1)We shall not pause here to consider other possible stationary solutions of this type, e.g., nonlinear waves with periodically varying amplitude.

2)We shall consider only the branch of transverse acoustic vibrations that interacts strongly with the spin branch.

3)For simplicity we neglect the effect of demagnetization.

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Superconducting contacts with a nonequilibrium electron distribution function

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The effect of nonequilibrium electrons on the current-voltage characteristics of superconducting contacts is found. In the case of bulk superconductors, when the length of the contact $a \gg \xi\tau^{1/4}$ superconductivity is stimulated in the contact and the current through the contact increases considerably under small voltages. In film contacts nonequilibrium effects lead to suppression of the superconductivity. The current-voltage characteristic in this case has a portion with negative resistance and this leads to experimentally observable voltage discontinuities.

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Superconducting contacts (film bridges, point contacts, bulk superconductors, etc.) possess, in a number of cases, volt-ampere characteristics that differ from the hyperbolic dependence found for sufficiently short contacts.[1] For example, portions corresponding to voltage discontinuities at constant current can appear in the current-voltage characteristics of contacts.[2,3] A possible explanation of these effects is that the energy distribution function of the electrons in the contact is a nonequilibrium function.

At currents exceeding the critical value a normal component of current flows through the contact and gives rise to a change in the electron distribution function. As a result the superconducting order parameter and, correspondingly, the magnitude of the superconducting current through the contact change. The changes in the current-voltage characteristic of the contact which then arise depend substantially on the dimensions of the contact.

When a current flows through the contact, the order parameter and the gap in the electron spectrum are smaller in the region of the contact than outside the contact. Electrons whose energy is less than Δ_0, the value of the gap outside the region of the contact, cannot go beyond the boundaries of the contact. For these electrons the time τ_ε for establishment of thermal equilibrium is determined by the collisions with phonons and is very long at low temperatures. Therefore, the distribution function of these electrons is greatly changed even in a weak electric field.

If the dimensions of the contact are small, the change in the distribution function of these electrons does not lead to substantial changes in the current-voltage characteristic of the contact. However, if the size a of the contact exceeds the characteristic length $\eta = \xi\tau^{1/4}$ (here ξ is the "size" of a superconducting pair and $\tau = (T - T_c)/T_c$; T_c is the critical temperature), then this change in the electron distribution function leads to stimulation of superconductivity in the contact. As a result, even in a weak electric field, a large increase in the current through the contact arises.

Electrons whose energy is higher than the value of the gap outside the contact diffuse out of the contact. Their relaxation time is determined by the diffusion rate. The effect of these electrons on the volt-ampere characteristic of the contact is substantially different for bulk-superconductor contacts (the three-dimensional case) and for film bridges. In the three-dimensional case the change in the distribution function of such electrons is small and can be disregarded. In a film, however, diffusion of the electrons is made difficult because of the two-dimensional character of their motion, and the electron distribution function is proportional to the logarithm of the long energy-relaxation time.

The sharp change in the distribution function of the electrons in a film contact decreases the order parameter in a broad region outside the contact and leads to a reduction of the superconducting current through the contact. Regions of negative resistance, which can lead to observable voltage discontinuities, appear in the current-voltage characteristic.

1. STIMULATION OF SUPERCONDUCTIVITY IN A THREE-DIMENSIONAL CONTACT

For a nonequilibrium electron distribution function, an additional term appears in the Ginzburg-Landau equation for the order parameter Δ. This term can be found with the aid of the equation for the order parameter[4]

$$\Delta=\lambda\int_{\Delta}^{\infty}\frac{\Delta}{\sqrt{\varepsilon^2-\Delta^2}}f(\varepsilon)\,d\varepsilon, \tag{1}$$

in which λ is the interaction constant and the distribution function is equal to $\frac{1}{2}[1-f(\varepsilon)]$. Equation (1) is written for the case when Δ varies sufficiently slowly with the coordinates; in particular, it can be used for sufficiently long contacts, with $a\gg\eta=\xi\tau^{1/4}$. In this case the Ginzburg-Landau equation has the form

$$\frac{\hbar\pi D}{8T}\left[\frac{\partial^2\Delta}{\partial\mathbf{r}^2}-(\nabla\varphi)^2\Delta\right]-\tau\Delta-\frac{7\zeta(3)}{8\pi^2}\frac{\Delta^3}{T^2}+\Delta\int_{\Delta}^{\infty}\left[f(\varepsilon)-\operatorname{th}\frac{\varepsilon}{2T}\right]\frac{d\varepsilon}{\sqrt{\varepsilon^2-\Delta^2}}=0, \tag{2}$$

where Δ is the modulus of the order parameter, φ is its phase and $D=v_F l/3$ is the diffusion coefficient (it is assumed that the mean free path l of the electrons is small). For the distribution function we make use of the following equation[5]:

$$\frac{1}{\tau_\varepsilon}\left[f(\varepsilon)-\operatorname{th}\frac{\varepsilon}{2T}\right]\langle\overline{\varepsilon(\varepsilon^2-\Delta^2)^{-1/2}}\rangle=\frac{\partial}{\partial\varepsilon}\left(D_\varepsilon\frac{\partial f}{\partial\varepsilon}\right), \tag{3}$$

where the energy-diffusion coefficient D_ε is given by the formula

$$D_\varepsilon=-\overline{\left\langle\frac{\partial\theta}{\partial t}\left(D\frac{\partial^2}{\partial\mathbf{r}^2}\right)^{-1}\frac{\partial\theta}{\partial t}\right\rangle},\quad \theta=\sqrt{\varepsilon^2-\Delta^2}-\langle\sqrt{\varepsilon^2-\Delta^2}\rangle. \tag{4}$$

In these formulas the brackets $\langle\,\rangle$ denote averaging over that region of the contact in which $\Delta<\varepsilon$, and the bar denotes time averaging. The boundary condition for the determination of the inverse operator in formula (4) is that the derivative along the normal to the surface $\varepsilon=\Delta(\mathbf{r})$ be equal to zero.

For contacts of length $a\ll\xi$ the second and third terms in Eq. (2) are small,[1] while the nonequilibrium term can be substantial. We shall calculate it for not too small voltages across the contact. Then, because τ_ε is large, the left-hand side of Eq. (3) is small and, therefore,

$$f(\varepsilon)=C\int_0^{\varepsilon}D_\varepsilon^{-1}\,d\varepsilon, \tag{5}$$

where the constant C is found from the condition for matching of $f(\varepsilon)$ at $\varepsilon=\Delta_0$ with its value $\tanh(\Delta_0/2T)\approx\Delta_0/2T$ outside the contact. As will be seen below, $f(\varepsilon)$ for ε close to Δ_0 depends weakly on ε and is close to the value $\Delta_0/2T$. This permits us to calculate the nonequilibrium term in the Ginzburg-Landau equation, for Δ close to Δ_0:

$$\Phi(\Delta)=\Delta\int_{\Delta}^{\Delta_0}\left[f(\varepsilon)-\operatorname{th}\frac{\varepsilon}{2T}\right]\frac{d\varepsilon}{\sqrt{\varepsilon^2-\Delta^2}}=\frac{\sqrt{2}}{3}\frac{\Delta_0^2}{T}\left(1-\frac{\Delta}{\Delta_0}\right)^{3/2}. \tag{6}$$

Here the integral over ε is taken up to the value Δ_0, since for $\varepsilon>\Delta_0$ the electron distribution function differs little from the equilibrium function.

We shall assume that the thickness of the contact is small compared with its length, so that the order parameter depends only on the coordinate x along the contact. Equation (2) then takes the form

$$\frac{\hbar\pi D}{8T}\Delta''-\frac{\hbar T}{2\pi\rho^2e^2S^2D}\frac{J_s^2}{\Delta^3}+\Phi(\Delta)=0, \tag{7}$$

where ρ is the density of states, S is the cross-sectional area of the contact, and J_s is the magnitude of the superconducting current through the contact, which is related to the gradient of the phase by the formula

$$J_s=\pi e\rho DS\frac{\Delta^2}{2T}\nabla\varphi. \tag{8}$$

In short contacts $a\ll\eta$ the nonequilibrium term $\Phi(\Delta)$ in Eq. (7) is small, and this equation coincides with the order-parameter equation used earlier.[1] In contacts of length $a\gg\eta$, on the other hand, the first term is small. In this case the modulus of the order parameter Δ is constant in almost the whole region of the contact and depends on the longitudinal coordinate x only near the edges of the contact. The magnitude of the order parameter in the contact ($\tilde{\Delta}$) is determined from the equation

$$\frac{\hbar T}{2\pi\rho^2e^2DS^2}J_s^2=\tilde{\Delta}^3\Phi(\tilde{\Delta}). \tag{9}$$

The right-hand side of this equation vanishes for $\tilde{\Delta}=0$ and $\tilde{\Delta}=\Delta_0$, and has a maximum at a certain intermediate value. To estimate the maximum value J_s^0 at which Eq. (9) still has a solution we shall use formula (6) for the function $\Phi(\Delta)$. As a result we obtain

$$J_s^0\approx 0.4e\rho S\frac{D^{1/2}\Delta_0^{5/2}}{\hbar^{1/2}T}. \tag{10}$$

This formula has been obtained to within a numerical coefficient. To find the latter it is necessary not only to know the function $\Phi(\Delta)$ for Δ close to Δ_0 but also to take into account nonequilibrium terms in the expression (8) for the current density.

The expression (10) gives a value for the current that is a/η times greater than the critical current of the contact. This sharp increase in the current through the contact occurs at comparatively low voltages (voltages inversely proportional to the long energy-relaxation time τ_ε), at which the distribution function is already nonequilibrium and determined by formula (5). On further increase of voltage the current depends comparatively weakly on the voltage, and on the volt-ampere characteristic there is a plateau at the current value J_s^0 determined by formula (10). The slow increase in the current in this region is associated with the growth of the normal component of current through the contact.

In the region of the plateau, for most of the time, a superconducting current close to J_s^0 and a comparatively weak normal current flow through the contact. The magnitude of the superconducting current is determined by the difference χ in the phases of the order parameter at the edges of the contact. Near the maximum value the superconducting current J_s depends quadratically on the phase difference and we have for the total current J through the contact

$$J=\frac{1}{R}\frac{\hbar}{2e}\dot{\chi}+J_s^0-J_s^0\frac{\eta^2}{a^2}(\chi-\chi_{max})^2, \tag{11}$$

where χ_{max} is the value of the phase difference corresponding to the maximum current and R is the resistance of the contact in the normal state. We note that the numerical coefficients in this formula have been found in order of magnitude, as in formula (10) for J_s^0.

The solution of Eq. (11) has the form

$$\chi-\chi_{max}=\frac{a}{\eta}\left(\frac{J-J_s^0}{J_s^0}\right)^{1/2}\operatorname{tg}\left[\frac{2eR}{\hbar}\frac{\eta}{a}(J_s^0(J-J_s^0))^{1/2}t\right] \tag{12}$$

and corresponds to infrequent pulses of normal current, on the background of the superconducting current J_s^0, following each other with frequency

$$\omega=\frac{2e\overline{V}}{\hbar}=\frac{2e}{\hbar}\frac{\eta}{a}R\sqrt{J_s^0(J-J_s^0)}. \tag{13}$$

Thus, the current through the contact depends quadratically on the mean voltage.

The picture obtained for the stimulation of superconductivity in a sufficiently long contact is based on the assumption that the nonequilibrium electron distribution function $f(\varepsilon)$ in the contact is greater than its equilibrium value $\tanh(\varepsilon/2T)$ for $\varepsilon<\Delta_0$. In this case, $\Phi(\Delta)>0$ and Eq. (9) has a solution. In order to find $f(\varepsilon)$, we shall calculate the function D_ε with the aid of formula (4). Here it is necessary to know the dependence of Δ on the coordinates. Comparing the first and last terms in Eq. (7), we convince ourselves that changes in the order parameter by an amount $\Delta_0-\tilde{\Delta}$ near the edge of the contact occur over a distance

$$\Delta x\sim\eta(1-\tilde{\Delta}/\Delta_0)^{-1/4}. \tag{14}$$

Assuming that Δ^2 depends linearly on x in this region, with the aid of formula (4) we obtain for the function D_ε

$$D_\varepsilon=\frac{2\hbar}{27}\Delta_0\frac{1}{t_0}\int d\tilde{\Delta}\frac{\partial\tilde{\Delta}}{\partial t}\left(\frac{\partial}{\partial\tilde{\Delta}}\frac{(\varepsilon-\tilde{\Delta})^{3/2}}{(\Delta_0-\tilde{\Delta})^{3/4}}\right)^2, \tag{15}$$

where the period $t_0=\pi\hbar/e\overline{V}$. For ε close to Δ_0, values of $\tilde{\Delta}$ close to Δ_0 are important in the integral (15). In this case the current flowing through the contact is principally the normal current, which is related to the difference in the phases at the edges of the contact by the Josephson relation $\hbar\dot{\chi}=2eRJ$. On the other hand, using formula (8) we have

$$\frac{\chi}{a}=\nabla\varphi=\frac{2TJ_s}{\pi e\rho DS\tilde{\Delta}^2}. \tag{16}$$

Using the relation (9) connecting $\tilde{\Delta}$ and J_s, for the derivative $\partial\tilde{\Delta}/\partial t$ we find

$$\frac{\partial\tilde{\Delta}}{\partial t}=\dot{\chi}\frac{\partial\tilde{\Delta}}{\partial J_s}\frac{\partial J_s}{\partial\chi}\approx-2.3\frac{e}{\hbar}J_s^0R\frac{\eta}{a}\Delta_0\left(1-\frac{\tilde{\Delta}}{\Delta_0}\right)^{1/4}. \tag{17}$$

Substituting this expression into formula (15), for ε close to Δ_0, we find

$$D_\varepsilon\approx0.1\frac{\eta}{\hbar a}e^2J_s^0R\overline{V}\Delta_0\left(1-\frac{\varepsilon}{\Delta_0}\right)^{-1/4}. \tag{18}$$

Correspondingly, using formula (5), we have for $f(\varepsilon)$:

$$f(\varepsilon)=\frac{\Delta_0}{2T}\left[1-\left(1-\frac{\varepsilon}{\Delta_0}\right)^{5/4}\right]. \tag{19}$$

It can be seen from this formula that for ε close Δ_0 the distribution function is close to the value $\Delta_0/2T$, and is greater than $\tanh(\varepsilon/2T)$, as was assumed at the start.

We shall now find the law by which the current approaches the value J_s^0 from the low-voltage side. Formulas (5) and (19) give the limiting values of the distribution function at sufficiently high voltages. We now find the voltage-dependent correction f' to the distribution function. This correction can be found by perturbation theory, by substituting the limiting value of $f(\varepsilon)$ already found into the left-hand side of formula (3). As a result, to within a numerical coefficient, we obtain

$$f'=-\frac{\hbar a}{\eta e^2}\frac{1}{\tau_\varepsilon\overline{V}}\frac{\Delta_0^2}{TJ_sR}\left(1-\frac{\varepsilon}{\Delta_0}\right)^{5/4}. \tag{20}$$

Correspondingly, we find the correction to the function $\Phi(\Delta)$ in the Ginzburg-Landau equation and then the correction to the maximum value of the current:

$$J_s-J_s^0\approx-\frac{\hbar a\Delta_0}{\eta e^2R}\frac{1}{\tau_\varepsilon\overline{V}}. \tag{21}$$

From this it can be seen that with increase of the voltage the current approaches the value J_s^0 by a hyperbolic law. Formula (21) is valid for sufficiently high voltages, when the correction to J_s^0 is small:

$$e\overline{V}>\hbar T/\tau_\varepsilon\Delta_0.$$

In the opposite case there are several regions with different dependences of current on voltage. At low voltages the distribution function differs little from $\tanh(\varepsilon/2T)$, and we can put $f=\tanh(\varepsilon/2T)$ in the right-hand side of formula (3). To determine D_ε from formula (4) it is necessary to know the dependence of the modulus of the order parameter on the coordinates; in this case, this is found in the zeroth approximation from the linear Ginzburg-Landau equation:

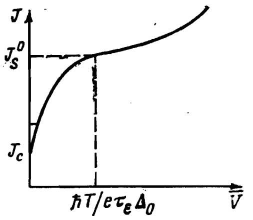

FIG. 1.

$$\Delta^2 = \frac{\Delta_0^2}{2}\left[1 + \left(\frac{x}{a}\right)^2 + \left(1 - \frac{x^2}{a^2}\right)\cos\chi\right], \tag{22}$$

where the difference χ in the phases at the edges of the contact is determined by the magnitude of the superconducting current $J_s = J_c \sin\chi$. As a result, for ε close to Δ_0, we obtain for $f(\varepsilon)$:

$$f - \operatorname{th}\frac{\varepsilon}{2T} = 5.8\frac{e^2JR}{\hbar T}\tau_\varepsilon\overline{V}\frac{a^2}{\eta^2}\left(1 - \frac{\varepsilon}{\Delta_0}\right)^{-1/2}. \tag{23}$$

For such a distribution function the nonequilibrium term $\Phi(\Delta)$ in the Ginzburg-Landau equation is given, for Δ close to Δ_0, by the formula

$$\Phi(\Delta) = 12.6\frac{e^2JR}{\hbar T}\frac{a^2}{\eta^2}\tau_\varepsilon\overline{V}\Delta_0. \tag{24}$$

Substituting the expression found for Φ into Eq. (7) for Δ, we can find the maximum superconducting current for each value of the voltage. Considering $\Phi(\Delta)$ as a perturbation, we obtain

$$J - J_c \approx 5.1\frac{e}{\hbar}J_c\tau_\varepsilon\overline{V}\frac{\Delta_0}{T}\left(\frac{a}{\eta}\right)^4. \tag{25}$$

In this region the current increases rapidly with voltage by a linear law. Formula (25) ceases to be applicable in the region of very low voltages, when the period of the current oscillations becomes longer than the energy-relaxation time τ_ε. In this case it is no longer possible to use formula (4) for D_ε, which presupposes averaging over a large number of oscillations in the time τ_ε.

Thus, for sufficiently long contacts ($a \gg \eta$), because of the long energy-relaxation time the current-ampere characteristic of the contact (Fig. 1) is found to differ from a hyperbola: the current increases rapidly at low voltages to the value J_s^0, and then varies comparatively slowly with voltage.

2. SUPERCONDUCTING FILM CONTACTS (BRIDGES)

In the two-dimensional case, because of the slow diffusion of the electrons, the distribution function at energies $\varepsilon > \Delta_0$ is also nonequilibrium. For sufficiently short contacts substantial changes in the order parameter and volt-ampere characteristic originate from electrons with energies $\varepsilon \sim T$ much greater than Δ_0. Using the kinetic equation for a normal metal we can obtain the diffusion equation for such electrons:

$$\frac{\partial f}{\partial t} = D\frac{\partial^2 f}{\partial \mathbf{r}^2} + e^2E^2D\frac{\partial^2 f}{\partial \varepsilon^2} - \frac{1}{\tau_\varepsilon}\left(f - \operatorname{th}\frac{\varepsilon}{2T}\right), \tag{26}$$

where, in the right-hand side, the first term corresponds to diffusion with respect to the coordinates, the second corresponds to diffusion with respect to the energies, and the third describes the energy relaxation.

In the region of superconductor that lies outside the contact and in which there is no electric field, the time-averaged solution of Eq. (26) in the two-dimensional case is given by Hankel functions of imaginary argument. In the region $r \ll \sqrt{D\tau_\varepsilon}$ in which substantial changes in the distribution function occur, this solution has the form

$$f - \operatorname{th}(\varepsilon/2T) = C\ln(r/2\sqrt{D\tau_\varepsilon}). \tag{27}$$

We note that the energy-relaxation time τ_ε appears only under the logarithm. This justifies the use of the simplified expression for the energy relaxation in Eq. (26).

The constant C in formula (27) is found by integrating Eq. (26) over the region in which there is an electric field:

$$C = -\frac{e^2}{2\pi}\frac{\partial^2\operatorname{th}(\varepsilon/2T)}{\partial\varepsilon^2}K, \quad K = \int\overline{E^2}d^2r. \tag{28}$$

Here, in determining C we have used the equilibrium function $\tanh(\varepsilon/2T)$, which ceases to be valid only at very high voltages, when normal current flows through the contact.

Thus, the deviation of the distribution function from the equilibrium function outside the contact falls off slowly with distance from the contact, in accordance with the law (27). Because of this, the value of the order parameter Δ_0 far from the contact also changes. To find Δ_0 we shall make use of Eq. (2), in which, for r greater than ξ, we can disregard the first term. Calculating the nonequilibrium term in this equation with the aid of formulas (27) and (28) for $f(\varepsilon)$, we obtain for the boundary value $\Delta_0 = \Delta/_{r\sim\xi}$ the expression

$$\Delta_0^2 = \Delta_{00}^2\left(1 - 0.3\frac{e^2}{|\tau|T^2}K\ln\frac{\sqrt{D\tau_\varepsilon}}{\xi}\right), \tag{29}$$

where Δ_{00} is the equilibrium value of the order parameter.

For short contacts, in the region of the contact the nonequilibrium term is small and the equations of the earlier paper are valid.[1] In particular, to find the volt-ampere characteristic of the contact we can use the formula

$$\overline{V}^2 = R^2(J^2 - J_c^2), \tag{30}$$

where, however, the critical current $J_c = \pi\Delta_0^2/4eTR$ itself already depends on the voltage. This dependence is determined with the aid of formula (29), in which it is necessary to calculate the expression for K. We shall assume that the contact is sufficiently narrow (the length a of the contact is considerably greater than its width b) and the voltage drop occurs principally in the region of the contact: $E = V/a$. In the two-dimensional case, for this it is necessary that the condition $a/b \gg \ln(\xi/b)$ be fulfilled. In the time-averaging we can

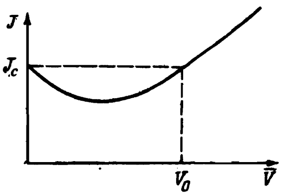

FIG. 2.

use the dependence $V(t)$ found earlier.[1] As a result we obtain $K = RJ\bar{V}b/a$, where $R = a/\sigma b^2$ is the resistance of the contact in the normal state.

For the volt-ampere characteristic of the contact we have

$$v^2 = i^2 - (1 - \alpha vi)^2; \quad \alpha = 15.2|\tau| \frac{b}{a} \ln \frac{\sqrt{D\tau_\varepsilon}}{\xi}. \tag{31}$$

Here we have introduced the dimensionless variables $v = \bar{V}/RJ_{c0}$ and $i = J/J_{c0}$, where J_{c0} is the critical current of the contact. The parameter α contains a large logarithm and, generally speaking, may be large. The current-voltage characteristic of the contact, described by formula (31), has a portion with negative resistance (Fig. 2). Therefore, on increase of the current the voltage increases discontinuously, at the critical value J_{c0}, to the value $V_0 = 2\alpha RJ_{c0}/(\alpha^2 + 1)$.

Formula (31) is valid in the region $\alpha vi > 1$. When this condition is not fulfilled, the expression (29) for Δ_0^2 becomes incorrect, since Δ_0^2 cannot be negative. For small values of Δ_0, in the Ginzburg-Landau equation (2) it is necessary to take into account the term with the derivative, which leads to an exponential dependence $\Delta(r)$ at distances $r \gtrsim \xi$. In this case, the current flowing through the contact is principally normal current, and the superconducting current is exponentially small:

$$J_s \sim \exp\left\{ -\left(\frac{e^2 RJ\bar{V}b}{\hbar DTa} \right)^{1/2} r_0 \right\}, \tag{32}$$

where the distance r_0 is determined in order of magnitude from the relation

$$|\tau| = 0.3 \frac{e^2 RJ\bar{V}b}{T^2 a} \ln \frac{\sqrt{D\tau_\varepsilon}}{r_0}.$$

We note also that the resistance R to normal current is determined by the region in the contact, of size $\sim r_0$, in which the superconductivity is destroyed, and can depend on the current and temperature.

We now find the current-voltage characteristic of long contacts in the two-dimensional case. In bridges of length $a \gg \eta$ the initial part of the characteristic is the same as in three-dimensional contacts: the current increases comparatively rapidly to the value J_s^0 determined by formula (10). This effect occurs because of the change in the distribution function of the electrons with energy $\varepsilon < \Delta_0$ inside the contact. Substantial changes in the current on further increase in voltage can arise on account of the electrons with energy $\varepsilon > \Delta_0$, which, in the two-dimensional case, change the boundary value of the order parameter. It is necessary, therefore, to find the nonequilibrium electron distribution function for $\varepsilon > \Delta_0$.

In the case of a long contact the greatest changes in the distribution function of such electrons occur at energies $\varepsilon \sim \Delta_0 \ll T$. In this case, in the diffusion equation (26) the term corresponding to diffusion with respect to the energies is now determined, in accordance with formula (3), by the expression

$$\frac{\partial}{\partial \varepsilon}\left(D_\varepsilon \frac{\partial f}{\partial \varepsilon} \right)$$

where D_ε is given by formula (4). The distribution function is then found from formula (27), where the constant C has the form

$$C = -\frac{1}{2\pi D} \frac{\partial}{\partial \varepsilon}\left(\frac{\partial f}{\partial \varepsilon} \int D_\varepsilon d^2 r \right)$$
$$= -\frac{1}{48\pi^2} \frac{e\bar{V}}{\hbar D^2} \frac{a^2 b}{T} \int \left(\frac{\partial \bar{\Delta}}{\partial t} \right)^2 \frac{\partial}{\partial \varepsilon} \left(\frac{\partial}{\partial \bar{\Delta}} \sqrt{\varepsilon^2 - \bar{\Delta}^2} \right)^2 dt. \tag{33}$$

Using formulas (6), (9) and (17) to calculate the order parameter and its time derivative, we can determine the nonequilibrium distribution function from formulas (27) and (33), and, with its help, calculate the nonequilibrium term in the Ginzburg-Landau equation. As a result we find that the boundary value of the order parameter is decreased:

$$\Delta_0^2 - \Delta_{00}^2 \approx -\frac{e^2 \bar{V} JRT}{\Delta_0} \frac{b}{a} \left(\frac{a}{\eta} \right)^{10/3} \ln \frac{\sqrt{D\tau_\varepsilon}}{\xi}. \tag{34}$$

As in the case of a short contact, this leads to a decrease of the current with increase of the voltage and, correspondingly, to the appearance of a negative-resistance part in the volt-ampere characteristic of the contact (Fig. 3). Thus, in the case of a long bridge, with increasing current the voltage first increases slowly and then, at a current of the order of J_s^0, increases discontinuously.

3. DISCUSSION OF THE RESULTS

Figures 1–3 show the possible types of volt-ampere characteristics of short ($a < \xi$) superconducting contacts with a nonequilibrium electron distribution function.

In the case of bulk-superconductor contacts the nonequilibrium electrons have energy $\varepsilon < \Delta_0$. Then the characteristic time $\hbar/\Delta_0$ determines the distance $\eta = \xi\tau^{1/4} = (D\hbar/\Delta_0)^{1/2}$ over which they can diffuse from the contact. If this distance is short compared with the

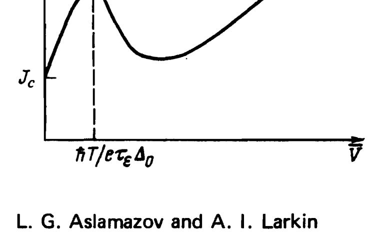

FIG. 3.

length of the contact ($a > \eta$), superconductivity is stimulated in the contact, and the current-voltage characteristic has the form shown in Fig. 1. In the opposite case ($a < \eta$), the nonequilibrium effects for three-dimensional contacts are small.

In film bridges the nonequilibrium electrons with energies $\varepsilon > \Delta_0$ lead to suppression of the superconductivity in a wide region of the contact. The dimensions of the region are determined by the distance over which the electrons diffuse in the energy-relaxation time τ_ε. This length

$$r^* \sim \sqrt{D\tau_\varepsilon} \sim \xi \sqrt{T\tau\tau_\varepsilon}$$

is usually considerably greater than the size ξ of a pair. The volt-ampere characteristic of the contact in this case has a falling part and is shown in Fig. 2.

As in the three-dimensional case, nonequilibrium electrons with energies $\varepsilon < \Delta_0$ lead to a change in the current-voltage characteristic of the bridge only when the contact is sufficiently long ($a > \eta$). As a result, such bridges can possess current-voltage characteristics of the type in Fig. 3.

In the experiments of[2,3] current-voltage characteristics of bridges, with the voltage discontinuities to which characteristics of the types in Figs. 2 and 3 lead, have been observed. This was explained by the increase in the temperature of the electrons in the contact and by the appearance of a region with the normal phase.[2]

The calculations lead to the same qualitative results as in the present paper. However, the thermal-conduction equation used in the calculations presupposes the existence of quasi-local equilibrium (slow variation of the distribution function in a region of size $\sim r^*$). This becomes invalid for contacts of small size and for not too large currents, when the size of the normal region is less than r^*. In the present paper, we use a kinetic equation for the distribution function that correctly describes the diffusion of electrons even in a short contact.

We note also that it has been assumed in the present work that the temperature of the phonon thermostat is constant. In short contacts of bulk superconductors this condition is always fulfilled. In bridges, it is necessary for this that the characteristic heat-transfer time be shorter than τ_ε. In the opposite case the volt-ampere characteristic is again given by formula (31), but the parameter α is then proportional to the logarithm of the long heat-transfer time.

The phenomenon, found in this work, of the stimulation of superconductivity in a contact would be interesting to observe experimentally. For this, however, it is necessary that a fairly restrictive condition on the length of the contact be fulfilled: $\xi\tau^{1/4} < a < \xi$. Then a considerable increase in the current through the contact sets in at a small voltage $V \sim \hbar/e\tau_\varepsilon \tau^{1/2}$.

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