Spin relaxation of electrons due to scattering by holes

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We evaluate the electron spin relaxation time connected with the exchange and annihilation interaction between electrons and free and bound holes. We show that this mechanism can play a determining role under conditions of optical orientation of electrons in semiconductors. We construct a theory of the electron spin relaxation when holes are strongly scattered by impurities and taking into account the fast relaxation of the hole spin. We show that the rate of electron spin relaxation due to holes is proportional to the time of interaction with the holes, i.e., the time during which the distance between them is less than the electron wavelength. Under conditions when this time equals the time of diffusion of holes through the interaction region, strong scattering of holes leads to a decrease in the electron spin relaxation time. On the other hand, under conditions when the hole spin relaxation time becomes less than the interaction time, strong hole spin relaxation leads to an increase in the electron spin relaxation time due to an efficient averaging of the hole spin.

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1. INTRODUCTION

In experiments on the optical orientation of electrons in a number of crystals of the A_2B_6 and A_3B_5 groups one normally uses samples with a high density of equilibrium holes, thanks to which it is possible to observe a rather strong luminescence (see the surveys[1, 2]). Under those conditions the main mechanism for the electron spin relaxation may be their scattering by holes involving spin flip. The way the electron spin relaxation time depends on the hole density, in particular, the appreciable decrease of the electron spin relaxation time in p-type samples as compared to n-type samples[3] which has been observed in a number of papers indicates the important role played by this mechanism.

We show below that the scattering of electrons by holes involving spin flip may be caused both by exchange and by annihilation interactions which lead also to a longitudinal-transverse splitting of the exciton levels. Both these interactions have a contact character.[4]

We shall consider the scattering of electrons by holes involving spin flip in the Born approximation and we shall assume that the wavefunction of the electron-hole pair may differ from a plane wave due to the Coulomb interaction.

The Hamiltonian for the interaction between an electron and a hole which involves spin-flip can be written in the form

$$\mathcal{H}=\pi a_B^3\hat{D}\delta(\mathbf{r})\delta_{\mathbf{K},\mathbf{K}'},\qquad(1)$$

where $\mathbf{r} = \mathbf{r}_e - \mathbf{r}_h$ is the difference in the coordinates of the electron and the hole, $a_B = \varkappa_0/e^2m$ is the exciton Bohr radius (everywhere in this paper $\hbar = 1$), $m^{-1} = m_e^{-1} + m_h^{-1}$, $\varkappa_0$ is the static dielectric constant, $\mathbf{K} = \mathbf{k} + \mathbf{p}$ is the total momentum of the electron-hole pair, $\mathbf{k}$ the electron momentum, and $\mathbf{p}$ the hole momentum. The operator $\hat{D}$ depends on the electron and hole spin operators and for the different cases has the form:

Exchange interaction

a) in the case of simple bands with axial symmetry[5]

$$D=\Delta_{\parallel}\sigma_z s_z+2\Delta_{\perp}(\sigma_+s_-+\sigma_-s_+),\qquad(2)$$

where σ and $\mathbf{s}$ are the electron and hole Pauli matrices, $\sigma_\pm = 1/2(\sigma_x \pm i\sigma_y)$. For A_2B_6 crystals for holes in the Γ_9 band $\Delta_\perp = 0$ and for cubic symmetry $\Delta_\perp = \Delta_\parallel = \Delta$.

In the case of the Γ_8 band for cubic crystals

$$D=\Delta_1(\mathbf{J}\sigma),\qquad(3)$$

where $\mathbf{J}$ is the hole angular momentum operator ($J = 3/2$). (We dropped in (3) the small anisotropic term $\Sigma_i J_i^3\sigma_i$.) We note that according to (1) the constants $\Delta_\parallel$, $\Delta_\perp$, and Δ_1 determine the exchange splitting of the exciton ground state.

Annihilation interaction

According to[4, 5] in that case

$$\pi a_B^3 D_{ij}(\mathbf{K})=\frac{4\pi e^2}{m_0^2\varkappa_\infty E_g^2}\frac{(\mathbf{P}_{0i}\mathbf{K})(\mathbf{P}_{0j}\mathbf{K})^+}{K^2},\qquad(4)$$

where the matrix elements of the momentum operator $\mathbf{P}$ are taken between the initial state (vacuum) and a state with one electron-hole pair, $\varkappa_\infty$ is the dielectric constant at the exciton excitation frequency, E_g the width of the forbidden band, and m_0 is the mass of a free electron.

We note that the macroscopic long-range interaction (4) leads to a spin flip of electrons and holes only thanks to the spin-orbit splitting of the valence band (or the conduction band) as only in that case do the momentum matrix elements depend on the spin states of the pair. It is just this fact which is connected with the possibility for optical orientation of the electrons. Together with the considered spin relaxation mechanisms connected with the exchange of spin between an electron and a hole, there is also another mechanism for electron spin relaxation which is connected with the spin-orbit interaction in the Coulomb field of a hole.[4] This mechanism equally also determines the spin relaxation involving scattering by impurities[6] where the contribution from the holes connected with the spin-orbit mechanism is less than the contribution connected with the scattering by charged impurities. Estimates given in Sec. 4 show that in uncompensated samples the contribution from the exchange mechanism considered by us exceeds appreciably the contribution from the spin-orbit mechanism connected with the scattering by impurities.

2. EVALUATION OF THE ELECTRON SPIN RELAXATION TIME

The electron spin relaxation drift time $\tau_{se}^{dr}(\mathbf{k})$, which we denote by $2\tau_{se}(\mathbf{k})$, is given in the Born approximation by the formula

$$\frac{1}{2\tau_{se}(\mathbf{k})}=2\pi^3 a_B^6\sum_{\substack{\alpha\alpha'\\ \mathbf{p},\mathbf{q}}}|\psi(0)|^4|D_{\uparrow\alpha,\downarrow\alpha'}(\mathbf{p},\mathbf{p}+\mathbf{q})|^2 \times f_p(1-f_{p+q})\delta(\varepsilon_k+\varepsilon_p-\varepsilon_{k-q}-\varepsilon_{p+q}). \tag{5}$$

The arrows indicate here the electron spin states, while α and α' are the hole spin indexes. We note once again that here $\epsilon_{\mathbf{k}}$ and $\mathbf{k}$ are the electron energy and momentum, ϵ_p and p the hole energy and momentum, $f_p \equiv r(\epsilon_p)$ the hole distribution function, and $\psi(r)$ the wave function of the relative motion of the electron and the hole.

We shall in what follows consider the case when the hole effective mass is much larger than the electron effective mass, which is usually the case in semiconductors in which optical orientation is studied.[1)] It is then clear that the momentum transfer and the electron momentum are small for $\epsilon_{\mathbf{k}} \lesssim \epsilon_p$ compared to the hole momentum and $K \approx p$. This means that $D(\mathbf{p}, \mathbf{p}+\mathbf{q})$ depends only on the direction of the vector p both for the annihilation interaction and for the exchange interaction in the hole band.

However, in this case in the case of Fermi hole degeneracy the hole velocity v_h may be either less or larger than the electron velocity v_e.

We consider in what follows two limiting cases: a) $v_h \ll v_e$ and b) $v_h \gg v_e$. In those cases the velocity of the relative motion is determined either by the electron velocity (a) or by the hole velocity (b). The quantity $|\psi(0)|^4$ then depends only on the absolute magnitude of the momentum or the relative motion mv.

It is well known that the value of $|\psi(0)|^2$ for the Coulomb potential is determined by the Sommerfeld factor.[7] For a screened potential this quantity depends strongly on the screening radius l_d.[8] When $l_d \ll a_B$ we have $|\psi(0)|^2 = 1$. We consider various limiting cases.

1) If the holes are not degenerate, using the fact that the energy transfer is small compared to ϵ_k when $m_e \ll m_h$, we find from (5)

$$\frac{1}{2\tau_{se}}=\frac{1}{\tau_0}\frac{v_k}{v_B}|\psi(0)|^4, \tag{6}$$

where

$$\frac{1}{\tau_0}=\frac{1}{4}Nv_Bs_B\frac{\overline{D_s^2}}{E_B^2}=\frac{\pi}{2}(Na_B^3)\frac{\overline{D_s^2}}{E_B}, \qquad \overline{D_s^2}=\frac{1}{4\pi}\int d\Omega_p D_s^2(\mathbf{p}),\quad D_s^2(\mathbf{p})=\frac{1}{2}\sum_{\alpha,\alpha'}|D_{\uparrow\alpha,\alpha'\downarrow}|^2. \tag{7}$$

Here

$$E_B=1/2ma_B^2,\quad s_B=\pi a_B^2,\quad v_B=1/ma_B, \tag{8}$$

N is the hole density.

2) If the holes are strongly degenerate so that the Fermi energy $\epsilon_F \gg \epsilon_k$, T it is convenient to introduce in (5) the energy transfer as the variable:

$$\omega=\varepsilon_k-\varepsilon_{k-q}=\varepsilon_{p+q}-\varepsilon_p=qv_F\cos\theta+q^2/2m_h,$$

where $\cos\theta = \cos(\mathbf{p}, \mathbf{q})$. After that, replacing everywhere except in the δ-function p by p_F and bearing in mind that when $\epsilon_F \gg T$

$$\int_0^\infty d\varepsilon_p f(\varepsilon_p)[1-f(\varepsilon_p+\omega)]=\omega/(1-e^{-\omega/T}),$$

we get

$$\frac{1}{2\tau_{se}}=\frac{3\pi}{2}\frac{N}{\varepsilon_F}\sum_q\int_{-\infty}^{+\infty}d\omega\frac{\omega}{1-e^{-\omega/T}}\int\frac{d\Omega_p V_s^2(\mathbf{p})}{4\pi} \times\delta\left(\omega-qv_F\cos\theta+\frac{q^2}{2m_h}\right)\delta(\omega-\varepsilon_k+\varepsilon_{k-q}), \tag{9}$$

where

$$V_s^2(\mathbf{p})=\frac{\pi}{2m^2}s_B\frac{D_s^2(\mathbf{p})}{E_B^2}.$$

For a strongly degenerate hole gas we have then $|\psi(0)|^4 \approx 1$. If the electron velocity is much larger than the hole velocity, i.e., $(\epsilon_F m_e/\epsilon_k m_h) \ll 1$, we find that for $m_e\epsilon_F\epsilon_k/m_hT^2 \ll 1$ the energy transfer $\omega \ll T, \epsilon_k$. We then get (see (60))

$$\frac{1}{2\tau_{se}}=\frac{1}{\tau_0}\frac{3}{2}\frac{v_k}{v_B}\frac{T}{\varepsilon_F}. \tag{10}$$

If the hole velocity is much larger than the electron velocity, i.e., $\epsilon_F m_e/\epsilon_k m_h \gg 1$, we have $\cos\theta \approx T/qv_F \sim v_e/v_h \ll 1$, i.e., $\mathbf{p} \perp \mathbf{q}$. If D_s^2 depends on the direction of p, $1/2\tau_{se}$ will in this case depend on the direction of k relative to the electron spin direction or the crystal axis direction. The quantity $\langle 1/2\tau_{se}\rangle$ which is averaged over all directions of the electron motion is as before determined by $\overline{D_s^2}$ from (7). In the case of fast holes we get for $\langle 1/2\tau_{se}\rangle$ (see (61))

$$\left\langle\frac{1}{2\tau_{se}}\right\rangle=\frac{1}{\tau_0}\frac{v_F}{v_B}\left(\frac{T}{\varepsilon_F}\right)^2\frac{m_h}{m_e}I_1\left(\frac{\varepsilon_k}{T}\right), \tag{11}$$

where

$$I_1(z)=\frac{\pi^2}{16}\left\{1+\frac{6}{\pi^2}z^2\int_0^1\frac{dy(1-y)\sqrt{y}}{1-e^{-z(1-y)}}\right\}.$$

It follows from (11) that the electron spin relaxation time is determined in that case by the larger of the quantities T or ϵ_k: we have $\langle 1/\tau_{se}\rangle \sim (T/\epsilon_F)^2$ when $\epsilon_k < T$ and $\langle 1/\tau_{se}\rangle \sim (\epsilon_k/\epsilon_F)^2$ when $\epsilon_k \gg T$.

Electron spin relaxation when the holes are strongly scattered

The formulae given above are valid only under the conditions where the electron-hole interaction time τ_{int} is much smaller than the hole scattering time τ and the hole spin flip time τ_{sh}. The interaction time $\tau_{int} = \lambda/v$, where λ is the size of the interaction region, equal to the inverse of the momentum transfer (for a short-range potential) while v is the relative velocity. When $m_e \ll m_h$ and $\epsilon_k \lesssim \epsilon_p$ we have $\lambda \sim 1/k$ and when $v_e > v_h$ when the interaction time is determined by the electron velocity and $\tau_{int} = \tau_{int}^e \sim \epsilon_k^{-1}$, the condition $\tau_{int} < \tau$ reduces to $\epsilon_k\tau > 1$. When $v_e \ll v_h$ the time τ_{int} is determined by the hole velocity and equal to $\tau_{int}^h = 1/kv_F$ and the condition $\tau_{int} \ll \tau$ reduces to $kl \gg 1$ where $l = \tau v_F$ is the hole mean free path. The range of values of $\epsilon_k\tau$ and $m_e\epsilon_k l^2 = l^2k^2$, where the condition $\tau_{int} < \tau$ is satisfied, i.e., $\epsilon_k\tau \gg 1$ or $lk \gg 1$ and where Eqs. (10) and (11) are valid are indicated in Fig. 1 by the letters a and c, respectively. The solid line corresponding to the condition $v_e = v_h$ separates the regions of slow and fast holes. When $\tau_{int} \gg \tau$, i.e., when $\epsilon_k\tau \ll 1$ and $kl \ll 1$, a hole manages to be scattered repeatedly during the interaction time and its motion in the region $\lambda \sim 1/k$ is thus diffusive in nature and it traverses this region in a time $\tau_{int}^h = 1/Dk^2$, where $D = 1/3v_F^2\tau$ is the hole diffusion coefficient. The boundary between the slow and fast hole regions in the quantum region, i.e., when $\epsilon_k\tau \ll 1$ and $kl \ll 1$ is the line $\tau_{int}^e/\tau_{int}^h = 6m_eD = 1$, shown in Fig. 1 by the dot-dash line.

When $\tau_{int} \ll \tau_{sh}$ the hole spin retains its direction during the interaction time. When $\tau_{int} \gg \tau_{sh}$, i.e., when the conditions $\epsilon_k\tau_{sh} \ll 1$ and $Dk^2\tau_{sh} = l_{sh}^2k^2 \gg 1$ are

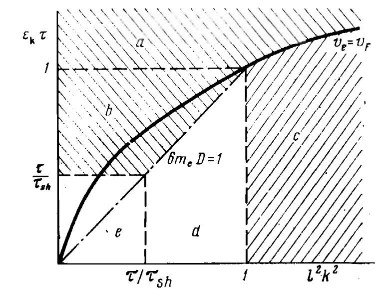

FIG. 1. Regions of applicability of the formulae for τ_{se}. a, b) slow-hole regions, given by (10), c) fast-hole region given by (11), d) diffusion region, given by (12), e) region of strong-hole spin relaxation, given by (13).

simultaneously satisfied, the hole spin manages to change its direction during the interaction many times and as a result the probability that an electron is scattered with spin flip is decreased.

In Sec. 5 below we evaluate the electron spin relaxation time when the holes are strongly scattered by impurities, when $T\tau < 1$, but $\epsilon_F\tau \gg 1$. In that section we give the final formula for various limiting cases.[2)]

In accordance with the calculations the probability for the scattering of an electron with spin flip which is determined by the quantity $\langle 1/\tau_{se}\rangle$ turns out to be proportional to the interaction time τ_{int}. For instance, in the above considered cases of slow and fast holes the ratio of these probabilities is according to (10) and (11) equal to $v_F/v_k = \tau_{int}^e/\tau_{int}^h$. In the case of strong hole scattering in the slow hole region, i.e., when $\tau_{int}^e/\tau_{int}^h = 6m_eD < 1$ and $\epsilon_k\tau_{sh} < 1$ (region b in Fig. 1) Eq. (10) remains valid, when, as in region a, $\tau_{int} = \tau_{int}^e = \epsilon_k^{-1}$. In the region $\tau_{int}^e/\tau_{int}^h = 6m_eD > 1$ and $l_{sh}^2k^2 \gg 1$ (region d in Fig. 1) the probability for spin flip decreases in comparison to (10) by a factor $\sim 6m_eD$ and is given by the equation (see (63))

$$\left\langle\frac{1}{2\tau_{se}}\right\rangle=\frac{1}{m_eD}\frac{\varepsilon_k v_k}{\varepsilon_F v_B}I_2\left(\frac{\varepsilon_k}{T}\right)\frac{1}{\tau_0}, \tag{12}$$

where

$$I_2(z)=\frac{3}{8\pi}\int_{-1}^{\infty}\frac{y\,dy}{e^{zy}-1}\ln\left|\frac{\sqrt{y+1}+1}{\sqrt{y+1}-1}\right|.$$

When $\epsilon_k/T \gg 1$ we have $I_2(\epsilon_k/T) \approx 1/4\pi$.

When $\epsilon_k\tau_{sh} \ll 1$ and $l_{sh}^2k^2 \ll 1$ (region e in Fig. 1) the effective interaction time is equal to τ_{sh} and the quantity $\langle 1/\tau_{se}\rangle$ decreases as compared to (10) by an amount $\epsilon_k\tau_{sh}$ and is given by the formula (see (65))

$$\left\langle\frac{1}{2\tau_{se}}\right\rangle=\frac{1}{\tau_0}\frac{\varepsilon_k}{\varepsilon_F}\frac{v_k}{v_B}\tau_{sh}\varepsilon_k I_3\left(\frac{\varepsilon_k}{T}\right), \tag{13}$$

where

$$I_3(z)=\frac{3}{2\pi}\int_{-1}^{\infty}\frac{y\,dy\sqrt{1+y}}{e^{zy}-1}.$$

When $\epsilon_k/T \gg 1$ we have $I_3 \approx 2/5\pi$.

Spin relaxation of electrons which are scattered by bound holes

At low temperatures and not too high hole densities the latter are bound to acceptors. Since the original Hamiltonian (1) is written in the **K**-representation it is convenient for the calculation of the matrix element to expand the hole wave function in a Fourier integral in p. We restrict ourselves here to the case where the hole wave function is spherically symmetric and the acceptor center is not excited during the scattering, while $\epsilon_k\tau_{sh} \gg 1$. Using the fact that $k \ll a_B^{-1}$ and $K \approx p$ when $m_e \ll m_h$ and $\epsilon_k \lesssim E_B$, we get then after summing over **K** and **K**′

$$\frac{1}{2\tau_{se}}=\frac{1}{\tau_0}\frac{v_k}{v_B}. \tag{14}$$

Here τ_0 is determined by the matrix (7) in which $N = N_a$ is the acceptor density,

$$\overline{D_s^2}=\frac{1}{g}\sum_{mm'}\int|D_{m\uparrow,m'\downarrow}|^2\frac{d\Omega_p}{4\pi},$$

g is the degree of degeneracy of the acceptor center ground state.

3. EVALUATION OF THE EFFECTIVE SPIN-SPIN INTERACTION CONSTANT

We now evaluate $\overline{D_S^2}$ for different cases, expressing it in terms of the exchange or the longitudinal-transverse annihilation splitting of the excitons.

In hexagonal crystals for a $\Gamma_7 \times \Gamma_7$ electron-hole pair it follows from (2) that

$$\overline{D^2}_{S\,exch}=2\Delta_\perp^2={}^1/_8\Delta_{exch}^2, \tag{15}$$

where Δ_{exch} is the exchange splitting between the Γ_1 and Γ_2 exciton states.

For a $\Gamma_7 \times \Gamma_9$ pair $\Delta_\perp = 0$ and $\overline{D_S^2}_{exch} = 0$, i.e., the exchange mechanism does not lead to scattering with spin flip.

Equation (15) is valid also for nondegenerate bands in a cubic crystal for which Δ_{exch} is the singlet-triplet splitting.

In a degenerate Γ_8 valence band of a cubic crystal the hole wave functions transform in the spherical approximation, i.e., when $D = \sqrt{3}B$, like the spherical harmonics $Y_m^{3/2}$ with $m = \pm 1/2$ or $m = \pm 3/2$ with the quantization axis parallel to p. (When the constants A, B, and D have the same sign, the states with $m = \pm 3/2$ correspond to the heavy holes.)

In the spherical approximation the averaging over the direction of p is equivalent to averaging over the direction of J. Bearing in mind that the J_i under rotations transform as the components of a vector we get for holes with $m = \pm 3/2$ and $m = \pm 1/2$.

$$\overline{D_s^2}_{exch}=\frac{3}{2}\Delta_1^2=\frac{3}{32}\Delta_{exch}^2, \tag{16}$$

where Δ_{exch} is the exchange splitting between the exciton states with J = 1 and J = 2.

We now consider the annihilation interaction. For nondegenerate bands the matrix elements $\mathbf{P}_{mn}$ are independent of the direction of p and after averaging we get: for a $\Gamma_7 \times \Gamma_9$ in hexagonal crystals

$$\overline{D_s^2}_{ann}=\frac{1}{15}\Delta E_\perp^2; \tag{17}$$

for a $\Gamma_7 \times \Gamma_7$ electron-hole pair

$$\overline{D_s^2}=\frac{1}{15}\Delta E_\perp^2+\frac{1}{30}\Delta E_\parallel\Delta E_\perp+\frac{1}{40}\Delta E_\parallel^2. \tag{18}$$

Here $\Delta E_\perp$ and $\Delta E_\parallel$ are the longitudinal-transverse splitting of the corresponding excitons with $K \parallel z$ and $K \perp z$, where z is a principal axis of the crystal. $\Delta E_\parallel = 0$ for a $\Gamma_7 \times \Gamma_9$ exciton.) In the quasi-cubic approximation when the crystal splitting is much smaller than the spin-orbit one, we have for the upper Γ_7 band (the one closest to the Γ_9 band) $\Delta E_{\parallel}^{(7)} = 4\Delta E_{\perp}^{(7)} = 4/3\Delta E_{\perp}^{(9)}$, where $\Delta E_{\perp}^{(9)}$ is the corresponding splitting for the $\Gamma_7 \times \Gamma_9$ exciton, and it is clear from (17) and (18) that in that case $\overline{D_s^2}_{ann}$ is the same for the neighboring Γ_7 and Γ_9 bands. For the Γ_7 band which due to the spin-orbit interaction is split off we have in this approximation $\Delta E_{\parallel} = \Delta E_{\perp} = 2/3\Delta E_{\perp}^{(9)}$ and according to (18)

$$\overline{D_s^2}_{ann} = {}^1/_8\, \Delta E_{ann}^2, \quad (19)$$

where ΔE_{ann} is the difference in energy of the longitudinal and the transverse excitons. Equation (19) is also valid for nondegenerate bands in cubic crystals.

The calculation shows that for degenerate bands in a cubic crystal, in contrast to the exchange interaction, the annihilation interaction is appreciably different for heavy and light holes with $m = \pm 3/2$ and $m = \pm 1/2$.

$$\overline{D_s^2}_{ann} = {}^1/_8\, \Delta E_{ann}^2 R. \quad (20)$$

Here ΔE_{ann} is the difference in energy of the longitudinal and the transverse $\Gamma_7 \times \Gamma_8$ excitons. For light holes ($m = \pm 1/2$) in the spherical approximation $R = 1$, and for heavy holes $R = 0$ in the spherical approximation. If we use the exact hole functions to evaluate $\overline{D_s^2}_{ann}$ we can show that for heavy holes

$$R<10^{-2}(D/\sqrt{2}B-1)^4.$$

The annihilation interaction between electrons and heavy holes contributes therefore practically nothing to the scattering of electrons involving spin flip.

For holes which are bound to an acceptor the constants $\overline{D_s^2}$ are the same in the case of a nondegenerate band for the exchange interaction as for free holes.

For a degenerate valence band when the acceptor wave function is given by a single smooth s-type function,

$$\overline{D_s^2}_{exch} = \frac{1}{4}\Delta_1^2 \sum_{mm'} |(J_+)_{mm'}|^2 = \frac{5}{2}\Delta_1^2 = \frac{5}{32}\Delta_{exch}^2. \quad (21)$$

In the case of annihilation radiation and when the acceptor wave function is spherically symmetric we get for $\overline{D_s^2}_{ann}$

$$\overline{D_s^2}_{ann} = \left(\frac{4}{3}\frac{e^2}{m_0^2 E_g^2 \varkappa_\infty a_B^3}\right)^2 \frac{1}{g}\sum_{mm'} |(\mathbf{P}_{m\downarrow i}\mathbf{P}_{\uparrow m'})|^2. \quad (22)$$

We note that the contribution of the annihilation interaction to the exchange constants of a bound exciton is determined by the same matrix (7) which determines in (22) the contribution from this interaction to $\overline{D_s^2}_{ann}$.

As according to (15) $\overline{D_s^2} \sim \Delta_{\perp}^2$ and as it follows from general considerations for a $\Gamma_7 \times \Gamma_9$ exciton that $\Delta_{\perp} = 0$, in scattering by bound holes neither the exchange nor the annihilation interaction contribute to $\overline{D_s^2}$, i.e., the electron spin does not change in elastic scattering of an electron by bound Γ_9 holes.

For a pair from the $\Gamma_7 \times \Gamma_7$ bands we have from (22) in a hexagonal crystal

$$\overline{D_s^2}_{ann} = \frac{1}{72}\Delta E_{\parallel}^2, \quad (23)$$

for nondegenerate bands in a cubic crystal

$$\overline{D_s^2}_{ann} = \frac{1}{72}\Delta E_{ann}^2, \quad (24)$$

and for a degenerate Γ_8 valence band

$$\overline{D_s^2}_{ann} = \frac{5}{288}\Delta E_{ann}^2. \quad (25)$$

We note that in those cases where the annihilation and exchange interactions give approximately the same contribution to $|D_S|^2$ we must take into account the cross terms, i.e., add the transition matrix elements and afterwards average the whole thing. For instance, for the scattering by bound holes for a pair from the $\Gamma_7 \times \Gamma_7$ bands we have thus in agreement with (34a) from[9]

$$\overline{D_s^2} = {}^1/_8(\Delta_{exch} + {}^1/_3\Delta E_{\parallel})^2. \quad (26)$$

4. DISCUSSION OF THE RESULTS AND NUMERICAL CALCULATIONS

It is clear from the formulae given above that owing to changes in the nature of the scattering the hole density and temperature dependence of the spin relaxation rate depends on the degree of degeneracy of the holes and their spin relaxation time and turns out to be different for free and for bound holes.

It is well known that at not too high acceptor densities, when the screening radius $l_d = (\pi a_B^a/4p_F)^{1/2}$ is larger than the acceptor Bohr radius a_B^a, the holes get bound to impurities when the temperature is lowered and at high temperatures the hole gas is nondegenerate. In that case, taking into account scattering by free and by bound holes we get from (6) and (14)

$$\frac{1}{2\tau_{se}} = \frac{1}{\tau_0}\varkappa\left[1+\frac{N}{N_a}(|\psi(0)|^4-1)\right], \quad (27)$$

where $\kappa = \sqrt{(\epsilon_k/E_B)}$ and N/N_a is the degree of ionization of the acceptors. Here τ_0 is determined according to (14) by the acceptor density. At low temperatures $1/\tau_{se} \propto T^{1/2}$, but already at low degrees of ionization of the acceptors it increases faster as at low temperatures

$$|\psi(0)|^4 = \left|\frac{2\pi}{\varkappa}\frac{1}{1-e^{-2\pi/\varkappa}}\right|^2 \gg 1.$$

At an appreciable ionization of the impurities, the decrease in $|\psi(0)|^4$ with increasing temperature compensates for the increase in κ. It is clear from (27) that $1/\tau_{se} \propto N_a$ both for a low and for a high degree of ionization of the impurities, but in the intermediate region the increase with density is slower.

At high impurity densities, when $l_d \lesssim a_B^a$, the impurities are always ionized and for not too high temperatures the holes are degenerate.

For a given hole density and temperature the applicability of the obtained formulae for $1/\tau_{se}$ is, as we noted earlier, determined by the point with the coordinate $\tau\epsilon_k$ and $l_h^2 k^2$ in Fig. 1.

For a fixed density this point moves with increasing temperature ($\epsilon_k \sim T$) along a straight line through the origin with slope $\Theta = \epsilon_k \tau / l_h^2 k^2 = 1/6 m_e D$. If $\Theta < 1$, the point intersects successively with increasing temperature the regions e, d, c, and a where Eqs. (13), (12), (11), and (10) apply, respectively. The temperature dependence of $1/\tau_{se}$ in these regions has, respectively, the form $1/\tau_{se} \propto T^{5/2}$ (e), $T^{3/2}$ (d), T^2 (c), and $T^{3/2}$ (a).

If, as in the Γ_8 band, $\tau_{sh} \approx \tau$, there is practically no region d. If $\Theta > 1$, the point goes directly from the region e to the regions b and a, in which Eq. (13) is valid.

For fixed temperature and τ = constant this point moves along a horizontal line. When $\tau T > 1$ it moves from the region a where $1/\tau_{se} \propto N^{1/3}$ into the region c, where $1/\tau_{se}$ is independent of N. When $\tau T < 1$ the point moves from the region b or e, where $1/\tau_{se} \propto N^{1/3}$, into the region d where according to (12) $1/\tau_{se}$ decreases with increasing N, and after that enters the region c, i.e., in that case the N-dependence of $1/\tau_{se}$ may be nonmonotonic.

As an example we have evaluated the value of $1/\tau_{se}$ for GaAs which is often used in optical orientation experiments. The basic parameters of this crystal are given in the Table. The critical density corresponding to the condition $l_d = a_B^a$ equals 1×10^{18} cm^{-3} for GaAs. We made the calculations for two densities. $N_1 = 10^{17}$ cm^{-3} and $N_2 = 4 \times 10^{19}$ cm^{-3}.

For the density $N_1 = 10^{17}$ cm^{-3} the temperature dependence of $1/\tau_{se}$ was calculated from (27). For $N = N_2 = 4 \times 10^{19}$ cm^{-3} $1/\tau_{se}$ was given by Eq. (13) in the temperature range 0 to 20 K, by Eq. (11) from ≈ 20 to ≈ 100 K, and by Eq. (10) for $100 \text{ K} \lesssim T \lesssim 350$ K.

In the calculations we took the hole spin relaxation time in the degenerate band to be the same as the transport time τ which for $N = N_2$ equals $\tau = 2.4(E_B^a)^{-1}$. The condition $\epsilon_k \tau > 1$ is for $N = N_2$ satisfied when $\epsilon_k \sim T > 10$ K. In Fig. 2 we have plotted the ratio $\tau_0^*/2\tau_{se}$, where τ_0^* is the value of τ_0 for $N = 10^{17}$ cm^{-3}. According to (7) and (16) for GaAs

$$1/\tau_0^* \approx 8 \cdot 10^6 (\Delta_{exch}/10^{-5} \text{ eV})^2,$$

where Δ_{exch} is the exchange splitting of the exciton, i.e., the difference in energy of the states with J = 1 and J = 2. (As we mentioned earlier, the annihilation interaction does not contribute significantly in this case to D_s^2.)

There are no reliable data on the magnitude of Δ_{exch} in GaAs. If we take in accordance with[10, 11] $\Delta_{exch} = 5 \times 10^{-5}$ eV, we get $1/\tau^* = 3 \times 10^8$ s^{-1}. According to Fig. 2 the quantity $1/\tau_{se}$ then exceeds 3×10^{10} s^{-1} for $N = 4 \times 10^{19}$ cm^{-3}, while for $N = 10^{17}$ cm^{-3} we have $1/2\tau_{se} \approx 3 \times 10^9$ s^{-1}, i.e., in both cases the mechanism considered leads to short electron spin relaxation times which may be comparable with or even less than the life time. Estimates show that the spin relaxation time τ_{se}^{so}, which is connected with the scattering by impurities and caused by the spin-orbit interaction, turns out for a screened Coulomb potential to be in GaAs about two orders of magnitude larger than the spin relaxation time τ_{se}^{exch} connected with the exchange interaction and becomes less than τ_{se}^{exch} for $\Delta_{exch} < 3 \times 10^{-6}$ eV. In semiconductors with a nondegenerate valence band the spin relaxation time connected with the annihilation interaction is always less than τ_{se}^{so} when $m_e < m_h$.

m_e/m_0	m_h/m_0	$\varkappa_0$	E_B^{ex}, meV	E_B^a, meV	a_B^{ex}, Å	a_B^a, Å	$N = 4 \cdot 10^{19}$ cm^{-3}	
							E_F, meV	K_F, cm^{-1}
$6.6 \cdot 10^{-2}$	0.45	16.4	4	27	150	19	96	10^7

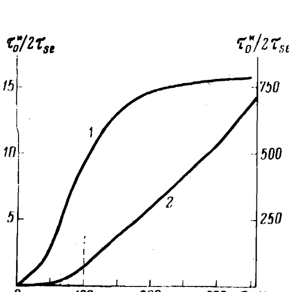

FIG. 2. Temperature dependence of the electron spin relaxation time in GaAs for hole concentrations $N_1 = 10^{17}$ cm^{-3} (curve 1, left-hand scale) and $N_2 = 4 \times 10^{19}$ cm^{-3} (curve 2, right-hand scale), τ_0^* is the value of τ_0 for $N = N_1$ from (7).

5. EVALUATION OF THE ELECTRON SPIN RELAXATION TIME WHEN HOLES ARE STRONGLY SCATTERED BY IMPURITIES

In this section we derive a kinetic equation to describe the electron spin relaxation due to holes and give a derivation of Eqs. (12) and (13). We shall assume that $\tau\epsilon_F \gg 1$ and $\tau_{ei}\epsilon_k \gg 1$, where τ_{ei} is the electron relaxation time due to impurities. As $m_e \ll m_h$, the quantity τT can be small here.

We use Keldysh's technique.[12] We consider the case of simple spherical bands with a hole-impurity interaction Hamiltonian of the form

$$\mathscr{H}_{int} = u_0(\mathbf{r}) + u_1(\mathbf{r})(\mathbf{SJ}), \tag{28}$$

S is the impurity spin operator and **J** the hole spin operator. We shall assume the holes to be strongly degenerate.

We use (9) to introduce the electron and hole Green function matrices $\hat{G}_{\beta\beta'}(x_1, x_2)$ and $\mathscr{G}_{\alpha\alpha'}(x_1, x_2)$. We shall assume that only the electrons are polarized as far as the spin is concerned and that $\hat{G}_{\beta\beta'} = 0$, if $\beta \neq \beta'$ while $\hat{\mathscr{G}}_{\alpha\alpha'} \propto \delta_{\alpha\alpha'}$. According to (9) each of the components $G_\beta \equiv G_{\beta\beta}$ is a matrix

$$\hat{G}_\beta(x_1, x_2) = \begin{pmatrix} G_\beta^{(c)}(x_1, x_2) & G_\beta^{(+)}(x_1, x_2) \\ G_\beta^{(-)}(x_1, x_2) & \tilde{G}_\beta^{(c)}(x_1, x_2) \end{pmatrix}. \tag{29}$$

The matrix $\mathscr{G}_\alpha(x_1, x_2)$ has a similar form. In zeroth order in the interaction the electron Green functions $G_\alpha^{(\pm)}(k)$ are of the form

$$G_\beta^{(12)}(k) = G_\beta^{(+)}(k) = 2\pi i f_{\mathbf{k}\beta} \delta(\varepsilon_e - \varepsilon_\mathbf{k}),$$
$$G_\beta^{(21)}(k) = G_\beta^{(-)}(k) = -2\pi i (1 - f_{\mathbf{k}\beta}) \delta(\varepsilon_e - \varepsilon_\mathbf{k}), \tag{30}$$
$$k \equiv \varepsilon_e, \mathbf{k}.$$

The Dyson equation for the electrons when they are scattered by holes have the form

$$\frac{i}{2} \tau_z \frac{\partial}{\partial t} \hat{G}_\beta(k, t) = 1 + i \hat{\Sigma}_\beta(k, t) \hat{G}_\beta(k, t). \tag{31}$$

Here the τ_i are Pauli matrices acting in the space of the Green functions (29). In (31) we have changed to the average time $t = (t_1 + t_2)/2$ and the coordinate differences $t_- = t_1 - t_2$, $\mathbf{r}_- = \mathbf{r}_1 - \mathbf{r}_2$ and Fourier transformed with respect to the time and coordinate differences. A graphical expression for $\hat{\Sigma}$ is given in Fig. 3.

According to[12] it is sufficient to write down the real part of the Dyson equation for $G_\beta^{(+)}(k)$ and integrate it over ϵ_e to obtain the kinetic equation for the electrons:

$$\frac{\partial f_{\mathbf{k}\beta}}{\partial t} = \mathrm{Im} \int \frac{d\varepsilon_e}{2\pi} [\Sigma_\beta^{(+)}(k) G_\beta^{(-)}(k) - \Sigma_\beta^{(-)}(k) G_\beta^{(+)}(k)]. \tag{32}$$

To evaluate $\Sigma_\alpha^{(+)}(k)$ it is necessary to find the explicit form of the function $\hat{\mathscr{G}}(p)$ taking into account the scattering by impurities. The Green functions $\mathscr{G}^{(c)}(p)$ and $\mathscr{G}^{(\pm)}(p)$ are connected with the retarded function $\mathscr{G}^{(r)}(p)$ and the function $\mathscr{G}^{(\rho)}(p) = \mathscr{G}^{(+)}(p) + \mathscr{G}^{(-)}(p)$ through the relations

$$\mathscr{G}^{(c)}(p) = \mathrm{Re}\, \mathscr{G}^{(r)}(p) + {}^1/_2 \mathscr{G}^{(\rho)}(p),$$
$$\mathscr{G}^{(\pm)}(p) = {}^1/_2 \mathscr{G}^{(\rho)}(p) \mp i\, \mathrm{Im}\, \mathscr{G}^{(r)}(p). \tag{33}$$

If we, therefore, perform a unitary transformation on

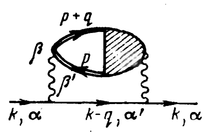

FIG. 3

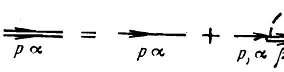

FIG. 4

FIG. 3. Self-energy part for the electron-hole interaction.
FIG. 4. Dyson equation for the hole Green functions.

the hole Dyson equation (Fig. 4) using[9] $\hat{U} = (1 + i\tau_y)/\sqrt{2}$, we get

$$\mathscr{G}^{(r)}(p)=\frac{1}{\varepsilon-\xi_p-i\Sigma^{(r)}(p)},\tag{34}$$

where $\xi_p = (p^2 - p_F^2)2m_h$, and $\Sigma^{(r)}(p)$ satisfies the equation

$$\Sigma^{(r)}(p)=-iN_i\sum_{p'}|w(\mathbf{p}-\mathbf{p}')|^2\frac{1}{\varepsilon-\xi_{p'}-i\Sigma^{(r)}(p')},\tag{35}$$

$$|w(\mathbf{p}-\mathbf{p}')|^2=|u_0(\mathbf{p}-\mathbf{p}')|^2+{}^1/_3S(S+1)|u_1(\mathbf{p}-\mathbf{p}')|^2,\tag{36}$$

N_i is the total impurity density.

The solution of (35) is

$$\Sigma^{(r)}=-\frac{1}{2\tau},\quad \frac{1}{\tau}=\frac{3\pi}{2}\frac{N}{\varepsilon_F}N_i\int\frac{d\Omega}{4\pi}|w(\theta)|^2,\tag{37}$$

where τ is the hole relaxation time. We included the imaginary part of $\Sigma^{(r)}$, which is practically independent of p, in the renormalization of the chemical potential.

Thus

$$\mathscr{G}^{(r)}(p)=\frac{1}{\varepsilon-\xi_p+i/2\tau}.\tag{38}$$

The equation for $\mathscr{G}^{(\rho)}(p)$ is of the form

$$\mathscr{G}^{(\rho)}(p)=i\Omega(p)|\mathscr{G}^{(r)}(p)|^2,\tag{39}$$

where

$$\Omega(p)=-iN_i\sum_{p'}|w(\mathbf{p}-\mathbf{p}')|^2\mathscr{G}^{(\rho)}(p').\tag{40}$$

From (38) to (40) we find an equation for $\mathscr{G}^{(\rho)}(p)$:

$$\mathscr{G}^{(\rho)}(\mathbf{p},\varepsilon)=\frac{N_i}{(\varepsilon-\xi_p)^2+1/4\tau^2}\sum_{p'}|w(\mathbf{p}-\mathbf{p}')|^2\mathscr{G}^{(\rho)}(\mathbf{p}',\varepsilon),\tag{41}$$

the solution of which is of the form

$$\mathscr{G}^{(\rho)}(p\varepsilon)=2\pi i[2f(\varepsilon)-1]\Delta(\varepsilon-\xi_p),\qquad \Delta(\varepsilon-\xi_p)=\frac{(2\pi\tau)^{-1}}{(\varepsilon-\xi_p)^2+1/4\tau^2},\tag{42}$$

$f(\epsilon)$ is an arbitrary function of ϵ. This function is determined by the inelastic collisions with phonons. When there are no external fields $f(\epsilon)$ is the equilibrium distribution function $f(\epsilon) = (e^{\epsilon/T} + 1)^{-1}$. Using Eq. (33) we get

$$\mathscr{G}^{(+)}(p)=2\pi if(\varepsilon)\Delta(\varepsilon-\xi_p),\quad \mathscr{G}^{(-)}(p)=-2\pi i[1-f(\varepsilon)]\Delta(\varepsilon-\xi_p).\tag{43}$$

We note that, using (33), (38), and (43), we can write the Green function matrix in the form

$$\hat{\mathscr{G}}(p)=\begin{pmatrix}f(\varepsilon) & f(\varepsilon)\\ f(\varepsilon)-1 & f(\varepsilon)-1\end{pmatrix}\mathscr{G}^{(a)}(p)-\mathscr{G}^{(r)}(p)\begin{pmatrix}f(\varepsilon)-1 & f(\varepsilon)\\ f(\varepsilon)-1 & f(\varepsilon)\end{pmatrix}=\hat{A}\mathscr{G}^{(a)}(p)-\hat{B}\mathscr{G}^{(r)}(p).\tag{44}$$

We need this representation in what follows, $\mathscr{G}^{(a)}$ is the advanced Green function, $\mathscr{G}^{(a)}(p) = \mathscr{G}^{(r)*}(p)$.

To find the vertex part which describes the interference of the scattering of holes by impurities and by an electron we must sum the diagrams shown in

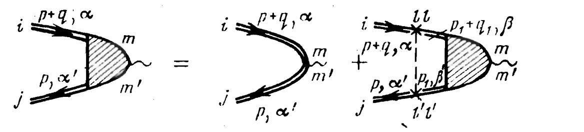

FIG. 5. Diagram equation for the vertex part.

Fig. 5. We can then neglect the diagrams with intersecting impurity lines as they are small in the parameter $1/\tau\varepsilon_F \ll 1$. The integral equation for the vertex function $K^{(imm'j)}(p + q, \alpha|p, \alpha')$ is of the form

$$K^{(imm'j)}(p+q,\alpha|p,\alpha')=\mathscr{G}^{(im)}(p+q)\tau_z^{(mm')}\mathscr{G}^{(m'j)}(p)V_0(\mathbf{q})(\sigma\mathbf{J}_{\alpha\alpha'})$$
$$+N_i\sum_{p_1,\beta,\beta'}\mathscr{G}^{(il)}(p+q)\tau_z^{(ll)}K^{(imm'l')}(p_1+q,\beta|p_1,\beta')\tau_z^{l'l'}\mathscr{G}^{(l'j)}(p)\tag{45}$$
$$\times\{|u_0(\mathbf{p}-\mathbf{p}_1)|^2\delta_{\alpha\beta}\delta_{\alpha'\beta'}+|u_1(\mathbf{p}-\mathbf{p}_1)|^2S_{\alpha\beta}S_{\beta'\alpha'}\}.$$

As the inhomogeneous term of the equation has the spin structure $\sigma\mathbf{J}_{\alpha\alpha'}$ the solution must be of the form

$$K^{(imm'j)}(p+q,\ \alpha|p,\ \alpha')=V_0(q)M^{(imm'j)}(\sigma\mathbf{J}_{\alpha\alpha'}).\tag{46}$$

Substituting (46) into (45) we get the equation

$$M^{(imm'j)}(p+q|p)=\mathscr{G}^{(im)}(p+q)\tau_z^{(mm')}\mathscr{G}^{(m'j)}(p)$$
$$+N_i\sum_{p_1ll'}\mathscr{G}^{(il)}(p+q)\tau_z^{ll}M^{(imm'l')}(p_1+q|p_1)\tau_z^{l'l'}\mathscr{G}^{(l'j)}(p)\tag{47}$$
$$\times\{|u_0(\mathbf{p}-\mathbf{p}_1)|^2-{}^1/_3S(S+1)|u_1(\mathbf{p}-\mathbf{p}_1)|^2\}.$$

The matrix element $M^{(2112)}(p + q|p)$ which is summed over p occurs in the kinetic equation. As the integral of two retarded functions vanishes we can look for the solution for $M^{(imm'j)}(p + q|p)$ as only term proportional to products of an advanced and a retarded function. It is thus convenient to write the solution of (47) in the form

$$M^{(imm'j)}(p+q|p)=A_{im}(\varepsilon+\omega)B_{m'j}(\varepsilon)\tau_z^{mm'}M_1(p+q|p)\mathscr{G}^{(a)}(p+q)\mathscr{G}^{(r)}(p)$$
$$+B_{im}(\varepsilon+\omega)A_{m'j}(\varepsilon)\tau_z^{mm'}M_2(p+q|p)\mathscr{G}^{(r)}(p+q)\mathscr{G}^{(a)}(p).\tag{48}$$

Using for $\hat{\mathscr{G}}(p)$ the representation (44), bearing in mind that $\hat{A}\tau_z\hat{A} = \hat{A}$, $\hat{B}\tau_z\hat{B} = -\hat{B}$, while $\hat{A}\tau_z\hat{B} = \hat{B}\tau_z\hat{A} = 0$, and dropping terms containing products of only retarded or advanced functions, we get two equations for $M_1(p + q|p)$ and $M_2(p + q|p)$:

$$M_1(p+q|p)=-\mathscr{G}^{(a)}(p+q)\mathscr{G}^{(r)}(p)\Big\{1-N_i\sum_{p_1}M_1(p_1+q|p_1)$$
$$\times[|u_0(\mathbf{p}-\mathbf{p}_1)|^2-{}^1/_3S(S+1)|u_1(\mathbf{p}-\mathbf{p}_1)|^2]\Big\},\tag{49}$$

$$M_2(p+q|p)=-\mathscr{G}^{(r)}(p+q)\mathscr{G}^{(a)}(p)\Big\{1-N_i\sum_{p_1}M_2(p_1+q|p_1)$$
$$\times[|u_0(\mathbf{p}-\mathbf{p}_1)|^2-{}^1/_3S(S+1)|u_1(\mathbf{p}-\mathbf{p}_1)|^2\Big\}.\tag{50}$$

It follows from (49) and (50) that $M_1(p + q|p) = M_2^*(p + q|p)$. The matrix element has the form

$$M^{(2112)}(p+q|p)=-[1-f(\varepsilon+\omega)]f(\varepsilon)\cdot 2\,\mathrm{Re}\,M_1(p+q|p).\tag{51}$$

As

$$\Sigma_\beta^{(-)}(k)=-2i\sum_{\substack{pq\\ \alpha\alpha'\beta'}}|V_q|^2\int\frac{d\omega}{2\pi}\int\frac{d\varepsilon}{2\pi}|\sigma_{\beta\beta'}\cdot\mathbf{J}_{\alpha\alpha'}|^2M^{(2112)}(p+q|p)G_{\alpha'}^{(21)}(k-q),\tag{52}$$

we get, substituting (52) into (30) and using (30), (42), (48), and (51), summing over the hole spin states α, α' and integrating over ϵ, the following expression for the drift term in the collision integral:

$$\left(\frac{\partial f_{k\beta}}{\partial t}\right)_{\text{coll}}=-2\pi\sum_{\mathbf{q},\beta'\neq\beta}|V_s(\mathbf{q})|^2\int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}\frac{1}{1-e^{-\omega/T}}$$
$$\times\mathrm{Im}\,\chi(\omega,\mathbf{q})f_{k\beta}(1-f_{k-q,\beta'})\delta(\omega-\varepsilon_k+\varepsilon_{k-q}),\tag{53}$$

where

$$|V_s(\mathbf{q})|^2 = |V_0(\mathbf{q})|^2 \sum_{\alpha\alpha'} (\sigma_{\beta\beta'} J_{\alpha\alpha'}), \qquad (54)$$

and

$$\chi(\omega, \mathbf{q}) = -i\frac{2\omega}{\pi}\sum_{\mathbf{p}} M_1(p+q|p).$$

When obtaining (53) we used the fact that Σ_pRe $M_1(p+q|p)$ is independent of ϵ and depends only on ω,q.

To solve Eq. (49) we shall assume that the potential for the scattering of a hole by impurities is a short-range one so that the scattering amplitude is independent of the angles. One can then easily solve Eq. (49) and after simple calculations we get for Im$_\chi(\omega, \mathbf{q})$ the following expression:

$$\operatorname{Im}\chi(\omega, \mathbf{q}) = \frac{3}{2}\frac{N}{\varepsilon_F}\omega\eta(\omega, \mathbf{q}), \qquad (55)$$

where

$$\eta(\omega, \mathbf{q}) = \operatorname{Re}[T_{\omega q}^{-1} - \tau^{-1} + \tau_{sh}^{-1}]^{-1}, \qquad (56)$$

$$T_{\omega q} = \frac{1}{2}\int_{-1}^{+1}\frac{dx}{-i(\omega - qv_F x) + 1/\tau} = \frac{1}{2iqv_F}\ln\frac{1 - i(\omega\tau - ql)}{1 - i(\omega\tau + ql)}. \qquad (57)$$

Here

$$\frac{1}{\tau_{sh}} = \frac{\pi}{2}N_i S(S+1)\frac{N}{\varepsilon_F}|u_1|^2$$

is the hole spin relaxation time due to impurities.

We turn now to the calculation of the average electron spin relaxation time. For a non-degenerate electron gas we get from (53) and (56)

$$\left\langle\frac{1}{2\tau_{se}}\right\rangle = \frac{3}{2}\frac{N}{\varepsilon_F}|V_s|^2\sum_{\mathbf{q}}\int_{-\infty}^{+\infty} d\omega \frac{\omega}{1 - e^{-\omega/T}}\eta(\omega, \mathbf{q})\delta(\omega - \varepsilon_{\mathbf{k}} + \varepsilon_{\mathbf{k}-\mathbf{q}}). \quad (58)$$

As $\tau \to \infty$ and $\tau_{sh} \to \infty$

$$\eta(\omega, \mathbf{q}) = \operatorname{Re} T_{\omega q} = \frac{\pi}{2}\int_{-1}^{+1}\delta(\omega - qv_F x)\,dx. \qquad (59)$$

Substituting (59) into (58) we get a formula which is the same as Eq. (9) introduced above (we have here in (58) and (59) dropped the term $q^2/2m_h$ which occurs in (9) as it is small compared to ω or qv_F in the parameter m_e/m_h when $v_F \gg v_e$, or in the parameter k/p when $v_F \ll v_e$). When $v_F \ll v_e$, we can when $|\omega| \ll \epsilon_k$, T let in (58) and (59) $\omega \to 0$, i.e., assume $\eta = \pi\delta(\omega)$. We then get, integrating over ω and changing from a summation over q to a summation over $k' = \mathbf{k} - \mathbf{q}$,

$$\left\langle\frac{1}{2\tau_{se}}\right\rangle = \frac{3}{2}\pi N\frac{T}{\varepsilon_F}|V_s|^2\sum_{\mathbf{k}'}\delta(\varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{k}}), \qquad (60)$$

whence (10) follows after integration.

When $v_F \gg v_e$ in (58) and (59) $x \approx v_e/v_F \ll 1$ are the important values. We can thus extend the integration over x in (59) from $-\infty$ to $+\infty$. Then $\eta = \pi/2qv_F$ and

$$\left\langle\frac{1}{2\tau_{se}}\right\rangle = \frac{3\pi}{4}\frac{N}{\varepsilon_F v_F}|V_s|^2\sum_{\mathbf{k}'}\frac{\varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{k}}}{|\mathbf{k}' - \mathbf{k}|}\left\{\exp\left(\frac{\varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{k}}}{T}\right) - 1\right\}^{-1}, \quad (61)$$

from which (11) follows after simple transformations.

We now turn to the general Eqs. (58) and (57) and make more precise the regions in Fig. 1 where Eqs. (9) to (11) are valid and consider the case when these formulae are inapplicable.

We consider first the region where $l^2k^2 \gg 1$, i.e., $qv_F \gg 1/\tau$. In the case of fast holes in that region $qv_F/\omega \sim v_F/v_e \gg 1$. We can thus neglect in (57) all terms except qv_Fx. It then follows from (57) and (56) that $\eta = T_{\omega q} = \pi/2qv_F$ whence follows (61). Hence, Eqs. (61) and (11) are valid in the whole fast-hole region, independent of the values of τT and τ_{sh}T. In the slow-hole case $\epsilon_k \gg qv_F \approx \omega \gg 1/\tau$ for $l^2k^2 \gg 1$. In this region $\eta(\omega, \mathbf{q})$ is thus determined by Eq. (59) and τ_{se} by Eqs. (60) and (10).

We consider now the region $l^2k^2 \ll 1$.

Expanding Eq. (57) in a series in $ql/(1 - i\omega\tau)$ we get

$$\eta(\omega, \mathbf{q}) = \operatorname{Re}\left[-i\omega + \frac{Dq^2}{1+\omega^2\tau^2} + \tau_{sh}^{-1}\right]^{-1}, \qquad (62)$$

where $D = 1/3v_F^2\tau$. When $1 \gg l^2k^2 \gg \tau/\tau_{sh}$ or $Dq^2 \gg \tau_{sh}^{-1}$ we can neglect the term $1/\tau_{sh}$ in this formula. When $6m_eD \gg 1$, i.e., in region d where $\omega\tau \sim T\tau \ll 1$, $Dq^2 \approx Dk^2 \gg \omega \approx \epsilon_k$, $\eta = 1/Dq^2$,

$$\left\langle\frac{1}{2\tau_{es}}\right\rangle = \frac{3}{2}\frac{N}{\varepsilon_F}\frac{|V_s|^2}{D}\sum_{\mathbf{k}'}\frac{\varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{k}}}{|\mathbf{k}' - \mathbf{k}|^2}\left\{\exp\left(\frac{\varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{k}}}{T}\right) - 1\right\}^{-1}, \quad (63)$$

whence follows (12).

When $6m_eD \ll 1$ the characteristic values $\omega \approx Dk^2 \ll \epsilon_k$ and then $\omega\tau \approx l^2k^2 \ll 1$. Therefore, in the whole band $\tau/\tau_{sh} \ll l^2k^2 \ll 1$ we may assume, independent of the magnitude of τT, that $\eta = \pi\delta(\omega)$ in (58) and (62) as a result of which we are again led to (60).

In the region where $l^2k^2 \ll \tau/\tau_{sh}$ we can, on the other hand, neglect the term Dq^2 in comparison with τ_{sh}^{-1} and, hence, in this region

$$\eta = \operatorname{Re}(-i\omega + 1/\tau_{sh})^{-1}. \qquad (64)$$

When $\tau_{sh} \ll 1/\omega \sim 1/T$ we can also drop in (64) the term $i\omega$ and assume that $\eta \approx \tau_{sh}$. We then get from (58)

$$\left\langle\frac{1}{2\tau_{se}}\right\rangle = \frac{3}{2}\frac{N}{\varepsilon_F}|V_s|^2\tau_{sh}\sum_{\mathbf{k}'}(\varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{k}})\left\{\exp\left(\frac{\varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{k}}}{T}\right) - 1\right\}^{-1}, \quad (65)$$

Whence follows (13).

When $\tau_{sh} \gg 1/T$ the important values are $\omega \sim \tau_{sh}^{-1} \ll T$ and, hence, again $\eta \approx \pi\delta(\omega)$ as a result of which we get again (60), independent of the magnitude of τT. Equations (60) and (10) are thus valid in the whole of the slow hole regions (a) and (b).

In conclusion the authors thank V. I. Perel' and É. I. Rashba for useful discussions.

[1)]In semiconductors with a degenerate valence band we take into account in this case only the scattering by the heavy holes, since the light-hole density is small.

[2)]In Sec. 5 we consider the scattering of holes by a short-range potential, equivalent to a strongly screened Coulomb potential when the drift time for relaxation is then the same as the transport time. When the scattering is by a not strongly screened Coulomb potential, we shall assume that τ is the transport time.

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