Particle creation in the vortex cosmological model

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A solution is found to the problem of particle creation near the singularity and the resulting reaction of the metric in the homogeneous vortex model considered by one of us earlier (Lukash, 1974). A complete picture of the evolution of the vortex model with allowance for matter production is constructed. It is shown that particle creation near the singularity for $t \gtrsim t_{pl}$ has the following effects: a) It strongly reduces the primeval vortex velocity of the matter, which is then quite inadequate for the vortex theory of the origin of rotation of galaxies; *b*) it does not lead to compensation of the total angular momentum of unit volume of matter by the angular momentum of the created free particles (gravitons); the angular momentum of the gravitons oscillates with an increasing amplitude. Bounds are deduced on the parameters of the vortex model from observations of the chemical composition of prestellar matter and the isotropic microwave background.

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1. INTRODUCTION

In this paper, we consider the spontaneous process of matter production in vortex cosmological models near the singularity and the effect of this process on the evolution of the models. The interest in vortex motions of matter in cosmological models is due to theories of galaxy formation from primeval vortices. These theories were already developed by von Weizsäcker, [1] Gamow, [2] and Oort. [3]

The development of vortex motions of matter as small perturbations of the Friedmann model (filled with a perfect fluid) was considered by Lifshitz. [4] The theory of the formation of galaxies in a hot model of the Universe from primeval vortices was developed in detail by Ozernoĭ, Chernin, Chibisov, Rees, Tomita, et al. [5-9] One of the main difficulties of this theory was a fact established by Lifshitz [4]: Despite the constancy of the vortex velocity of matter with equation of state $p = \epsilon/3$, the vortex perturbations of the metric increase when the singularity in the past is approached.[1)] This means that in the past the vortex perturbations of the metric must have been large and the cosmological model near the singularity was not the isotropic Friedmann model but something quite different.

It has become necessary to solve the problem of constructing a cosmological model that is essentially non-Friedmann near the singularity but during expansion goes over into a Friedmann model with vortex motions. One model of this type was constructed by one of the present authors in [11]; in this model, the choice of the vortex velocity profile in the form of a circularly polarized wave made it possible to preserve the spatial homogeneity at all times, including near the singularity, which greatly simplifies the mathematical investigation.

In the present paper, we solve the problem of cosmological expansion with allowance for particle creation near the singularity and the problem of the reaction of the metric to the particle creation in such a homogeneous vortex model. The importance of matter production in the vortex model was pointed out by Zel'dovich. [12] He conjectured that particle production at $t \sim t_{pl} \approx 10^{-43}$ sec would rapidly isotropize the cosmological expansion and transform the solutions to the so-called (see [13]) quasi-isotropic solution (see also [14]).

The process of matter production in anisotropic models without vortex motions was investigated in a preceding paper, [15] in which it was shown that the influence of the gravitation of the created particles on the metric leads to a rapid isotropization of the expansion. It is of particular interest to consider matter production in the presence of vortex motions in connection with Chibisov's conjecture of "zero vortices". [16] In principle, this conjecture permits one to have fairly strong vortex motions of the matter at the onset of galaxy formation but at the same time avoid the dilemma of a non-Friedmann start of the cosmological expansion. This is achieved as follows. It is assumed that near the singularity the vortex motion of matter with hydrodynamic energy-momentum tensor is exactly compensated by the oppositely directed vortex motion of free particles (gravitons). The total vortex is therefore zero and the metric of the Friedmann model is not destroyed. During expansion, this compensation is conserved in practice until the vortex dimension λ is greater than the horizon length t (we assume throughout $8\pi G = c = \hbar = 1$). After this time, gravitons from different vortices are mixed, their distribution becomes isotropic, and the vortex motions of matter with hydrodynamic tensor remain. However, these vortices have little influence on the metric since $\lambda < t$. In this model, the compensation of the vortex motions of the matter by the motion of the gravitons is specified as an initial condition.

Chibisov conjectured that this compensation (vanishing of total angular momentum because of the counterstreaming flux of gravitons) arose automatically during the spontaneous creation of particles (and, in particular, gravitons) near the singularity. On the face of it, such a conjecture has serious arguments in favor of it; for we know that particle creation due to a strongly anisotropic deformation near the singularity rapidly eliminates the anisotropy of the deformation. It was assumed by analogy that the presence of vortex motions modifies the particle creation in such a way that the particles acquire a motion opposite to the motion of the primeval particles and that the process continues until the total vortex is zero. Since the created moving flux includes gravitons (as well as interacting particles), vanishing of the total flux would entail the existence of compensated counterstreaming fluxes of ordinary matter (the primeval flux plus the created interacting particles) and the created gravitons, i.e., Chibisov's idea would be realized.

In this paper, the process of particle creation in vortex models is calculated on the basis of the existing theory of particle creation. [14] We find that in the vortex models the matter is produced on the average virtually at rest (see Sec. 4 for details), and the primeval vortex is not compensated. The later evolution of the model leads to oscillatory variations of the total vortex (in which the sign reverses) with an ever increasing amplitude.[2)] Thus, a process which in a certain sense is the opposite of Chibisov's vortex compensation idea occurs.

Conclusions from calculations of the evolution of anisotropic cosmological models with allowance for matter production are compared with astrophysical data. It is shown that for all initial data in the models the vortex motions of matter at the time of galaxy formation are very weak and quite inadequate to explain the formation of galaxies in the vortex theory.[3)]

In Sec. 2 we write down the metric and the equations that describe the evolution of the homogeneous vortex model.[11,17] In Sec. 3, we consider the evolution of the vortex model without allowance for particle creation. In Sec. 4, we construct the complete picture of the evolution of the vortex model with allowance for matter production near the singularity. In Sec. 5, astrophysical conclusions are drawn.

2. METHOD AND EQUATIONS OF THE EVOLUTION OF THE HOMOGENEOUS VORTEX MODEL

We shall consider a homogeneous model with vortex motion of the matter. The matter velocity field is a circularly polarized harmonic: As one moves along the z axis parallel to the wave vector $\mathbf{k} = \{k_\alpha\} = \{0, 0, k\}$, the velocity vector, which lies in the xy plane, is rotated through an angle $k\Delta z$. In the late stage of evolution, this model is a flat Friedmann model containing vortex motions of the matter (vector perturbations in Lifshitz's classification[4]). The metric of this model is Binachi type VII$_0$ with $T_3^0 \equiv 0$ (T_i^k is the energy-momentum tensor) and can be written in the form[11,17]

$$ds^2=dt^2-g_{\alpha\beta}dx^\alpha dx^\beta,\quad x^1\equiv x,\quad x^2\equiv y,\quad x^3\equiv z;\quad g_{\alpha\beta}=\gamma_{ab}e_\alpha{}^a e_\beta{}^b;$$
$$\gamma_{ab}(t)=\begin{pmatrix}\lambda_1 & 0 & 0\\ 0 & \lambda_2 & \nu\lambda_2\\ 0 & \nu\lambda_2 & \lambda_3+\nu^2\lambda_2\end{pmatrix}$$
$$e_\alpha{}^a(\mathbf{x})=\begin{pmatrix}\cos kz & \sin kz & 0\\ -\sin kz & \cos kz & 0\\ 0 & 0 & 1\end{pmatrix};\qquad (1)$$

k = const is the wave number, and the size of the vortex is $\lambda = 2\pi\bar{\lambda}$, $\bar{\lambda} = \lambda_3^{1/2}/k$.

The curvature of three-dimensional space is characterized by $\mu \equiv \frac{1}{2}\ln(\lambda_1/\lambda_2)$. If $\mu = 0$, the section t = const is flat. The principal directions of the metric tensor $g_{\alpha\beta}$ are oriented along the moving frame.[4)] The function $\nu(t)$ uniquely determines the orientation of the moving frame relative to the fixed frame e_α^a (see [17]). The metric tensor when projected onto the moving frame has diagonal form: $\{\lambda_{ab}(t) = \mathrm{diag}\{\lambda_a\}$ ($\lambda_{1,2}$ are defined to within an arbitrary common factor).

If ν = const, a transformation of the coordinate system reduces the metric (1) to diagonal form[17]:

$$g_{\alpha\beta}=\lambda_{ab}e_\alpha{}^a(\tilde{x})e_\beta{}^b(\tilde{x}),\quad \tilde{x}=x-(\nu/k)\cos kz,\quad \tilde{y}=y+(\nu/k)\sin kz,\quad \tilde{z}=z \qquad (2)$$

(in what follows, we assume $\nu \to 0$ as $t \to \infty$). Therefore, the rotation of the moving frame is characterized by the function L(t) related to the derivative of $\nu(t)$ by[11,17]

$$L=\frac{\dot{\nu}}{2}\frac{\lambda_2}{\lambda_3}\gamma^{1/2}=\frac{\nu'}{2}\frac{\lambda_2}{\lambda_3}. \qquad (3)$$

The dot denotes the derivative with respect to t and the prime denotes the differential operator $d/d\tau = \gamma^{1/2}d/dt$.

In the metric (1), the following nondiagonal components of the energy-momentum tensor of the matter are nonzero:

$$\gamma^{1/2}T_1{}^0=kL, \qquad (4a)$$
$$\gamma^{1/2}T_2{}^3=-L. \qquad (4b)$$

Thus, L is the total angular momentum of the matter in the scale of the wavelength $\lambda = 2\pi\lambda_3^{1/2}/k$ per unit Lagrangian volume.

Finally, we write down the diagonal Einstein equations projected onto the moving frame ($\gamma = \det\{g_{\alpha\beta}\} = \lambda_1\lambda_2\lambda_3$, $T = T_i^i$):

$$(\ln\lambda_1)''+k^2(\lambda_1^2-\lambda_2^2)=\gamma(T-2T_1{}^1), \qquad (5)$$
$$(\ln\lambda_2)''-4L^2\frac{\lambda_3}{\lambda_2}+k^2(\lambda_2^2-\lambda_1^2)=\gamma(T-2T_2{}^2), \qquad (6)$$
$$(\ln\lambda_3)''+4L^2\frac{\lambda_3}{\lambda_2}-k^2(\lambda_1-\lambda_2)^2=\gamma(T-2T_3{}^3), \qquad (7)$$
$$\frac{\lambda_1'}{\lambda_1}\frac{\lambda_2'}{\lambda_2}+\frac{\lambda_2'}{\lambda_2}\frac{\lambda_3'}{\lambda_3}+\frac{\lambda_3'}{\lambda_3}\frac{\lambda_1'}{\lambda_1}-4L^2\frac{\lambda_3}{\lambda_2}-k^2(\lambda_1-\lambda_2)^2=4\gamma T_0{}^0. \qquad (8)$$

Equations (8) and (4a) are first integrals of Eqs. (4b)–(7), which completely determine the evolution of the model.[5)] The Bianchi identities are obtained by differentiating Eq. (8) and eliminating the function L from Eqs. (4):

$$\frac{(\gamma^{1/2}T_0{}^0)'}{\gamma^{1/2}}=\frac{\lambda_1'}{2\lambda_1}T_1{}^1+\frac{\lambda_2'}{2\lambda_2}T_2{}^2+\frac{\lambda_3'}{2\lambda_3}T_3{}^3+2L\frac{\lambda_3}{\lambda_2}T_2{}^3, \qquad (9)$$
$$(\gamma^{1/2}T_1{}^0)'/\gamma^{1/2}=-k\gamma^{1/2}T_2{}^3.$$

In a Pascal fluid without viscosity, $T_2^3 \equiv 0$ and $L \equiv$ const; the presence of viscosity ($T_2^3 \neq 0$) changes L and the matter flux $J = T_1^0/\lambda_1^{1/2} = kL/\gamma^{1/2}\lambda_1^{1/2}$ related to it, the change in the flux being always collinear to the initial flux [see Eq. (4)].[11,17]

To explain the physical meaning of the metric (1), it is helpful to consider the case when it differs little from a homogeneous Friedmann model.[6)] In this case, apart from the vortex mode of perturbations (in accordance with Lifshitz's classification), which is characterized by velocity v_F at the start of the Friedmann stage [for $p = \epsilon/3$, the velocity $v = \beta/(1-\beta^2)^{1/2}$ is constant, where β is the 3-velocity], there also exist perturbations of the type of standing gravitational waves on the background of the flat Friedmann model. These waves are circularly polarized like the vortex mode with wave vector **2k** parallel to the z axis; in addition, there is also an axisymmetric (relative to z) anisotropic perturbation[7)] with infinite wavelength.[11,17,21]

Perturbations of the type of gravitational waves can be written in the form

$$\mu\approx A\frac{\sin\eta}{t^{1/2}}+B\frac{\cos\eta}{t^{1/2}},\quad \eta=\int_0^t\omega\,dt, \qquad (10)$$

and of axial type in the form

$$\lambda_3/(\lambda_1\lambda_2)^{1/2}\approx\mathrm{const}\,(1+C/t^{1/2});$$

$\omega = 2k/\lambda_3^{1/2}$ is the frequency of the gravitational wave, and A, B, C are constants.

Thus, for small deviations from the Friedmann model, the model (1) is characterized by four constants; v_F (the vortex motions of the matter), A and B (gravitational waves) and C (axial mode). The constants must be small but are otherwise arbitrary and can, in particular, be zero. If A = B = C = 0, there remains only the vortex mode of perturbations. It is natural to call this the "purely vortex" model.

Let us turn to the general case $A \neq 0$, $B \neq 0$, $C \neq 0$, $v_F \neq 0$. We shall consider the early stages of expansion of the models, when the deviations from the Friedmann model are large. In this case, all effects are nonlinear and it is no longer possible to make the subdivision into "gravitational-wave," "vortex," and "axial" modes. But the number of physically arbitrary constants remains the same. Considering the early stages in the evolution of the models, we shall classify as "purely vortex" those models that have A = B = C = 0 during the evolution in the nearly Friedmann stage. It is clear that these models represent a strongly degenerate case among the complete set of models (1).

If in addition to $v_F \neq 0$ at least one of A, B, or C is nonzero, we shall simply say that we have a vortex model (as opposed to a purely vortex model).

3. EVOLUTION OF THE VORTEX MODEL WITHOUT ALLOWANCE FOR MATTER PRODUCTION

In this section, we shall consider the evolution of the vortex model under the assumption that the velocity of the matter relative to the system (1) is low, $v^* \ll 1$. As will become clear from what follows (see Sec. 4), it is only this case that is of interest when allowance is made for particle creation near the singularity. Except for a special case (which we shall point out), we shall also assume that the energy—momentum tensor is hydrodynamic and $p = \epsilon/3$.

The picture of the evolution of the model is shown in Fig. 1. In the most general case, there is a "vacuum stage" near the singularity for which the terms with T_i^k can be ignored in Eqs. (4b)—(8). The solution in the vacuum stage has an oscillatory nature (like the Belinskiĭ-Lifshitz-Khalatnikov model [22]), and the amplitudes of the oscillations of the functions λ_1/λ_2 and λ_1/λ_3 decrease very rapidly as one moves away from the singularity [11,17-19,21-23]; in the intervals of monotonic variation of λ_a we have a Kasner solution. [24]

Let t^* be the time at which the vacuum solution ends. At this time, $\epsilon \sim 1/t^2$. We shall assume that at t^* the amplitude of the oscillations is still large and, in addition, the exponent in the dependence $\lambda_3 \sim t^{2q_3}$ is not particularly close to unity: $q_3 \neq 1$. We regard the fulfillment of these conditions as the most general requirement [18,19] and shall not consider other cases here.

It follows from Eqs. (5)—(8) that at $t \sim t^*$

$$\lambda v e^{\mu}/t \lesssim 1. \tag{11}$$

We shall assume that $\lambda \gg t$ at t^*. After the time $t \sim t^*$ the influence of the gravitation of the matter rapidly isotropizes the expansion. If $\mu_0 = |\mu^*| \gg 1$ at this time, the anisotropy of the spatial curvature is large. However, it still does not affect the expansion and the solution in the first approximation is the so-called quasi-isotropic solution [$\mu(t) \approx \mu^* = \text{const}$; $\lambda_a \sim t$]. We emphasize that the solution is not exactly quasi-isotropic (see [13]) but differs from it by small vortex corrections.

The influence of strong anisotropy of the spatial curvature begins to be manifested at the time $t \sim t_D$, which is determined by the condition $e^{\mu_0} \approx \lambda/t \gg 1$, and alters the rate of cosmological expansion in such a way that the anisotropy of the curvature and the deformation decrease in a power manner (the "damping" stage). [18,19,21] During the damping stage, $te^{|\mu|}/\lambda = \text{const} \approx 1$, and this stage continues until $\lambda \sim t(|\mu| \sim 1)$, after which there begins the Friedmann stage of expansion (with small perturbations), in which the gravitation of matter plays the principal role ($\Omega \approx 1$). We write down the laws of expansion during the damping stage ($a^2 = \lambda_{1,2} \gg b^2 = \lambda_{2,1}$, $c^2 = \lambda_3$) under the assumption that at the time $t \sim t_D$ the Universe is filled by:

1) a perfect fluid with $\epsilon_\gamma = 3p_\gamma$ (the index γ denotes all interacting particles); then

$$a \sim t^{1/4}, \quad b,\ c \sim t^{5/8}, \quad \gamma^{1/2} \approx abc \sim t^{3/2}; \quad te^{|\mu|}/\lambda \approx \sqrt{3/4}, \quad \varepsilon_\gamma \approx 21/32t^2; \tag{12}$$

2) by free massless particles (gravitons) ϵ_g with distribution function that is isotropic at this time. Then

$$a \sim t^{1/3}, \quad b,\ c \sim t^{2/3}, \quad \gamma^{1/2} \approx abc \sim t^{5/3}; \quad te^{|\mu|}/\lambda \approx 2\sqrt{2}/3, \quad \varepsilon_g \approx (-T_a^a)_g \approx 2/3t^2 \gg (-T_b^b)_g \approx (-T_c^c)_g. \tag{13}$$

During the Friedmann stage ($\lambda < t$), one can assume that $\lambda_1 = \lambda_2 \sim \lambda_3 \sim t$, $\epsilon_g/\epsilon_\gamma \approx$ const in the principal approximation. We shall consider this stage in more detail.

In Table I we give the values of the vortex velocity v at different stages of the cosmological expansion. It is important to note that v_F does not depend on the equation of state, i.e., on the law of evolution in the damping stage [see (12) and (13)] and is uniquely related to v^* and μ_0.

The different stages of evolution following the epoch t^* listed above occur when at the time $t \sim t^*$ the anisotropy of the curvature is large, $\mu_0 = |\mu^*| \gg 1$. But if $\mu_0 \sim 1$, then the Friedmann stage follows immediately after t^*. If the purely vortex model is to be realized, then besides the condition $\mu^* \sim -1$, which removes the wave A, one

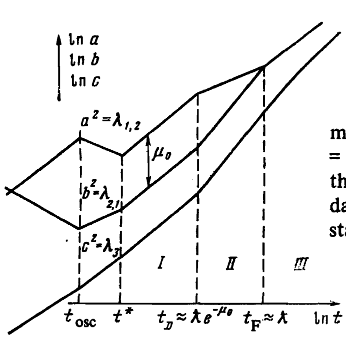

FIG. 1. Evolution of vortex model in the case $v^* \ll 1$ ($\mu_0 = |\mu^*| = \ln(a/b) \lesssim \ln(\lambda/t)^*$); I is the quasi-isotropic stage, II is the damping stage, III is the Friedmann stage.

TABLE I

	$t^* < t < t_D$	$t_D < t < t_F$	$t_F < t$
$a^2 = \lambda_1 (\mu_0 = \mu^*)$	$v^* \lesssim \left(\frac{t}{\lambda}\right)^* e^{-\mu_0}$	$v^* e^{2(\mu_0 - \mu)/3}$	$v_F = v^* e^{2\mu_0/3} \lesssim \left(\frac{t}{\lambda}\right)^* e^{-\mu_0/3} \lesssim \left(\frac{t}{\lambda}\right)^*$
$b^2 = \lambda_1 (\mu_0 = -\mu^*)$	$v^* \lesssim \left(\frac{t}{\lambda}\right)^* e^{\mu_0} \lesssim 1$	$v^* e^{-(\mu_0 + \mu)/3}$	$v_F = v^* e^{-\mu_0/3} \lesssim \left(\frac{t}{\lambda}\right)^* e^{2\mu_0/3} \lesssim \left(\frac{t}{\lambda}\right)^{*1/3}$

must also remove the wave B or the perturbation C [see (10)]. For this, one must require that at $t \sim t^*$

$$v\bar{\lambda}/t \sim 1, \qquad (14)$$

and the values of the Kasner exponents in the dependence $\lambda_a \sim t^{2q_a}$ must have the following form as t^* is approached from the left [see (19) and (20) below]:

$$q_1 \approx 1/3 - 17\pi v/8\sqrt{3}, \quad q_2 \approx 1/3 - 1/\sqrt{3} + 17\pi v/16\sqrt{3},$$
$$q_3 \approx 1/3 + 1/\sqrt{3} + 17\pi v/16\sqrt{3}. \qquad (15)$$

During the stage $\mu < 1$ ($t \gtrsim t^*$) this solution is of the form

$$\mu \approx -4\left(\frac{\bar{\lambda}}{t}v\right)^2 + 17\frac{\bar{\lambda}}{t}v^2\left(\sin\eta\int_\infty^\eta \frac{\cos\eta}{\eta}d\eta - \cos\eta\int_\infty^\eta \frac{\sin\eta}{\eta}d\eta\right);$$

$$\eta = \int_0^t \omega\, dt \approx 4t/\bar{\lambda}, \quad v \ll 1. \qquad (16)$$

For $t \gg \bar{\lambda}$, it follows from (16) that

$$\mu \approx 1/4(\bar{\lambda}v/t)^2, \qquad (17)$$

i.e., A = B = 0; see (10). For $t^* \approx \bar{\lambda}v < t \ll \bar{\lambda}$

$$\mu \approx -4\left(\frac{\bar{\lambda}}{t}v\right)^2 + \frac{17\pi}{2}\frac{\bar{\lambda}}{t}v^2 + 68v^2\left(C - \ln\frac{\bar{\lambda}}{4t}\right), \qquad (18)$$

where C = 0.57... is Euler's constant.

The first two terms in (18) are determined solely by the rotation [the terms $\sim L_2$ in (5)–(8)]; the corrections associated with the curvature and gravitation of the matter flux are small [$\sim v^2$ in (18)]. Therefore, during the stage $t_{osc} \approx \lambda v^r \ll t \ll \bar{\lambda}$ the solution in the principal approximation has the form

$$L\left(\frac{\lambda_3}{\lambda_2}\right)^{1/2} = \frac{\Lambda_1}{\mathrm{ch}\, 2\Lambda_1(\tau - \tau_0)}, \quad e^{2\mu} = \frac{\Lambda_0^2 \exp(-2\Lambda_2\tau)}{\mathrm{ch}\, 2\Lambda_1(\tau-\tau_0)},$$

$$p\gamma = \Lambda^2/9\, \mathrm{sh}^2(\Lambda\tau/3), \qquad (19)$$

where Λ, Λ_0, Λ_1, Λ_2 are constants, $\Lambda^2 = 3\Lambda_1^2 + \Lambda_2^2$, $r = (3 + \sqrt{3})/4$.

From the conditions of fitting to (18) [it is also necessary to remember that the function $\lambda_3/(\lambda_1\lambda_2)^{1/2}$ does not contain corrections proportional to $1/\sqrt{t}$, i.e., $\sim\tau$, during the Friedmann stage; C = 0, see (10)] we obtain

$$\Lambda_2 \approx 17\pi\, 3^{1/2}\Lambda v/16 \ll \Lambda,$$
$$-\Lambda_1\tau_0 \approx 17\pi v/32 \ll 1,$$
$$\Lambda_0 \approx 1 \quad (\Lambda_1\tau \approx 2\bar{\lambda}v/t < 1). \qquad (20)$$

For $\Lambda\tau \ll -1$ ($t < t^*$) Eqs. (19) describe the Kasner stage with exponents (15) to terms of order $\sim v^2$. At the time $\bar{\lambda}v/t \sim 1$ ($-\Lambda\tau \sim 1$) the Kasner stage is replaced by the Friedmann stage (16)–(20). At the time $t \sim t_{osc} \approx \bar{\lambda}e^\mu \approx \bar{\lambda}v^r \ll t^*$ we have the first oscillation ($t \to 0$) of the oscillatory vacuum asymptotic behavior (see [11] and [17]), $-\mu_{osc} \approx r\ln(1/v) \gg 1$; see Fig. 2. During the stage $t > t_{osc}$, $v \approx v_F = \mathrm{const} \ll 1$.

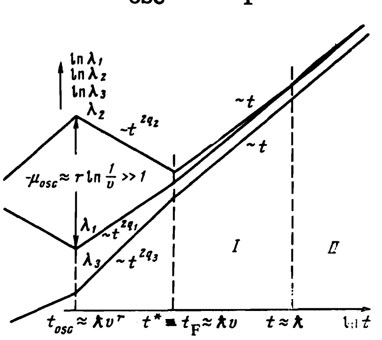

FIG. 2. Evolution of the purely vortex model with $v_F \equiv v^* \approx (t/\bar{\lambda})^* \ll 1$ ($v \approx v_F$ for $t > t_{osc}$). I and II are the Friedmann stage, $r = (3 + \sqrt{3})/4$, $q_1 = 1/3$, $q_2 = -(\sqrt{3} - 1)/3$, $q_3 = (\sqrt{3} + 1)/3$.

Thus, we have constructed the complete solution for the purely vortex model subject to the condition $v \ll 1$ during the Friedmann stage. We emphasize once more that to eliminate the quasi-isotropic mode (A) of gravitational waves from the general solution (1) it is necessary to specify an amount of matter in the model which is such that the time at which the vacuum stage ends, $t^* = t_F$, coincides with a definite value of the curvature anisotropy: see Eqs. (19) and (20), $\mu \sim -1$ for $\tau \sim \tau_F = \Lambda^{-1}$. To eliminate the Kasner modes (B and C) of gravitational waves it is necessary to make a special choice of the rate of cosmological expansion during the Kasner stage (15), (19). The amplitude of the B waves [at $t \lesssim \bar{\lambda}$, $\mu_B \approx B/\sqrt{t}$; see (10)] is directly related to the Kasner exponents during the stage (19), (20) (A = C = 0):

$$\mu_B \approx 8v\frac{\bar{\lambda}}{t}\left(q - \frac{17\pi}{16}v\right), \qquad (21)$$

$$q = \frac{\sqrt{3}(1/3 - q_1)}{2[1 - 9/4(1/3 - q_1)^2]^{1/2}}, \quad |q| < 1.$$

4. EVOLUTION OF THE VORTEX MODEL WITH ALLOWANCE FOR PARTICLE PRODUCTION

We consider the production of particle pairs near the singularity in the metric (1). Of course, the very formulation of the problem of quantum effects in an external classical gravitational field is correct only when $t > t_{pl} = (G\hbar/c^5)^{1/2} \approx 10^{-43}$ sec (then curvature invariants of the type $R_{iklm}R^{iklm}$ have values less than t_l^{-4}). In order to use the results obtained earlier in [14] about the effect of production in a given external metric without allowance for the reaction of the metric to the created particles, we use, as in the foregoing paper [15], the following formal device: We shall assume that for $t < t_0$, where

$$t_0 \gg t_{pl}, \qquad (22)$$

there is no particle production; then, for $t = t_0$, when we can now correctly determine the vacuum state (for example, by diagonalizing the Hamiltonian of the quantum fields), the production effect is switched on. One can then, as was done earlier in ref. 15, calculate the expectation value $\langle T_i^k\rangle_p$ of the energy-momentum tensor of the quantum fields for all $t \geq t_0$. The result obtained for $\langle T_i^k\rangle_p$, which is a functional of the given metric (1), is substituted into the right-hand side of Einstein's equations. Thus, for $t \geq t_0$ one seeks a solution of the equations

$$R_i^k - 1/2 R\delta_i^k = \langle T_i^k\rangle_p + T_i^{k(0)} \qquad (23)$$

with the initial condition $\langle T_i^k\rangle_p = 0$ for $t = t_0$. The admissibility of the classical description of spacetime is here guaranteed by the fulfillment of the condition (22). We shall assume that the classical primeval matter $T_i^{k(0)}$ needed if the vortex in the metric (1) is already to have existed prior to the time t_0 at which the production begins has the equation of state $p = \epsilon/3$. The total angular momentum L of the matter is equal to the sum of the angular momenta of the primeval (L_0 = const) and the produced (L_p) matter: $L = L_0 + L_p$.

The result obtained by solving the system of equations (23) will depend on t_0 as a parameter. In reality, one will expect that $t_0 \sim t_{pl}$; therefore, in the result obtained we set $t_0 \sim t_{pl}$, but $t > t_{pl}$. At the same time, we shall still be at the limit of applicability of the theory we have constructed, and we can therefore hope that the results remain qualitatively correct.

The program described above for solving Einstein's equations with allowance for particle production and the reaction of the metric to the created particles was implemented in [15] for the case of the Bianchi type I model, in which the spatial curvature is identically equal to zero. The calculations of Zel'dovich and one of the authors [14] also referred to this model. It was found that the main contribution to $\langle T_i^k \rangle_p$ is made by particles created at the earliest possible time, i.e., at $t \sim t_0$, these having had energy $\omega_p \sim t_0^{-1}$ at that time. In other words, the wavelength of the particles that make the main contribution to $\langle T_i^k \rangle_p$ is of the order of the horizon at time $t \sim t_0$. It follows that if we consider the metric (1) with initial condition

$$\lambda \gg t \text{ for } t=t_0 \tag{24}$$

(and it is a condition of this type that is physically interesting)[8] then the production in such a metric will take place in accordance with the same laws as in the Bianchi type I metric [to within small terms of the form $(t/\lambda)^2$ and less]. Therefore, we can use the results obtained previously in [15]. In particular, we can assume that the production process ends at $t \sim 3-5t_0$; the created particles do not yet affect the metric. The reaction of the metric to the created particles becomes important only at $t^* \gg t_0$; if the production occurred during the Kasner era with exponents $q_1 \leq q_2 \leq q_3$, then $t^* \approx t_0(t_0/t_{pl})^{2/(1+|q_1|)}$. At $t \sim t^*$, quantum effects can be ignored and the tensor $\langle T_i^k \rangle_p$ has purely classical form; in particular, when $t \sim t^*$, the tensor $\langle T_i^k \rangle_p$ satisfies the energy dominance condition, which is violated at $t \sim t_0$.

Because the scalar curvature vanishes, $R = 0$, for the metric we are considering (by hypothesis, the primeval classical matter has $T = T_i^{i(0)} = 0$), the absence of conformal covariance for gravitons noted by Grishchuk [25] is unimportant for the creation process. One must therefore expect that the gravitons are produced in about the same amount as the matter particles.

The energy density of the created particles is $\sim t_{pl}^2 t_0^{-4}$ at $t \sim t_0$. Estimates of the cross sections [15] show that for all particles except gravitons $\sigma n t \gtrsim 1$ when $t \sim t_0$, and that then $\sigma n t$ increases. We shall therefore assume that when $t \gg t_0$ all matter particles are described by the equation of state $p = \epsilon/3$; at the same time, the created matter is mixed with the primeval classical matter described by the energy-momentum tensor $T_i^{k(0)}$.

For gravitons, $\sigma n t < (t_{pl}/t_0)^2 \ll 1$ and $\sigma n t$ decreases with increasing t, so that the gravitons always remain free. Their energy density is also $\sim t_{pl}^2 t_0^{-4}$ at $t \sim t_0$, and the distribution function can be assumed to be approximately isotropic, as we already assumed in [15]. The exact structure of the graviton distribution function is not important here; it is only important that this function decreases rapidly at energies $\omega_g(t_0) > t_0^{-1}$, i.e., when the wavelength of the created gravitons is less than the horizon.

How does the vortex velocity of the matter change when new particles are created? Note first that if the created particles are at rest on the average in the system (1) (i.e., their center of mass in unit volume does not move, $L_p \equiv 0$), then once they have been mixed (have interacted) with the primeval matter the final velocity of the matter as a whole will be less than the primeval velocity. The change in the velocity can be readily calculated if one knows the ratio of the densities of the created and the primeval particles. At the same time, the total angular momentum L in unit volume of space is conserved.[9]

The variation of L during the particle production and during the subsequent evolution of the model is a nontrivial problem. It is difficult to calculate this variation. If L becomes equal to zero as a result of particle production and subsequent evolution, this would mean that the total angular momentum is compensated by the counterstreaming motions of the matter and the gravitons and that Chibisov's idea is realized.

Let us consider the change of L during the particle production process. As we have already said, the production process continues from the switching-on time t_0 for a time that is, basically, of order t_0. We shall show that as a result of particle production the total angular momentum changes by an amount that is of the order of the primeval angular momentum L_0, i.e., that immediately after the particle production process has ended the total angular momentum is $L^* = L_0 + L_p \sim L_0$. The mechanism by which L varies reduces to the following: In the production process, viscous off-diagonal strains appear in the energy-momentum tensor ($T_2^3 \sim \dot{L}$), changing L. During the production process, the Bianchi identities (4) and (9) are satisfied. In addition, it is easy to show that for $\lambda \gg t$

$$T_2{}^3 \approx -\nu \frac{\lambda_2}{\lambda_3} T_2{}^2 \approx \nu \frac{\lambda_2}{\lambda_3} \frac{t_{pl}{}^2}{t^4}$$

[we have assumed everywhere $\nu_0 = \nu(t_0) = 0$; for T_2^2 we have used the estimate obtained in [14]. Hence,

$$\dot{L} = -\gamma^{1/2} T_2{}^3 \approx -\nu \frac{\lambda_2}{\lambda_3} \gamma^{1/2} \frac{t_{pl}{}^2}{t^4}, \quad \Delta L \sim L_0 \left(\frac{t_{pl}}{t_0}\right)^2 \ll L_0. \tag{25}$$

Thus, during the production time $\Delta t \sim t_0$ the angular momentum hardly changes. If we take $t_0 \sim t_{pl}$, then $\Delta L \sim L_0$. However, analysis of the equation for L [see (26) below] shows that in the general case one cannot assert that $|L_0 - \Delta L| \ll L_0$, so that total compensation of the primeval angular momentum L_0 during the time $\Delta t \sim t_0$ never occurs. Moreover, the sign of ΔL is not always opposite to that of L_0, but depends on the sign of T_2^2, which, in its turn, depends on the axis along which contraction takes place at the time $t \sim t_0$. Thus, under the condition $\lambda \gg t$ the created particles are almost at rest on the average. This enables us to determine the number of created particles on the basis of the results already obtained in [15] and, as we have pointed out above (in footnote 9), to calculate the change in the vortex velocity. In this way, we determine all the initial parameters for the subsequent integration of the Einstein equations (4)—(8) without any quantum effects.

Thus, the solution of the self-consistent problem (4)—(8), (23) for $t > 3-5t_0 \approx t^*$, when the production processes can be already ignored, reduces to a solution of the Einstein equations on whose right-hand side there is a sum of the energy-momentum tensors of the matter $(T_a^b)_\gamma$ with equation of state $\epsilon_\gamma = 3p_\gamma$ (created particles except for gravitons plus the primeval classical matter) and the free massless particles $(T_a^b)_g$ (gravitons).[10] The total angular momentum when $t > t^*$ is $L = L_\gamma + L_g$.

Here, L_γ is the angular momentum of the interacting particles with Pascal pressure and L_g is the angular momentum of the free particles (gravitons); $L_\gamma = \text{const}$ in the subsequent evolution [since $(T_2^3)_\gamma \equiv 0$], and L_g varies because of the kinetics of the noninteracting particles. Thus, the total angular momentum $L(t)$ changes only because L_g does. When $t \sim t^*$, $L \sim L^* \sim L_0$. The function $\nu(t)$, whose derivative determines the function $L(t)$ [see Eqs. (3) and (4)], can be found from a second-order equation (which does not depend on **k**) from the known metric $\lambda_a(t)$:

$$\ddot{\nu}+\dot{\nu}\left(\ln\frac{\lambda_2\gamma^{1/2}}{\lambda_3}\right)-2\nu(T_2^2)_g=0. \tag{26}$$

We recall that $v^* < 1$ when $t^* \sim t_0 \sim t_{pl}$ (see footnote 9). The evolution of the metric with these initial conditions was constructed in Sec. 3. Thus, the problem reduces to integrating Eq. (26) with the initial conditions $L \sim L^* \sim L_0$ for $t \sim t^* \sim t_{pl}$. Depending on the parameter $\alpha = (\epsilon_g/\epsilon_\gamma)^*$, the fraction of free particles, or gravitons (during the quasi-Friedmann stage the graviton distribution function is isotropic), one can have different laws of variation of the total angular momentum $L(t)$:

$$\alpha>\frac{1}{7},\quad \nu\sim\frac{t^{-1/4}}{\beta}\sin\xi,\quad L\sim t^{1/4}\left(\cos\xi-\frac{1}{\beta}\sin\xi\right),$$

$$\alpha<\frac{1}{7},\quad \nu\sim\frac{t^{-1/4+\beta/4}}{2\beta}\left[1-\left(\frac{t}{t_1}\right)^{-\beta/2}\right],$$

$$L\sim\frac{t^{1/4-\beta/4}}{2\beta}\left[1-\frac{1-\beta}{1+\beta}\left(\frac{t}{t_1}\right)^{\beta/2}\right],\quad (-T_2^2)_g=\frac{\alpha}{4(\alpha+1)t^2}, \tag{27a}$$

$$\xi=\frac{\beta}{4}\ln\frac{t}{t_1},\quad \beta=(|1-7\alpha|/1+\alpha)^{1/2},$$

$$t^*<t<t_D,\qquad t_1=\text{const}.$$

In no cases are the vortex fluxes compensated: Although at the initial time $t > t^*$ a vortex flux does arise in the graviton distribution function, and this flux completely compensates the vortex motions of the matter at a certain time $t = t_{eq}$, at this time the viscosity $(T_2^3)_g$ is maximal $[\dot{L} \sim (T_2^3)_g \neq 0]$, and at subsequent times the total vortex flux in the gravitons oscillates with increasing amplitude (see Fig. 3). During the damping stage ($t_D < t < t_F$) the graviton distribution function in the momentum space takes the form of an ellipsoid of revolution that becomes ever more prolate in the a direction (see Fig. 1). (At the end of the damping stage, the pressure of the gravitons in the direction a is $e^{2\mu_0}$ greater than in the perpendicular direction.) Therefore, the

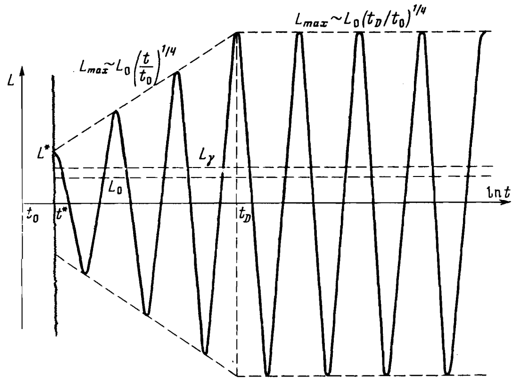

FIG. 3. Evolution of the total angular momentum $L = L_\gamma + L_g$ for $\alpha = (\epsilon_g/\epsilon_\gamma)^* \sim 1$ during the stage $t_{pl} < t^* < t < t_F \sim \lambda\!\!\!^-$ (see Fig. 1, $\lambda_1 = b^2$).

graviton energy $\epsilon_g \approx (-T_a^a)_g \gg (-T_b^b)_g \approx (-T_c^c)_g$ increases in this stage compared with the energy ϵ_γ of the radiation. If $a^2 = \lambda_1 > \lambda_2$, then $L \approx \text{const}$ since in this case $(-T_2^2)_g$ decreases rapidly compared with $\epsilon_\gamma(t)$ [i.e., the viscosity $T_2^3 = (T_2^3)_g$ can be ignored]. In the opposite case, $a^2 = \lambda_2 > \lambda_1$, $(-T_2^2)_g \approx \epsilon_g \approx 2/3t^2 > \epsilon_\gamma$ [see (13)] and from Eq. (26) we obtain

$$\nu\sim\sin\left(\frac{2}{\sqrt{3}}\ln t+\varphi_0\right),$$
$$L\approx L_{max}\cos\left(\frac{2}{\sqrt{3}}\ln t+\varphi_0\right), \tag{27b}$$

where φ_0 and L_{max} are constants.

Here the viscosity plays an important role since the powerful counterstreaming flux of free particles $(-T_2^2)_g$ in the direction perpendicular to the vectors $\mathbf{k} = \{0, 0, k\}$ and $\mathbf{v} = \{v, 0, 0\}$ in the presence of rotation in the 2–3 plane ($\nu - \nu^* \sim \nu\Delta t \sim L_0 \neq 0$; we recall that we assume throughout $\nu_0 = 0$) leads to the appearance of a component of the flux in the direction 3 (the direction of **k**)—to the appearance of a strong oblique flux in the 2–3 plane $(T_3^2)_g \approx \nu(-T_2^2)_g$.

Let us dwell in somewhat more detail on the physical nature of the resulting angular momentum L_g (the interpretation is due to Zel'dovich). The occurrence of the component T_2^3 in the energy-momentum tensor signifies the appearance of oblique counterstreaming fluxes of free particles in the 2–3 plane, i.e., the principal directions of the energy-momentum tensor of the gravitons are not oriented along the axes 2 and 3 (see Fig. 4). Suppose that at $t = t_0$ there are no directed particle fluxes in the graviton distribution function (all the fluxes are counterstreaming, of equal magnitude in opposite directions), $(T_\alpha^0)_0 = 0$. Suppose, for simplicity, that the gravitons move toward one another in the 2–3 plane perpendicular to the axis 1 at an angle θ to axis 2 (see Fig. 4), $\{k_2 = \pm k_0\cos\theta,\ k_3 = \pm k_0\sin\theta\}$ are the covariant components of the wave vector of the gravitons. To terms of order $\sim (t/\lambda\!\!\!^-)^2$, k_0 and θ are integrals of the motion and

$$(-T_2^2)_g=\frac{\varepsilon_g}{1+\lambda_2\,\text{tg}^2\theta/\lambda_3},\quad (-T_3^3)_g=\frac{\varepsilon_g}{1+\lambda_3\,\text{ctg}^2\theta/\lambda_2},$$

$$(-T_2^3)_g=\frac{\varepsilon_g\,\text{ctg}\,\theta}{1+\lambda_3\,\text{ctg}^2\theta/\lambda_2}; \tag{28}$$

$$\varepsilon_g=(T_0^0)_g=\frac{n\omega}{\gamma^{1/2}}=\frac{n}{\gamma^{1/2}}k_0\left(\frac{\sin^2\theta}{\lambda_3}+\frac{\cos^2\theta}{\lambda_2}\right)^{1/2},$$

where ϵ_g is the energy density of the gravitons, and $n = \text{const}$ is the number of particles in unit Lagrangian volume.

It is easy to calculate the energy flux density J_g of the gravitons that arises at an arbitrary point of space (A). At time t at the point A ($z = 0$) particles arrive that at $t = t_0$ were at points with coordinates $\pm z$ (see Fig. 4), where

$$z=\left|\int_{t_0}^{t}\frac{k^3}{\omega}dt\right|=\int_{t_0}^{t}\left(1+\frac{\lambda_3}{\lambda_2}\text{ctg}^2\theta\right)^{-1/2}\frac{dt}{\lambda_3^{1/2}}. \tag{29}$$

Obviously, the uncompensated part of the momentum carried by these particles to the point A is always collinear with the velocity **v** and equal to $k_1 = k_0 kz\cos\theta$.

Thus [see (28) and (29)]

$$J_g = \frac{(T_1{}^0)_g}{\lambda_1{}^{1/2}} = \frac{k_0 kz\cos\theta}{\lambda_1{}^{1/2}}\frac{n}{\gamma^{1/2}} = \frac{k}{(\gamma\lambda_1)^{1/2}}\cdot\int_{t_0}^{t}\varepsilon_g\gamma^{1/2}\left(\frac{\sin^2\theta}{\lambda_3}+\frac{\cos^2\theta}{\lambda_2}\right)^{-1/2}$$

$$\times\cos\theta\left(1+\frac{\lambda_3}{\lambda_2}\operatorname{ctg}^2\theta\right)^{-1/2}\frac{dt}{\lambda_3{}^{1/2}} = -\frac{k}{(\lambda_1\gamma)^{1/2}}\int_{t_0}^{t}\gamma^{1/2}(T_2{}^3)_g\,dt. \qquad (30)$$

This equation completely recalls the physical meaning of Eqs. (4), which describe the evolution of the angular momentum L. It only remains to add that the occurrence of $(T^3_2)_g$ is due to the rotation in the 2–3 plane [$\dot{\nu} \neq 0$, see (3)], i.e. the primeval vortex velocity **v**:

$$(T_2{}^3)_g \approx -(\nu-\nu_0)\frac{\lambda_2}{\lambda_3}(T_2{}^2)_g,$$

so that $\nu(t) - \nu(t_0)$ has the meaning of $-\tan\theta$ in Eqs. (28).

5. ASTROPHYSICAL APPLICATIONS

In the foregoing sections, we have constructed the evolution of the vortex model subject to the condition that the spatial homogeneity is preserved. This last condition necessarily restricts direct applications of the model to processes in the real Universe. Nevertheless, the model does enable one to draw important conclusions.

First of all, as we have already emphasized in Secs. 2 and 3, the purely vortex model is extremely degenerate even in the framework of the homogeneous model. The purely vortex model differs little from the Friedmann model if $ƛv/t < 1$. If allowance is made for matter production at the time $t_0 \sim t_{pl}$ and the conditions for the existence of the purely vortex model are satisfied, the Friedmann stage begins, as we have shown, at the time t_0 and continues until $t = \infty$. However, the vortex velocities which result are then extremely small [see (34) below]. In the general case of homogeneous vortex models, the Friedmann model can be continued into the past only to the time t_F, which is determined from the condition $ƛ \sim t$. Before this, the anisotropic stage occurred. This condition in conjunction with the observed degree of isotropy of the microwave background imposes strong restrictions on the parameters of the model (see the end of this section).

The circumstance that we have succeeded in constructing a complete solution with vortex motion from t_{pl} to the contemporary t allows us to consider the formation of chemical elements in the model. Comparison with observations gives a new restriction on the parameters of the vortex motion.

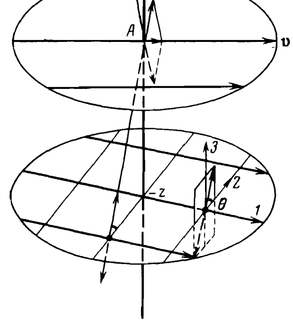

FIG. 4. Spatial structure of the vortex model.

We introduce the following notation: l is the contemporary value of $ƛ$ in centimeters, κ is the ratio of the density of gravitons to the density of photons at the contemporary time: $\kappa = \epsilon_g/\epsilon_\gamma = \alpha e^{2\mu_0}/3$; $|\mu| \approx \mu_0 = \text{const}$ during the quasi-Friedmann stage ($t_{pl} < t < t_D$), see Fig. 1.

The evolution considered in Sec. 3 with allowance for the initial conditions at $t_0 \sim t_{pl}$ in Sec. 4 enables us to calculate ($\kappa \gtrsim 1$)

$$t_F \approx 10^9 \kappa^{1/2}(l/10^{25})^2 \text{ sec}, \qquad (31)$$

$$t_D \approx \kappa^{-9/2} t_F \approx e^{-3\mu_0} t_F. \qquad (32)$$

In the estimate we have assumed that at t_{pl} the number of created gravitons is of the order of the number of interacting particles, $\alpha = (\epsilon_g/\epsilon_\gamma)_0 \approx 1.$[11] If $t_D > 1$ sec, chemical elements are formed during the stage of quasi-isotropic expansion and the outcome of the nuclear reactions is the same as in the Friedmann model. About 30% He^4 by weight is formed. If $t_D < 1$ sec, the chemical elements are formed during the anisotropic stage (13). The amount of He^4 can be estimated by the method indicated earlier in[28]. For there to be less than 50% of He^4 (the contrary would strongly contradict observations) it is necessary that $t_D > 10^{-3}$ sec, and from (32) we find that

$$\kappa < 10^3 (l/10^{25})^{1/4}. \qquad (33)$$

[We recall that Eqs. (31)–(33) apply for $\kappa \gtrsim 1$ since $\alpha \sim 1$.]

Thus, the energy density in the gravitons can at the contemporary epoch be of the order of the matter density in galaxies, which still does not contradict observations of the chemical composition of matter. We recall that according to the estimates of Shvartsman[29] there is a much stronger restriction on κ in the case of isotropic expansion: $\kappa < 3$–5. In our case, κ becomes much greater than unity during the damping stage. For $t > t_F$, the gravitons are distributed anisotropically: Up to the time $t < t_F e^{2\mu_0} \approx t_F \kappa^3$ the distribution function has the form of an ellipsoid elongated in the direction a (see Fig. 1); by the time $t \sim t_F e^{2\mu_0}$ the anisotropy of the distribution in the 1–2 plane is equalized out, after which the distribution function in the momentum space has the form of an ellipsoid of revolution that is oblate in the direction of the wave vector **k** (the direction **c**) with semiaxis ratio $\approx e^{\mu_0} \approx \kappa^{3/2}$.

From the inequality (33) and Table I we obtain an upper bound on the vortex velocity v_F during the quasi-Friedmann stage:

$$v_F \leqslant \left(\frac{t}{ƛ}\right)^{*}\kappa \approx \left(\frac{t_{pl}}{t_F}\right)^{1/2}\kappa^{7/4} \approx 10^{-26}\left(\frac{l}{10^{25}}\right)^{-1}\kappa^{3/2} < 10^{-21}\left(\frac{l}{10^{25}}\right)^{-1/4}, \qquad (34)$$

which is obviously quite inadequate for the vertex theory of the origin of rotation of galaxies.

Note also that the upper bound on κ that follows from the fact that at the contemporary time we undoubtedly have $\Omega = \epsilon/\epsilon_c < 4$ and therefore $\kappa < 10^5$ is weaker than the bound (33) associated with the chemical composition.

We now turn to the restrictions that follows from observations of the isotropy of the microwave background radiation. For a model with gravitational waves ($\mathbf{v} = 0$) with $\kappa = 0$ the expressions for $\Delta T/T$ obtained earlier in [20] holds. Let z_l be the red shift at the time when the Universe became transparent for the radiation. From observations there follow bounds on the large-scale anisotropy of the radiation:

$$\Delta T/T \lesssim 10^{-3}. \qquad (35)$$

According to our earlier results, [18-20] we obtain the following bound at the time t_F: if $z_l \sim 10^3$, then $t_F \sim t_{pl}$.

For the purely vortex model $\Delta T/T \lesssim v_l^2$. [Recall that in the models we are considering one always has $\Omega \approx 1$ and the anisotropy of the background radiation has a quadrupole nature.]

In the case when $\kappa \gtrsim 1$ one can use Eq. (31) and for $\Delta T/T$ we then obtain (for simplicity, the graviton distribution function is approximated by a counterstreaming flux in the direction a; see Fig. 1)

$$\frac{\Delta T}{T} \lesssim \left(\frac{l}{10^{25}}\right)^{1/2} \begin{cases} \varkappa^{3/4}\cdot 10^{-2}(z_l/10^3)^{3/2}, & z_l < 10^4/\varkappa \\ \varkappa^{1/4}\cdot 10^{-1}(z_l/10^3)^{1/2}, & z_l > 10^4/\varkappa \end{cases} . \qquad (36)$$

Obviously, this model can be reconciled with the observations only if κ and l are small.

We are very grateful to Ya. B. Zel'dovich, at whose initiative this work was carried out, for numerous discussions.

1)Zel'dovich and one of the present authors [10] have shown clearly that this growth of the perturbations is due to the growth in the anisotropy of the deformation of the model.

2)The total vortex is reversed because of the reversal of the vortex of free particles (gravitons) on account of the kinetics of their motion (see Sec. 4). The vortex of the interacting particles remains unchanged.

3)We assume that among the created particles gravitons do not form the overwhelming fraction of all particles. If this is not so, the velocity of the vortex motion need not be small in the late epoch. However, in this case the energy density of gravitons at our epoch would exceed by many orders the density of the microwave background and the density of ordinary matter, which in incompatible with the estimate of the age of the Universe.

4)The transformation $\mathbf{S} = \{S_a^b\}$ ($\det \mathbf{S} = 1$) determines the moving frame $\mathbf{e}' = \{e^a_{\alpha'}\}$ relative to the fixed frame $\mathbf{e} = \{e^a_\alpha\}$, $\mathbf{e}' = \mathbf{eS}$, if it leaves invariant the structure constants $C^c_{\cdot ab} = e^\alpha_a e^\beta_b (\partial_\beta e^c_\alpha - \partial_\alpha e^c_\beta)$ reduces the metric tensor to the diagonal form $\boldsymbol{\gamma} = \mathbf{S}\lambda\mathbf{S}^T$, where $\boldsymbol{\gamma} = \{\gamma_{ab}\}$, $\lambda = \mathrm{diag}\{\lambda_a\}$ (see [17]). For the metric (1), the following components are nonzero: $C^1_{\cdot 23} = C^2_{\cdot 31} = -k$, $S^1_1 = S^2_2 = S^3_3 = 1$, $S^2_3 = \nu$.

5)Note that the function $\nu(t)$ does not occur in the basic equations (4)–(8) and is determined independently from (3). This splitting of the equations holds for all homogeneous metrics. [17]

6)During the evolution, all models of this type approach the Friedmann model with critical matter density. [11,17-20] In what follows, we shall assume that the matter is described by a hydrodynamic energy–momentum tensor with $p = \epsilon/3$.

7)If $\mu \equiv \nu \equiv 0$, the three-dimensional section $t = \text{const}$ is flat and the metric (1) describes an axisymmetric model of type I. The perturbations in the Friedmann stage, $g_{\alpha\beta} \sim t(\delta_{\alpha\beta} - h_{\alpha\beta})$, are of the form $h_{\alpha\beta} = CP_{\alpha\beta}/t^{1/2}$, where $P_{\alpha\beta} = \mathrm{diag}\{1/3; 1/3; -2/3\}$. The tensor $P_{\alpha\beta}$ can be obtained by a passage to the limit ($n \to 0$) from either the scalar mode (and then $\delta\epsilon_n \to 0$, $\nu_n \to 0$):

$$P_{\alpha\beta} = \lim_{n\to 0}\left(\frac{1}{3}\delta_{\alpha\beta} - \frac{n_\alpha n_\beta}{n^2}\right) e^{inz}, \qquad \mathbf{n} = \{n_\alpha\} = \{0, 0, n\},$$

or from the two tensor modes:

$$P_{\alpha\beta} = \lim_{n\to 0}\left[{}^1/_3 e^{in_1 x}\,\mathrm{diag}\{0, 1, -1\} + {}^1/_3 e^{in_2 y}\,\mathrm{diag}\{1, 0, -1\}\right];$$

$$\mathbf{n}_1 = \{n_1, 0, 0\}, \quad \mathbf{n}_2 = \{0, n_2, 0\}, \quad n_1 = n_2 = n.$$

8)As we shall see in Sec. 5 (see [20]), if we take $t_0 \sim t_{pl}$ and $ƛ \sim t_0$ for $t = t_0$ and $\mu_0 \sim 1$, then at the present time ($t \sim 10^{18}$ sec) $ƛ \sim 10^{-1}$ cm. But if we take $ƛ$ equal to the mean distance between galaxies, $ƛ \sim 3$ Mpc $\sim 10^{25}$ cm, then $(t/ƛ)_0 \sim 10^{-26}$.

9)From the condition that the total angular momentum L of unit volume does not change appreciably during the production process, $\Delta t \sim t_0$, see (25), one can readily estimate an upper bound for the vortex velocity of the matter after the end of the production processes of particles and their mixing with the primeval matter. For $t \ll \lambda$, we obtain from Eqs. (4)–(8) the inequalities

$$\epsilon\nu(1 + \nu^2)^{1/2}\lambda t e^{\mu} \lesssim 1, \quad t e^{|\mu|} \lesssim \lambda,$$

and, in addition, when $t_0 \approx t_{pl}$ we have $\epsilon \sim 1/f_{pl}^2$, i.e., $t^* \sim t_{pl}$, and to estimate v^* it is sufficient to use Eq. (11):

$$\nu^* \lesssim t e^{-\mu}/\lambda \lesssim 1, \quad t \sim t^* \sim t_{pl}.$$

10)Note that when $t \gg t_0$ the wavelength of the gravitons that make the main contribution to the energy-momentum tensor is much less than the horizon and therefore these gravitons are well defined classical entities, i.e., short gravitational waves on the background of the metric (1), and they can be treated, for example, by Isaacson's method. [26]

11)According to generally adopted theories, α does not differ strongly from unity. [27] Estimates for arbitrary α can be readily obtained from Eqs. (12) and (13).

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