ferent temperature dependences for the two terms. (Notice, for comparison, that the polaron parameter $\Phi(T)-\Phi(0)$ in the case when the acoustic phonons predominate depends on T at low T also like $(T/\omega_D)^4$, while at $T>\omega_D$ we have $\Phi(T)-\Phi(0)\propto T$.)

The fluctuational barrier lowering effect should play a very important role in subbarrier diffusion of atomic particles whose mass is large compared to the electron mass. The exponential dependence of the tunneling amplitude on the particle mass m for a fixed barrier and the virtual nondependence on m of the polaron exponent of the exponential function clearly lead to a situation in which the fluctuations of the barrier play a more and more decisive role as the mass of the diffusing particle increases. The effect can become so strong that it virtually amounts to a coherent preparation of a "hole" in the atomic configuration, especially for heavy particles or for "soft" lattices with an appreciable zero-point vibration amplitude. We emphasize the coherent nature of the "hole" (or barrier) preparation, bearing in mind its virtual character and the return of the lattice to the normal (with respect to the phonon number) state. Here we do not, naturally, postulate the actual creation of a vacancy. In this sense, the considered subbarrier-transfer mechanism differs radically from the vacancy mechanism, which has been widely discussed in connection with the problem of diffusion in quantum crystals (see, for example, [6]), although in both cases there arises an exponential growth of the coherent-diffusion coefficient with increasing T (in the case of the vacancy mechanism this growth is connected simply with the growth of the number of real vacancies). In the case of diffusion of He^3 in He^4, such a growth has been experimentally observed[4,5] (see also[6]).

As the temperature increases further, the incoherent or phonon-stimulated diffusion mechanism begins to play a greater and greater role, going over subsequently into the quasiclassical over-the-barrier diffusion. Not dwelling on the picture that arises in the fixed-potential-profile model and, in particular, on the role, discussed in detail in[2], of the many-level nature of the individual potential wells, we only note that, as follows from Sec. 3, the fluctuations of the barrier can sharply enhance the incoherent transfer mechanism. It is significant that such a transfer mechanism becomes predominant at sufficiently high T and in the total absence of the polaron effect, leading to a change in the exponential growth in comparison with $D_{\rm coh}(T)$. The qualitative picture of the dependence $D(T)$ naturally remains the same as shown in Fig. 2.

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Two-dimensional Heisenberg model with weak anisotropy

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A low-temperature phase transition in a two-dimensional Heisenberg model with weak anisotropy is considered. The transition is associated with spontaneous breaking of the symmetry group $O(2)$ of the Hamiltonian. The scaling dimension of the spin operator and the correlation length of the spin fluctuations that take the spin out of the easy-magnetization plane are calculated in the leading logarithmic approximation.

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1. INTRODUCTION

The question of the existence of a phase transition in two-dimensional degenerate systems (the XY-model and Heisenberg model) has been considered in the papers.[1–4] Berezinskiĭ showed that a phase transition exists in the XY-model, even though the spontaneous magnetic moment is equal to zero at all finite temperatures. Arguments in favor of the existence of a phase transition in the Heisenberg model were adduced in[2]. However, it was made clear in[3] that the screening of the interaction which takes place in this case was not taken into account in[2]. The question of the existence of a phase transition in the isotropic Heisenberg model remains open at the present time, although there are reasons to think that a transition does not occur at finite temperatures[4]. In[2-4] the isotropic Heisenberg model was considered. In the present paper we consider a low-temperature phase transition in a Heisenberg model with weak anisotropy. It is shown that for arbitrarily small anisotropy there is a phase transition. The transition temperature tends to zero when the anisotropy disappears.

In Sec. 2 we consider the asymptotic forms of the correlation functions. It is shown that at low temperatures the spin correlations have a power-law decay, and this implies spontaneous breaking of the symmetry group $O(2)$.[1] In Sec. 3 the temperature dependence of the exponent Δ is found in the leading logarithmic approximation. In Sec. 4 the results are discussed and a comparison with the experimental data of[5] is given.

2. ASYMPTOTIC FORMS OF THE CORRELATION FUNCTIONS

The Hamiltonian of the system of spins with anisotropic interaction has the form

$$H=\frac{Js^2}{2}\sum_{(\mathbf{x},\mathbf{a})}(\mathbf{n}_{\mathbf{x}+\mathbf{a}}-\mathbf{n}_{\mathbf{x}})^2+\frac{\lambda s^2}{2}\sum_{(\mathbf{x})}(n_{\mathbf{x}}^{(3)})^2, \tag{1}$$

where $\mathbf{n}_{\mathbf{x}}$ is the direction of the spin vector at the point $\mathbf{x}$ (here and everywhere below, $\mathbf{n}^2=1$), the summation over $\mathbf{x}$ runs over all the sites of a two-dimensional lattice and that over $\mathbf{a}$ runs over nearest neighbors. Two types of transition are possible, depending on whether λ is positive or negative. If $\lambda<0$ (easy axis), at low temperatures there is a spontaneous magnetic moment directed along the 3-axis. The transition in this case will occur at $T\sim 4\pi Js^2/\ln(J/\lambda)$, and the nature of the singularities in the thermodynamic quantities will be the same as in the Ising model. In the case $\lambda>0$ (easy plane), there is no spontaneous moment at any nonzero temperature, and the transition is associated with the appearance of a "transverse stiffness," as in the XY-model. Below we shall consider the case $\lambda>0$.

For the elucidation of the character of the asymptotic forms of the correlation functions at low temperatures $T\ll J$, only the long-wavelength fluctuations of $\mathbf{n}_{\mathbf{x}}$, the scales of which are much greater than the distance between neighboring spins, are important. For these long-wavelength excitations, in formula (1) we can go over from the summation to an integration and replace the difference by a derivative:

$$H=\frac{Js^2}{2}\int d^2\mathbf{x}[(\partial_\mu\mathbf{n})^2+m^2(n^{(3)})^2], \tag{2}$$

where $m^2=\lambda/J|\mathbf{a}|^2$ ($|\mathbf{a}|$ is the distance to the nearest neighbor).

In order to elucidate whether or not there is a phase transition, it is necessary, following[1], to find the asymptotic forms of the correlation functions

$$G_{ab}(\mathbf{x}_1-\mathbf{x}_2)=Z^{-1}\int\prod_{\mathbf{x}}d\mathbf{n}_{\mathbf{x}}e^{-\beta H}n_a(\mathbf{x}_1)n_b(\mathbf{x}_2), \tag{3}$$

$$Z=\int\prod_{\mathbf{x}}d\mathbf{n}_{\mathbf{x}}e^{-\beta H} \tag{4}$$

at large distances. In formulas (3) and (4) the integration over $\mathbf{n}$ runs over the unit sphere; $\beta=T^{-1}$ (T is the temperature). In calculating (3) it is convenient to parametrize the unit vector $\mathbf{n}$ in the following way:

$$n^{(3)}=\varphi,\quad -1\leqslant\varphi\leqslant 1; \tag{5}$$

$$n_\pm=n^{(1)}\pm in^{(2)}=\sqrt{1-\varphi^2}\,e^{\pm i\alpha},\quad -\pi\leqslant\alpha\leqslant\pi. \tag{6}$$

In this parametrization, the Hamiltonian (2) has the form

$$H=\frac{Js^2}{2}\int d^2\mathbf{x}[(\partial_\mu\varphi)^2+m^2\varphi^2+(\partial_\mu\sqrt{1-\varphi^2})^2+(1-\varphi^2)(\partial_\mu\alpha)^2]. \tag{7}$$

To calculate the asymptotic forms of the correlation functions at low temperatures $T\ll J$ we shall make use of the low-temperature expansion in the parameter $g^2=T/Js^2$. We rewrite the definition (3), (4) of the correlation functions in the parameters φ and α by making the replacement $\varphi\to g\varphi$, $\alpha\to g\alpha$:

$$G_{33}(\mathbf{x}_1-\mathbf{x}_2)=\frac{g^2}{Z}\int_{-1/g}^{1/g}\prod_{\mathbf{x}}d\varphi_{\mathbf{x}}\int_{-\pi/g}^{\pi/g}\prod_{\mathbf{x}}d\alpha_{\mathbf{x}}e^{-H}\varphi(\mathbf{x}_1)\varphi(\mathbf{x}_2), \tag{8}$$

$$G_{+-}(\mathbf{x}_1-\mathbf{x}_2)=Z^{-1}\int_{-1/g}^{1/g}\prod_{\mathbf{x}}d\varphi_{\mathbf{x}}\int_{-\pi/g}^{\pi/g}\prod_{\mathbf{x}}d\alpha_{\mathbf{x}}e^{-H}$$
$$\cdot\exp\{ig(\alpha_{\mathbf{x}_1}-\alpha_{\mathbf{x}_2})\}[(1-g^2\varphi_{\mathbf{x}_1}^2)(1-g^2\varphi_{\mathbf{x}_2}^2)]^{1/2}, \tag{9}$$

where

$$H=\frac{1}{2}\int d^2\mathbf{x}[(\partial_\mu\varphi)^2+m^2\varphi^2+(\partial_\mu\sqrt{1-g^2\varphi^2})^2+(1-g^2\varphi^2)(\partial_\mu\alpha)^2]. \tag{10}$$

In the calculation of (8) and (9) the finite size of the region of integration over φ and α leads to corrections that are exponential in g, and these, as shown in[1], do not change the character of the asymptotic forms of the correlation functions if the correlations without allowance for the corrections have a power-law decay ($G_{+-}(R)\propto R^{-2\Delta}$). Therefore, in calculating (8) and (9) we can integrate over φ and α between infinite limits and use ordinary perturbation theory.

We begin by calculating the asymptotic form of $G_{33}(\mathbf{x})$. From the Dyson equation for

$$G_{33}(\mathbf{q}^2)=\int d^2\mathbf{x}\,e^{i\mathbf{q}\mathbf{x}}G_{33}(\mathbf{x})$$

it follows that

$$G_{33}^{-1}(\mathbf{q}^2)=\mathbf{q}^2+m_R^2-\Sigma_R(\mathbf{q}^2,m_R^2), \tag{11}$$

$$G_{33}(\mathbf{x})=\text{const}\cdot\exp\{-m_R|\mathbf{x}|\}, \tag{12}$$

where $\Sigma_R=\Sigma(\mathbf{q}^2,m_R^2)-\Sigma(0,m_R^2)$, Σ is the sum of Feynman graphs that cannot be cut into two by cutting one line, $m_R^2=m^2-\Sigma(0,m^2)$. The expansion of $\Sigma(\mathbf{q}^2,m^2)$ goes in powers of g^2; therefore, for $g^2=(T/Js^2)\ll 1$ we shall have $m_R^2>0$ and G_{33} falls off exponentially in the region $|\mathbf{x}|\gg m_R^{-1}$. In first order in g^2,

$$m_R^2=m^2\left[1-\frac{g^2}{4\pi}\ln\frac{J}{\lambda}\right]. \tag{13}$$

We note also that the fluctuations of the field φ are small at low temperatures: $\langle\varphi^2\rangle \sim g^2\ln(J/\lambda)$, i.e., in the integrals (8) and (9) the principal contribution is made by those distributions of $\mathbf{n}_\mathbf{x}$ in which the vectors $\mathbf{n}_\mathbf{x}$ lie in the easy plane.

We turn now to the calculation of the asymptotic form of $G_{+-}(\mathbf{x})$ in the region $|\mathbf{x}| \gg m_R^{-1}$. If we neglect the finite size of the limits of integration, the integration over α can be performed since the integral (9) over the variable α becomes Gaussian:

$$\int_{-\infty}^{+\infty}\prod_{\mathbf{x}} d\alpha_\mathbf{x} \exp\left\{-\frac{1}{2}\int d^2\mathbf{x}[(1-g^2\varphi^2)(\partial_\mu\alpha)^2]+ig(\alpha_{\mathbf{x}_1}-\alpha_{\mathbf{x}_2})\right\}$$
$$=\exp\{F[\varphi^2]-g^2[G_{\mathbf{x}_1\mathbf{x}_2}(\varphi^2)-{}^1/_2 G_{\mathbf{x}_1\mathbf{x}_1}(\varphi^2)-{}^1/_2 G_{\mathbf{x}_2\mathbf{x}_2}(\varphi^2)]\}. \qquad (14)$$

Here $F[\varphi^2]$ is the sum of the one-loop diagrams and $G_{\mathbf{x}_1\mathbf{x}_2}(\varphi^2)$ is the correlation function $\langle\alpha_{\mathbf{x}_1}\alpha_{\mathbf{x}_2}\rangle$, depending on φ. The next step in the calculation of G_{+-} is to average over φ. The main contribution to the asymptotic form is then given by the expression

$$G_{+-}(\mathbf{x}_1-\mathbf{x}_2)=A\exp\{-g^2\langle[G_{\mathbf{x}_1\mathbf{x}_2}(\varphi^2)-{}^1/_2G_{\mathbf{x}_1\mathbf{x}_1}(\varphi^2)-G_{\mathbf{x}_2\mathbf{x}_2}(\varphi^2)]\rangle_\varphi\} \qquad (15)$$

or

$$G_{+-}(\mathbf{x}_1-\mathbf{x}_2)=A\exp\left\{-g^2\int\frac{d^2\mathbf{q}}{(2\pi)^2}\langle|\alpha_\mathbf{q}|^2\rangle(e^{-i\mathbf{q}\mathbf{x}}-1)\right\}. \qquad (16)$$

where A is a constant. The graphs not taken into account in (15), (16) fall off no more slowly than $|\mathbf{x}|^{-2}\ln|\mathbf{x}|$ at large distances and therefore lead only to corrections to the asymptotic form. For $\mathbf{q}^2\to 0$,

$$\langle|\alpha_\mathbf{q}|^2\rangle=Z_c(g^2, m_R)/\mathbf{q}^2. \qquad (17)$$

It is easy to convince oneself of this directly, by examining the perturbation theory; we, however, shall not do this, since (17) follows from the Goldstone theorem. It can be seen from (16) and (17) that for $|\mathbf{x}_{12}|\to\infty$

$$G_{+-}(\mathbf{x})\propto|\mathbf{x}|^{-2\Delta}. \qquad (18)$$

where $\Delta = g^2Z_c/4\pi$.

Thus, as in the XY-model, at low temperatures the spin correlations have a power-law decay. In first order in the temperature,

$$\Delta=T/4\pi Js^2, \qquad (19)$$

which coincides with the result of Berezinskiĭ.[1]

3. CALCULATION OF $\Delta(T)$

We turn now to the calculation of $\Delta(T)$. First we find the next term in the expansion of Δ in powers of the temperature. It can be seen from (17), (18) that for this it is necessary to find the first correction to $Z_c(g^2, m_R)$, which is easily calculated and is found to be equal to

$$Z_c=1+\frac{g^2}{4\pi}\ln\left(\frac{J}{\lambda}\right). \qquad (20)$$

In (20) we have neglected terms of the form $(\lambda/J)^n$. The appearance of $\ln(J/\lambda)$ in (20) is not accidental and is connected with the fact that the correlation function $\langle\varphi_0\varphi_r\rangle$ to first order in g^2 is equal to $(g^2/4\pi)\ln(mr)$ in the region $|\mathbf{a}|\ll r\ll m^{-1}$. In this region fluctuations of the field φ turn out to be important. In the approximation we are considering, with the Hamiltonian (10), the whole of the difference of Z_c from unity is connected with these fluctuations, and therefore it is clear that in the expansion of $Z_c(g^2, m_R)$ there will be terms of the form $[g^2\ln(J/\lambda)]^n$; the terms associated with the discreteness of the lattice will be of order g^{2n} and so we can neglect them. We also neglect terms of the form $g^2[g^2\ln(J/\lambda)]^n$ and so on, assuming that

$$g^2\ll 1, \quad g^2\ln(J/\lambda)\lesssim 1.$$

Thus, we arrive at the problem of summing the principal logarithmic terms $[g^2\ln(J/\lambda)]^n$. The separation of the "leading logarithms" in the perturbation-theory series in g^2 in the two-dimensional case encounters difficulties associated with the fact that all the Feynman graphs make a contribution. However, this difficulty can be circumvented using the method developed in[1,2]. We give here the arguments of Berezinskiĭ and Blank,[2] since in our case their method makes it possible to calculate $m_R(T)$, i.e., the correlation length of the field φ.

At low temperatures $T\ll J$ the spins of large regions fluctuate weakly about the overall direction.

We divide up the system of spins in the plane into regions of dimensions $|\mathbf{a}|\ll R\ll m^{-1}$. We integrate in formula (4) over the spins inside these regions, for a fixed distribution of spins on the boundaries. Then,

$$Z=N^{-1}\int\prod_{\mathbf{x}\in\Gamma}d\mathbf{n}_\mathbf{x}\prod_j Z\{\mathbf{n}_{\Gamma_j}\}, \qquad (21)$$

where Γ is the totality of boundaries and $Z\{\mathbf{n}_{\Gamma_j}\}$ is the partition function of the region S_j with a specified distribution of spins on the boundary Γ_j of the region. It is well-known that in two-dimensional systems with a continuous symmetry group the spontaneous moment is equal to zero.[6] This is correct, however, only in the thermodynamic limit, i.e., for a system of infinite dimensions. If, however, we consider a system of finite dimensions, its spontaneous moment is nonzero and is determined by the distribution of the spins on the boundary. Corresponding to a given distribution of the spins $\mathbf{n}_\Gamma$ on the boundary there is a distribution of the $\mathbf{n}_j$, i.e., of the directions of the spontaneous moment (j labels the region S_j). In (21), therefore, instead of integrating over distributions of the spins on the boundaries we can integrate over the directions and magnitudes in the spontaneous-moment distribution. In this case,

$$Z=\tilde{N}^{-1}\int\prod_j d\mathbf{n}_j P\{\mathbf{n}_j\}, \qquad (22)$$

where $P\{\mathbf{n}_j\}$ is the probability distribution of the directions of the spontaneous moments of the regions S_j. In the calculation of $P\{\mathbf{n}_j\}$ for a division into regions with characteristic dimensions $a\ll R_j\ll m^{-1}$, following[2], we write the distribution of the spins in the following form

$$n_x = n_0(x)(1-\varphi_a^2(x))^{1/2} + e_a(x)\varphi_a(x), \tag{23}$$

where $\mathbf{n}_0(\mathbf{x})$ is the direction of the spontaneous moment in the region S_j to which the point $\mathbf{x}$ belongs; $\mathbf{e}_a$ is a system of unit vectors ($\mathbf{e}_a \cdot \mathbf{n}_0 = 0$, $\mathbf{e}_a \cdot \mathbf{e}_b = \delta_{ab}$); summation over repeated indices is implied, and each index takes the values 1 and 2; $\varphi_a(\mathbf{x})$ is the deviation of the vector $\mathbf{n}$ from the direction of the spontaneous moment. Then,

$$P\{\mathbf{n}_0(\mathbf{x})\} = \langle \exp\{-\beta H(\mathbf{n}_0, \varphi)\}\rangle_{\varphi_a}, \tag{24}$$

where H is the Hamiltonian (2) in which $\mathbf{n}_x$ is parametrized by formula (23). With logarithmic accuracy, the averaging over the fields φ_a in (24) is performed over those

$$\varphi_a(\mathbf{q}) = \int d^2\mathbf{x} e^{i\mathbf{q}\mathbf{x}} \varphi_a(\mathbf{x}),$$

for which $R^{-1} < |\mathbf{q}| < a^{-1}$. The fluctuations of the fields with such $\mathbf{q}$ are small if

$$g^2 \ln (R/a) \ll 1.$$

For regions whose dimensions satisfy this inequality,

$$P\{\mathbf{n}_x\} = \exp\{-H(\mathbf{n})/T_R\}, \tag{25}$$

$$T_R^{-1} = T^{-1}\left[1 - \frac{T}{2\pi J s^2} \ln \frac{R}{a}\right], \tag{26}$$

$$H(\mathbf{n}) = \frac{Js^2}{2} \sum_{(\mathbf{x},\mathbf{a})} (n_{\mathbf{x}+\mathbf{a}} - \mathbf{n}_\mathbf{x})^2 + \frac{m_R^2}{2} \sum_{(\mathbf{x})} (\mathbf{n}_\mathbf{x}^{(3)})^2, \tag{27}$$

$$m_R^2 = m^2 \left[1 - \frac{T}{2\pi J s^2} \ln \frac{R}{a}\right]. \tag{28}$$

The sum over $\mathbf{x}$ in (27) denotes a sum over the regions, and the sum over $\mathbf{a}$ is over the neighboring regions. In (25)–(28) we have neglected terms of the order of $m^2 T \ln(R/a)$, $T^2 \ln(R/a)$, and so on. It can be seen from (25)–(28) that, after integrating over part of the spins, we have arrived at the original problem, but with changed parameters T and m and with a new lattice, in which the nearest-neighbor distance is of the order of R.

Repeating the reasoning, we go from the lattice with spacing R to a lattice with spacing R' ($R \ll R' \ll m_R^{-1}$). The arguments can be repeated for so long as the distance between neighboring "spins" $R \ll m_R^{-1}$. When the distance R becomes of the order of m_R^{-1}, the correlations of $n_\mathbf{x}^{(3)}$ can be neglected, since the spontaneous moment of regions with such dimensions lies in the plane and deviations that take it out of the easy plane can, with logarithmic accuracy, be neglected. Then,

$$P\{\mathbf{m}_x\} = \exp\left\{-\frac{Js^2}{2T} \sum_{(\mathbf{x},\mathbf{a})} (\mathbf{m}_{\mathbf{x}+\mathbf{a}} - \mathbf{m}_\mathbf{x})^2\right\}, \tag{29}$$

where $\mathbf{m}$ is a two-component unit vector;

$$T_R^{-1} = T^{-1}\left[1 - \frac{T}{4\pi J s^2} \ln \frac{J}{\lambda}\right]. \tag{30}$$

The distribution $P\{\mathbf{m}\}$ coincides with the distribution of spins in the XY-model (cf. [1]) and, therefore, the correlation function

$$\langle \mathbf{m}_0 \mathbf{m}_R \rangle = C(T)(a/R)^{T_R/4\pi\rho_s(T_R)}, \tag{31}$$

where $\rho_s(T_R)$ is the "superfluid density" in the XY-model (cf. [1,7]).

Thus, comparing (18) and (31), we find, with logarithmic accuracy,

$$\Delta(T) = \frac{T}{4\pi J s^2 \rho_s(T_R)} \left[1 - \frac{T}{4\pi J s^2} \ln \frac{J}{\lambda}\right]^{-1}, \tag{32}$$

$$m_R^2 = m^2 \left[1 - \frac{T}{4\pi J s^2} \ln \frac{J}{\lambda}\right]. \tag{33}$$

4. DISCUSSION OF THE RESULTS AND COMPARISON WITH EXPERIMENT

Thus, in Sec. 1 we have ascertained that the spin correlations at low temperatures in the two-dimensional Heisenberg model with weak anisotropy of the "easy-plane" type have a power-law decay. As shown in [1], this means that at low temperatures the symmetry group $O(2)$ of the Hamiltonian is spontaneously broken. This, in its turn, means that when the temperature is raised a second-order phase transition occurs in the system. The transition temperature is easily estimated. It follows from (32) that

$$T_c \approx 4\pi J s^2 / \ln (J/\lambda). \tag{34}$$

As T_c is approached, the index $\Delta(T)$ increases. The correlation length $R_c = m_R^{-1}$ also increases.

The behavior of the system above the transition point is not known, since it is not known whether a phase transition exists in the isotropic magnet. If there is not such a transition, then above T_c all the correlation functions will decay exponentially.

In the paper [7] by Pokrovskiĭ and Uĭmin a simple theory of layer magnets, based on scale-invariance considerations, was developed. Of special interest is the case when the intraplanar couplings are ferromagnetic and the interplanar couplings are antiferromagnetic. An example of a substance possessing such properties is the compound $(C_2H_5NH_3)CuCl_4$, which was investigated by de Jongh, van Amstel and Miedema. [5]

In [7] it was shown that, for such magnets, $\chi_\parallel / \chi_\perp = \Delta(T)$, where $\chi_\parallel$ is the susceptibility parallel to the magnetic field and $\chi_\perp$ is the transverse part of the magnetic-susceptibility tensor. A comparison of our results (formulas (32) and (34)) with the results of the experiments of [5] shows that formula (32) gives not a bad description of the experimental susceptibility data. (In comparing with experiment it is necessary to take into account that $\chi_\parallel = \chi_\perp \Delta(T)$ is fulfilled when $\Delta \leqslant 1$.) According to the estimate of the paper [5], the anisotropy maintaining the spins in the plane is $(J/\lambda) \sim 10^{-3}$, and, therefore, $T_c/J \sim 0.5$ (the experimental value of T_c/J is 0.548 [5]).

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Influence of pressure on the Fermi surface of cadmium

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Measurements were made of the areas of the extremal sections of the Fermi surface of cadmium and their pressure dependences were determined for magnetic field directions lying in the crystallographic planes (10$\bar{1}$0) and (11$\bar{2}$0). These experimental results were used in a calculation—within the local pseudopotential model—of the matrix elements of the pseudopotential and their pressure derivatives.

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1. The present paper reports an investigation of the Fermi surface of cadmium and the influence of hydrostatic pressure on this surface. Cadmium is characterized by a strong compressibility anisotropy, so that one can expect large changes under pressure. Investigations of the influence of pressure on the dimensions of the various parts of the Fermi surface make it possible to interpret more reliably the observed oscillation frequencies and to calculate, within the framework of the adopted model, the pressure dependences of the matrix elements of the pseudopotential. In these calculations use is made of the local pseudopotential model whose matrix elements W_q are governed only by the reciprocal lattice vectors q_i and are identical at all points in the Brillouin zone.

Like other hexagonal metals of the second group in the periodic table, cadmium has a Fermi surface which can be described qualitatively but satisfactorily by the model of almost-free electrons. This means that if we use the local pseudopotential approximation and the OPW method, we can calculate relatively simply and in a clear manner the areas of extremal sections of the Fermi surface and their pressure dependences, and to determine the matrix elements of the pseudopotential and their pressure dependences from a comparison of the calculated and experimental results. Such data can be used to explain the behavior of some macroscopic properties under pressure, such as the electrical resistivity, magnetoresistance, etc., and to estimate the pressure at which the topology of the Fermi surface changes. Investigations of the influence of pressure on the matrix elements of the pseudopotential are also important from the point of view of the pseudopotential theory because they can give information on the dependence of the pseudopotential on the wave vector.

In the one-wave approximation the Fermi surface of cadmium is of the same form as the Fermi surfaces of other divalent hexagonal metals,[1] but the radius of the Fermi sphere of free electrons k_F is less than the side a of the hexagonal base of the prism and, consequently, there are no needles or electron surfaces in the third Brillouin zone (Fig. 1).

The lattice potential alters not only the dimensions but also the shape of the Fermi surface of cadmium: the horizontal arms of the monster in the second Brillouin zone are broken, and the butterfly in the third Brillouin zone as well as the cigar in the fourth zone are absent, as shown by calculations of Falicov and Stark.[2]

Thus, according to the model of Tsui and Stark[3] the Fermi surface of cadmium consists of two hole pockets (α) in the first Brillouin zone and of the residue of the monster in the second Brillouin zone, and also of one electron lens (β) at the center of the third zone. In the extended-zone scheme the monster is a corrugated cylinder elongated along the [0001] axis and it has a minimal section β in the ALH plane in the region of the point H in the Brillouin zone. The section β includes the pocket α and it is separated from it by the spin–orbit gap. The maximum section of the monster γ in the plane ΓMK of the Brillouin zone is formed by residues of three monsters in contact and it consists of three sheets. As shown in Refs. 3 and 4, magnetic breakdown in various fields gives either the total cross section S_γ (in strong and weak fields) or $\frac{1}{3}S_\gamma$ and $\frac{2}{3}S_\gamma$ (in the range of intermediate magnetic fields). In addition to the frequencies of the oscillations corresponding to these sections, other