the chemical potential $\langle\Phi\rangle$. To do this we introduce the flux

$$\mathbf{i} = \int d\tau_p \mathbf{v} \frac{|\xi_p|}{\varepsilon_p} f_p. \quad (31)$$

Using (14) and (16) we have

$$\mathbf{i} = -D\nabla \left\{ \langle\Phi\rangle + \frac{\gamma}{\rho w^2} S \right\} \frac{\partial N}{\partial \mu} + \frac{D}{N} \frac{\partial N}{\partial \mu} \Gamma_s S + \frac{1}{e^2} \sigma_n \dot{\mathbf{p}}_s \frac{N_n}{N}, \quad (32)$$

where $\sigma_n = e^2 \tau_{tr} N/m$ is the conductivity of the normal metal. At the superconductor-vacuum boundary $\mathbf{v}_{sn} = 0$. Since condition (29) is satisfied at the superconductor-vacuum boundary, we also have $\mathbf{i}_n = 0$. From (32) and the vanishing of the normal component of the superfluid velocity follows that

$$\nabla \left\{ \langle\Phi\rangle + \frac{\gamma}{\rho w^2} S \right\} \bigg|_n = \frac{\Gamma_s S}{N} \bigg|_n . \quad (33)$$

At the boundary of two superconductors with widely differing gaps we have because of (30)

$$\frac{\gamma}{\rho w^2} S + \langle\Phi\rangle|_{x=0} = 0. \quad (34)$$

Using the expression for the flux $\mathbf{i}$ we can write Eq. (32) as a continuity equation:

$$\operatorname{div} \mathbf{i} + \delta n/\tau_Q = 0,$$

where $\delta n = -\langle\Phi\rangle\, \partial N/\partial\mu$ is the change in the electron density due to the change in the chemical potential.

We now find the distribution of the acousto-electric field in an open superconductor sample along which a sound wave propagates along the x-axis. The solution of Eq. (23) with the boundary condition (34) is trivial and is of the form

$$E_{ac} = -\frac{1}{e} \frac{\partial \langle\Phi\rangle}{\partial x} = \frac{S_0 \Gamma_s}{eN} \frac{1}{1-(L\Gamma_s)^2} \left[\exp\left(-\frac{x}{L}\right) - (L\Gamma_s)^2 \exp(-\Gamma_s x) \right] - \frac{1}{e} \frac{\gamma \Gamma_s}{\rho w^2} \exp(-\Gamma_s x). \quad (35)$$

The electric field thus decreases exponentially with distance over two lengths L and Γ_s^{-1}. It is interesting that at the point

$$x_0 = -\frac{2L}{1+L\Gamma_s} \ln(L\Gamma_s)$$

the electric field changes sign (when we neglect the contribution due to the non-linear interaction of the wave). It is clear that ν_s behaves similarly; because the total current vanishes the velocity $\mathbf{v}_s$ is not uniform over the sample and therefore takes on the value corresponding to the absence of an electric field and the ν_s obtained by Gal'perin *et al.*[6] only at distances $x \gg L$.

3. PHOTO-ELECTRIC FIELDS IN A SUPERCONDUCTOR

In this section we construct a theory for the appearance of stationary electric fields in a superconductor when it is irradiated by a microwave field. The physical reason for the appearance of these fields lies in the fact that the high-frequency currents induced near the surface give under the action of the magnetic microwave field a stationary Hall current of the excitations in the bulk of the superconductor as a result of which an opposite gradient of ν_s occurs which cancels that current. At the same time there appears a gradient in the chemical potential Φ as a consequence of the absence of space charges, and an electrostatic field $\mathbf{E} = -e^{-1}\nabla\Phi$.

If a microwave of frequency $\omega < \Delta$ is incident upon the superconductor-vacuum boundary, the correction to the distribution function is in the linear approximation

$$f_{\mathbf{p}}^{(1)}(\mathbf{q},\omega)=\frac{i\omega}{-i\omega+i\mathbf{q}\mathbf{v}\xi_{\mathbf{p}}/\varepsilon_{\mathbf{p}}+\hat{I}_{\mathbf{p}}}(\mathbf{p}_{s0}\mathbf{v})\frac{\partial n_0}{\partial\varepsilon_{\mathbf{p}}}, \tag{36}$$

where, as before, the zeroth-order distribution function is $n_0(\tilde{\varepsilon}_{\mathbf{p}})$. We shall use the index zero to indicate the values of the fields obtained from the solution of the linear problem.

To obtain the equation to determine the stationary fields we average (8) over the time and Fourier transform over the coordinates. In that case

$$\langle\Phi\rangle_k+\left\langle\frac{p_{s0}^2}{2m}\right\rangle_{\mathbf{k}}=\frac{\partial\mu}{\partial N}\int d\tau_{\mathbf{p}}\frac{\xi_{\mathbf{p}}}{\varepsilon_{\mathbf{p}}}\langle f_{\mathbf{p}}^{(2)}\rangle_{\mathbf{k}}. \tag{37}$$

For the function $f_{\mathbf{p}}^{(2)}$ we get from the kinetic equation

$$\left(i\mathbf{k}\mathbf{v}\frac{\xi_{\mathbf{p}}}{\varepsilon_{\mathbf{p}}}+\hat{I}_{\mathbf{p}}\right)\langle f_{\mathbf{p}}^{(2)}\rangle_{\mathbf{k}}=-\left\langle\mathbf{v}_{s0}\frac{\partial f_{\mathbf{p}}^{(1)}}{\partial\mathbf{r}}\right\rangle_{\mathbf{k}}$$
$$+\left\langle\frac{\partial}{\partial\mathbf{r}}(\mathbf{p}\mathbf{v}_{s0})\frac{\partial f_{\mathbf{p}}^{(1)}}{\partial\mathbf{p}}\right\rangle_{\mathbf{k}}-(\mathbf{p}\dot{\mathbf{v}}_s)\frac{\partial n_0}{\partial\varepsilon}-\langle I^{(1)}\{f_{\mathbf{p}}^{(1)}\}\rangle_{\mathbf{k}}. \tag{38}$$

Using for $\langle I^{(1)}\{f_{\mathbf{p}}^{(1)}\}\rangle_{\mathbf{k}}$ its expression in terms of the linearized collision operator for scattering by impurities (see Appendix) and introducing the function

$$\psi_{\mathbf{p}}=f_{\mathbf{p}}^{(2)}-\frac{\varepsilon_{\mathbf{p}}}{\xi}\mathbf{p}_{s0}\frac{\partial f_{\mathbf{p}}}{\partial\mathbf{p}}, \tag{39}$$

we get the following equation for $\langle\psi_{\mathbf{p}}\rangle_{\mathbf{k}}$:

$$\left\{i\mathbf{k}\mathbf{v}\frac{\xi_{\mathbf{p}}}{\varepsilon_{\mathbf{p}}}+\hat{I}_{\mathbf{p}}\right\}\langle\psi_{\mathbf{p}}\rangle_{\mathbf{k}}=-(\mathbf{p}\dot{\mathbf{v}}_s)\frac{\partial n_0}{\partial\varepsilon}+$$
$$+\left\langle\mathbf{p}_{s0}\frac{\partial}{\partial\mathbf{p}}\left(\frac{\varepsilon_{\mathbf{p}}}{\xi_{\mathbf{p}}}\frac{\partial f_{\mathbf{p}}}{\partial t}\right)\right\rangle_{\mathbf{k}}+\frac{e}{c}\left\langle[\mathbf{v}\times\mathbf{H}_0]\frac{\partial f_{\mathbf{p}}}{\partial\mathbf{p}}\right\rangle_{\mathbf{k}}, \tag{40}$$

which is valid up to terms of order $\lambda/\tau_{ph}v_F \ll 1$, where λ is the penetration depth of the high-frequency field. This parameter was used by us in deriving (40) and enabled us to drop the term linear in the wave amplitude in the expansion of the phonon-scattering operator.

The subsequent procedure for obtaining the equation is fully analogous to how we derived the corresponding equation for sound. We therefore only give the final expression, bearing in mind that microwave fields are transverse, i.e., that $\mathbf{k}\perp\mathbf{p}_{s0}$, $\mathbf{H}_0$:

$$\left(-\nabla^2+\frac{1}{L^2}\right)\left\langle\Phi+\frac{p_s^2}{2m}\right\rangle=\operatorname{div}\frac{1}{Nc}\langle[(\mathbf{J}_0-eN_s\mathbf{v}_{s0})\times\mathbf{H}_0]\rangle. \tag{41}$$

Here

$$\mathbf{J}_0(\omega)=-i\omega\sigma(\omega)\mathbf{p}_{s0}/e+eN_s\mathbf{v}_{s0}. \tag{42}$$

It is necessary to note that Eq. (41) is valid only in the normal skin effect case when the penetration depth of the microwave field into the superconductor is much longer than the mean free path. This is just the reason why in Eq. (42) for the current the uhf conductivity at $\mathbf{q}=0$ occurs,

$$\sigma_{ik}(\omega)=-e^2\int d\tau_{\mathbf{p}}v_i\frac{1}{-i\omega+\hat{I}_{\mathbf{p}}}v_k\frac{\partial n_0}{\partial\varepsilon_{\mathbf{p}}}. \tag{43}$$

To obtain the boundary conditions for Eq. (41) we proceed similarly to what we did before, viz.: we evaluate the flux $\mathbf{i}$. We have

$$\mathbf{i}=\frac{1}{e^2}\sigma_n\dot{\mathbf{p}}_s\frac{N_n}{N}-D\frac{\partial N}{\partial\mu}\left\{\nabla\left\langle\Phi+\frac{p_{s0}^2}{2m}\right\rangle+\frac{1}{Nc}\langle[(\mathbf{J}_0-eN_s\mathbf{v}_{s0})\times\mathbf{H}_0]\rangle\right\} \tag{44}$$

The boundary condition is obtained from the requirement that on the boundary $\mathbf{i}_n=0$ and $\mathbf{v}_{sn}=0$:

$$\nabla\left\langle\Phi+\frac{p_{s0}^2}{2m}\right\rangle\Big|_n=-\frac{1}{Nc}\langle[(\mathbf{J}_0-eN_s\mathbf{v}_{s0})\times\mathbf{H}_0]\rangle|_n\,. \tag{45}$$

in the case $\omega\tau_{tr}\gg 1$

$$\mathbf{J}_0(\omega)-eN_s\mathbf{v}_{s0}=eN_n\mathbf{v}_{s0}.$$

As a result (41) takes a simple form:

$$\left(-\nabla^2+\frac{1}{L^2}\right)\left\langle\Phi+\frac{p_{s0}^2}{2m}\right\rangle=\frac{e}{c}\frac{N_n}{N}\operatorname{div}\langle[\mathbf{v}_{s0}\times\mathbf{H}_0]\rangle \tag{41a}$$

with the corresponding boundary condition

$$\nabla\left\langle\Phi+\frac{p_{s0}^2}{2m}\right\rangle\Big|_n=-\frac{e}{c}\frac{N_n}{N}\langle[\mathbf{v}_{s0}\times\mathbf{H}_0]\rangle|_n\,. \tag{45a}$$

If, however, $\omega\tau_{tr}\ll 1$, the imaginary part of the conductivity is small compared to the real part and

$$\mathbf{J}_0-eN_s\mathbf{v}_{s0}=\operatorname{Re}\sigma(\omega)\mathbf{E}_0(\omega).$$

The expression for $\operatorname{Re}\sigma(\omega)$ has been analyzed in detail, for instance, in[7] and close to T_c it is, with logarithmic accuracy, the same as the conductivity of the normal metal.

The electric field distribution obtained from the solution of Eq. (41) with boundary condition (45) has a form similar to the distribution for the acousto-electric field, with only that difference that Γ_s^- is replaced by the penetration depth of the microwave field into the superconductor which at low frequencies is the London penetration depth and at higher frequencies is the skin penetration depth. The equation determining the degree of imbalance has the form (28). There occurs then in the expression for the current a term connected with the drag of the excitations by the microwave field and analogous to the acoustic-electric current. Bearing in mind that the scale of the change in ν_s is much longer than the mean free path we have for the photo-electric current

$$\langle J^{\rm photo}\rangle=\frac{e^2}{mc}\int d\tau_p\,\tau_{tr}(\varepsilon)\langle[\mathbf{v}\times\mathbf{H}_0]f_p^{(1)}\rangle, \quad (46)$$

where $\tau_{tr}(\varepsilon)$ is the total transport relaxation time:

$$\frac{1}{\tau_{tr}}=\frac{1}{\tau_{imp}}\frac{|\xi_p|}{\varepsilon_p}+\frac{1}{\tau_p};$$

$1/\tau_p \propto T^5/\Theta_D^4$ is the time for the relaxation of the momentum by phonons. When $\omega\tau_{tr}\gg 1$ we have $f_p^{(1)} \approx -(\mathbf{p}\mathbf{v}_{s0})\partial n_0/\partial\varepsilon_p$ and

$$\langle J^{\rm photo}\rangle=\frac{1}{c}\sigma_s(0)\langle[\mathbf{v}_{s0}\times\mathbf{H}_0]\rangle, \quad (47)$$

where $\sigma_s(0)$ is the static conductivity of the superconductor given by Eq. (43).

Near the transition point

$$\frac{\sigma_s}{\sigma_n}\approx 1+\frac{\Delta}{2T}\ln\frac{\tau_p}{\tau_{imp}}.$$

When $\omega\tau_{tr}\ll 1$

$$\langle J^{\rm photo}\rangle=\frac{\sigma_n\mu_H^s}{c}\langle[\mathbf{E}_0\times\mathbf{H}_0]\rangle, \quad (48)$$

μ_H^s is the Hall mobility in the superconductor. As $T\to T_c$[1])

$$\mu_H^s\approx\mu_n\left\{1+\frac{\Delta}{2T}\left[\frac{\tau_p}{2\tau_{imp}}+\ln\frac{\tau_p}{\tau_{imp}}\right]\right\}.$$

4. THERMOELECTRIC FIELD IN SUPERCONDUCTORS

We have already noted that the problem of a thermoelectric field in a superconductor was discussed by Artemenko and Volkov.[3] In this section we wish to make more precise a number of physical points.

When there is a temperature gradient present, the kinetic equation for the correction to the distribution function takes the form

$$\mathbf{v}\frac{\xi_p}{\varepsilon_p}\frac{\partial f_p}{\partial \mathbf{r}}+\hat{I}_pf_p=(\mathbf{v}\nabla T)\frac{\xi_p}{T}\frac{\partial n_0}{\partial\varepsilon_p}-(\mathbf{v}\dot{\mathbf{p}}_s)\frac{\partial n_0}{\partial\varepsilon_p}. \quad (49)$$

Averaging (49) over the direction of $\mathbf{p}$ we get

$$\overline{\mathbf{v}\frac{\xi_p}{\varepsilon_p}\frac{\partial f_p}{\partial \mathbf{r}}}+\overline{I_{ph}f_p}=0. \quad (50)$$

Substracting (50) from (49) and iterating in terms of $kl\ll 1$ and $\tau_{imp}\ll\tau_{ph}$ we get

$$f_p=\bar{f}_p-I_{imp}^{-1}\mathbf{v}\frac{\xi_p}{\varepsilon_p}\frac{\partial \bar{f}_p}{\partial\mathbf{r}}+I_{imp}^{-1}\left\{(\mathbf{v}\nabla T)\frac{\xi_p}{T}\frac{\partial n_0}{\partial\varepsilon_p}-(\mathbf{v}\dot{\mathbf{p}}_s)\frac{\partial n_0}{\partial\varepsilon_p}\right\}. \quad (51)$$

Substituting (51) into (50) we have for $\bar{f}_p$ the equation

$$\left\{-D\nabla^2\frac{|\xi|}{\varepsilon}+\hat{I}_{ph}\right\}\bar{f}_p=-\frac{\partial}{\partial r_i}\tau_{tr}v_i\,\mathrm{sign}\,\xi\left[(\mathbf{v}\nabla T)\frac{\xi}{T}-(\mathbf{v}\dot{\mathbf{p}}_s)\right]\frac{\partial n_0}{\partial\varepsilon}. \quad (52)$$

If we work close to the transition point we can, as before, obtain a closed equation for Φ by introducing the relaxation time for the disbalance τ_Q. To do this we multiply (52) by signξ and integrate over all ξ. As a result we have

$$\left(-\nabla^2+\frac{1}{L^2}\right)\Phi=-\frac{N_n}{N}\mathrm{div}\,\dot{\mathbf{p}}_s+e\nabla^2T\alpha_n\frac{6}{\pi^2T^2}\int_0^\infty\xi^2d\xi\frac{\partial n_0}{\partial\varepsilon}. \quad (53)$$

Here α_n is the differential thermo emf of the normal metal.

In the spatially uniform case the last term vanishes. In the stationary state $\dot{\mathbf{p}}_s$ also vanishes. We therefore obtain the equation[3]

$$(-\nabla^2+1/L^2)\Phi=0. \quad (54)$$

As before, the boundary condition arises from the vanishing of the flux $\mathbf{i}$. Without giving the detailed calculations we give only the final result

$$\nabla\Phi|_n=-e\alpha_n\nabla T\frac{6}{\pi^2T^2}\int_0^\infty\xi^2d\xi\frac{\partial n_0}{\partial\varepsilon}. \quad (55)$$

We evaluate the current when a temperature gradient is present using the distribution function (51):

$$\mathbf{J}=eN_s\mathbf{v}_s-D\nabla\nu_s-\eta_s\nabla T+\sigma_s(\mathbf{E}+e^{-1}\nabla\Phi). \quad (56)$$

Here η_s is the thermoelectric coefficient[8]:

$$\eta_s=\eta_n\frac{6}{\pi^2T^2}\int_0^\infty\varepsilon\xi\frac{\partial n_0}{\partial\varepsilon}d\xi,$$

where η_n is the thermoelectric coefficient in a normal metal. We note that the expression for the current is valid for any temperature in contrast to the equation for ν_s which is valid only close to T_c:

$$(-\nabla^2+1/L^2)\nu_s=0. \quad (57)$$

The boundary conditions are the vanishing of the normal components of $\mathbf{v}_s$ and of the total current. In the stationary case

$$\nabla\nu_s|_n=-\frac{\eta_s}{D}\nabla T=-e\frac{\partial N}{\partial\mu}\alpha_n\nabla T\frac{6}{\pi^2T^2}\int_0^\infty\varepsilon\xi\frac{\partial n_0}{\partial\varepsilon}d\xi. \quad (58)$$

It follows from (54), (55), (57), and (58) that

$$\nabla\nu_s=\frac{\partial N}{\partial\mu}\nabla\Phi\int_0^\infty\varepsilon\xi\frac{\partial n_0}{\partial\varepsilon}d\xi\Big/\int_0^\infty\xi^2\frac{\partial n_0}{\partial\varepsilon}d\xi. \quad (59)$$

Notwithstanding the fact that close to the transition point the ratio of the integrals is close to unity, the difference between Φ and ν_s is one of principle and is important far from the transition point.

As $\Delta\to 0$, the problem arises how the expressions obtained, for instance for the current, change to the corresponding ones for a normal metal. We discuss this using the example of the thermoelectric effect. As $\Delta\to 0$ we have $N_s\to 0$ and the superfluid current therefore vanishes. There arises between the conductivity and the diffusion coefficient in the normal metal the Einstein relation, and we can thus neglect $\nabla\Phi$, introducing a new chemical potential:

$$\nabla\mu=\nabla\Phi-\frac{\partial N}{\partial\mu}\nabla v_s.$$

As a result, as $\Delta\to 0$ (53) takes on the usual form of the expression for the current in a normal metal when the Ohmic current is proportional to the gradient of the electrochemical potential. In conclusion we dwell upon the ways of observing electric fields in superconductors. It has been noted already[3] that the thermoelectric fields, like the acousto-electric ones can be measured using Tinkham and Clarke's method.[2] In that experiment the quantity Φ is measured directly. On the other hand, as the superfluid velocity reaches its stationary value only at distances much larger than L we can directly measure the spatial dependence of v_s by studying the way the phase difference which arises, for instance, due to the acousto-electric effect[6] on the sample size.

Yet another method for measuring Φ consists in studying the frequency shift of the microwave radiation in a Josephson junction. It was shown in[1], where the corresponding measurement scheme is described, that the frequency shift of the Josephson radiation, when one of the superconductors is in equilibrium while in the other one the value of the chemical potential Φ is non-vanishing, is

$$\Delta\omega(x)=2\Phi(x)/\hbar. \tag{60}$$

We assume that the contact is a point contact and one can therefore measure the chemical potential in a given point. We estimate the change in the frequency of the radiation. According to (35)

$$\Delta\omega_{max}=\frac{2I_0}{N\hbar w}\frac{L\Gamma_s}{1+L\Gamma_s}, \tag{61}$$

where $I_0=wS_0$ is the sound energy flux density and w is the sound velocity. If $I_0\sim 1\ \mathrm{W/cm^2}$, $N\sim 10^{22}\ \mathrm{cm^{-3}}$, $L\sim 10^{-2}$ cm, $\Gamma_s\sim 10\ \mathrm{cm^{-1}}$, $w=3\times 10^5$ cm/s, we have $\Delta\omega\sim 10^6\ \mathrm{s^{-1}}$ and can easily be measured.

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APPENDIX

We show how the term linear in the wave amplitude in the expansion of the collision operator reduces to Eq. (13) in the case of impurity scattering.

We have

$$\langle\hat I^{(1)}_{imp}\{f^{(1)}_p\}\rangle=\left\langle p_{s0}\int\frac{d^3p'}{(2\pi)^3}A_{pp'}(v-v')\left[\frac{\partial}{\partial\varepsilon_p}\delta(\varepsilon_p-\varepsilon_{p'})\right](f^{(1)}_p-f^{(1)}_{p'})\right\rangle$$
$$+\left\langle\Phi_p\int\frac{d^3p'}{(2\pi)^3}\left\{\frac{\partial}{\partial\xi_p}[A_{pp'}\delta(\varepsilon_p-\varepsilon_{p'})]+\frac{\partial}{\partial\xi_{p'}}[A_{pp'}\delta(\varepsilon_p-\varepsilon_{p'})]\right\}(f^{(1)}_p-f^{(1)}_{p'})\right\rangle. \tag{A.1}$$

Here

$$A_{pp'}=2\pi N_i\frac{(2\pi)^4}{m^2}|f_{pp'}|^2(u_pu_{p'}-v_pv_{p'})^2, \tag{A.2}$$

N_i is the impurity density and $f_{pp'}$ is the amplitude for the scattering of the electrons in the normal metal by the impurities.

By a simple partial integration we can write Eq. (A.1) in the form

$$\langle\hat I^{(1)}_{imp}\{f^{(1)}_p\}\rangle=\left\langle\Phi_p\frac{\partial}{\partial\xi_p}\hat I_{imp}\{f^{(1)}_p\}\right\rangle-\hat I_{imp}\left\{\left\langle\Phi_p\frac{\partial f^{(1)}_p}{\partial\xi_p}\right\rangle\right\}$$
$$-\left\langle p_{s0}\int\frac{d^3p'}{(2\pi)^3}\delta(\varepsilon_p-\varepsilon_{p'})\left[\frac{\partial}{\partial p}\left(\frac{\varepsilon_p}{\xi_p}A_{pp'}\right)+\frac{\partial}{\partial p'}\left(\frac{\varepsilon_{p'}}{\xi_{p'}}A_{pp'}\right)\right](f^{(1)}_p-f^{(1)}_{p'})\right\rangle$$
$$-\hat I_{imp}\left\{\frac{\varepsilon_p}{\xi_p}\left\langle p_{s0}\frac{\partial f^{(1)}_p}{\partial p}\right\rangle\right\}+\left\langle p_{s0}\frac{\partial}{\partial p}\left[\frac{\varepsilon_p}{\xi_p}\hat I_{imp}\{f^{(1)}_p\}\right]\right\rangle. \tag{A.3}$$

One verifies easily that if we neglect terms in $p_{s0}/p_F\ll 1$ which arise when differentiating the scattering amplitude $f_{pp'}$ we find the following relation

$$p_{s0}\frac{\partial}{\partial p}\left(\frac{\varepsilon_p}{\xi_p}A_{pp'}\right)=0. \tag{A.4}$$

Indeed, using the equality $\varepsilon_p=\varepsilon_{p'}$ we have

$$\frac{\partial}{\partial p}\left[\frac{\varepsilon_p}{\xi_p}\left(1+\frac{\xi_p\xi_{p'}-\Delta^2}{\varepsilon_p\varepsilon_{p'}}\right)\right]=0.$$

We then finally get

$$\langle I^{(1)}_{imp}\{f^{(1)}_p\}\rangle=\left\langle\left[\Phi_p\frac{\partial}{\partial\xi_p}+p_{s0}\frac{\partial}{\partial p}\frac{\varepsilon_p}{\xi_p}\right]\hat I_{imp}\{f^{(1)}_p\}\right\rangle$$
$$-\hat I_{imp}\left\{\left\langle\Phi_p\frac{\partial f^{(1)}_p}{\partial\xi_p}+\frac{\varepsilon_p}{\xi_p}p_{s0}\frac{\partial f^{(1)}_p}{\partial p}\right\rangle\right\}. \tag{A.5}$$

1)We note that the strong dependence of the Hall mobility in a superconductor on the phonon relaxation time is connected with the fact that this mobility is proportional to τ^2_{tr} as in a normal metal. Since the impurity relaxation time increases as $\xi_p\to 0$, excitations with $\xi_p\sim\Delta\tau_{imp}/\tau_p$ make a considerable contribution to the Hall mobility. At the same time the criterion for the applicability of the initial equations is the condition $\xi\tau_{imp}\ll 1$. The criterion for the applicability for the last expression has thus the form $\Delta\tau_{imp}\gg\tau_p/\tau_{imp}$, i.e., it is exceedingly rigid. However, even when this criterion is violated, the phenomenological expression for the photo-electric current has, of course, the form (48), but with another definition of the quantity μ^s_H.

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Dynamics of a *Z*-pinch in *n*-InSb under impact ionization conditions

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A nonstationary theory of a Z-pinch in *n*-InSb is developed by taking into account quadratic volume recombination and electron-hole scattering. The time scans of the electric field intensity are calculated for a sample under prescribed current conditions. It is shown that the scans are oscillatory as a result of excitation of an ionization domain in the crystal under pinch conditions. If electron-hole scattering is taken into account the oscillations can arise only in a definite current range corresponding to a strongly developed pinch. The frequency and amplitude of the oscillations are calculated as functions of current and of the recombination and scattering parameters. The pinch radius, pinch time, and shape of the current-voltage characteristics are determined. It is shown that the dependence of the pinch radius on current is nonmonotonic (possesses a minimum) under electron-hole scattering conditions. The theoretical results are compared with available experimental data, some of which are explained for the first time. Most of the calculations were carried out with a computer.

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1. The Z pinch phenomenon in a gas plasma wherein the plasma is compressed by the magnetic field of its own current, is well known.[1,2] Much later, Glicksman and Steele[3] observed this phenomenon in the electron-hole plasma of a semiconductor by investigating the anomalies of the resistance of InSb samples with electronic conductivity (n-InSb) in the impact-ionization (interband breakdown) regime at low lattice temperatures ($T_c = 77\,^\circ$K). It turned out that at large currents ($I > 5$ A) the plasma resistance decreased when the samples were located in a longitudinal magnetic field ($\mathbf{H} \parallel \mathbf{E}$, where $\mathbf{E}$ is the electric field intensity at the sample) comparable in strength with magnetic field of the current (H_φ). Glicksman and Steele have advanced the hypothesis that the anomalous resistance of the samples, which occurs at $H = 0$, is due to pinching of the plasma. When the plasma contracts its resistance increases, since the processes of quadratic volume recombination and electron-hole scattering, which decrease the number of particles[4] and the electron mobility,[5] become stronger. The pinch effect can become weaker and the plasma resistance can decrease in a longitudinal magnetic field. Glicksman and Steele's guess was soon confirmed[6,7] by a series of experiments in which this phenomenon was revealed by using direct procedures for observing strong contraction of the plasma. It turned out[8] that the destruction of the pinch in a longitudinal magnetic field is due to the development of a screw instability,[9] which leads to an anomalous escape of plasma to the sample surface. A number of subsequent studies[10–12] have shown that the plasma contraction in n-InSb under given-current conditions evolves in time in a rather complicated manner. Thus, for sufficiently large current the time sweeps of the field intensity (E) at the sample have an oscillatory character. It was shown in[10] that the frequency and damping time of these oscillations increase with the current. On the other hand, it is noted in[11] that these oscillations appear in the current transition region to the state of strong plasma contraction. It is shown in[12] that the pinch oscillations appear in a larger range of currents when the pinch is already developed, but the oscillations vanish at larger values of I.

It will be shown here that this phenomenon has indeed a complex character and depends strongly on the initial crystal parameters that determine directly or indirectly the role of the electron-hole scattering (impurity concentration, mobilities and lifetimes of the non-equilibrium carriers, etc.). At present there are two (theoretically justified) points of view concerning the nature of these oscillations. According to[13] the oscillations of the field E can be due to the magnetothermal character of the pinch, when the heating of the lattice and the thermal ionization in the region of strong contraction play an essential role. In the given-current regime, periodic heating and cooling of the lattice in the pinch channel can take place, and the sample conductivity oscillates correspondingly. This concept, however, can hardly be used to explain the experiments performed with short pulses ($\tau_{\text{pulse}} \lesssim 10^{-6}$ sec) and relatively weak currents ($I < 20$ A), when the lattice is

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