Anomalous penetration of an electromagnetic field into a metal with a mirror boundary

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A theory is constructed for anomalous penetration (AP) of an electromagnetic wave into a metal located in a magnetic field parallel to the surface of the metal. The reflection of electrons by the metal-vacuum interface is assumed to be specular. AP of the field occurs along a chain of electron trajectories. It is shown that under anomalous skin-effect conditions, at distances equal to one or two cyclotron diameters, there are singularities in the field distribution. At greater depths the field has a quasiharmonic character. In the radiofrequency range, and also on the left wing of the cyclotron resonances (on the weak-field side), the singularities of the field have the form of spikes with a marked spatial structure. On the right wing of the resonance lines, the spikes disappear, and a short cyclotron wave is excited in the metal. The possibility of experimental detection of these effects is discussed.

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1. INTRODUCTION

It is well known that in a number of cases a metal placed in a constant magnetic field **H** may prove to be transparent for electromagnetic radiation. One of the mechanisms of anomalous penetration (AP) of a wave into a metal is so-called trajectory transport, which is realized under anomalous skin-effect conditions. The essence of it is as follows. A high-frequency field **E** is absorbed in the metal by electrons whose trajectories pass through the skin layer δ. Since in the plane perpendicular to **H** their motion is finite and periodic, an electron transports the energy obtained in the skin layer into the depth of the specimen at a distance equal to the diameter D of the Larmor orbit ($D \gg \delta$). The greatest contribution to AP is made by electrons of extremal sections of the Fermi surface. They form, at a distance D_{extr} from the metal boundary, a secondary current plane—a spike of electromagnetic field. The skin layer produced at depth D_{extr} serves as a source of energy for the next spike, etc. Thus there occurs in the metal a whole series of spikes of electromagnetic field (see Fig. 1). An AP effect of the trajectory type under cyclotron-resonance conditions was predicted by Asbel'.[1] In general, however, trajectory transport is not dependent on resonance conditions and takes place over a wide range of frequencies.[2] The most important theoretical and experimental results obtained in investigation of various AP mechanisms are contained in a review by Gantmakher and one of the authors.[3]

We emphasize that the basic skin layer δ_0 that generates the subsequent spikes of electromagnetic field **E** is formed by electrons located near the metal boundary. Therefore the physical picture of AP along the chain of trajectories depends on the nature of the interaction of the electrons with the surface of the specimen. Nevertheless this fact has been disregarded in all the theoretical researches carried out so far. It has been assumed that AP effects are determined by electrons that do not collide with the boundary. This approximation can give only a qualitative description of the situation in the case when the reflection of the electrons from the surface of the metal is diffuse or nearly so. Recently, because of the possiblity of obtaining materials with nearly ideal surfaces,[4–7] great interest has been aroused in investigations of the high-frequency properties of metals with specular reflection of the electrons by the specimen surface.

The chief peculiarity of specular reflection in a parallel magnetic field **H** consists in the fact that in addition to the electronic states that exist in an unbounded metal (volume states), there appears a group of electrons connected with the surface–surface electrons. To this group belong electrons which, moving in the magnetic field **H**, are on each volution reflected by the specimen boundary. The trajectories of both electronic groups are depicted in Fig. 1.

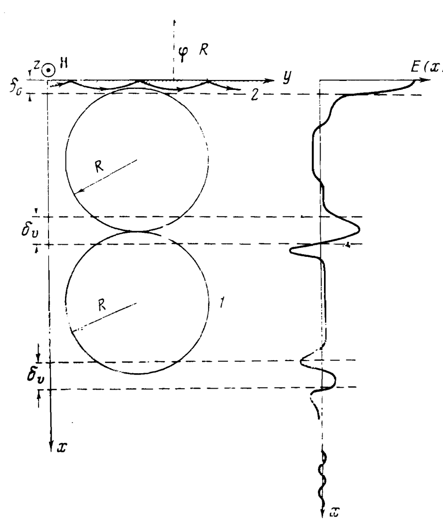

FIG. 1. Picture of the anomalous penetration of an electromagnetic field into a metal with a specular boundary. On the left are shown trajectories of the groups of electrons that participate in AP: 1) volume electrons, 2) grazing electrons (of grazing angle $\varphi \sim (\delta_0/R)^{1/2}$). On the right is shown a schematic form of the field distribution in the specimen.

The presence of surface electrons leads to a number of interesting effects. They are responsible, for example, for oscillations of the surface impedance in weak magnetic fields, discovered by Khaĭkin,[8] and for a change of character of the anomalous skin effect[9] and of the cyclotron resonance.[10] It is natural to expect that the trajectory AP effect in a metal with a mirror boundary will also change as compared with the diffuse case. With diffuse reflection, the wave field interacts chiefly with the volume electrons that form the initial skin layer δ_0. But they transport the electromagnetic field from this skin layer into the depth of the metal by producing spikes of current. In the case of specular scattering, the determining role in the production of a screening current is played by surface electrons whose trajectories lie completely within the skin depth δ_0 (grazing electrons). In Fig. 1 the trajectory of grazing electrons is marked with the index 2. The specific property of trajectory penetration in the specular case consists in the fact that current produced by grazing electrons is transported into the depth of the metal by volume electrons.

In the present paper, a theory is constructed for AP of an electromagnetic field along a chain of trajectories into a metal located in a magnetic field **H** parallel to the boundary. The reflection of the electrons by the metal-vacuum interface is specular. It is shown that the first two spikes of the electromagnetic field are well described within the framework of the theory of the anomalous skin effect. Study of the behavior of the field at greater distances from the surface requires consideration of the long-wave components in the spectral decomposition of **E**. This indicates absence of pronounced peculiarities in the distribution of the high-frequency field at such depths. In fact, experiments carried out on bismuth, rubidium, and indium (see[3]), and also on cadmium,[11] give evidence that at distances from the surface exceeding the depth of the second spike, the field has a quasiharmonic character.

2. FORMULATION OF THE PROBLEM

We shall consider a metallic half-space located in a constant and uniform magnetic field **H**. The vector **H** lies in the plane of the metal-vacuum interface. We choose the following system of coordinates: we place the y and z axes on the surface of the metal (the plane $x=0$), the z axis parallel to **H**, and the x axis parallel to the normal into the interior of the specimen (Fig. 1).

Let there be incident on the boundary $x=0$ a plane monochromatic wave of frequency ω, whose **E** vector is polarized perpendicular to the constant field **H** ($\mathbf{E}\parallel y$). The direction of propagation of the wave coincides with the x axis. The electric field $\mathbf{E}=\{0, E, 0\}$ inside the metal, $x\geq 0$, depends only on the coordinate x; that is, $E=E(x)\exp(-i\omega t)$ We introduce Fourier transformations according to

$$E(x)=\frac{1}{\pi}\int_0^\infty dk\mathscr{E}(k)\cos(kx),\quad \mathscr{E}(k)=2\int_0^\infty dxE(x)\cos(kx). \tag{2.1}$$

Maxwell's equation for the Fourier component $\mathscr{E}(k)$ of the field has the form

$$k^2\mathscr{E}(k)+2E'(0)=4\pi i\omega c^{-2}j(k). \tag{2.2}$$

Here $j(k)$ is the Fourier component of the current density, c is the velocity of light, and the prime denotes differentiation with respect to the argument.

Formulas connecting the Fourier component $j(k)$ of the current density with the field $\varepsilon(k)$ in a metal with a mirror boundary were given in[9]. Because of spatial inhomogeneity, this relation is an integral relation:

$$j(k)=\mathscr{K}(k)\mathscr{E}(k)-\frac{1}{\pi}\int_0^\infty dk'Q(k,k')\mathscr{E}(k'). \tag{2.3}$$

Here $\mathscr{K}(k)$ is the Fourier component of the conductivity of an unbounded metal, due to volume electrons. The integral kernel of the electrical conductivity operator $Q(k,k')$ contains contributions both of volume and of surface electrons.

We shall not write down the exact formulas for the conductivity of a metallic half-space $x\geq 0$, which are contained in[9]. We shall give only the asymptotic expressions, needed hereafter, for the kernels $\mathscr{K}(k)$ and $Q(k,k')$ in the anomalous skin-effect limit.[1)] For simplicity, we shall restrict ourselves to consideration of the case of a quadratic and isotropic law of dispersion of the conduction electrons (an alkali metal). The results obtained below, however, are also applicable when the cross section of the Fermi surface in a plane perpendicular to **H** is a convex closed curve of arbitrary shape.

The asymptotic form of the conductivity $\mathscr{K}(k)$ of an unbounded metal is

$$\mathscr{K}(k)\cong\frac{3\omega_0^2}{16kv}\left[\coth(\pi\gamma)-\sqrt{\frac{2}{\pi}}\sinh^{-1}(\pi\gamma)\,\frac{\sin(2kR-\pi/4)}{(2kR)^{1/2}}\right]. \tag{2.4}$$

Here

$$\omega_0=\left(\frac{4\pi Ne^2}{m}\right)^{1/2},\quad \gamma=\frac{\nu-i\omega}{\Omega},\quad R=\frac{v}{\Omega},\quad \Omega=\frac{eH}{mc};$$

ω_0 is the plasma frequency of the metal; N is the concentration, e is the absolute value of the charge, m is the effective mass, and v is the Fermi velocity of the electrons; R is the maximum Larmor radius of the electronic orbit, Ω is the cyclotron frequency of rotation in magnetic field **H**, and ν is the frequency of collision of an electron with the scatterers.

The integral kernel of the conductivity $Q(k,k')$ asymptotically approaches the sum of three terms:

$$\begin{aligned}Q(k,k')\cong&-\frac{9\pi}{20\cdot 2^{1/3}\Gamma^2(1/3)}\frac{\omega_0^2}{\Omega\gamma R^{1/3}}\frac{|k-k'|^{-2/3}-(k+k')^{-2/3}}{(kk')^{1/2}}\\&+\frac{3\omega_0^2}{16\pi v}\coth(\pi\gamma)\frac{\ln(k/k')}{k^2-k'^2}\\&-\frac{3\omega_0^2}{4(2\pi)^{1/2}v}\frac{\mathrm{sh}^{-1}(\pi\gamma)}{k^2-k'^2}\left[\frac{\cos(2kR-\pi/4)}{(2kR)^{1/2}}-\frac{\cos(2k'R-\pi/4)}{(2k'R)^{1/2}}\right].\end{aligned} \tag{2.5}$$

The first term in (2.5) is the conductivity of grazing electrons with characteristic grazing angles $\varphi\propto(\delta_0/$

$R)^{1/2}$ (see Fig. 1). The remaining terms of (2.5), like (2.4), represent a contribution of volume electrons. It is the second term in (2.4) and the third term in (2.5), containing factors that oscillate over $2kR$, that insure an AP effect on the trajectories of electrons of the central cross section of the Fermi surface.

The conductivity of the metallic half-space $x \geq 0$ is described asymptotically by the expressions (2.4) and (2.5) under conditions of strong spatial dispersion.

$$|\gamma \coth(\pi\gamma)|^2 \ll kR \qquad (2.6)$$

with specular scattering of the electrons by the specimen boundary. By "specular reflection" one must understand closeness of the reflection coefficient ρ to unity:

$$1-\rho \ll |\gamma|/(kR)^{1/2}. \qquad (2.7)$$

The condition (2.6) includes a whole series of inequalities, namely $kR \gg |\gamma|$, $kR \gg 1$, and $kR \gg |\gamma|^2$. The first of these is simply the condition for anomalousness of the skin effect and is equivalent to the requirement $v/|\nu - i\omega| \gg k^{-1} \sim \delta$. The second inequality implies anomaly with respect to the magnetic field, $R \gg \delta$. These two conditions are sufficient to determine the asymptotic behavior (2.4) of the conductivity $\mathscr{K}(k)$ of an unbounded metal. For calculation of the asymptotic expression (2.5), it is necessary also that the third condition contained in (2.6) be satisfied, namely $kR \gg |\gamma|^2$. The physical meaning of this condition is that the characteristic arc length $(8R\delta_0)^{1/2}$ of a trajectory of grazing electrons is much smaller than the effective length $v/|\nu - i\omega|$ of the free path. Fulfillment of this inequality is necessary in order that the surface electrons may make a significant contribution to the current density $j(k)$. We note that the condition $kR \gg |\gamma|^2$ implies also the absence of a retardation effect in cyclotron resonance.[12]

By virtue of the inequality (2.6), the conductivity of grazing electrons (the first term in formula (2.5) is larger by a factor $(kR)^{1/2}/|\gamma \coth(\pi\gamma)| \gg 1$ than the smooth terms in the volume conductivity (the first term in the expression (2.4) and the second in (2.5)). The AP terms in the electrical conductivity of volume electrons (2.4) and (2.5) contain, as compared with the smooth terms, an additional smallness parameter $(kR)^{-1/2}$, equal to the relative number of volume electrons that participate effectively in trajectory transport. This means that the high-frequency current in the skin layer δ_0 is formed chiefly by grazing electrons.

For convenience in the subsequent discussion we shall introduce the symbols

$$\mathscr{E}(k) = -2E'(0)k_0^{-2}F(k/k_0),$$
$$k_0 = \left(\frac{9\pi\sqrt{2}}{10\Gamma^2(1/4)} \frac{\omega\omega_0^2}{c^2\Omega\gamma R^{1/2}}\right)^{2/5}. \qquad (2.8)$$

The value of k_0 determines the depth of the primary skin layer δ_0 produced by grazing electrons ($\delta_0 \equiv |k_0|^{-1}$).

We substitute the asymptotic expressions (2.4) and (2.5) for the electrical conductivity of the metal in formula (2.3) for the current. Then in Maxwell's equation (2.2), with the current density (2.3), we transform from the Fourier component $\mathscr{E}(k)$ to the function $F(k/k_0)$. We also introduce the dimensionless wave number $\xi = k/k_0$. As a result we obtain the following integral equation:

$$\xi^2 F(\xi) - i\int_0 dx\, F(\xi x) \frac{|1-x|^{-1/2} - (1+x)^{-1/2}}{(\xi x)^{1/2}}$$
$$-\frac{5i\Gamma^2(1/4)}{6}\frac{\gamma \coth(\pi\gamma)}{(2k_0R)^{1/2}}\xi^{-1}\left[F(\xi) - \frac{1}{\pi^2}\int_0^\infty dx\, F(\xi x)\frac{\ln x}{x^2-1}\right] = 1 \qquad (2.9)$$
$$-\frac{10i\Gamma^2(1/4)}{3\pi^{1/2}}\frac{\gamma}{\sinh(\pi\gamma)}(2k_0R)^{-1}\xi^{-3/2}\Big[F(\xi)\sin(2k_0R\xi - \pi/4)$$
$$-\frac{1}{\pi}\int_0^\infty dx\, F(\xi x)\frac{x^{-1/2}\cos(2k_0R\xi x - \pi/4) - \cos(2k_0R\xi - \pi/4)}{x^2-1}\Big].$$

Solution of equation (2.9) determines the function $F(\xi)$ and consequently also $\mathscr{E}(k)$. Knowing $\mathscr{E}(k)$, we shall by means of (2.1) investigate the spatial distribution of the electromagnetic field $E(x)$.

3. SOLUTION OF THE INTEGRAL EQUATION

The AP effect along a chain of trajectories represents a spatial resonance in the interaction of electrons with those Fourier harmonics of the field whose wave vectors are multiples of the reciprocal of the diameter $2R$ of the extremal orbit. Therefore it is natural to seek a solution of Eq. (2.9) in the form of an expansion in a "Fourier series of spatial harmonics":

$$F(\xi) = F_0(\xi) + \frac{5i\Gamma^2(1/4)}{6}\frac{\gamma \coth(\pi\gamma)}{(2k_0R)^{1/2}}\Delta F(\xi)$$
$$+\sum_{n=1}^{2}\left[\frac{10i\Gamma^2(1/4)}{3\pi^{1/2}}\frac{\gamma}{\sinh(\pi\gamma)}(2k_0R)^{-1/2}\right]^n \qquad (3.1)$$
$$\times\{f_{+n}(\xi)\exp(2ink_0R\xi - in\pi/4) + f_{-n}(\xi)\exp(-2ink_0R\xi + in\pi/4)\}.$$

The first two terms in (3.1) represent the zero-order harmonic—the smooth part of the function $F(\xi)$. The terms with $n = 1$, oscillating over $2k_0R$ (the first harmonic), determine the anomaly of the field in the vicinity of the first spike. Correspondingly, the second harmonic (the terms with $n = 2$, oscillating over $4k_0R$) in the spectral resolution of $E(x)$ describes the character of the field distribution in the region of the second spike. The monotonic terms in the expression (3.1) are due chiefly to the conductivity of grazing electrons and to the smooth part of the volume conductivity from (2.4) and (2.5). In general, the oscillating terms in (2.4) and (2.5) also make a contribution to the zero-order harmonic of the field. It is easy to show, however, that the corresponding terms may be neglected because of their smallness. The first and second harmonics in the spectrum of $E(x)$ arise because of the AP terms in the conductivity of volume electrons. The small value of the volume conductivity, in comparison with the surface, permits us to seek the function $F(\xi)$ in the form of the perturbation-theory series (3.1) in the parameter $|\gamma \coth(\pi\gamma)/(2k_0R)^{1/2}| \ll 1$.

In the smooth part of the function $F(\xi)$ it proves convenient to separate the two terms $F_0(\xi)$ and $\Delta F(\xi)$, corresponding to fields produced by grazing and by volume electrons, respectively. The first of these, $F_0(\xi)$, is the solution of the unperturbed problem and is determined from Maxwell's equation (2.2) with the current density of grazing electrons; that is,

$$\xi^2 F_0(\xi)-i\int_0^\infty dx\,F_0(\xi x)\frac{|1-x|^{-1/2}-(1+x)^{-1/2}}{(\xi x)^{1/2}}=1. \qquad (3.2)$$

Equation (3.2) was first solved in[9]. According to[9], the function $F_0(\xi)$ can be represented in the form of a Mellin contour integral

$$F_0(\xi)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}dz\,M(z)\,\xi^z, \quad -2<c=\mathrm{Re}\,z<\frac{1}{2}. \qquad (3.3)$$

The Mellin transform $M(z)$ is regular in the vertical strip $-2<\mathrm{Re}\,z<\frac{1}{2}$ of the complex z plane. At the points $z=-2$ and $z=\frac{1}{2}$, the function $M(z)$ has simple poles with residues 1 and $-i/4$ respectively. $M(z)$ satisfies the difference equation

$$\frac{M(z-{}^5/_2)}{M(z)}=i\sqrt{2\pi}\,\frac{\Gamma(z+{}^1/_2)\cos[\pi(z-{}^1/_2)/2]}{\Gamma(z+1)\cos(\pi z/2)} \qquad (3.4)$$

and is determined by the following expression:

$$M(z)=\exp\left\{\frac{z+2}{5}\left[i\pi+\ln\frac{({}^2/_5)^4}{2\pi}\right]\right\}\cos\left(\frac{\pi z}{2}\right)\frac{\Gamma(z+1)}{\Gamma({}^7/_5)}\times\Gamma\left(\frac{1}{5}-2\frac{z}{5}\right)\Gamma\left(\frac{3}{5}-2\frac{z}{5}\right). \qquad (3.5)$$

The only singularities of $M(z)$ are simple poles located on the real axis. From formula (3.3) it follows that

$$F_0(\xi)\cong\xi^{-2} \text{ as } \xi\to\infty \text{ and } F_0(\xi)\cong i\xi^{1/2}/4 \text{ as } \xi\to 0. \qquad (3.6)$$

The second term in formula (3.1) is due to the contribution to the current density $j(k)$ from the smooth part of the volume conductivity, which in the present case plays the role of a perturbation operator. Therefore $\Delta F(\xi)$ is the solution of the equation

$$\xi^2\Delta F(\xi)-i\int_0^\infty dx\,\Delta F(\xi x)\frac{|1-x|^{-1/2}-(1+x)^{-1/2}}{(\xi x)^{1/2}}=\frac{F_0(\xi)}{\xi}-\frac{1}{\pi^2\xi}\int_0^\infty dx\,F_0(\xi x)\frac{\ln x}{x^2-1}. \qquad (3.7)$$

Here we have omitted small smooth terms due to the first and second harmonics in (3.1). Equation (3.7) was solved in[10]. For the purposes of the present work, knowledge of the explicit form of the function $\Delta F(\xi)$ is not obligatory, since it enters into the formula for the field $E(x)$ in the form of an insignificant correction to the large monotonic term from $F_0(\xi)$. Therefore we shall not discuss the solution of (3.7).

We move on to the calculation of the amplitudes $f_{+n}(\xi)$. The equations for the functions $f_{+n}(\xi)$ are obtained by equating the coefficients of the corresponding rapidly oscillating exponentials after substitution of the expansion (3.1) in the integral equation (2.9). It turns out that the amplitudes $f_{+1}(\xi)$ are determined solely by the solution $F_0(\xi)$ of the unperturbed problem. The contribution of the smooth correction $\Delta F(\xi)$ and the effect of the second harmonic on the first are small by virtue of the inequality (2.6). The coefficients of the second harmonic $f_{+2}(\xi)$ are expressed in terms of $f_{+1}(\xi)$. Physically this means that the field in each successive spike is determined solely by the field amplitude in the preceding currenty layer, while the effect of the inverse relation is inappreciable.

In the process of solution of (2.9), there occur on its left side several characteristic integrals of rapidly oscillating functions. It is necessary to obtain asymptotic estimates of these integrals in the limit when $2|k_0|R\xi\gg 1$. On supposing that the amplitudes $f_{+n}(\xi)$ are smooth functions of their argument, we have

$$\int_0^\infty dx\,f_{\pm n}(\xi x)\frac{|1-x|^{-1/2}-(1+x)^{-1/2}}{(\xi x)^{1/2}}\exp(\pm 2ink_0R\xi x)\cong\left(\frac{\pi}{nk_0R}\right)^{1/2}\exp(\pm 2ink_0R\xi)f_{\pm n}(\xi)/\xi. \qquad (3.8)$$

The integral containing the logarithm in (2.9) introduces a small correction to the monotonic part of the function $F(\xi)$. For example, for the first harmonic it is asymptotically equal to the following expression:

$$\int_0^\infty dx\,f_{\pm 1}(\xi x)\frac{\ln x}{x^2-1}\exp(\pm 2ik_0R\xi x)\cong\pm if_{\pm 1}(0)\frac{C\mp i\pi+\ln(2k_0R\xi)}{2k_0R\xi}, \qquad (3.9)$$

where C is Euler's constant. The contribution of this integral to the expression for $\Delta F(\xi)$, like the omitted smooth term in (3.8) (the contribution of the point $x=0$) may be neglected because of its smallness. A similar conclusion may be drawn about the smooth contribution of the second harmonic. Thus in calculating the coefficients of the first and second harmonics, among the integrals of the left side of (2.9) it is necessary to take into account only the oscillating term (3.8).

On the right side of (2.9) there also occur integrals of oscillatory functions, which we shall estimate asymptotically under the condition $2|k_0|R\xi\gg 1$. It is easy to show that

$$\frac{2}{\pi}\int_0^\infty dx\,F_0(\xi x)\frac{x^{-1/2}\exp(\pm 2ik_0R\xi(x-1))-1}{x^2-1}=\pm iF_0(\xi)-\Phi_0(\xi). \qquad (3.10)$$

Here $\Phi_0(\xi)$ is determined by the equation

$$\Phi_0(\xi)=\frac{2}{\pi}\int_0^\infty dx\frac{F_0(\xi x)}{x^2-1}=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}dz\,\xi^z M(z)\,\mathrm{tg}(\pi z/2), \quad -1<c=\mathrm{Re}\,z<{}^1/_2. \qquad (3.11)$$

From formula (3.11) follows

$$\Phi_0(\xi)\cong 2M(-1)/\pi\xi \text{ as } \xi\to\infty \text{ and } \Phi_0(\xi)\cong i\xi^{1/2}/4 \text{ as } \xi\to 0. \qquad (3.12)$$

The behavior of $\Phi_0(\xi)$ in the limit of small ξ coincides with the corresponding asymptotic behavior of the function $F_0(\xi)$ (see (3.6)). Integrals of the type (3.10), but with substitution in the integrand of the first harmonic $f_{\pm 1}(\xi)\exp(\pm 2ik_0R\xi)$ instead of the function $F_0(\xi)$, having the form

$$\frac{1}{\pi}\int_0^\infty dx\,f_{\pm 1}(\xi x)\frac{x^{-1/2}\exp(\pm 4ik_0R\xi(x-1))-\exp(\pm 2ik_0R\xi(x-1))}{x^2-1},$$

and zero in the approximation being considered.

The result of the calculation of the coefficients $f_{\pm 1}(\xi)$ looks like this:

$$f_{\pm 1}(\xi)=-\frac{i\Phi_0(\xi)\pm 3F_0(\xi)}{4(2\pi\xi)^{1/2}}\left[1+\frac{5\Gamma^2(1/4)\,\gamma\coth(\pi\gamma)}{6(2\pi)^{1/2}}+i\xi^3\left(\frac{k_0R}{\pi}\right)^{1/2}\right]^{-1} \tag{3.13}$$

The amplitudes $f_{\pm 2}(\xi)$ of the second harmonic can be expressed in terms of $f_{\pm 1}(\xi)$ as follows:

$$f_{\pm 2}(\xi)=\mp\frac{f_{\pm 1}(\xi)}{2(\pi\xi)^{1/2}}\left[1+\frac{5\Gamma^2(1/4)\,\gamma\coth(\pi\gamma)}{6\pi^{1/2}}+i\xi^3\left(\frac{2k_0R}{\pi}\right)^{1/2}\right]^{-1}. \tag{3.14}$$

The expressions $[\ldots]^{-1}$ in formulas (3.13) and (3.14) consist of three terms. The third term is the contribution of the vortical part in Maxwell's equation (the first term on the left side of (2.9)). The unity terms in the expressions (3.13) and (3.14) originate from the conductivity of grazing electrons, whereas the second term is produced by volume electrons (the second and third terms, respectively, on the left side of (2.9)). Thus in contrast to the zero-order harmonic, which is determined chiefly by grazing electrons, the contributions of volume and surface electrons in the denominators of the oscillatory terms of the expansion (3.1) can in general prove to be of the same order.

In the solution of (2.9) we have restricted ourselves to the first three harmonics in the spectral representation of the function $F(\xi)$. This was not done accidentally, since the procedure presented above for finding $F(\xi)$ is correct only for the first two spikes. The amplitudes of even the third harmonic cannot be found by this method, because a logarithmic divergence at small ξ occurs in the integrals on the right side of (2.9). This indicates the inapplicability of the anomalous-skin-effect approximation (2.6), and consequently the absence of any marked singularities in the field distribution at distances of three or more cyclotron diameters. This fact is corroborated experimentally, as was indicated in the Introduction. The reason for this behavior of the wave field lies in the fact that with increasing distance from the metal surface $x=0$, information about the distribution of the field $E(x)$ is determined by smaller and smaller values of the wave number k. Study of the structure of the field at such depths requires an accurate taking into account of the long-wave asymptotic behavior of the conductivity tensor and constitutes a very complicated problem. Its solution does not fall within the purposes of the present work, and therefore we shall restrict ourselves to consideration of just the first two spikes of $E(x)$.

4. THE FIELD DISTRIBUTION

The spatial structure of the electromagnetic field $E(x)$ in the metal is determined by means of the solution $F(\xi)$ of the integral equation (2.9) according to the formula

$$E(x)=-\frac{2E'(0)}{\pi k_0^2}\int_0^\infty dk\,F\left(\frac{k}{k_0}\right)\cos(kx). \tag{4.1}$$

Since the function $F(k/k_0)$ is the sum of three spatial harmonics, the field $E(x)$ also consists of three terms, each of which corresponds to a definite harmonic:

$$E(x)=E_0(x)+\frac{2E'(0)}{\pi k_0^2\delta_v}\frac{\pi\gamma}{\sinh(\pi\gamma)}\left(\frac{\delta_v}{4\pi R}\right)^{1/2}\left[\Psi_1\left(\frac{x-2R}{\delta_v}\right)+\Psi_1\left(-\frac{x+2R}{\delta_v}\right)\right]$$
$$+\frac{8}{\pi}\frac{E'(0)}{k_0^2\delta_v}\left[\frac{\pi\gamma}{\sinh(\pi\gamma)}\left(\frac{\delta_v}{4\pi R}\right)^{1/2}\right]^2\left[\Psi_2\left(\frac{x-4R}{\delta_v}\right)+\Psi_2\left(-\frac{x+4R}{\delta_v}\right)\right]. \tag{4.2}$$

Here $E_0(x)$ is the monotonic component of the field distribution, due principally to the zero-order harmonic $F_0(x)$. The second and third terms in (4.2) determine the singularities of the field in the vicinity of the first $(x=2R)$ and second $(x=4R)$ spikes, respectively. The value of δ_v characterizes the scale of variation of the field at the spikes and is

$$\delta_v=(3\omega\omega_0^2/4c^2R\,|\nu-i\omega|)^{-1/3}. \tag{4.3}$$

It is interesting to note that δ_v coincides with the depth of the skin layer produced by volume electrons under anomalous skin-effect conditions. The relation between the thickness of the skin layer of grazing electrons, $\delta_0=|k_0|^{-1}$, and that of volume electrons, δ_v, is obtained very simply from (2.8) and (4.3):

$$k_0\delta_v=[6\pi\sqrt{2\pi}/5\Gamma^2(1/4)]^{1/3}(k_0R/\pi)^{1/6}\exp[i(\pi-2\varkappa)/6]$$
$$=[6\pi\sqrt{2\pi}/5\Gamma^2(1/4)]^{2/5}(R/\pi\delta_v)^{1/5}\exp[i(\pi-2\varkappa)/5],$$

where $\varkappa=\cot^{-1}(\nu/\omega)$. It is obvious that $|k_0|\delta_v\gg 1$. The resulting renormalization of the value of the skin depth at the spikes with respect to the original skin depth δ_0 is quite natural. Physically it is a consequence of the fact that the spikes are formed exclusively by volume electrons.

The function $\Psi_1(x)$ has the following form:

$$\Psi_1(x)=\int_0^\infty\frac{dt}{t^{1/2}}\frac{[3F_0(t/k_0\delta_v)-\Phi_0(t/k_0\delta_v)]\cos(xt)}{t^3\exp(i\varkappa)+\pi\gamma\coth(\pi\gamma)+6\pi(2\pi)^{1/2}/5\Gamma^2(1/4)}$$
$$+\int_0^\infty\frac{dt}{t^{1/2}}\frac{[3F_0(t/k_0\delta_v)+\Phi_0(t/k_0\delta_v)]\sin(xt)}{t^3\exp(i\varkappa)+\pi\gamma\coth(\pi\gamma)+6\pi(2\pi)^{1/2}/5\Gamma^2(1/4)} \tag{4.4}$$

The expression for $\Psi_2(x)$ looks like this:

$$\Psi_2(x)=\int_0^\infty\frac{dt}{t}\frac{\Phi_0(t/k_0\delta_v)\cos(xt)-3F_0(t/k_0\delta_v)\sin(xt)}{t^3\exp(i\varkappa)+\pi\gamma\coth(\pi\gamma)+6\pi(2\pi)^{1/2}/5\Gamma^2(1/4)}$$
$$\times[t^3\exp(i\varkappa)+\pi\gamma\coth(\pi\gamma)+6\pi^{3/2}/5\Gamma^2(1/4)]^{-1}. \tag{4.5}$$

We shall consider in detail the structure of each of the terms in formula (4.2).

The monotonic component $E_0(x)$ of the field is formed by grazing electrons. To obtain it, it is necessary in formula (4.1) to substitute the function $F_0(k/k_0)$ instead of $F(k/k_0)$. The corrections to $E_0(x)$ for the volume group of electrons are small. By use of the representation (3.3) for the function $F_0(k/k_0)$, it is easy to obtain $E_0(x)$ in the form of a contour integral:

$$E_0(x)=-\frac{2E'(0)}{\pi k_0}\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}dz\,(k_0x)^{-z}M(z-1)\Gamma(z)\cos(\pi z/2), \tag{4.6}$$

$0<c=\operatorname{Re}z<\frac{3}{2}$. The integral (4.6) is equal to the sum of the residues at the poles of the integrand located in the left half-plane $\mathrm{Re}\,z<0$; that is,

$$E_0(x)=E_0(0)\sum_{n=0}^{\infty}\frac{(-k_0x)^n}{\Gamma(3/5)(n!)^2}\Gamma\left(\frac{3}{5}+\frac{2n}{5}\right)\Gamma\left(1+\frac{2n}{5}\right)\times\exp\left\{-\frac{n}{5}\left[i\pi+\ln\frac{(2/5)^4}{2\pi}\right]\right\}. \quad (4.7)$$

The behavior of the field $E(x)$ at great distances from the surface, $|k_0|x\gg 1$, is described by an asymptotic expression which consists of the residues of the integrand of (4.6), with sign reversed, at the simple poles $z=\frac{3}{2}$ and $z=\frac{5}{2}$:

$$E_0(x)=E_0(0)\frac{-i\sqrt{2\pi}}{16M(-1)}(k_0x)^{-5/2}\left[1+\frac{9M(-1)}{2\pi}(k_0x)^{-1}\right]. \quad (4.8)$$

The power-law nature of the attenuation of the field $E_0(x)$ with distance from the metal surface $x=0$ is a consequence of the anomalousness of the skin effect. Mathematically, it is due to the presence of the branch point $k^{-1/2}$ in the integral operator of the conductivity of grazing electrons (the first term in (2.5)). In contrast to (4.8), the field of the principal skin layer in the diffuse case decreases in inverse proportion to the square of the distance.

The analysis of the second and third terms in formula (4.2) we shall carry out separately for the low-frequency case and for frequencies in the neighborhood of the cyclotron resonances.

1. *Low frequencies, strong magnetic fields*, $\omega\ll\nu\ll\Omega$. In this range $\varkappa=\pi/2$, the wave number k_0 is real, and $\pi\gamma\coth(\pi\gamma)\cong 1$, since $|\gamma|\ll 1$. The terms containing $\Psi_1(-(x+2R)/\delta_v)$ and $\Psi_2(-(x+4R)/\delta_v)$ are small monotonic corrections to $E_0(x)$ and may be neglected. The asymptotic expressions for the function $\Psi_1(x)$ near the center of the spike (small $|x|$) and at its edges (large $|x|$) have the following form:

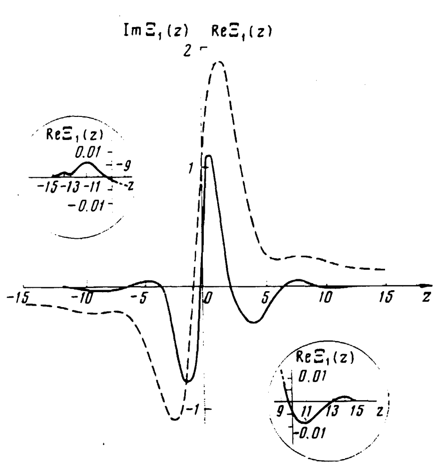

FIG. 2. Line shape of the first spike. The solid curve shows the behavior of $\mathrm{Re}\,\Xi_1(z)$, the dotted curve that of $\mathrm{Im}\,\Xi_1(z)$; $z=(x-2R)\alpha^{1/3}/\delta_v$.

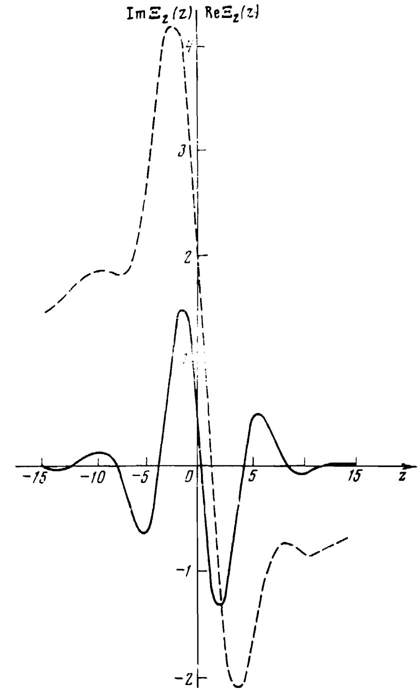

FIG. 3. Line shape of the second spike. The solid curve shows the behavior of $\mathrm{Re}\,\Xi_2(z)$, the dotted curve that of $\mathrm{Im}\,\Xi_2(z)$; $z=(x-4R)\alpha^{1/3}/\delta_v$.

$$\Psi_1(x)\cong\frac{\alpha^{-2/3}}{2(k_0\delta_v)^{1/2}}\begin{cases}\dfrac{2\pi}{3^{3/2}}\exp\left(\dfrac{\pi i}{3}\right)\left[1+2x\alpha^{1/3}\exp\left(-\dfrac{\pi i}{6}\right)\right], & |x|\ll\alpha^{-1/3}\\ \dfrac{2i}{x\alpha^{1/3}}+\dfrac{6}{x^4\alpha^{4/3}}, & \alpha^{-1/3}\ll|x|.\end{cases} \quad (4.9)$$

Here $\alpha=1+6\pi\sqrt{2\pi}/5\Gamma^2(\frac{1}{4})\approx 1.7$. The asymptotic forms of the function $\Psi_2(x)$ are also obtained without difficulty and are determined by the formula

$$\Psi_2(x)\cong\frac{i\alpha^{-11/6}}{4(k_0\delta_v)^{1/2}}\begin{cases}a_0\exp(-\pi i/12)-a_1x\alpha^{1/3}\exp(-\pi i/4), & |x|\ll\alpha^{-1/3}\\ \dfrac{b_0(1-3\,\mathrm{sgn}\,x)}{|x|^{1/2}\alpha^{1/6}}-i\dfrac{b_1(1+3\,\mathrm{sgn}\,x)}{|x|^{7/2}\alpha^{7/6}}, & \alpha^{-1/3}\ll|x|.\end{cases} \quad (4.10)$$

Here

$$a_0=\frac{5\pi}{9\beta}F\left(1,\frac{1}{6};2;\frac{(1-\alpha)(\sqrt{2}-1)}{\alpha+2^{1/2}-1}\right)\approx 1.97;$$

$$a_1=\frac{\pi}{2\beta}F\left(1,\frac{1}{2};2;\frac{(1-\alpha)(\sqrt{2}-1)}{\alpha+2^{1/2}-1}\right)\approx 1.71;$$

$$b_0=\frac{\sqrt{\pi}}{2^{1/2}\beta}\approx 1.43;\qquad b_1=\frac{15\sqrt{\pi}}{8\cdot 2^{1/2}}\frac{1+\beta}{\beta^2}\approx 5.96;$$

$\beta=(\alpha+\sqrt{2}-1)/\alpha\sqrt{2}\approx 0.88$; $F(a,b;c,d)$ is the hypergeometric function; the sign function $\mathrm{sgn}\,x=1$ for $x>0$ and $\mathrm{sgn}\,x=-1$ for $x<0$.

Figures 2 and 3 show graphs of the real and imaginary parts of the functions

$$\Xi_1(z)=\int_0^\infty dt\frac{\cos(zt)+2\sin(zt)}{t^3-i}\cong 2\alpha^{2/3}(k_0\delta_v)^{1/2}\Psi_1(z\alpha^{-1/3}) \quad (4.11)$$

and

$$\Xi_2(z)=i\int_0^\infty\frac{dt}{t^{1/2}}\frac{3\sin(zt)-\cos(zt)}{(t^3-i)(t^3-i\beta)}\cong 4\alpha^{11/6}(k_0\delta_v)^{1/2}\Psi_2(z\alpha^{-1/3}),$$

constructed by means of an electronic computer. These numerical calculations, together with the analytic formulas (4.9) and (4.10), give exhaustive information about the field distribution in the vicinity of the first and second spikes.

The line shape of the spikes in a metal with a mirror surface differs importantly from that that occurs in the diffuse case.[2,3] The important fact is that the field distribution $E(x)$ inside the spikes reflects the character of the principal skin layer δ_0 (see formulas (4.4) and (4.5)). Near the centers of both spikes, $x=2R$ and $x=4R$, the field changes linearly with distance. This is a general rule of behavior of $E(x)$ and is independent of the nature of the reflection. On increase of distance from the centers, the behavior of the second and third terms in (4.2) is different. Thus in the first spike, $\mathrm{Im}E(x)/E'(0)$ decreases in inverse proportion to the distance from the center (meanwhile $\mathrm{Re}E(x)/E'(0)$ changes according to the law $(x-2R)^{-4}$). In the second spike, the change of $\mathrm{Im}E(x)/E'(0)$ is slower—inversely proportional to the square root of this distance ($\mathrm{Re}E(x)/E'(0)\sim|x-4R|^{-7/2}$). Such a spreading out of the spikes with increase of the spike number indicates the absence of singularities in the distribution $E(x)$ at distances of three or more cyclotron diameters from the metal surface $x=0$. Besides the change of shape of the spikes, specular scattering leads to a diminution of their intensity in comparison with the diffuse case. The ratio of the amplitudes of the spikes in (4.2) to the corresponding values for diffuse reflection is proportional to the small quantity $(|k_0|\delta_v)^{-5/2}\ll 1$. Physically this is due to the fact that the impedance of the metal in the specular case is less than in the diffuse by a factor $|k_0|\delta_v \gg 1$. It is interesting to note that the ratio of the second term to the first in (4.2) is independent of the character of the scattering of electrons at the boundary and is in order of magnitude equal to $[\pi\gamma/\sinh(\pi\gamma)](\delta_v/4\pi R)^{1/2}\ll 1$. This result is quite natural, because the field $E_0(x)$ of the principal skin layer δ_0, which depends on the law of interaction of the electrons with the surface $x=0$, affects both spikes in the same way.

2. *High frequencies, proximity to cyclotron resonance*, $\nu\ll\Omega$ and $|\omega-n\Omega|\ll\Omega(n=1, 2, 3,\ldots)$. In this range $\varkappa=0$, the wave number $k_0=|k_0|\exp(i\pi/5)$, and $\pi\gamma\coth(\pi\gamma)\cong\omega/(\omega-n\Omega+i\nu)$. The contribution of grazing electrons in expressions (4.4) and (4.5) (the third terms in the denominators) in the resonance region are negligibly small in comparison with the contribution of volume electrons, $\pi\gamma\coth(\pi\gamma)$, and may be disregarded. Furthermore, the inequality (2.6) permits replacement of the functions $F_0(t/k_0\delta_v)$ and $\Phi_0(t/k_0\delta_v)$ by their asymptotic expressions for small values of the argument. As a result of all these simplifications we have

$$\Psi_1(x)=\frac{t}{2(k_0\delta_v)^{1/2}}\int_0^\infty dt\frac{\cos(xt)+2\sin(xt)}{t^3-(\Delta-i\tilde{\nu})^{-1}}, \tag{4.12}$$

$$\Psi_2(x)=\frac{i}{4(k_0\delta_v)^{1/2}}\int_0^\infty\frac{dt}{t^{1/2}}\frac{\cos(xt)-3\sin(xt)}{[t^3-(\Delta-i\tilde{\nu})^{-1}]^2}. \tag{4.13}$$

Here the following symbols have been introduced: $\tilde{\nu}=\nu/\omega$, the reciprocal of the quality factor, and $\Delta=(n\Omega-\omega)/\omega$, the relative "detuning" of the resonance ($\tilde{\nu}\ll 1$ and $|\Delta|\ll 1$).

In the immediate neighborhood of the resonance ($\tilde{\nu}\gg|\Delta|$) and also on its left wing ($\Delta<0$, $|\Delta|\gtrsim\tilde{\nu}$), formulas (4.12) and (4.13) can be analyzed by a method similar to that used in the preceding section. At distances of the order $x=2R$ and $x=4R$ there remain sharp spikes of the field. In contrast to the low-frequency case, the line shape of the spikes in the neighborhood of cyclotron resonance becomes complicated because of the presence of phase in the wave number k_0. There fore the real and imaginary parts of $E(x)/E'(0)$ in this frequency range are determined by combinations of $\mathrm{Re}\Psi_1$ and $\mathrm{Im}\Psi_1$ in the region of the first spike and of $\mathrm{Re}\Psi_2$ and $\mathrm{Im}\Psi_2$ in the region of the second. The role of the quantity α is now played by the large complex parameter $(\Delta-i\tilde{\nu})^{-1}$. The resonance condition $|\Delta-i\tilde{\nu}|\ll 1$ leads to a compression of the spike picture along the x axis as compared with the low-frequency field distribution. Physically this change of scale is due to the sharp diminution of the thickness of the skin layer of volume electrons under cyclotron-resonance conditions:

$$\delta_{\text{рез}}=\delta_v|\Delta-i\tilde{\nu}|^{1/3}\ll\delta_v. \tag{4.14}$$

The structure of the field in the specimen changes radically on transition to the right wing of the resonance, $\Delta>0$. If the detuning is sufficiently large, that is if

$$\tilde{\nu}\ll\Delta\ll 1, \tag{4.15}$$

then in the metal there can occur a propagation of weakly attenuating electromagnetic oscillations–cyclotron waves.[13] The wavelength λ of these excitations and their attenuation distance Λ, in the transparency region (4.15) of the metal in the short-wave range (2.6), are determined by the formulas

$$\lambda=\delta_v\Delta^{1/3},\quad \Lambda=3\lambda\Delta/\tilde{\nu}\gg\lambda. \tag{4.16}$$

From the expressions (4.12) and (4.13) it follows that in the metallic half-space, when conditions (4.15) are satisfied, an unusual cyclotron wave is excited. Mathematically this corresponds to the appearance in the integrands of (4.12) and (4.13) of a pole $t=\Delta^{-1/3}(1+i\tilde{\nu}/3\Delta)$ whose real part is much larger than its imaginary. The centers of excitation of the wave are the points $x=2R$ and $x=4R$. We shall consider the structure of Ψ_1 and Ψ_2 in the region (4.15) of existence of cyclotron waves. The function Ψ_1 can be expressed as the sum of two terms:

$$\Psi_1\left(\frac{x-2R}{\delta_v}\right)=-\frac{\Delta^{2/3}}{2(k_0\delta_v)^{1/2}}\Big\{i\int_0^\infty dt\frac{t^3+2\,\mathrm{sgn}(x-2R)}{t^6+1}\exp\left(-\frac{|x-2R|}{\lambda}t\right)$$
$$+\frac{\pi\sqrt{5}}{3}\exp[|x-2R|(i\lambda^{-1}-\Lambda^{-1})-i\,\mathrm{sgn}(x-2R)\tan^{-1}3]\Big\}. \tag{4.17}$$

The second term in (4.17), which oscillates with period $2\pi\lambda$, describes the field of a cyclotron wave that propagates in the vicinity of the point $x=2R$. The first term in (4.17) is a monotonic function of $|x-2R|$ and decays over a distance of the order of the wavelength λ from the center of excitation $x=2R$. It is the mean value of the field of the cyclotron wave. The asymptotic forms of this term are easily calculated, and therefore we shall not present them. The geometric center of the wave $x=2R$ is distinguished by the fact that the phase of the wave at this point undergoes a discontinuity of amount $2\tan^{-1}2$, while the mean value of the field, which experiences a jump, is different from zero. It is interesting that the amplitude of the wave is not modulated.

The expression for the function Ψ_2 looks like this:

$$\Psi_2\left(\frac{x-4R}{\delta_v}\right)=-\frac{\Delta^{11/6}}{4(k_0\delta_v)^{1/2}}\left\{\frac{i}{\sqrt{2}}\int_0 \frac{dt}{t^{1/2}}\exp\left(-\frac{|x-4R|}{\lambda}t\right)\cdot\frac{(t^6-1)[1-3\,\mathrm{sgn}(x-4R)]-2t^3[1+3\,\mathrm{sgn}(x-4R)]}{(t^6+1)^2}\right.$$
$$\left.-\frac{\pi\sqrt{10}}{18}\left(1-2i\frac{|x-4R|}{\lambda}\right)\exp[|x-4R|(i\lambda^{-1}-\Lambda^{-1})+i\,\mathrm{sgn}(x-4R)\tan^{-1}2]\right\}. \quad (4.18)$$

The meanings of the first and second terms in (4.18) are the same as for the corresponding terms of formula (4.17). The discontinuity of phase of the wave at the center $x=4R$ is $2\tan^{-1}3$.

Thus we have shown that excitation of a cyclotron wave by an external source of field occurs in the vicinity of the points $x=2R$ and $x=4R$. If the wave is propagated over a distance $\Lambda\ll 4R$, then in formula (4.2) we may neglect the terms containing

$$\Psi_1\left(-\frac{x+2R}{\delta_v}\right)\text{and}\,\Psi_2\left(-\frac{x+4R}{\delta_v}\right).$$

In the reverse case $\Lambda\gg 4R$, the terms containing

$$\Psi_1\left(-\frac{x+2R}{\delta_v}\right)\text{and}\,\Psi_2\left(-\frac{x+4R}{\delta_v}\right)$$

give the same kind of contribution to the wave-field distribution as do the terms containing, respectively,

$$\Psi_1\left(\frac{x-2R}{\delta_v}\right)\text{and}\,\Psi_2\left(\frac{x-4R}{\delta_v}\right).$$

The structure of the field of the cyclotron wave in the metal depends strongly on the nature of the scattering of the electrons by the specimen surface $x=0$. The excitation of cyclotron waves by the trajectory AP effect in a metal with a diffuse boundary was investigated qualitatively in[14]. The distribution of the field $E(x)$ obtained in[14] differs significantly from the corresponding expressions (4.2), (4.17), and (4.18). The different forms of the field in the specular and in the diffuse cases may make it possible to determine the character of the scattering of electrons by the specimen surface in experiments on wave transmission.

The phenomena considered in the present paper may be detected in experiments with metallic plates. In one-sided excitation of a plate of thickness d, the contribution of spikes to the impedance is negligibly small. Therefore it is possible to detect them only by analyzing the field $E(d)$ that passes through the metal under conditions such that a given spike emerges at the second specimen boundary $x=d$. The fact that in the case (4.15) the field in the spikes is a short cyclotron wave may facilitate experiments on detection of such waves. Usually difficulties are connected with the fact that the amplitude of a short cyclotron wave passing through a plate is exponentially small. If one uses the current plane $x=2R$ or $x=4R$ as a generator of cyclotron waves, then it is possible to choose conditions under which $d-2R<\Lambda$ (or $d-4R<\Lambda$), and the wave field will emerge at the second metal surface $x=d$ not too much weakened. In experiments with two-sided excitation, the effect of the second surface of the plate reduces to the fact that the formulas obtained in the present paper for the field $E(x)$ at $x=d$ determine the line shape of the dimensional effect.

1)The method of obtaining the asymptotic behavior of $\mathscr{K}(k)$ and $Q(k,k')$ is described in sufficient detail in[9,10]; therefore there is no need here to go into the details of the calculation.

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