Investigation of nonadiabatic effects of the motion of charged particles in a dipole trap in the presence of adiabatic perturbations

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The number of particles in a dipole trap is found to change under the action of a reversible adiabatic magnetic perturbation. The critical value of the adiabaticity parameter for the perturbing field is found: This value defines the boundary between the reversible and irreversible changes in the particle flux. The pitch-angle dependence of the critical value of the adiabaticity parameter for a static field is presented. The dependence of the particle density in the trap on the amplitude and duration of the perturbation is determined.

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Theoretical and experimental investigations of the motion of charged particles in magnetic traps show that, depending on the magnitude of the ratio, $\chi = R_L/R_c$, of the Larmor radius to the radius of curvature of the magnetic lines of force, the motion can become unstable as a result of irreversible changes that occur in the magnetic moment during multiple reflections from the mirrors. The intensity of the nonadiabatic processes then depend on both the magnitude of the nonadiabaticity parameter χ and the geometry of the lines of force. [1] On the basis of the existence of the two types of motion—stable and unstable—in static traps, we can expect that adiabatic magnetic perturbations that decrease the initial field can lead to additional particle losses as a result of the appearance or enhancement of the nonadiabaticity of the motion.

In view of this, further investigation of the nonadiabatic effects in magnetic traps with variable—in time—fields is of interest. The present paper is devoted to the investigation of the effect of reversible magnetic perturbations on particle dynamics. The obtained data may be useful for the understanding of certain effects of the dynamics of the earth's radiation belts during magnetic storms.

The experiment was performed with an apparatus[2] that allows the production of $\sim 5\times 10^{-10}$-Torr vacuum in an effective volume of ~ 1 m^3. The static field was produced by a uniformly magnetized sphere of diameter 8 cm and magnetic moment $M \approx 1.95\times 10^4$ G-cm^3. A pulsed magnetic field was produced by discharge through a coil of diameter 35 cm by a capacitor bank with a maximum energy supply of ~600 J. The coil was positioned coaxially with the dipole in the midplane ($\theta = \pi/2$) (a spherical coordinate system (R, θ, φ) with origin on the dipole and axis parallel to the magnetic moment **M** is used here). The amplitude of the perturbations was ~10 Oe; the minimum time of growth of the field to the peak value was chosen from the adiabaticity condition, and was $\sim 10^{-4}$ sec. The characteristic particle-motion times corresponding to all the three periodicities—the Larmor revolution, the oscillation of the guiding center between the points of reflection, the precession around the magnet—were respectively equal to $\tau_1 \sim 10^{-8}$ sec, $\tau_2 \sim 10^{-7}$ sec, and $\tau_3 \sim 10^{-6}$ sec.

For the capture of the electrons by the dipole field, internal injection with the aid of crossed E and H fields was used. [2,3] The particles were detected with electron channel multipliers, as previously described. [2] The detectors were located on two field lines $R = R_e \sin^2\theta$; $\varphi = \text{const}$ (R_e was equal to 8 and 10 cm) at an angular distance of $\theta \approx 50°$. Here R_e is the distance from the coordinate origin to the point of intersection of the magnetic line of force with the midplane ($\theta = \pi/2$).

The principal quantity investigated was the relative number of registered particles as a function of the amplitude and duration of the magnetic perturbation

$$n = \int_{t_3}^{\infty} N(t, W, b, \alpha_e, \tau)\, dt \Big/ \int_{t_3}^{\infty} N(t, W, \alpha_e)\, dt, \qquad (1)$$

$$b = h_e/H_e,$$

where $N(t, W, \alpha_e)$ is the number of electrons of a given energy detected at the moment of time t; H_e and h_e are the intensity of the dipole field and the amplitude of the magnetic perturbation at the distance R_e; α_e is the initial value of the pitch angle in the midplane; τ is the pulse halfwidth; and t_3 is the time lag of the start-up of the counter array behind the injection. The perturbation was switched on after the injection and ceased in a time $< t_3$.

As a preliminary, using the previously described technique, [2] we measured the pitch-angle dependence of the critical value of the adiabaticity parameter for the static case. As the sought quantity, we chose the maximum value of the χ_c component in a direction perpendicular to a line of force, i.e., of $\chi_{\perp c}(\alpha_e, \theta = \pi/2)$. The results of the $\chi_{\perp c}$ measurements in the constant dipole field are shown in Fig. 1.

During the action of the magnetic perturbation there occur a displacement of the drifting shell (a change in the radius of precession around the sphere), a change in the pitch-angle distribution, and a change in the

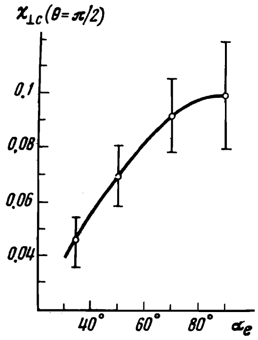

FIG. 1. The dependence $\chi_{\perp c}(\alpha_e, \theta=\pi/2)$ for the constant dipole field.

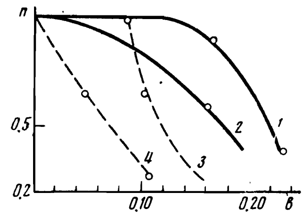

FIG. 3. The dependence $n(b)$ for different W and α_e at $\tau \approx 750$ μsec. For $\alpha_e = \pi/2$ (solid curves): 1) $W \approx 5$ eV, 2) $W \approx 9$ eV; for $\alpha_e \approx 34°$ (the dashed curves): 3) $W \approx 4$ eV, 4) $W \approx 6$ eV.

particle energy. Correspondingly, the adiabaticity parameter changes. Since the particle motion in the dipole trap is described by three adiabatic invariants, [4] we shall compute the indicated quantities in the axially-symmetric, perturbed field on the basis of the laws of conservation of these invariants.

From the invariance of the magnetic flux through the drifting trajectory follows the equation

$$MR_e + R_e \int R_e' h(R_e') dR_e' - MR_e' = 0. \tag{2}$$

Here and below the prime indicates that the corresponding quantities pertain to the perturbed state. For $R_e/R_b \leq 0.75$ (R_b is the radius of the coil), a good approximation for h is $h = h_0(1 - 3.91\times 10^{-2}R_e + 8.82\times 10^{-8}R_e^2)$. The solution to (2) as a function of the amplitude of the perturbation is shown in Fig. 2 (the curve 1).

The change in the angular distribution is found from the condition of conservation of the magnetic moment $W\sin^2\alpha/H$ and the longitudinal invariant $\oint mv\cos\alpha\, ds$, where the integral is evaluated along a line of force over a complete oscillation. A numerical integration shows that the angle, θ_0, of the points of reflection varies according to the law $\theta_0' \approx \theta_0 R_e/R_e'$ ($\alpha_e \gtrsim 30°$). Consequently, the particles are drawn towards the mid-plane during the perturbation. Since the lifetime of the particles increases with θ_0, [1] this effect will not affect the quantity n. Furthermore, it turns out that the total energy ($W = W_\perp + W_\parallel$) of the electrons with points of reflection $\theta_0 \gtrsim 60°$ ($\alpha_e \gtrsim 30°$) varies in the same way as the quantity $W_\perp$, which is determined by the conservation of the magnetic moment, i.e., that

$$W' = W\frac{H_e' - h(R_e')}{H_e}, \quad H_e' = \frac{M}{R_e'^3}, \tag{3}$$

where R_e' is found from Eq. (2).

The adiabaticity parameter for the perturbed magnetic field can, with allowance for (3), be written in the form

$$\chi_\perp(\alpha_e, \theta=\pi/2) = 3.4\left(\frac{W}{H_e}\right)^{1/2} \sin\alpha_e \frac{[3H_e' + R_e'\partial h(R_e')/\partial R_e']}{R_e'[H_e' - h(R_e')]^{3/2}}, \tag{4}$$

where W is measured in eV. The dependence of χ' on h is shown in Fig. 2 (the curve 2). The theoretical results presented in Fig. 2 were used in the analysis of the experimental data. It should be emphasized that the Eqs. (2) and (3) are valid only for the case when $\chi \lesssim \chi_c$; for $\chi > \chi_c \ll 1$ they can be used for approximate calculations. For $\chi \sim 1$ these equations cease to have any meaning, since the magnetic moment changes in a single reflection by an amount $\Delta\mu \sim \mu$.

The measurement of the quantity n was carried out simultaneously by two detectors. The readings of the first detector ($R_e = 8$ cm) are shown in Figs. 3 and 4. As can be seen from Figs. 1 and 3, the nonadiabatic effects have a threshold dependence on the quantity h for particles with an initial parameter $\chi < \chi_c$. To each value of W corresponds a definite critical value h_c separating the reversible changes in n from the irreversible changes. To the various values of $h_c(W)$ corresponds roughly one and the same value of the parameter $\chi_c' \sim \chi_c$ at which the decrease of n is observed (Fig. 4). On the second detector ($R_e = 10$ cm) we observed the increase of n with increasing b. This effect can be ex-

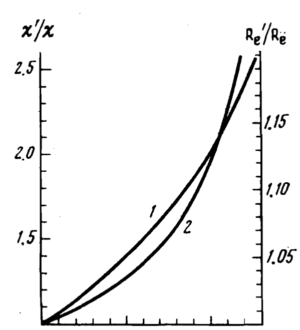

FIG. 2. The dependence of R_e'/R_e on b (the curve 1) and of χ'/χ on b (the curve 2).

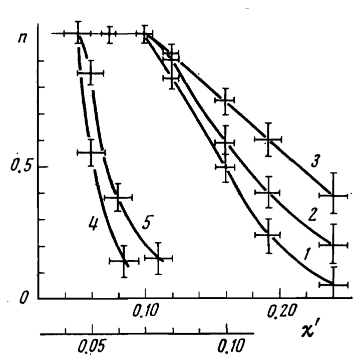

FIG. 4. The dependence $n(\chi')$ for different τ and α_e. For $\alpha_e = \pi/2$: 1) $\tau \approx 10^4\tau_2$, 2) $\tau \approx 5\times 10^3\tau_2$, 3) $\tau \approx 2.5\times 10^3\tau_2$; for $\alpha_e \approx 34°$: 4) $\tau \approx 10^4\tau_2$, 5) $\tau \approx 2.5\times 10^3\tau_2$ (the lower scale).

plained as follows. The particle density in the unperturbed field decreases sharply when $R_e > 8$ cm ($N(R_e = 10\text{ cm})/N(R_e = 8\text{ cm}) \lesssim 10^{-1}$). During the perturbation the drifting shells expand outwards to $R_e \sim 10$ cm and the particles go over from the region of absolute confinement, $\chi < \chi_c$, into the unstable state, $\chi' > \chi'_c$, which leads to an increase in the count of the second detector. The analytic expression of the functional dependence $n = n(\chi', \tau)$ (Fig. 4) can be represented in the form

$$n=\exp(-\bar{D}\tau), \quad \bar{D}\approx 4\cdot 10^4 \exp(-a/\chi_\perp'), \quad a=0.7, \quad \chi_\perp' \geqslant 0.1, \quad \alpha_e=\pi/2, \quad a=0.22, \quad \chi_\perp' \geqslant 0.045, \quad \alpha_e \approx 34°, \tag{5}$$

where $\chi'_\perp$ is the peak value of the adiabaticity parameter. The quantity $\bar{D}$ is the effective coefficient of pitch-angle diffusion, and describes the rate of nonadiabatic losses due to "scattering" by the inhomogeneities of the magnetic field during the perturbation.

Thus, the adiabatic reversible variation in time of the magnetic field ($\tau_3 \mathring{H}/H \ll 1$) gives rise to a reversible evolution of the particles if by chance the condition $\chi' \lesssim \chi_c$ is fulfilled during the perturbation. The virtual coincidence of the critical value of $\chi_\perp$ for the static dipole field[2] with $\chi'_{\perp c}$ for the perturbed state is apparently explained by the fact that the inhomogeneity of the resultant field is determined primarily by the dipole term ($\nabla(H'_e - h) \approx \nabla H'_e$).

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Stationary Langmuir spectra in a non-uniform plasma

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We consider the effect of a weak inhomogeneity in the plasma on the structure of the Langmuir spectra, taking into account the induced scattering by ions. We determine, in the framework of a one-dimensional model, the general properties of the solution for the spectral density of the Langmuir oscillations with a source and a sink for plasmons. In the case when the role of the induced scattering is relatively small we obtain numerical solutions.

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The theory of a weakly turbulent plasma shows that the processes of induced scattering of Langmuir oscillations by ions lead to a plasmon energy spectral transfer to the small wavenumber region and in the case of a one-dimensional plasma to the formation of the so-called "Langmuir condensate" at $k = 0$.[1] By virtue of the plasmon-number conservation ($\omega_l^2 = \omega_p^2 + 3k^2 v_{Te}^2 = \text{const}$), their propagation in a non-uniform plasma is accompanied by drift in wavenumber space which prevents the formation of a condensate, so that, as we shall show below, there may exist stationary solutions for the spectral energy distribution of the Langmuir oscillations.

We consider the one-dimensional stationary problem with an external source which generates Langmuir waves in the direction of the density gradient in the plasma.[1)] Taking the direction of the z-axis in the direction opposite to that of the density gradient in the plasma and assuming, for the sake of argument, that the plasma is isothermal and that the Langmuir oscillations are generated in a narrow wavenumber interval close to $k_z = -k_0$ ($r_D^{-1} \gg k_0 \gg (m_e/m_i)^{1/2} r_D^{-1}$) we get, neglecting the linear damping, thermal noise, and spontaneous scattering, for the one-dimensional spectral density W of the Langmuir oscillations[2)] (see, e.g., [3]):

$$\frac{1}{L}\frac{\partial W}{\partial x} = P\delta(x+x_0) + AW\int W(x')\psi(x+x')\,\text{sign}(x'-x)\,dx'; \tag{1}$$

$$x = k_z/\Delta k, \quad x_0 = k_0/\Delta k, \quad \Delta k = {}^2/_3 (m_e/m_i)^{1/2} r_D^{-1},$$

$$L = \left| r_D \frac{\partial \ln \omega_p}{\partial z} \right|^{-1} \gg 1, \quad A = \frac{\pi}{18}\frac{m_e}{m_i},$$

$$\psi(y) = \frac{y}{\sqrt{2\pi}} a(y) \exp\left(-\frac{y^2}{2}\right),$$

$$a(y) = \left| 1 - \frac{y}{2} e^{-y^2/2} \int_{i\infty}^{y} e^{\tau^2/2}\, d\tau \right|^{-2},$$

$$r_D \int W(x(k_z))\, dk_z = U_l/nT,$$

where U_l is the energy density of the Langmuir oscil-